Teacher Energized Resource Manual

Class : 10th
Subject : Mathematics

CENTRAL BOARD OF SECONDARY EDUCATION

Preface

In consonance with the move towards outcome-based education where focus is on developing competencies in students, the Central Board of Secondary Education is delighted to share the *Teacher Energized Resource Manual* that will aid teachers in aligning their classroom transaction to a competency framework.

Each chapter of the Resource Manual corresponds to the respective chapters in the NCERT textbooks. The chapters have been chunked by concept; these concepts have been linked to the NCERT Learning Outcomes; and an attempt has been made to delineate Learning Objectives for each concept. Every chapter has a set of assessment items, where two items have been provided as examples for each Learning Objective. Teachers can use these to assess if the learner has acquired the related concept. Needless to say, the items are illustrative examples to demonstrate how competency-based items can be prepared to measure Learning Objectives and Outcomes. The variety in item forms is suggestive of the ways in which a particular concept can be assessed to identify if the learner has attained different competencies. We trust and hope that teachers would be able to generate many more similar test items for use in practice.

Your observations, insights and comments as you use this Resource Manual are welcome. Please encourage your students to voice their suggestions as well. These inputs would be helpful to improve this Manual as these are incorporated in the subsequent editions. All possible efforts have been made to remove technical errors and present the Manual in a form that the teachers would find it easy and comfortable to use.

Acknowledgements

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HOW TO USE THIS MANUAL

The goal of the Teacher Energized Resource Manual (TERM) is to provide teachers with competency-based education resources aligned to NCERT textbooks that would support them in the attainment of desired Learning Outcomes and development of requisite competencies of the learner. The TERM has equal number of corresponding chapters as NCERT Textbooks with listing of Concepts, Learning Outcomes developed by NCERT and Learning Objectives. Competency based test items for each corresponding Learning Objective and sample activities for enrichment have been provided.

Learning Objectives:

Each chapter has a *Learning Objectives* table. The table also lists the Concepts covered in the chapter. Learning Objectives are broken down competencies that a learner would have acquired by the end of the chapter. They are a combination of skills and what the learner would use this skill for. For example, the first Learning Objective in the table below relates to the skill of *application* and the students will use this competency to obtain the highest common factor of 2 positive integers. Teachers can use these specific Learning Objectives to identify if a student has acquired the associated skill and understands how that skill can be used.

Concepts	Learning Objectives	Learning Outcomes
Euclid's Division	Apply Euclid Division Algorithm in order to obtain HCF of 2 positive integers in the context of the given problem	Generalises properties of numbers and relations among them studied earlier, to evolve results, such as, Euclid's division algorithm, fundamental theorem of arithmetic in order to apply them to solve problems related to real life contexts
	Apply Euclid Division Algorithm in order to prove results of positive integers in the form of ax+b where a and b are integers	
Fundamental Theorem of Arithmetic	Use the Fundamental Theorem of Arithmetic in order to calculate HCF and LCM of the given numbers in the context of the given problem	
Irrational Numbers	Recall the properties of irrational number in order to prove that whether the sum/difference/product/quotient of 2 numbers is irrational or not	
	Apply theorems of irrational number in order to prove whether a given number is irrational or not	
Decimal Representation of Irrational Numbers	Apply theorems of rational numbers in order to find out about the nature of their decimal representation and their factors	

Concepts:

The important concepts in a particular chapter are listed in the first section. Most often, they follow a logical order and present a sequence in which these are likely to be covered while teaching. In case, your teaching strategy is different and presents them in a different order, you need not worry. Teach the way, you consider the best. You only need to ensure their understanding and the attainment of desired learning objectives.

Learning Outcomes (NCERT):

A mapping of Learning Outcomes developed by the NCERT and Learning Objectives is provided in last column of the table. The Learning Outcomes have been developed by the NCERT. Each Learning Objective is mapped to NCERT Learning Outcomes and helps teachers to easily identify the larger outcome that a learner must be able to demonstrate at the end of the class/ chapter.

Test items:

For each Learning Objective, at least two competency-based test items have been provided. Although, the items in this resource manual are multiple choice questions, which assess developed competencies of a student rather than only knowledge, it must be kept in mind that there can be different kinds of assessment that can easily align with competency-based education. Teachers can use these items to assess if a learner has achieved a particular Learning Objective and can take necessary supportive actions. Teachers are also encouraged to form similar questions which assess skills of students.

LOB: Apply Euclid Division Algorithm in order to obtain HCF of 2 given numbers in the context of the given problem

1. A worker needs to pack 350 kg of rice and 150 kg of wheat in bags such that each bag weighs the same. Each bag should either contain rice or wheat. Which option shows the correct steps to find the greatest amount of rice/wheat the worker can pack in each bag?

   Option 1: Step 1: $350 = 2(150) + 50$
   Step 2: $150 = 3(50) + 0$
   Step 3: Greatest amount: 50 kg

   Option 2: Step 1: $350 = 2(150) + 50$
   Step 2: $150 = 2(50) + 0$
   Step 3: Greatest amount: 50 kg

   Option 3: Step 1: $350 = 2(150) + 50$
   Step 2: $150 = 3(50) + 0$
   Step 3: Greatest amount: 150 kg

   Option 4: Step 1: $350 = 2(150) + 50$
   Step 2: $150 = 2(50) + 0$
   Step 3: Greatest amount: 150 kg

   Correct Answer: Option 1

Fig: 3

Suggested Teacher Resources

At the end of each chapter, certain activities have been suggested which can be carried out by the teachers with learners to explain a concept. These are only samples and teachers can use, adapt, as well as, create activities that align to a given concept.

TABLE OF CONTENTS

C2Q4Y8

1. REAL NUMBERS

QR Code:

L8Y6G8

Learning outcome and Learning Objectives:

Content area / Concept	Learning Objectives	Learning Outcome
Euclid's Division	Apply Euclid Division Algorithm in order to obtain HCF of 2 positive integers in the context of the given problem	Generalises properties of numbers and relations among them studied earlier, to evolve results, such as, Euclid's division algorithm, fundamental theorem of arithmetic in order to apply them to solve problems related to real life contexts
	Apply Euclid Division Algorithm in order to prove results of positive integers in the form of ax+b where a and b are integers	
Fundamental Theorem of Arithmetic	Use the Fundamental Theorem of Arithmetic in order to calculate HCF and LCM of the given numbers in the context of the given problem	
Irrational Numbers	Recall the properties of irrational number in order to prove that whether the sum/difference/product/quotient of 2 numbers is irrational or not	
	Apply theorems of irrational number in order to prove whether a given number is irrational or not	
Decimal Representation of Irrational Numbers	Apply theorems of rational numbers in order to find out about the nature of their decimal representation and their factors	

Test items

LOB: Apply Euclid Division Algorithm in order to obtain HCF of 2 given numbers in the context of the given problem

1. A worker needs to pack 350 kg of rice and 150 kg of wheat in bags such that each bag weighs the same. Each bag should either contain rice or wheat. Which option shows the correct steps to find the greatest amount of rice/wheat the worker can pack in each bag?

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Option 3: Step 1: $350 = 2(150) + 50$
Step 2: $150 = 3(50) + 0$
Step 3: Greatest amount: 150 kg

Option 4: Step 1: $350 = 2(150) + 50$
Step 2: $150 = 2(50) + 0$
Step 3: Greatest amount: 150 kg

Correct Answer: Option 1

2. Pranay wants to stack few one-rupee coins and some five-rupee coins in such a way that:

a. Each stack has the same number of coins.
b. There is least number of stacks.
c. Each stack either has one rupee or five-rupee coins.
d. No coins are left over after creating stacks.

His first step to find the number of coins that should be in each stack is 195=1(180) +15. Given that he has more of five-rupee coins than one-rupee coins, how many one-rupee coins stack can he make?

Option 1: 12
Option 2: 13
Option 3: 15
Option 4: 25

Correct Answer: Option 1

LOB: Apply Euclid Division Algorithm in order to prove results of positive integers in the form of ax+b where a and b are constants

1. Given that p is a non-negative integer, which of these gives positive integers that are multiple of 5?

Option 1: 10p and 10p+2.
Option 2: 10p and 10p+3.
Option 3: 10p and 10p+4.
Option 4: 10p and 10p+5.
Correct Answer: Option 4

2. In the equation below, a, b, q, r are integers, 0≤r<b, a is a multiple of 3 and b=9.
a=bq+r
Which of the following forms represent a?

Option 1: Only 9q and 9q+3, as only these forms when divided by 3 gives r=0.
Option 2: Only 9q+1and 9q+4, as only these forms when divided by 3 gives r=1.
Option 3: Only 9q, 9q+3, and 9q+6, as only these forms when divided by 3 gives r=0.
Option 4: Only 9q+1, 9q+4, and 9q+7, as only these forms when divided by 3 gives r=1.
Correct Answer: Option 3

LOB: Use the Fundamental Theorem of Arithmetic in order to calculate HCF and LCM of the given numbers in the context of the given problem

1. Rahul has 40 cm long red and 84 cm long blue ribbon. He cuts each ribbon into pieces such that all pieces are of equal length. What is the length of each piece?

Option 1: 4 cm as it is the LCM of 40 and 84
Option 2: 4 cm as it is the HCF of 40 and 84
Option 3: 8 cm as it is the LCM of 40 and 84
Option 4: 8 cm as it is the HCF of 40 and 84

Correct Answer: Option 2

2. Three bulbs red, green and yellow flash at intervals of 80 seconds, 90 seconds and 110 seconds. All three flash together at 8:00 am. At what time will the three bulbs flash altogether again?

Option 1: 9:00 am
Option 2: 9:12 am
Option 3: 10:00 am
Option 4: 10:12 am

Correct Answer: Option 4

LOB: Recall the properties of irrational number in order to prove that whether the sum/difference/product/quotient of 2 numbers is irrational or not

1. Which of the following is an irrational number?

Option 1: $\frac{\sqrt{2}}{\sqrt{8}}$
Option 2: $\frac{\sqrt{3}}{3\sqrt{5}}$
Option 3: $\frac{\sqrt{5}}{\sqrt{20}}$
Option 4: $\frac{\sqrt{63}}{\sqrt{7}}$

Correct Answer: Option 2

2. A teacher creates the question "Which of the following could be the sum of two rational numbers?". She now needs to create three incorrect choices and one correct answer. Which option shows the choices that the teacher should create?

Option 1: First choice: π; Second choice: 20+16; Third choice: 50-1; Correct Answer: 49
Option 2: First choice: 227; Second choice: 25+16; Third choice: 64; Correct Answer: 5
Option 3: First choice: 125; Second choice: 36+42; Third choice: 81; Correct Answer: 169
Option 4: None of them

Correct Answer: Option 1

LOB: Apply theorems of irrational number in order to prove whether a given number is irrational or not

1. Which of the following is NOT an irrational number?

Option 1: $2 \times \frac{1}{\sqrt{2}}$
Option 2: $\sqrt{2} \times \frac{1}{2}$
Option 3:$\sqrt{2} \times \frac{1}{\sqrt{3}}$
Option 4: $\sqrt{2} \times \frac{1}{\sqrt{2}}$

Correct Answer: Option 4

2. Is $9 + \sqrt{2}$ an irrational number?

Option 1: No, because if $9 + \sqrt{2} = \frac{a}{b}$, where a and b are integers and $b \neq 0$, then $\sqrt{2} = \frac{9b-a}{b}$, but $\sqrt{2}$ is an irrational number. So, $9 + \sqrt{2} \neq \frac{a}{b}$.

Option 2: No, because if $9 + \sqrt{2} = \frac{a}{b}$, where a and b are integers and $b \neq 0$, then $\sqrt{2} = \frac{9b+a}{b}$, but $\sqrt{2}$ is an irrational number. So, $9 + \sqrt{2} \neq \frac{a}{b}$.

Option 3: Yes, because if $9 + \sqrt{2} = \frac{a}{b}$, where a and b are integers and $b \neq 0$, then $\sqrt{2} = \frac{9b-a}{b}$, but $\sqrt{2}$ is an irrational number. So, $9 + \sqrt{2} \neq \frac{a}{b}$.

Option 4: Yes, because if $9 + \sqrt{2} = \frac{a}{b}$, where a and b are integers and $b \neq 0$, then $\sqrt{2} = \frac{9b+a}{b}$, but $\sqrt{2}$ is an irrational number. So, $9 + \sqrt{2} \neq \frac{a}{b}$.

Correct Answer: Option 2

LOB: Apply theorems of rational numbers in order to find out about the nature of their decimal representation and their factors

1. Which of the following is equivalent to a decimal that terminates?

   Option 1: $\frac{1}{5^2 2^2}$

   Option 2: $\frac{1}{2^2 3}$

   Option 3: $\frac{1}{5^2 7}$

   Option 4: $\frac{1}{5^2 11^2}$

   Correct Answer: Option 1

2. The fractions $\frac{3}{a}$ and $\frac{7}{b}$ are equivalent to decimals that terminate. Which best describes the product of a and b?

   Option 1: It is a prime number.

   Option 2: It cannot be an odd number.

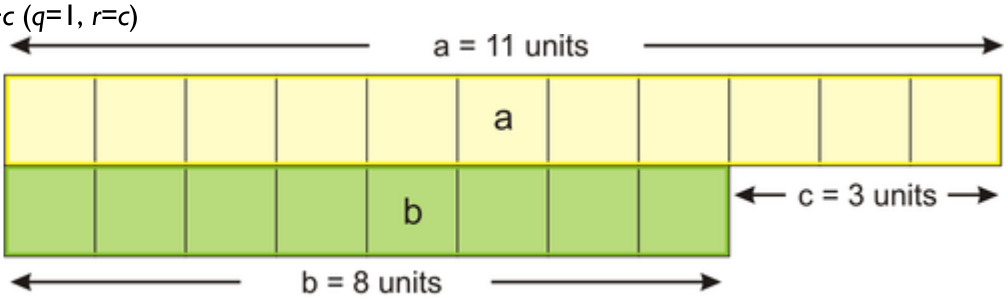
   Option 3: It is of the form $21k$, where k could be multiples of 2 or 5.

   Option 4: It is of the form $21k$, where k could be multiples of 7 or 9.

   Correct Answer: Option 3

Suggested Teacher Resources

I Lesson Plan

Objective	Apply Euclid Division Algorithm in order to obtain HCF of 2 given numbers in the context of the given problem
Prerequisite	Equation solving
Vocabulary words	Lemma, Algorithm
Materials required	Sheets and graph papers
Procedure	Activity: Opening **Setup:** Students will be doing this activity with their peers. Provide them graph sheets. Clarify 1 unit is one box on the graph paper. **Procedure** 1. Cut one strip of length "*a*" unit (let a=11 *cm*) and one strip of length "*b*" units (let *b*=8 *cm*) of width1 cm each (*a*>*b*). Shown in Fig 1. 2. Paste the strips of length "*a*" unit above the strip of length "*b*" units aligning them from length as shown in fig 1. The remaining length is *c cm* (*c*=3 *cm*, *b*>*c*). 3. Cut another strip of length *b* unit and 2 strips of *c* units. Paste the strip of *b* units above the strips of c *unit* aligning them from left as shown in fig 2. The remaining length is *d* cm. (*d*=2 *cm*, *d*<*c*) 4. Repeat the process till the length proceeding strips covered completely and second strip which covers the proceeding strips is the HCF of given number (fig 4). **Observation:** By Euclid's division lemma $a = b \times q + r$, $0 \leq r < b$ Fig 1. Shows $a=b\times 1+c$ ($q=1$, $r=c$)  Fig (1)

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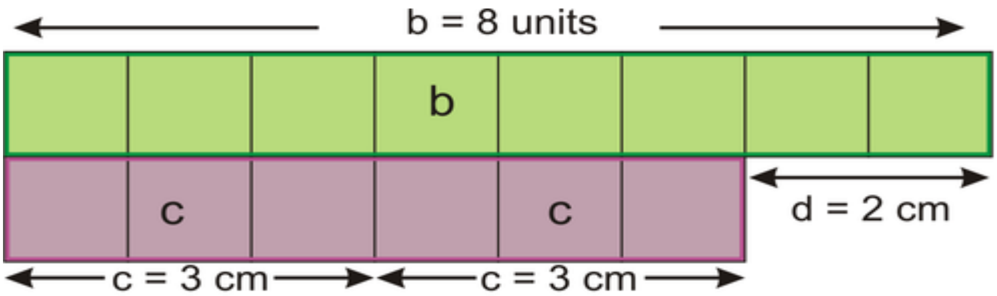


Fig (2)

Fig 2. Shows: $b=c\times 2+d$ ($q=2$, $r=d$)
Fig 3. Shows: $c=d\times 1+e$ ($q=1$, $r=e$)
Fig 4. Shows: $d=e\times 2+0$ ($q=2$, $r=0$)
H.C.F. of a and b is e.
Here, $a=11$ *cm*, $b=8$ *cm*, $c=3$ *cm*, $d=2$ *cm* and $e=1$ *cm*. H.C.F of 11 and 8 is 1.

Conclusion: Euclid's division lemma can be used for finding the HCF of two or more numbers.

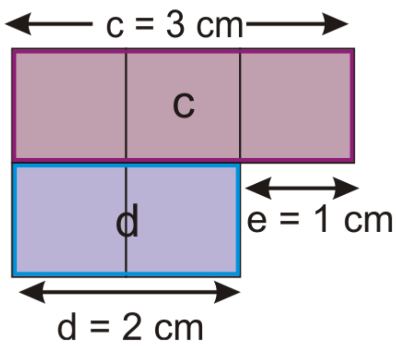


Fig (3)

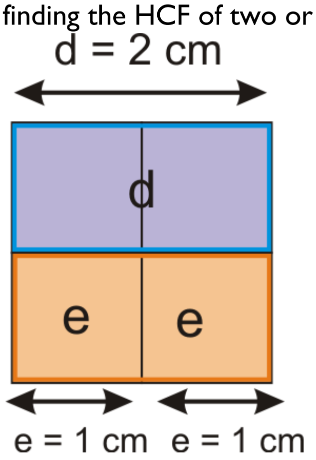


Fig (4)

Teacher will give an abstract representation for further understanding like solve for H.C.F of (25,15) using Euclid's division lemma.
</td></tr>
<tr><td>Source</td><td>https://www.ck12.org/book/CBSE-Maths-Book-Class-10/section/1.6/</td></tr>
</table>

2

Activity

Objective: SWBAT know when the sum or product of rational and irrational numbers is rational or irrational.
Setup: Recall-Ask students to answers the following questions. It will help the students recall the concepts.

ENTRANCE CARD
Rational or Irrational?
1.a. Write three rational numbers
b. Explain rational number in your own words
2.a. Write three irrational numbers
b. Explain irrational number in your own words

Teacher's Note: Students tend to define rational numbers as:" numbers that can be written as a fraction". The problem with this is that irrational numbers such as the square root of 3 are also fractions. A better definition is that rational numbers are "numbers that can be written as a fraction of integers".

Divide students into groups before moving to main activity.

Instructions:
Complete using addition:

+	6	1/2	0	√2	1/√2	π
6						
½						
0						
√2						
1/√2						
π						

Complete using multiplication:

×	6	1/2	0	√2	1/√2	π
6						
½						
0						
√2						
1/√2						
π						

1) Students need to work in pairs to complete above sheets within time.

Once done students will work on following sentences:
a. The sum of a rational number and a rational number is rational
b. The sum of a rational number and an irrational number is irrational
c. The sum of an irrational number and an irrational number is irrational
d. The product of a rational number and a rational number is rational
e. The product of a rational number and an irrational number is irrational
f. The product of an irrational number and an irrational number is irrational

2) The answers should conjecture if the statements are ALWAYS, SOMETIMES, or NEVER True.
3) Come up with examples and counter example to argue the answers.

Extension:
True/False
1) An expression containing √6 and π is IRRATIONAL. __________
2) Between two irrational numbers there is an irrational number. ____________
3) Between two rational numbers, there is an irrational number. __________
4) If the circumference of a circle is rational, then the area is rational. __________
5) If the area of a circle is rational, then the circumference is rational. ______
Source: https://betterlesson.com/lesson/435707/operation-rational-irrational-numbers?from=search

2. POLYNOMIALS

QR Code:

K2I7X1

Learning outcome and Learning Objectives:

<table>
<tr><th>Content area / Content area / Conceptss</th><th>Learning Objectives</th><th>Learning Outcome</th></tr>
<tr><td rowspan="2">Geometrical meaning of Zeroes of a Polynomial</td><td>Recall degree of polynomial in order to find the number of zeroes of polynomial</td><td rowspan="6">Uses algebraic and graphical method of finding zeroes of a polynomial in order to establish a relationship between them</td></tr>
<tr><td>Analyse the graph of the polynomials in order to find the number of zeroes of polynomial</td></tr>
<tr><td rowspan="2">Relationship between Zeroes and Coefficients of a Polynomial</td><td>Compute zeroes of the polynomials in order to verify the relationship between zeroes and the coefficients</td></tr>
<tr><td>Compute the sum and product of zeroes of the polynomial in order to find the quadratic polynomial</td></tr>
<tr><td rowspan="2">Division Algorithm for Polynomials</td><td>Divide the two given polynomials in order to verify the division algorithm</td></tr>
<tr><td>Divide the given polynomial with its known zero in order to find all the other zeroes of that polynomial</td></tr>
</table>

Test items

LOB: Recall degree of polynomial in order to find the number of zeroes of polynomial

1. Which of the following statements is correct?

   Option 1: A polynomial of degree 2 has three zeroes
   Option 2: A polynomial of degree 3 has two zeroes
   Option 3: A polynomial of degree 4 has four zeroes
   Option 4: A polynomial of degree 5 has ten zeroes

   Correct Answer: Option 3

2. Ravi claims that the polynomial $p(x) = mx^a + x^{2b}$ has $4b$ zeroes. For Ravi's claim to be correct, which of these must be true?

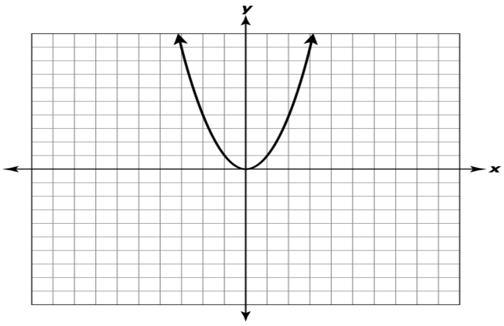
   Option 1: $a = 2b$ or $a = 4b$
   Option 2: $a = 2$ or $a = 4b$
   Option 3: $m = 2b$
   Option 4: $m = 4b$

   Correct Answer: Option 1

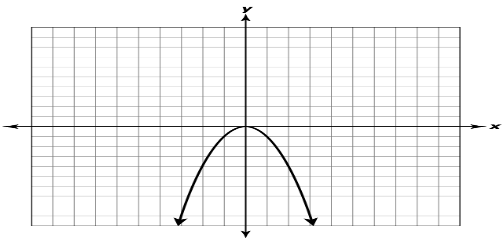
LOB: Analyse the graph of the polynomials in order to find the number of zeroes of polynomial

1. Which of the following graphs could be for the simple polynomial x^2 ?

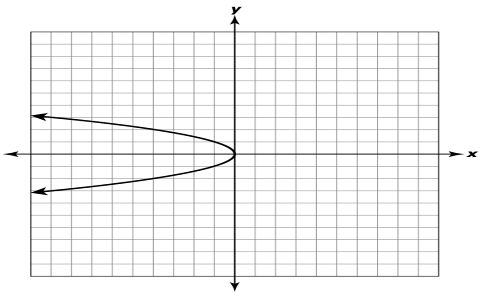
Option 1:



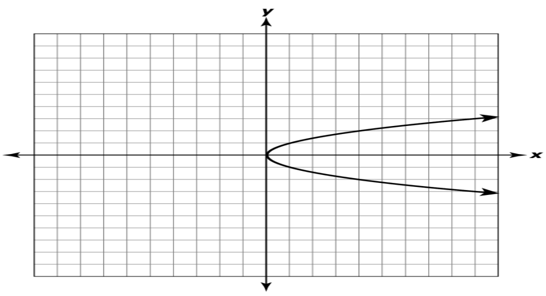
Option 2:



Option 3:



Option 4:



Correct Answer: Option 2

2. Consider the expression $x^{(m^2-1)} + 3x^{\frac{m}{2}}$, where *m* is a constant. For what value of *m*, will the expression be a cubic polynomial?

Option 1: -2
Option 2: -1
Option 3: 1
Option 4: 2

Correct Answer: Option 4

LOB: Compute zeroes of the polynomials in order to verify the relationship between zeroes and the coefficients

1. Consider the polynomial in z, $p(z) = z^4 - 2z^3 + 3$. What is the value of the polynomial at $z = -1$?

Option 1: 2
Option 2: 3
Option 3: 5
Option 4: 6

Correct Answer: Option 4

2. The polynomial in x, is $x^2 + kx + 5$, where *k* is a constant. At $x = 2$, the value of the polynomial is 15. What is the value of the polynomial at $x = 5$?

Option 1: 3
Option 2: 18
Option 3: 35
Option 4: 45

Correct Answer: Option 4

LOB: Compute the sum and product of zeroes of the polynomial in order to find the quadratic polynomial

1. Which of these is a zero of the polynomials $p(y) = 3y^3 - 16y - 8$?

Option 1: –8
Option 2: –2
Option 3: 0
Option 4: 2

Correct Answer: Option 2

2. Given that *m* + 2, where *m* is a positive integer, is a zero of the polynomial $q(x) = x^2 - mx - 6$. Which of these is the value of *m*?

Option 1: 1
Option 2: 2
Option 3: 3
Option 4: 4

Correct Answer: Option 1

LOB: Divide the two given polynomials in order to verify the division algorithm

1. The polynomial $q(z) = z^3 - 4z + a$ when divided by the polynomial $(z - 3)$ leaves remainder 5. What is the value of a?

Option 1: –10
Option 2: –3
Option 3: 3
Option 4: 10

Correct Answer: Option 1

2. The polynomial $p(x) = x^m + x$, where $m > 1$, when divided by $(x - a)$, leaves remainder 6. Given that *a* is a positive integer, what is the value of *m*?

Option 1: 2
Option 2: 3

Option 3: 5
Option 4: 6
Correct Answer: Option 1

LOB: Divide the given polynomial with its known zero in order to find all the other zeroes of that polynomial

1. Which of these is a factor of the polynomial $p(x) = x^3 + 4x + 5$?
   Option 1: $(x - 1)$
   Option 2: $(x + 1)$
   Option 3: $(x - 2)$
   Option 4: $(x + 2)$
   Correct Answer: Option 2

2. The polynomial$(x - a)$, where $a > 0$, is a factor of the polynomial$q(x) = 4\sqrt{2}x^2 - \sqrt{2}$. Which of these is a polynomial whose factor is $\left(x - \frac{1}{a}\right)$?
   Option 1: $x^2 + x + 6$
   Option 2: $x^2 - 5x + 4$
   Option 3: $x^2 + 4x - 3$
   Option 4: $x^2 + x - 6$
   Correct Answer: Option 4

Suggested Teacher Resources

1 Lesson Plan

<table>
<tr><td>Objectives</td><td>Students will be able to multiply polynomials using algebra tiles in order to solve polynomials related problems</td></tr>
<tr><td>Prerequisites</td><td>Plot points on the graph, solve quadratic equations for different values of (x,y)</td></tr>
<tr><td>Vocabulary</td><td>Zeros, polynomials, quadratic, parabola</td></tr>
<tr><td>Materials required</td><td>1. Algebra tiles (1 set for every two participants).
2.Flip charts or large pieces of newsprint for recording drawings and number sentences.
3.Crayons or coloured markers (for each group).</td></tr>
<tr><td>Procedure</td><td>

- Review the meaning of each algebra tile with students.

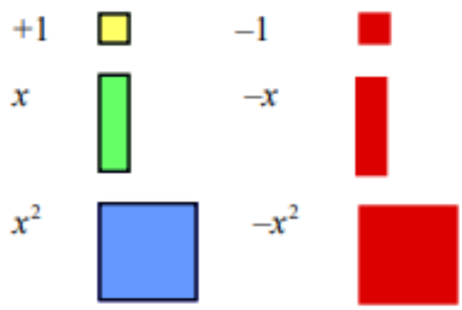


Fig1

- Ask students how the above table shows basic multiplication facts. Ask: How can we find the product 3 × 4 in this table? Ask: What is the product of 3 × 4?
- Help Students relate a basic multiplication table to a rectangular array to model the multiplication of polynomials. Ask: How can we model the multiplication of (x + 3)(x – 2)? ¨ Elicit the fact that algebra tiles that represent the first factor (x + 3) are placed on the vertical axis and algebra tiles that represent the second factor (x – 2) are placed on the horizontal axis.

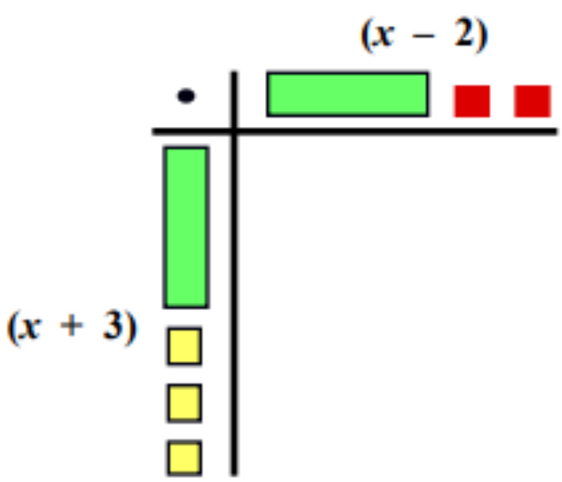


Fig2

</td></tr>
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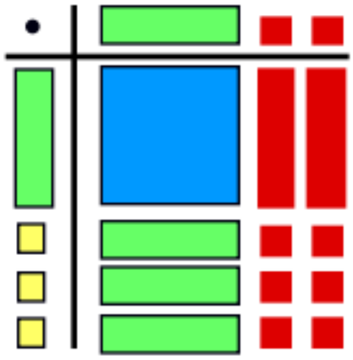
- Ask: How can we model (x + 3)(x – 2)? ¨ Give participants a chance to respond and model the multiplication with algebra tiles.



$(x + 3)(x - 2)$

- Place a transparency of fig1. on the board. Cover the “Model/Answer” column. Have participants use algebra tiles to model the following multiplications of polynomials. After each group has recorded its models and solutions, uncover the “Model/Answer” column and discuss the groups’ findings. (x – 1)(x – 4)
- Place a transparency of on the overhead projector. Cover the “Model/Answer” column. Have participants use algebra tiles to model the following multiplications of polynomials. After each group has recorded its models and solutions, uncover the “Model/Answer” column and discuss the groups’ findings. (–2x + 2) (x – 3).

</td></tr>
<tr><td>Source</td><td>https://betterlesson.com/lesson/595236/multiplying-polynomials-part-i?from=search</td></tr>
</table>

2 Activity

Materials: Polynomial cards; 4 or 6-sided die; Paper; Pencil

Answer sheet directions:

Students play in pairs, and each pair needs one set of polynomial cards and one answer sheet.

1. Both students draw one polynomial card.
2. One student rolls the die to determine what should be done with the two polynomials. A rolled one means add, two means subtract, three means multiply, and four means divide. If a five or six is rolled, the student simply rolls the die again until a one, two, three, or four is obtained.
3. Students perform the operation and write their answer on the answer sheet, according to what was rolled on the die. NOTE: In a division problem, let the higher-degree polynomial be the dividend, when applicable. Each team earns one point for every correct answer.
4. The used cards are now shuffled back into the deck of cards.
5. Students repeat steps 1–4 until time is called. u Who Wins? The pair with the highest number of points wins. u Tip: Remind students to write their answers in the correct form. Sums, differences, and products should be written in standard form. Answers to division problems should be written in the form q(x) d(x) + r(x) , when applicable.

Answer sheet for A Dice Polynomial Situation-:

Polynomial	Operation	Polynomial2	Answer

3. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

QR Code:

U8R9F1

Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Introduction and Properties	State the properties of linear equation in order to classify the given equations as linear or non linear	Uses graphical and other methods in order to finds solutions of pairs of linear equations in two variables
Graphical Method of Solution of a Pair of Linear Equations	Interpret the concepts of linear equations in order to represent any given situation algebraically and graphically	
	Demonstrate given two linear equations in order to comment on the nature/behaviour of the lines representing the linear equations	
Algebraic method for solving Linear Equations	Use different algebraic methods in order to solve a pair of linear equations	
	Use the most appropriate algebraic method in order to solve the given pair of linear equations	
	Use the concepts of pair of linear equations in two variables in order to represent any given situation algebraically and find its solution	
	Calculate the ratio of coefficients of linear equations in order to discuss the nature of pair of linear equations	
Equations Reducible to a Pair of Linear Equations in Two Variables	Rewrite the given equations (using substitution method) which are reducible to a pair of linear equations in order to find the solution of those equations	

Test items

LOB: State the properties of linear equation in order to classify the given equations as linear or non linear

1. Which of these is a linear equation in two variables?
   Option 1: 5x-2y=0
   Option 2: $x+x^2-2y+8=0$
   Option 3: $x-2y+10=x^2+y$
   Option 4: 3x-2y+1=0

Correct Answer: Option 1

2. If $x^{2n}-1+y^{m-4}=0$ is a linear equation, which of these is also a linear equation?
   Option 1: $x^n+y^m=0$
   Option 2: $x^{\frac{n}{5}}+y^{\frac{m}{5}}=0$
   Option 3: $x^{n+1/2}+y^{m+4}=0$
   Option 4: $x^{1/n}+y^{m/5}=0$

Correct Answer: Option 4

LOB: Interpret the concepts of linear equations in order to represent any given situation algebraically and graphically

1. Rohit earned ₹3550 by selling some bags each for ₹500 and some baskets each for ₹150. Aarav earned ₹3400 by selling the same number of bags each for ₹400 and the same number of baskets each for ₹ 200 as Rohit sold. Which of these equations relates the number of bags *x*, and the number of baskets, *y*?
   Option 1: 500x+150y=3550 and 400x+200y=3400
   Option 2: 500x+150y=3400 and 400x+200y=3550
   Option 3: 400x+150y=3550 and 500x+200y=3400
   Option 4: 500x+200y=3550 and 400x+150y=3400

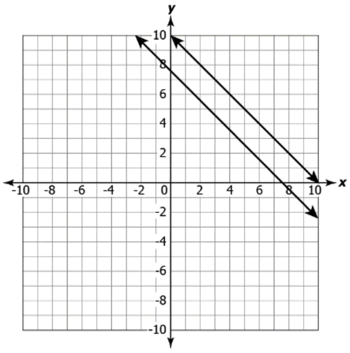
Correct Answer: Option 1

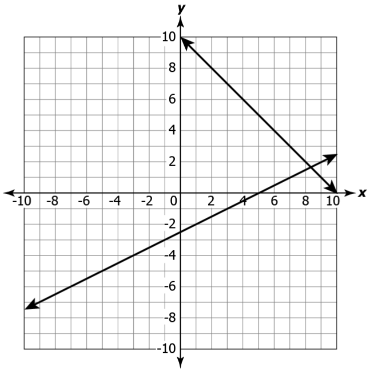
2. The cost of production per unit for two products, A and B, are ₹100 and ₹80 respectively. In a week, the total production cost is ₹32000. In the next week, the production cost reduces by 20%, and the total cost of producing the same number of units of each product is ₹25600. Which of these are the equations that can be used to find the number of units of A, *x*, and the number of units of B, *y*?
   Option 1: 100x+80y=25600 and 80x+64y=32000
   Option 2: 100x+64y=32000 and 80x+100y=25600
   Option 3: 80x+80y=32000 and 100x+64y=25600
   Option 4: 100x+80y=32000 and 80x+64y=25600
   Correct Answer: Option 4

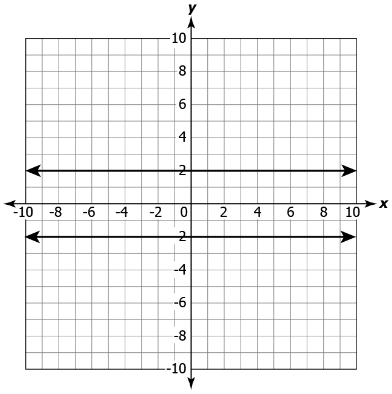
LOB: Demonstrate given two linear equations in order to comment on the nature/behaviour of the lines representing the linear equations

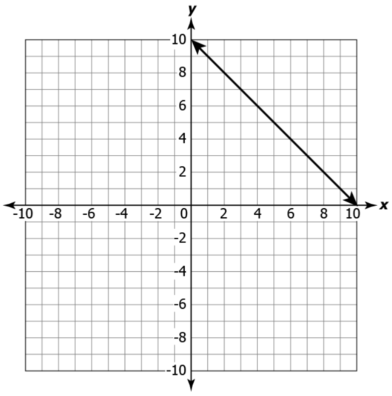
1. Which of these linear equations have a unique solution?

Option 1:



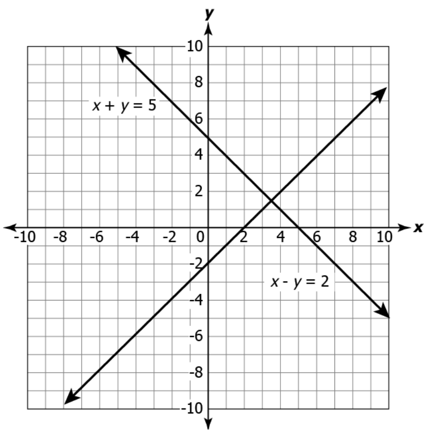
Option 2: 

Option 3: 

Option 4: 

Correct Answer: Option 2

2. Consider the graph shown.



Which of these is true about the given graph?

Option 1: These lines have a unique solution as they are intersecting at a point.
Option 2: These lines have infinitely many solutions as they lie in the same quadrant.
Option 3: These lines have a unique solution as the coefficient of x in both the equations is one.
Option 4: These lines have infinitely many solutions as they lie in the same quadrant.

Correct Answer: Option 1

LOB: Use different algebraic methods in order to solve a pair of linear equations

1. Consider the equations shown.

$$4x + 3y = 41$$
$$x + 3y = 26$$

Which of these is the correct way of solving the given pair of equations?

Option 1: $4x - x + 3y - 3y = 41 - 26$
Option 2: $4(x + 3y) + 3y = 41$
Option 3: $4x + x + 3y - 3y = 41 - 26$
Option 4: $4x + 3y + 3y = 41 - 26$

Correct Answer: Option 1

2. In the equations shown below, a and b are unknown constants.

$$3ax + 4y = -2$$
$$2x + by = 14$$

If (–3, 4) is the solution of the given equations, what are the values of a and b?

Option 1: a=2, b=5
Option 2: a=-2, b=5
Option 3: a=5, b=2
Option 4: a=5, b=-2
Correct Answer: Option 1

LOB: Use the most appropriate algebraic method in order to solve the given pair of linear equations

1. Consider the equations shown.$p + q = 5\ \& p - q = 2$
Which of these are the values of p and q?

Option 1: p=3.5, q=1.5
Option 2: p=1.5, q=3.5
Option 3: p=4, q=2
Option 4: p=2, q=4
Correct Answer: Option 1

2. Consider the equations shown: $ax + by = ab\ \& 2ax + 3by = 3b$
Which of these is the value of y in terms of a?

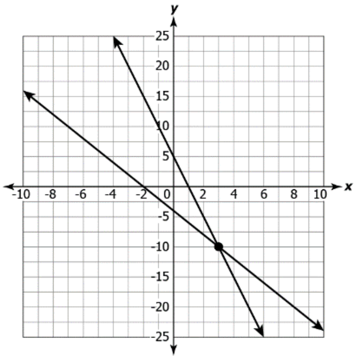
Option 1: $y = 2a - 35$
Option 2: $y = 2ab - 3b$
Option 3: $y = 9a - 35$
Option 4: $y = 3 - 2a$
Correct Answer: Option 4

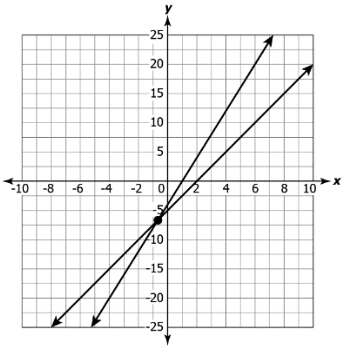
LOB: Use the concepts of pair of linear equations in two variables in order to represent any given situation algebraically and find its solution

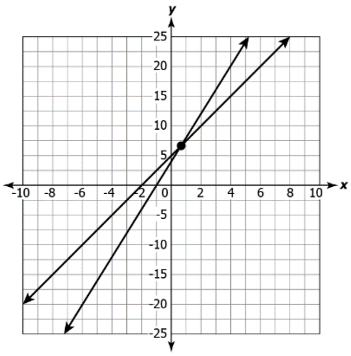
1. Shivi gave a note of ₹2,000 for a pair of jeans worth ₹500. She was returned 11 notes in denominations of ₹200 and ₹100. Which pair of equations can be used to find the number of ₹200 notes, x, and the number of ₹100 notes y? How many notes of ₹200 did she get?

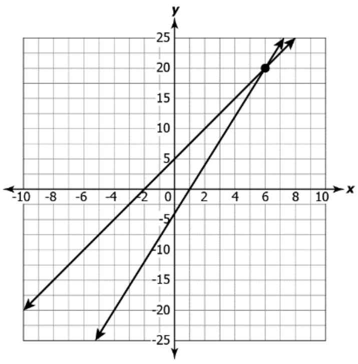
Option 1: $x + y = 11\ and\ 200x + 100y = 1500;\ 8$
Option 2: $y = x + 11\ and\ 200x + 100y = 2000;\ 4$
Option 3: $x + y = 11\ and\ 200x + 100y = 1500;\ 4$
Option 4: $x + y = 11\ and\ 100x + 200y = 2000;\ 10$
Correct Answer: Option 3

2. Arnav wants to plant some saplings in columns. If he increases the number of saplings in a column by 4, the number of columns decreases by 1. If he decreases the number of saplings by 5 in a column, the number of columns increased by 2.
Which of these graphs relates the number, x, of columns and the number, y, of plants in a column?

Option 1: 

Option 2: 

Option 3: 

Option 4: 

Correct Answer: Option 4

LOB: Calculate the ratio of coefficients of linear equations in order to discuss the nature of pair of linear equations

1. Consider the equations as shown:

$$9x + 6y = 5$$
$$3x + 2y = 7$$

Which of these is true about the given equations?

Option 1: This is a pair of coincident lines as $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Option 2: This is a pair of Intersecting lines as $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

Option 3: This is a pair of coincident lines as $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

Option 4: This is a pair of parallel lines as $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Correct Answer: Option 4

2. Consider the equations as shown:

$$(x-a)(y-b) = (x-2a)(y-\frac{b}{2})$$
$$x(x+\frac{1}{2b}) + y(y+\frac{a}{2}) - 2xy = 5 + (x-y)^2$$

On comparing the coefficients, a student says these pairs of equations is consistent. Is he/she correct? Which of these explains why?

Option 1: Yes; because they are intersecting lines.
Option 2: Yes; because they are parallel lines.
Option 3: No; because they are parallel lines.
Option 4: No; because they are intersecting lines.

Correct Answer: Option 1

LOB: Rewrite the given equations (using substitution method) which are reducible to a pair of linear equations in order to find the solution of those equations

1. Consider a pair of equations as shown.

$$\frac{7}{x+2} + \frac{2}{y-2} = \frac{33}{20}$$
$$\frac{2}{x+2} + \frac{6}{y-2} = \frac{23}{20}$$

Which of these pair of equations is equivalent to the given pair of equations?

Option 1: $7u + 2v = \frac{20}{33}$ and $2u + 6v = \frac{20}{23}$
Option 2: $7u + 6v = \frac{33}{20}$ and $2u + 2v = \frac{23}{20}$
Option 3: $7u + 2v = \frac{33}{20}$ and $2u + 6v = \frac{23}{20}$
Option 4: $2u + 6v = \frac{33}{20}$ and $7u + 2v = \frac{23}{20}$

Correct Answer: Option 3

2. Consider a pair of equations as shown.

$$\frac{5}{x+2} + \frac{7}{y+2} = \frac{31}{12}$$
$$\frac{4}{x+2} + \frac{3}{y+2} = \frac{17}{12}$$

What is the value of x and y?

Option 1: $x = 2$ and $y = 4$
Option 2: $x = 4$ and $y = 2$
Option 3: $x = 14$ and $y = 16$
Option 4: $x = 16$ and $y = 14$

Correct Answer: Option 2

Suggested Teacher Resources

1 Lesson Plan

Objectives	To demonstrate given two linear equations in order to comment on the nature/behaviour of the lines representing the linear equations
Prerequisites	Solving equations, graph plotting
Vocabulary	Solutions, zeros, polynomials, linear equations, variables, consistent, inconsistent, unique, infinite
Materials required	Two sheets of graph paper, ruler, pencil

Procedure

The teacher will start the class by introduction:
We shall consider a pair of linear equations in two variables of the type
$a_1x + b_1y = c_1$
$a_2x + b_2y = c_2$
Step 1: Let the first system of linear equations be
$x + 2y = 3$ … (i)
$4x + 3y = 2$ … (ii)
Step 2: From equation (i), we have
$y = \frac{1}{2}(3 - x)$.
Find the values of y for two different values of x as shown below.

x	1	3
y	1	0

Similarly, from equation (ii), we have
$y = 1/3(2 - 4x)$.
Then

x	-1	2
y	2	-2

Step 3: Draw a line representing the equation $x+2y = 3$ on graph paper 1 by plotting the points (1,1) and (3,0), and joining them.
Similarly, draw a line representing the equation $4x + 3y = 2$ by plotting the points (-1, 2) and (2, -2), and joining them.
Step 4: Record your observations in the first observation table.

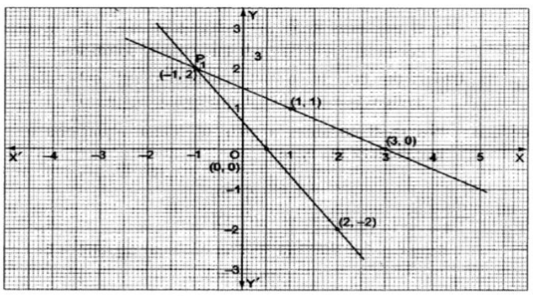


Fig. 1.1 Graph paper I

Step 5: Consider a second system of linear equations:

<table>
<tr>
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<td>
x – 2y = 3 … (iii)
-2x + 4y = -6 … (iv)
Step 6: From equation (iii), we get
<table>
<tr><td>x</td><td>3</td><td>1</td></tr>
<tr><td>y</td><td>0</td><td>-1</td></tr>
</table>
From equation (iv), we get
<table>
<tr><td>x</td><td>-3</td><td>-1</td></tr>
<tr><td>y</td><td>-3</td><td>-2</td></tr>
</table>
Draw lines on graph paper II using these points and record your observations in the second observation table.

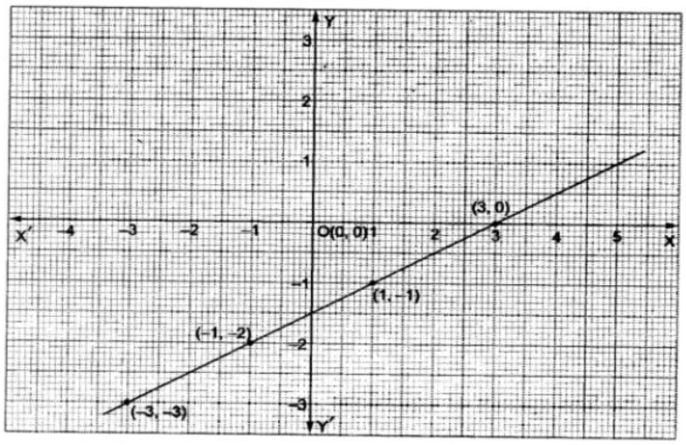

Fig. 1.2 Graph paper II

Next, the teacher asks students to conclude what they observed in these two graphs and equations.
Conclusion:

The first system of equations is represented by intersecting lines, which shows that the system is consistent and has a unique solution at x=-1, y=2.

The second system of equations is represented by coincident lines which shows that the system is consistent and has infinitely many solutions.
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<tr>
<td>Source</td>
<td>https://www.aplustopper.com/math-labs-with-activity-solve-the-system-of-linear-equations/</td>
</tr>
</table>

2

Activity

The teacher organizes the class into small groups of two or three students.
And presents them with the following situation:
A store sells pens at 2 Rupees and notebooks at 5 Rupees.
n = number of notebooks sold.
p = number of pens sold.
The following equations are true:
$4n = p$
$5n + 2p = 39$
Here is what Raj and Ria think the equations mean:

Now the teacher would ask students to share their solution with their partners and notice any similarities and differences in them.

Next the teacher will discuss:
Explain to students that this activity will enable them to decide which approach to collaboratively pursue. Once students have had a chance to discuss their work, hand out to each group a sheet of poster paper. In your groups agree on the best method for completing the School Fair task. Produce a poster that shows a joint solution that is better than your individual work. Students should now have another go at the task, but this time they will combine their ideas. Throughout this activity, encourage students to articulate their reasoning, justify their choices mathematically, and question the choices put forward by others. As students work you have two tasks, to note student approaches to their work and to support them working as a group.

Next, the teacher will ask:

1. What do the letters x and y represent?
2. Replace x and y in this equation by words and now say what the equation means.
3. Are there more bags of raisins or more bags of nuts? How do you know?
4. Do you have any values for x and y that work for the first equation?
5. How can you check to see if they also work for the second one? If these don't fit, what other values for x and y can you use?
6. Why have you chosen these values for x and y?
7. Suppose there are 5 bags of nuts, so x = 5. From the first equation, how many bags of raisins are there? [15.]
8. From the second equation, how many bags of raisins are there? [50.]
9. There cannot be both 15 and 50 bags of raisins! Can you find a value for x that will give the same answer in both cases?
10. How can you check that your answer is right?

4. QUADRATIC EQUATIONS

QR Code:

P8W1S5

Learning outcome and Learning Objectives:

<table>
<tr><th>Content area / Concepts</th><th>Learning Objectives</th><th>Learning Outcome</th></tr>
<tr><td rowspan="2">Introduction to Quadratic Equations</td><td>Write Quadratic Equation in order to represent the given situation algebraically</td><td rowspan="10">Demonstrates knowledge of application of various strategies in order to find roots and determine the nature of roots of a given equation</td></tr>
<tr><td>Rewrite the given equations in the standard form in order to check whether they are quadratic or not</td></tr>
<tr><td rowspan="2">Factorization Method</td><td>Solve quadratic equations through factorization in order to find its roots</td></tr>
<tr><td>Solve quadratic equations through middle term splitting in order to find its roots</td></tr>
<tr><td>Completing Square Method</td><td>Solve quadratic equations by completing the square in order to find its roots</td></tr>
<tr><td>Solving a Quadratic Equation</td><td>Use the quadratic formula in order to find the roots of quadratic equation</td></tr>
<tr><td>Roots of a Quadratic Equation</td><td>Substitute the value of the roots of a given equation in order to verify the roots of that quadratic equation</td></tr>
<tr><td rowspan="2">Nature of Roots</td><td>Examine the discriminant of quadratic equation in order to find out the nature of its roots</td></tr>
<tr><td>Describe the nature of the roots of a quadratic equation in order to determine that whether a given situation is possible or not</td></tr>
</table>

Test items

LOB: Write Quadratic Equation in order to represent the given situation mathematically

1. A sum of ₹4000 was divided among x persons. Had there been 10 more persons, each would have got ₹80 less. Which of the following represents the above situation?
   - **Option 1:** $x^2 + 10x - 500 = 0$
   - **Option 2:** $8x^2 + 10x - 400 = 0$
   - **Option 3:** $x^2 + 10x + 500 = 0$
   - **Option 4:** $8x^2 + 10x + 400 = 0$

Correct Answer: Option 1

2. The product of three consecutive integers is equal to 6 times the sum of the three integers. If the smallest integer is x, which of the following equations represent the above situation?
   - **Option 1:** $2x^2 + x - 9 = 0$
   - **Option 2:** $x^2 + 2x + 18 = 0$
   - **Option 3:** $x^2 + 2x - 18 = 0$
   - **Option 4:** $2x^2 - x + 9 = 0$

Correct Answer: Option 3

LOB: Rewrite the given equations in the standard form in order to check whether they are quadratic or not

1. Which of these is a quadratic equation?
   - **Option 1:** $x^2 - x = x^2 + 2$
   - **Option 2:** $x^2 + \frac{1}{x^2} + 1 = 0$
   - **Option 3:** $(x + 1)(x + 3) = (x - 1)(x - 4)$
   - **Option 4:** $x(x + 1) = 2x - 3$

Correct Answer: Option 4

2. Consider the equation $kx^2 + 2x = c(2x^2 + b)$

For the equation to be quadratic, which of these cannot be the value of k?
   - **Option 1:** c
   - **Option 2:** 2c
   - **Option 3:** 3c
   - **Option 4:** 2c + 2b

Correct Answer: Option 2

LOB: Solve quadratic equations through factorization in order to find its roots

1. What are the roots of the equation $4x^2 - 2x - 20 = x^2 + 9x$?
   - **Option 1:** $\frac{-4}{3}$ and -5
   - **Option 2:** $\frac{4}{3}$ and -5
   - **Option 3:** $\frac{-4}{3}$ and 5
   - **Option 4:** $\frac{4}{3}$ and 5

Correct Answer: Option 3

2. What is the smallest positive integer value of k such that the roots of the equation x^2-9x+18+k=0 can be calculated by factoring the equation?
   - **Option 1:** 1
   - **Option 2:** 2
   - **Option 3:** 3
   - **Option 4:** 4

Correct Answer: Option 2

LOB: Solve quadratic equations through middle term splitting in order to find its roots

1. A student is trying to find the roots of $3x^2 - 10x - 8 = 0$ by splitting the middle term as follows:

Step 1: $3x^2 - 10x - 8 = 0$
Step 2: $3x^2 - mx + nx - 8 = 0$
What could be the values of m and n?

Option 1: $m = 12$ and $n = 2$
Option 2: $m = -12$ and $n = -2$
Option 3: $m = 8$ and $n = 2$
Option 4: $m = -8$ and $n = -2$

Correct Answer: Option 1

2. Rahul follows the below steps to find the roots of the equation $3x^2 - 11x - 20 = 0$, by splitting the middle term.

Step 1: $3x^2 - 11x - 20 = 0$
Step 2: $3x^2 - 15x + 4x - 20 = 0$
Step 3: $3x(x - 5) + 4(x - 5) = 0$
Step 4: $(3x - 4)(x - 5) = 0$
Step 5: $x = 4/3 \ and\ 5$
In which step did Rahul make the first error?

Option 1: Step 2
Option 2: Step 3
Option 3: Step 4
Option 4: Step 5

Correct Answer: Option 3

LOB: Solve quadratic equations by completing the square in order to find its roots

1. A student simplified the equation $5x^2 + 3x - 1 = 0 \ as\ x^2 + \frac{3}{5}x - \frac{1}{5} = 0$

Which of these could be the next step to solve the equation using the method of completing the square?

Option 1: $x^2 + 5(2)x + \frac{3}{5}x - \frac{1}{5} = 0$
Option 2: $x^2 + 5(2)x + \frac{3}{5}x - \frac{3}{5}x + \frac{1}{5} = 0$
Option 3: $x^2 + 2\left(\frac{3}{5}x\right) + \left(\frac{2}{3}\right)^2 - \frac{1}{3} = 0$
Option 4: $x^2 + \left(\frac{3}{5}\right)x + \left(\frac{3}{10}\right)^2 - \frac{9}{100} - \frac{1}{5} = 0$

Correct Answer: Option 4

2. The roots of the equation $a^2x^2 - 3abx - 4b^2 = 0$ by completing the square method is

Option 1: $\frac{4b}{a}$ and $-\frac{b}{a}$
Option 2: $\frac{b}{2a}$ and $\frac{b}{a}$
Option 3: $\frac{2a}{b}$ and $\frac{a}{b}$
Option 4: $\frac{2a}{b}$ and $\frac{2b}{a}$

Correct Answer: Option 1

LOB: Use the most appropriate method in order to find the roots of quadratic equation

1. A teacher asked students to find the roots of the equation $\frac{x}{x+1} + \frac{x+1}{x} - \frac{34}{15} = 0$. Two students, Ravi and Ankit gave following answers. Ravi said one of the roots is 3/2. Ankit said one of the roots is (-5)/2. Who is correct?

Option 1: Ravi

Option 2: Ankit
Option 3: Both Ravi and Ankit
Option 4: Neither Ravi nor Ankit
Correct Answer: Option 3

2. Which is the best method to find the roots of the equation $2x^2 + 7x + 5 = 0$?
   Option 1: splitting the middle term $7x$ to $5x + 2x$ and then factorizing the equation
   Option 2: splitting the middle term $7x$ to $10x - 3x$ and then factorizing the equation
   Option 3: using the method of completing the square to get $x^2 + \frac{7}{2}x + 5 = 0$
   Option 4: using the method of completing the square to get $x^2 + 7x + 5 = 0$
Correct Answer: Option 1

LOB: Substitute the value of the roots of a given equation in order to verify the roots of that quadratic equation

1. Which is the correct way to verify that 2 and 3 are the roots of the equation $x^2 - 5x + 6 = 0$?
   Option 1: On substituting $x = 2$ and $x = 3$ on the left-hand side of the equation, the result should be 0
   Option 2: On substituting x^2 with 2 and x with 3 on the left-hand side of the equation, the result should be 0.
   Option 3: On substituting $x = 2$ on the left-hand side of the equation, the result should be 3.
   Option 4: On substituting $x = 3$ on the left-hand side of the equation, the result should be 2.
Correct Answer: Option 1

2. A student solved a quadratic equation and obtains the roots as –4 and 3. Part of the student's work to verify the root is shown: $(-4)^2 + 2(-4) - 9 = 0$. Based on the student's work, which of these is correct?
   Option 1: The student calculated the roots correctly.
   Option 2: The student calculated the roots correctly but should replace $2(-4)$ in his work with $2(3)$.
   Option 3: The student calculated the roots of the equation that can be obtained by adding 1 to the equation that the student solved.
   Option 4: The student calculated the roots of the equation that can be obtained by adding -1 to the equation that the student solved.
Correct Answer: Option 3

LOB: Examine the discriminant of quadratic equation in order to find out the nature of its roots

1. The roots of $ax^2 + bx + c = 0, a \neq 0$ are real and unequal. Which of these is true about the value of discriminant, D?
   Option 1: $D < 0$
   Option 2: $D > 0$
   Option 3: $D = 0$
   Option 4: $D \leq 0$
Correct Answer: Option 2

2. Consider the equation $px^2 + qx + r = 0$. Which conditions are sufficient to conclude that the equation have real roots?
   Option 1: $p > 0 , r < 0$
   Option 2: $p > 0 , r > 0$
   Option 3: $p > 0 , q > 0$
   Option 4: $p > 0 , q < 0$
Correct Answer: Option 1

LOB: Describe the nature of the roots of a quadratic equation in order to determine that whether a given situation is possible or not

1. For what value of k, are the roots of the quadratic equation $3x^2 + 2kx + 27 = 0$ real and equal?

    Option 1: $k = \pm 3$
    Option 2: $k = \pm 9$
    Option 3: $k = \pm 6$
    Option 4: $k = \pm 4$

Correct Answer: Option 2

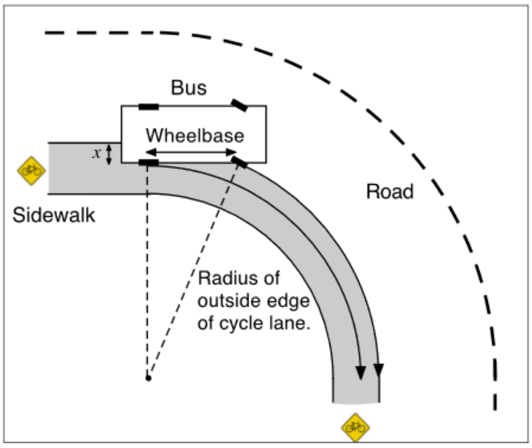
2. If the equation $x^2 - mx + 1 = 0$ does not possess real roots, then

    Option 1: $-3 < m < 3$
    Option 2: $-2 < m < 2$
    Option 3: $m > 2$
    Option 4: $m < -2$

Correct Answer: Option 2

Suggested Teacher Resources

Lesson Plan

Objective	1. Solve quadratic equations through factorization in order to find its roots 2. Solve quadratic equations through middle term splitting in order to find its roots 3. Solve quadratic equations by completing the square in order to find its roots
Prerequisite	Methods of finding roots of quadratic equations
Materials required	Cutting corners worksheet, coloured chalks
Procedure	Activity: Cutting Corners Setup: Students are divided into groups of 5 and need to solve the cutting corner problem. All group members must agree on the solution before presenting. Problem: When a bus turns a corner, it must swing out so that its rear wheels don't go into the cycle lane. In this picture, as the bus goes round the corner, the front wheel is on the edge of the cycle lane, but the rear wheel cuts into the cycle lane.  The diagram below shows the geometry of this situation  The distance between the front and the rear wheel is called wheelbase, w. Let r represent the radius of the outside edge of the cycle lane. The distance marked x shows the amount by which the rear wheels cuts into the cycle lane. Teacher's note: Students may find the dynamic situation hard to visualize. A physical demonstration using, for example, a bicycle, or a toy truck could be helpful. If the wheels donorship sideways, they will always be tangential to the circle they describe. The back-wheel changes direction as soon as the

front wheel does, a wheelbase length behind the front wheel. The back wheel is pulled forwards and inwards, turning a smaller circle than the front wheel.

Questions:
Q1) Use the diagram to show that $x^2 -2xr+w^2 = 0$.
Q2) Let w = 10 feet and r = 17 feet.
a. Figure out how much the rear wheel cuts into the cycle lane.
b. Figure out how far the front wheel must be from the outside edge of the cycle lane for the rear wheel not to cut into the cycle lane.

Teacher's note:
Relevant mathematics is not identified: For example: the students does not refer to Pythagorean Theorem (Q1).
Ask: What shape can they see in the diagram? What kind of equations are they studying?

Sample solution:

Ans 1)
The radius of the circle formed by the back wheel is r-x.
This radius is perpendicular to the tangent to the circle, the line segment formed by the wheelbase.
Thus r, r-x and w form a right-angled triangle.
Using the Pythagorean Theorem:
$r^2 = (r-x)^2 + w^2$ $r^2 = r^2 - 2xr + x^2 + w^2$
$x^2 - 2xr + w^2 = 0$

Ans 2.a)
Take w = 10 and r = 17
substitute into the equation: $x^2 -34x+100= 0$
There is a range of methods students might use to figure out the value for x.

Completing the square:
$(x- 17)^2 -172+ 100 = 0$
$(x- 17)^2-189 = 0$
$(x- 17)^2= 189$
$x- 17 = 13.75$
or
$x- 17 = -13.75$
(Taking the square root of both sides.)

$x= 30.75$
or
$x= 3.25$.
The outside edge of the cycle lane is a circle of radius only 17 feet, therefore x= 30.75 feet is not a possible solution in this context.

The bus cuts into the cycle lane by 3.25 feet (3 feet 3 inches.)

Using the quadratic formula:
The radius of the outside edge of the cycle lane is only 17 feet, therefore x= 30.75 feet is impossible.

The bus cuts into the cycle lane by 3.25 feet (3 feet 3 inches.)
Replacing $r-x$ with the variable a.
Note that a represents the radius of the circle described by the rear wheel.

$r^2 =a^2 + w^2$
$172 = a^2 + 102$
$a^2 =289 -100 = 189$

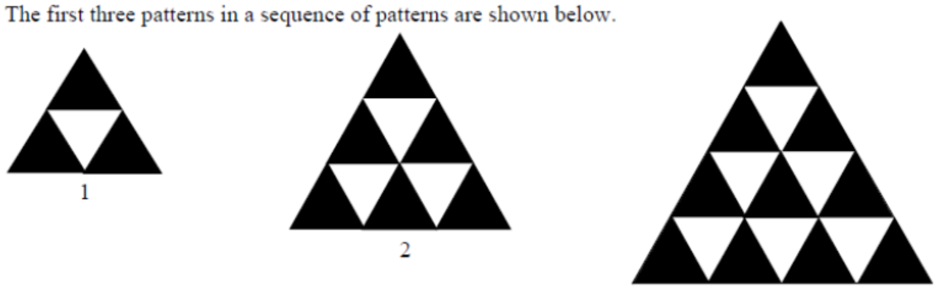
	$a = 13.75$ or $a= -13.75$. $x= r-a \Rightarrow$ $x= 17 -13.75$ or $x= 17 + 13.75$ $x=3.25$ or $x= 30.75$ The radius of the outside edge of the cycle lane is only 17 feet, therefore $x = 30.75$ feet is impossible. The bus cuts into the cycle lane by 3.25 feet (3 feet 3 inches.) Ans 2.b) The student now needs to reinterpret the situation, to find the value of r when x=0. Using the Pythagorean Theorem: b2=172+ 10=389 b=19.72 or b= −19.72. A negative value for bis impossible given the context, therefore the front wheel is: 19.72 −17 = 2.72 feet (approx. 2 feet 9inches) from the outside edge of the cycle lane. Students may also solve the quadratic equation: $(17 + y)^2 =17^2 + 10^2$ $289+ 34y+ y^2 =289+ 100$ $y^2 +34y-100= 0$. They could do this by either completing the square or using the quadratic formula. Alternatively, students could start with the equation from Q1 and, instead of substituting in $r = 17$, substitute $r = 17 + x$. Simplifying this leads to the same quadratic equation as above. Completing the square: $(y+ 17)^2 -289 -100 = 0$ $(y+ 17)^2= 389$ $y+ 17 = 19.72$ or $y+ 17 = -19.72$ $y = 19.72 -17= 2.72$ or $y =-19.72 -17 = -36.72$ A negative value for y is impossible, therefore the front wheel is 2.72 feet (approx. 2 feet 9 inches) from the outside edge of the cycle lane. Using the quadratic formula: Y = −b± √(b2−4ac)/2a (a = 1, b = 34, and c =−100) $y = [-34 \pm \sqrt{(34^2 - 4\cdot1\cdot(-100))}]/ (2\cdot1)$ $y = (-34 \pm \sqrt{1556})/2$ $y= 2.72$ or $y=-36.72$ A negative value for y is impossible, therefore the front wheel is 2.72 feet (approx. 2 feet 9 inches) from the outside edge of the cycle lane

Source	https://www.map.mathshell.org/download.php?fileid=1736

2 Activity

Objective: Finding the general term (Tn) of a quadratic pattern in many ways.
Learning Outcomes: At the end of this lesson students should be able to find the General Term of a Quadratic Pattern using more than one method.
Materials Required: growth patterns

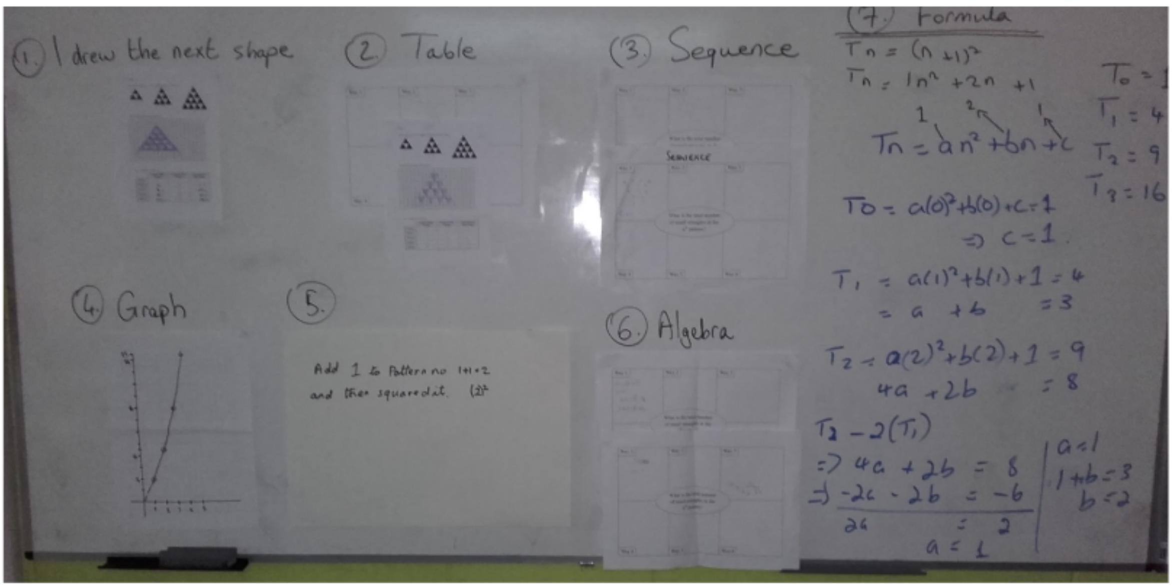
The first three patterns in a sequence of patterns are shown below.



Teacher's Note: Teacher can use website http://www.visualpatterns.org/ to search for more patterns
Setup:
Students will be divided into groups of 4 and work together to solve the following worksheet.
Q1) Draw the 4th pattern.
Q2) Complete the following table

	Number of black triangles	Number of white triangles	Total number of small triangles
Pattern 1	3	1	T1 = 4
Pattern 2			T2 =
Pattern 3			T3 =
Pattern 4			T4 =
Pattern 5			T5 =

Q3) How many small triangles would be there in pattern 100?
Q4) Write an expression in n for the total number of triangles in the nth pattern.
Comparing and Discussing:



Source: https://www.projectmaths.ie/documents/MathsCounts2017/TheGeneralTermOfAQuadratic.pdf

5. ARITHMETIC PROGRESSION

QR Code:

Q7C8H7

Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Arithmetic Progressions-Introduction	Produce patterns in order to observe that succeeding terms are obtained by adding a fixed number to the preceding terms.	Develops strategies in order to apply the concept of AP to daily life situations
	Distinguish between finite and infinite AP in order to determine the nature and write the last term of the given AP	
nth term of AP	Calculate the nth term of a given AP in order to find its terms and their nature	
nth term of AP Sum of an AP	Calculate the nth term of a given AP in order to solve for a real-life word problem	
	Calculate the sum of a given AP in order to get the solution for a real-life word problem	
Sum of an AP Last term of an AP	Calculate the sum of a given AP in order to solve for various questions related to progression	
	Calculate the last term of the given AP in order to find the solution for a real-life word problem	
Last term of an AP	Use appropriate formula to calculate the last term of the given AP	

Test items

LOB: Produce patterns in order to observe than succeeding terms are obtained by adding a fixed number to the preceding terms.

1. What are the missing numbers in the pattern given below?
-32, -19.5, -7, ___, 18, 30.5, ______

Option 1: 19.5 and 43
Option 2: 5.5 and 43
Option 3: 5 and 43
Option 4: 5 and 43.5

Correct Answer: Option 2

2. Which of the following two numbers do you think should be the succeeding terms of the pattern below?
-1/3, -1/12, 1/6, 5/12, ___, ___

Option 1: 2/3 and 11/12
Option 2: 4/6 and 11/4
Option 3: 4/6 and 12/11
Option 4: 1/2 and 24/22

Correct Answer: Option 1

LOB: Distinguish between finite and infinite AP in order to determine the nature and write the last term of the given AP

1. Which of the following is an infinite AP?

a) -2, -1, 0, 1, 2, 3, 4, 5, 6, 7…
b) -14, -12, -10, -8, ___, _____, _____, 0
c) First 40 even numbers
d) Odd Numbers

Option 1: a and b
Option 2: b and c
Option 3: a and d
Option 4: b and d

Correct Answer: Option 3

2. What would be the last term of an arithmetic progression with 10 terms whose second term is -23 and the third term is -35?

Option 1: 119
Option 2: -107
Option 3: -650
Option 4: -12

Correct Answer: Option 1

LOB: Calculate the nth term of a given AP in order to find its terms and their nature

1. If the first term of an AP is 2 and the common difference is (-1)/2, what would be the 12th term of the AP?

Option 1: $2 + 11\left(-\frac{1}{2}\right)$
Option 2: $2 + 12\left(-\frac{1}{2}\right)$
Option 3: $2 - 11\left(-\frac{1}{2}\right)$
Option 4: $2 - 12\left(-\frac{1}{2}\right)$

Correct Answer: Option 1

2. Find the nth term of the AP shown below.
_, _, -36, _, - 44, ………. up to n
Option 1: 4n – 32
Option 2: -40 + 4n
Option 3: 24 - 4n
Option 4: - 4n – 24
Correct Answer: Option 4

LOB: Calculate the nth term of a given AP in order to solve for a real-life word problem

1. Jessie needs ₹1,70,000 for her College admissions in the starting of January 2021.Her mother helped her by creating a fund of ₹12,000 in the end of January 2019. Thereafter she has been collecting ₹5500 in the starting of each month for Jessie's college fund. How much money will be collected in the fund before Jessie's admissions?
Option 1: ₹$(12000 + 11(5500))$
Option 2: ₹$(5500 + 11(12000))$
Option 3: ₹$(12000 + 23(5500))$
Option 4: ₹$(5500 + 23(12000))$

Correct Answer: Option 3

2. On an average Jane writes 12 short stories every year. Although in the first year she was able to write some more short stories. If after 12 years of her career she has written a total of 147 stories. How many had she written after 7 years of her writing?
Option 1: 75
Option 2: 84
Option 3: 87
Option 4: 90
Correct Answer: Option 3

LOB: Calculate the sum of a given AP in order to get the solution for a real-life word problem

1. Aamod loves to travel and he travels every year. He has seen 8 different cities in his first year. Thereafter every year he has seen 2 cities. If he had followed this pattern, how many cities did he see by the end of 10 years of travel?
Option 1: $\left(5(16 + 9(2))\right)$ cities
Option 2: $\left(5(8 + 9(2))\right)$ cities
Option 3: $\left(5(16 + 10(2))\right)$ cities
Option 4: $\left(5(8 + 10(2))\right)$ cities
Correct Answer: Option 1

2. Mr. Kapoor is collecting donations for building a new school building in a village. His wife starts by donating ₹125000. His brother also contributes by donating ₹54000. Thereafter every person who contributes in the donation pays ₹5250 more than the previous donor. How much amount has Mr. Kapoor able to collect after 23 donations?
Option 1: ₹164250
Option 2: ₹2400750
Option 3: 2525750
Option 4: ₹2570250
Correct Answer:

LOB: Calculate the sum of a given AP in order to solve for various questions related to progression

1. What is the sum of all the three-digit numbers divisible by 12?

Option 1: $\left(\frac{73}{2}(108+996)\right)$

Option 2: $\left(\frac{75}{2}(216+996)\right)$

Option 3: $\left(\frac{73}{2}(216+996)\right)$

Option 4: $\left(\frac{75}{2}(108+996)\right)$

Correct Answer: Option 4

2. The sum of first 76 terms of an AP is 21850 and the sum of first 40 terms is 7900. Find the sum of first 100 terms of this AP

   Option 1: 29750
   Option 2: 34155
   Option 3: 34750
   Option 4: 35350

Correct Answer: Option 3

LOB: Calculate the last term of the given AP in order to find the solution for a real-life word problem

1. Mr. Singh buys a property every year for 12 years. Every year he buys X acres more than the previous year. If in the 8th year he bought 45 acres of land and in the 5th year he bought 30 acres of land, how many acres did he buy in the last year?

   Option 5: $(10+11(5))$ Acres
   Option 6: $(10+12(5))$ Acres
   Option 7: $(5+11(10))$ Acres
   Option 8: $(5+12(10))$ Acres

Correct Answer: Option 1

2. Ranvijay likes to collect stamps. He has a total of 4290 stamps after 10 years of his stamp collection. He counted his total stamps in the 6th year as well and found that he had a total of 1830 stamps then. How many stamps did he collect in the last year of his collection?

   Option 1: 522
   Option 2: 646
   Option 3: 708
   Option 4: 1370

Correct Answer: Option 3

LOB: Use appropriate formula to calculate the last term of the given AP

1. The first term of an AP is -76 and the sum of first 45 terms is -9360. Which of the following is the last term of this AP?

   Option 1: 416 + 76
   Option 2: 416 - 76
   Option 3: -416 - 76
   Option 4: -416 + 76

Correct Answer: Option 4

2. The sum of n terms of an AP is 8558. If the 15 term is 97 and the 10th term is 32. What is the nth term?

   Option 1: 431
   Option 2: 474
   Option 3: 644
   Option 4: 7612

Correct Answer: Option 2

Suggested Teacher Resources

1

Lesson Plan

Objectives	SWBAT translate from algebraic to numerical and graphical representations of arithmetic sequences in order to understand the progression patterns
Prerequisite Knowledge	A.P. generation or general form of A.P; Finding first term, common difference and *n*th term.
Material Required	Activity sheet, strips
Procedure	Begin the class by do now activity as arithmetic sequences are the output of a linear function (with a domain restriction), we begin this lesson by practicing writing the equation of a line through two points.

Do Now

Write the equation of the line through the two points.

1. (2, 5) and (7, -5)
2. $\left(6,\frac{1}{3}\right) and \left(8,\frac{5}{3}\right)$
3. (-2, 5) and (6.5, 3)

- Here we focus our attention as a class on the analysis of arithmetic patterns. We will work together to learn the formal methods of defining patterns explicitly and recursively. We build on the sequence strips activity ([a), (d), and (f).

a. -3, 0, 6, 9, ________, ________, ________	
b. 1.5, 3, 6, 12, ________, ________, ________	
c. 1, 4, 9, 16, ________, ________, ________	
d. 2, 4.5, 6, 7.5, ________, ________, ________	
e. 1, 1, 2, 3, 5, 8, ________, ________, ________	
f. 9, 5, 1, -3, ________, ________, ________	

g. 48, 24, 12, ________, ________, ________	
h. 27, 9, 3, 1, ________, ________, ________	
i. 8, 2, 0, 2, 8, 18, ________, ________, ________	
j. 0.75, 1.5, 3, 6, ________, ________, ________	

- Ask students to work together to come up with a rule for each sequence that gives the term as a function of n, the term's position in the sequence
- Ask students to submit their sequence rule, which gives us a collection of expressions to consider. We check each idea together to see if substituting 1 for n yields the correct initial term, 2 for n yields the correct second term, etc.
- Our intent is to employ the students' observations in a compare and contrast discussion of notations for describing sequences and linear functions.
- To ensure that we make strong connections to student's Algebra knowledge, it is important to help students recognize an arithmetic sequence as a linear function with a restricted domain, the positive counting numbers. During this segment of the lesson, students generally come up with slope-intercept form of an equation easily. Occasionally, one or more groups share the arithmetic sequence formula as well.
- As we transition to the practice, give students the opportunity to take notes on the standard formula used to generate the nth term in an arithmetic sequence.
- Review some examples of how to use this formula to find a term, the term number, or the common difference. Also examine the graph of an arithmetic sequence and compare it to the graph of a linear function.
- Now students will work in table groups to practice with the arithmetic sequence formula. Stop to ask questions and give hints as necessary.

Q1. Name the first five terms of each arithmetic sequence described.

1. $a_n = 7 + 5(n-1)$

2. $a_n = a_{n-1} - 2$

3. $a_1 = \frac{3}{4}$, $d = -\frac{1}{4}$

Q2. Name the next four terms of each arithmetic sequence.

1. 21, 15, 9, … 2. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$

Q3. Find the explicit form and the indicated term for each arithmetic sequence.

1. −17, -13, -9, …

$a_n =$

$a_{12} =$

2. a_{10} for 8, 3, -2, …

$a_n =$

$a_{12} =$

Q4. Find the explicit form of the arithmetic sequence.

1. $a_4 = 34$, $a_8 = 6$ $a_n =$
2. $a_2 = -3$, $a_7 = 22$ $a_n =$

3. $a_5 = 6$, $a_6 = 15$ $a_n =$

- After students have worked for 10 or 15 minutes, ask them to pair up and check answers. When they are finished ask if anything needs to be resolved at the board. If so, ask for student volunteers to work out a problem at the board or if there is confusion around a particular problem solve it for them.

Assessment-:

Q1. Name the first five terms of each arithmetic sequence described.

1. $a_n = 2 + 5(n-1)$
2. $a_n = a_{n-1} - 2$
3. $a_1 = \frac{1}{2}, d = -\frac{1}{4}$

Q2. Name the next four terms of each arithmetic sequence.

1. 20, 11, 2, …
2. $\frac{1}{2}, \frac{5}{2}, \frac{7}{2}, \dots$

Q3. Find the explicit form and the indicated term for each arithmetic sequence.

1. -13, -9, -5…
   $a_n =$
   $a_{12} =$
2. a_{10} for 6, 4, 2, …
   $a_n =$
   $a_{12} =$

Q5. Find the explicit form of the arithmetic sequence.

1. $a_4 = 30, a_8 = 6$
   $a_n =$
2. $a_2 = 2, a_7 = 12$
   $a_n =$
3. $a_5 = 6, a_6 = 18$
   $a_n =$

Source	https://betterlesson.com/lesson/571165/arithmetic-sequences

2

Activity

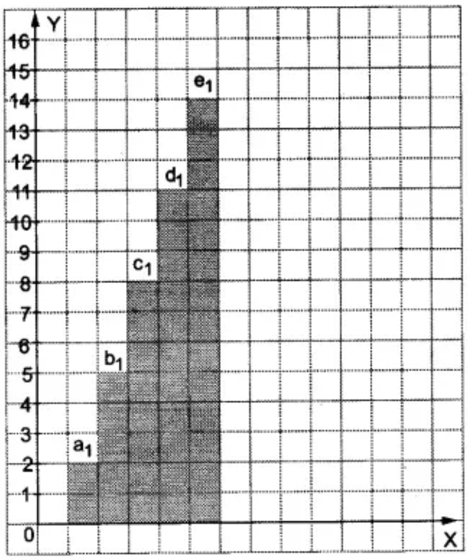
Objective-: To verify using a graphical method that a given sequence of numbers is an arithmetic progression.

Material Required-:

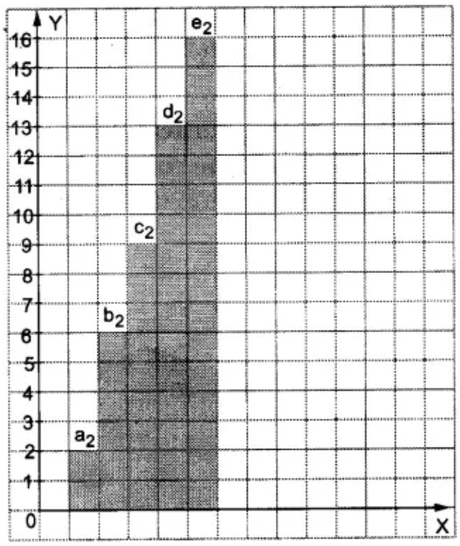
1.Two sheet of graph paper
2. Ruler
3.Glue
4.Pencil.
5.Paper tape(coloured).

Procedure-:

Step1: Mark both the sheet of graph paper with squares (each side=1 unit).
Step2: Mark x and y axis on each sheet of graph paper.
Step3: We will first test if the sequence 2,5,8,11,14 ….is an AP.
Step4: Cut the paper tape in rectangular strips of length 2 units, 5 units, 8 units,11 units, 14units….
Step5: Using the x axis as the base, paste the strips on graph paper and sequentially record the observation on the first observation table.



Step6: Now we will test if sequence 2,6,9,13, 16. is an A.P.
Step7: Cut the paper tape in rectangular strips of length 2 units, 6 units, 9 units,13 units, 16units…...



Step8: Using the x axis as the base, paste the strip on the second graph sequentially and record your observation in the second observation table.

Observations-:
For Fig1.

$b_1 - a_1$	$c_1 - b_1$	$d_1 - c_1$	$e_1 - d_1$	**Common difference** $d = b_1 - a_1 = c_1 - b_1 = d_1 - c_1 = e_1 - d_1$
3	**3**	**3**	**3**	**3**

For Fig2.

$b_2 - a_a$	$c_2 - b_2$	$d_2 - c_2$	$e_2 - d_2$	**Common difference** $d = b_2 - a_2 = c_2 - b_2 = d_2 - c_2 = e_2 - d_2$
4	**3**	**4**	**3**	**Does not exist**

<u>Conclusion-:</u>
1.The sequence 2,5,8,11,14 forms a uniform ladder having equal steps and has common difference of 3 units. Hence this sequence is an A.P.
2. The sequence 2,6,9,13,16 forms a uniform ladder having equal steps and has common difference of 3 units. Hence this sequence is an A.P.

<u>Source-:</u>
https://www.aplustopper.com/math-labs-activity-sequence-numbers-arithmetic-progression-ap/

6. TRIANGLES

QR Code:

B5U8J2

Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Similar figures	Distinguish between congruency and similarity in order to understand the concept of similar figures	Uses reasoning in order to differentiate between congruent and similar figures
	Compute the angles and ratio of sides of polygons in order to determine their similarity	Uses different geometric criteria established earlier such as basic proportionality theorem etc. in order to establish properties for similarity of two triangles
Similarity of triangles	Compute the angles and ratio of sides of triangles in order to determine their similarity	
	Apply basic proportionality theorem and its converse in order to determine the ratio of sides in the given triangle(s)	
Criteria for similarity of Triangles	Apply various criterions of similarity in order to prove whether given triangles are similar or not	
	Show similarity of triangles in order to solve for given real life word problems	
Areas of Similar Triangles	Compute the square of the ratio of the corresponding sides of triangles in order to find the area of similar triangles	
	Compute the area of similar triangles in order to find the relation between their sides, medians, mid points of the triangles	
Pythagoras Theorem	Apply the theorem that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle in order to prove Pythagoras Theorem and to each other.	
	Prove Pythagoras theorem and its converse in order to solve real life word problems and mathematical statements/questions	
	Apply Pythagoras theorem and its converse in order to determine whether a given triangle is a right-angled triangle or not	

Test items

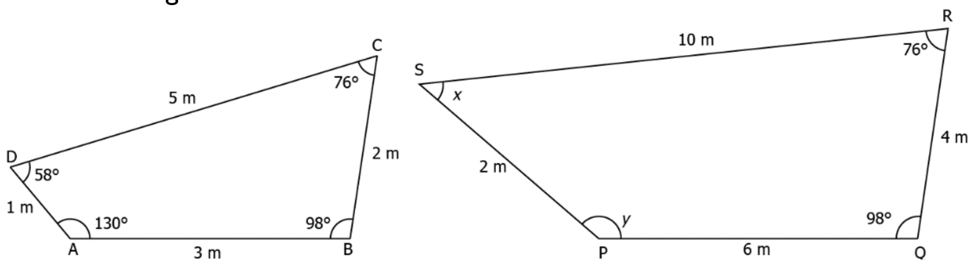
LOB: Distinguish between congruency and similarity in order to understand the concept of similar figures

1. Rahul claims that congruent figures are similar as well. Aman claims that similar figures are congruent as well. Who is/are correct?
   Option 1: only Rahul
   Option 2: only Aman
   Option 3: Both Rahul and Aman
   Option 4: Neither Rahul nor Aman
   Correct Answer: Option 1

2. Consider the statements below.
   Statement 1: All circles are similar.
   Statement 2: All squares are similar.
   Statement 3: All right triangles are congruent.
   Statement 4: All equilateral triangles are congruent.
   Which statement is/are correct?
   Option 1: Statement 1 and Statement 3
   Option 2: Statement 2 and Statement 4
   Option 3: Statement 1 and Statement 2
   Option 4: Statement 3 and Statement 4
   Correct Answer: Option 3

LOB: Distinguish between congruency and similarity in order to understand the concept of similar figures

1. Two similar figures are shown.

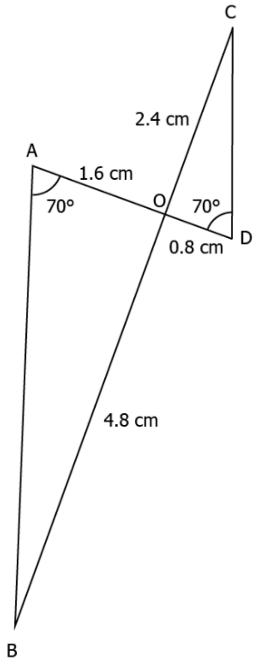


   What are the values of x and y?
   Option 1: $x = \mathbf{58}°, y = 130°$
   Option 2: $x = 98°, y = 76°$
   Option 3: $x = 82°, y = 84°$
   Option 4: $x = 130°, y = 84°$
   Correct Answer: Option 1

2. Two quadrilaterals are such that their diagonals bisect each other. What additional information is required to conclude that the quadrilaterals are similar?
   Option 1: Opposite sides are equal
   Option 2: Opposite angles are equal
   Option 3: Diagonals are equal and opposite sides are equal
   Option 4: Diagonals bisect at right angle and adjacent angles are equal
   Correct Answer: Option 4

LOB: Compute the angles and ratio of sides of triangles in order to determine their similarity

1. Consider the figure below.



Which of the following statement is correct about the triangles in the figure?

Option 1: $\Delta AOB \sim \Delta DOC$ because $\frac{AO}{DO} = \frac{BO}{CO}$.

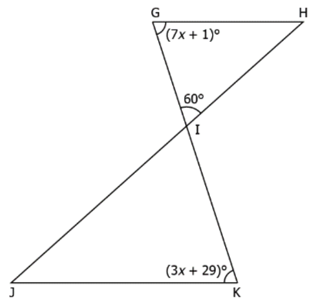
Option 2: $\Delta AOB \sim \Delta DOC$ because $\angle AOB = \angle DOC$.

Option 3: $\Delta AOB \sim \Delta DOC$ because $\frac{AO}{DO} = \frac{BO}{CO}$ and $\angle AOB = \angle DOC$.

Option 4: $\Delta AOB \sim \Delta DOC$ because $\frac{AO}{DO} = \frac{BO}{CO}$ and $\angle BAO = \angle CDO$.

Correct Answer: Option 3

2. Consider the figure below.



Which of the following information help proving that triangle GHI is similar to triangle KIJ?

Information 1: $\angle H = 70°$ Information 2: $\angle J = 70°$

Option 1: Each information alone is sufficient

Option 2: Information (1) alone is sufficient, but Information (2) alone is not sufficient.

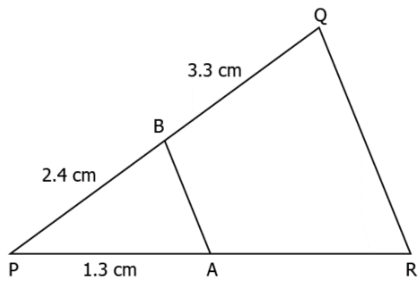
Option 3: Information (2) alone is sufficient, but Information (1) alone is not sufficient.

Option 4: Both information together is sufficient, but neither information alone is sufficient.

Correct Answer: Option 1

LOB: Apply basic proportionality theorem and its converse in order to determine the ratio of sides in the given triangle(s)

1. n the figure below, $BA \parallel QR$.

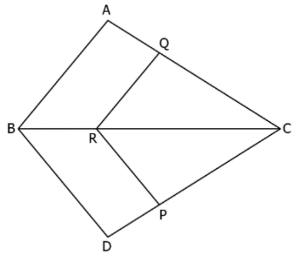


To the nearest tenth, what is the length of AR?

Option 1:1.8 cm
Option 2:1.7 cm
Option 3:1.4 cm
Option 4:2.2 cm

Correct Answer: Option 1

2. In the given figure, $QR \parallel AB$, $RP \parallel BD$, $CQ = x + 2$, $QA = x$, $CP = 5x + 4$, $PD = 3x$.



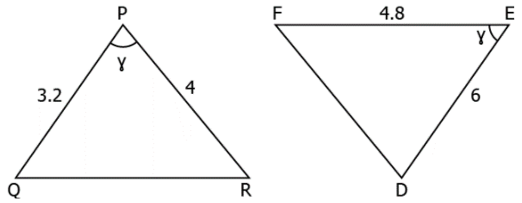
The value of x is __.

Option 1: 1
Option 2: 3
Option 3: 6
Option 4: 9

Correct Answer: Option 1

LOB: Apply various criterions of similarity in order to prove that whether given triangles are similar or not

1. Observe the two triangles shown below.

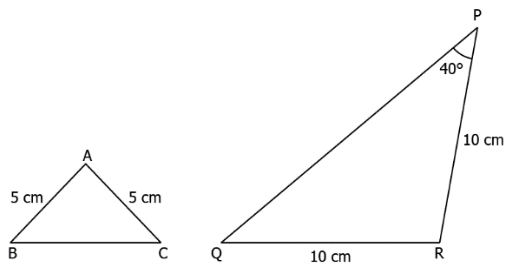


Which statement is correct?

Option 1: Triangles are similar by SAS
Option 2: Triangles are similar by SSA
Option 3: Triangles are not similar as sides are not in proportion
Option 4: No valid conclusion about similarity of triangles can be made as angle measures are not known.

Correct Answer: Option 1

2. Consider the triangles below.



Which statement is correct?

Option 1: Triangles are similar as all isosceles triangles are similar.
Option 2: Triangles are similar as corresponding sides of the triangles are in the ratio 1:2.
Option 3: For triangles to be similar, the measure of $\angle A = 40°$.
Option 4: For triangles to be similar, the measure of $\angle A = 100°$.

Correct Answer: Option 4

LOB: Show similarity of triangles in order to solve for given real life word problems

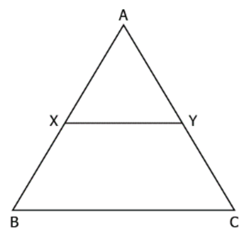
1. Nishant is 6 feet tall. At an instant, his shadow is 5 feet long. At the same instant, the shadow of a pole is 30 feet long. How tall is the tower?
   Option 1:24 feet
   Option 2:30 feet
   Option 3:36 feet
   Option 4:18 feet
   Correct Answer: Option 3

2. Rahul is 5 feet tall. He places a mirror on the ground and moves until he can see the top of a building. At the instant when Rahul is 2 feet from the mirror, the building is 48 feet from the mirror. How tall is the building?
   Option 1:120 feet
   Option 2:96 feet
   Option 3:240 feet
   Option 4:180 feet
   Correct Answer: Option 1

LOB: Compute the square of the ratio of the corresponding sides of triangles in order to find the area of similar triangles

1. The area of two similar triangles are a and k^2a. What is the ratio of the corresponding side lengths of the triangles?
   Option 1: $1:k^2$
   Option 2: $1:k$
   Option 3: $1:a^2$
   Option 4: $1:a$
   Correct Answer: Option 2

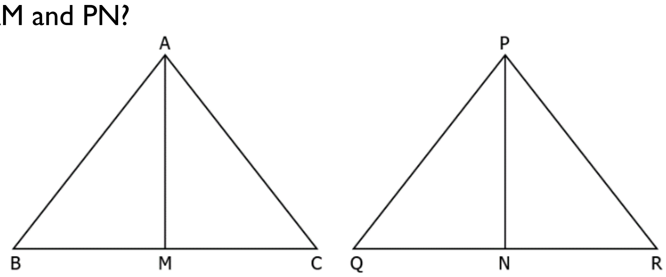
2. In the figure below, $\mathrm{XY} \parallel \mathrm{BC}$.



The ratio of the perimeter of triangle ABC to the perimeter of triangle AXY is 3:1. Given that the numerical value of the area of triangle AXY is a whole number, which of the following could be the area of the triangle ABC?
   Option 1: 28
   Option 2: 55
   Option 3: 60
   Option 4: 99
   Correct Answer: Option 4

LOB: Compute the area of similar triangles in order to find the relation between their sides, medians, mid points of the triangles

1. The ratio of the areas of two similar triangles, ABC and PQR shown below is 25:144. What is the ratio of their medians AM and PN?



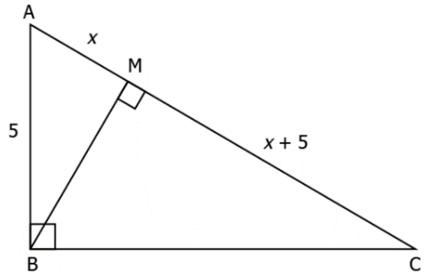
   Option 1: 5:12
   Option 2: 25:144

Option 3: 12:5
Option 4: 5:16
Correct Answer: Option 1

2. The ratio of the areas of two similar right triangles is 9:16. The length of one of the sides of the smaller triangle is 15 cm. How much longer is the length of the corresponding side of the larger triangle from smaller triangle?
Option 1: 3 cm
Option 2: 4 cm
Option 3: 5 cm
Option 4: 7 cm
Correct Answer: Option 3

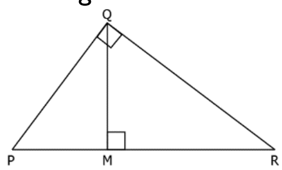
LOB: Apply the theorem that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle in order to prove Pythagoras Theorem and to each other.

1. Observe the right triangle ABC, right angled at B as shown below.



What is the length of MC?
Option 1:7.5 cm
Option 2:2.5 cm
Option 3:6.5 cm
Option 4:10.5 cm
Correct Answer: Option 1

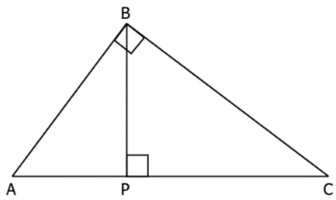
2. Observe the right triangle PQR, right angled at Q as shown below. If $QM \perp PR$, then which of the following is NOT correct?



Option 1: $\Delta PMQ \sim \Delta PQR$
Option 2: $QR^2 = RM \cdot PR$
Option 3: $PR^2 = PQ \cdot RQ$
Option 4: $\Delta PMQ \sim \Delta QMR$
Correct Answer: Option 3

LOB: Prove Pythagoras theorem and its converse in order to solve real life word problems and mathematical statements/questions

1. Consider the figure below.



Rahul follows the below step to prove $AB^2 + BC^2 = AC^2$.
Step 1: $\Delta APB \sim \Delta ABC$
Step 2: $\frac{AP}{AB} = \frac{AB}{AC}$

Step 3: $AB^2 = AP \cdot AC$

Which of these could be his next step?

Option 1: Prove $\Delta ABC \sim \Delta PAB$

Option 2: Prove $\Delta APB \sim \Delta CPB$

Option 3: Prove $\Delta BPC \sim \Delta ABC$

Option 4: Prove $\Delta APB \sim \Delta BPC$

Correct Answer: Option 3

2. From point X, Ankit walks 112 m east to reach at point Y. From point Y, Ankit walks 15 m toward north to reach point Z. What is the straight-line distance between position when he started and his position now?

Option 1:123 m

Option 2:127 m

Option 3:117 m

Option 4:113 m

Correct Answer: Option 4

LOB: Apply Pythagoras theorem and its converse in order to determine whether a given triangle is a right-angled triangle or not

1. Which set of lengths forms a right triangle?

Option 1: 5cm, 12 cm, 16 cm

Option 2: 7 cm, 24 cm, 25 cm

Option 3: 3 cm, 3 cm, 4 cm

Option 4: 6 cm, 7 cm, 9 cm

Correct Answer: Option 2

2. Consider the following three claims about a triangle JKL with side lengths m, n and r.

Claim 1: JKL is a right triangle provided $n^2 - m^2 = r^2$.

Claim 2: Triangle with side lengths $m + 2$, $n + 2$ and $r + 2$ is a right-angle triangle.

Claim 3: Triangle with side lengths $2m$, $2n$ and $2r$ is a right-angle triangle.

Which of these is correct?

Option 1: Claim 1 would be correct if $n > m$, $n > r$ and Claim 2 would be correct if JKL is a right triangle.

Option 2: Claim 1 would be correct if $r > m$, $r > n$ and Claim 2 would be correct if JKL is a right triangle.

Option 3: Claim 1 would be correct if $n > m$, $n > r$ and Claim 3 would be correct if JKL is a right triangle.

Option 4: Claim 1 would be correct if $r > m$, $r > n$ and Claim 3 would be correct if JKL is a right triangle.

Correct Answer: Option 3

Suggested Teacher Resources

1

Lesson Plan

Objective	• To explore and apply various criterions of similarity in order to prove whether given triangles are similar or not
Material Required	Student handouts with triangles, tracing paper, rulers
Prerequisite Knowledge	Similarity of triangles
Procedure	The teacher will start by revising previous topics such as congruent triangles. After this discussion, place students in pairs and assign one partner to collect materials. Remind students how to measure using centimetres. Be sure to point out that the edge of the ruler may not indicate zero centimetres; they need to begin measuring the side length at zero centimetres which may be slightly inward from the edge of the ruler. Circulate and help students to verify side and angle measures. As students discover properties of similarity, help them to generalize their thoughts and write sentences explaining their findings. • Ask the class: List all Triangle Congruence Postulates that you know. Draw a picture of congruent triangles with the corresponding parts indicated for each postulate. (Quickly review each postulate.)

SAS	AAS	ASA	SSS	RHS

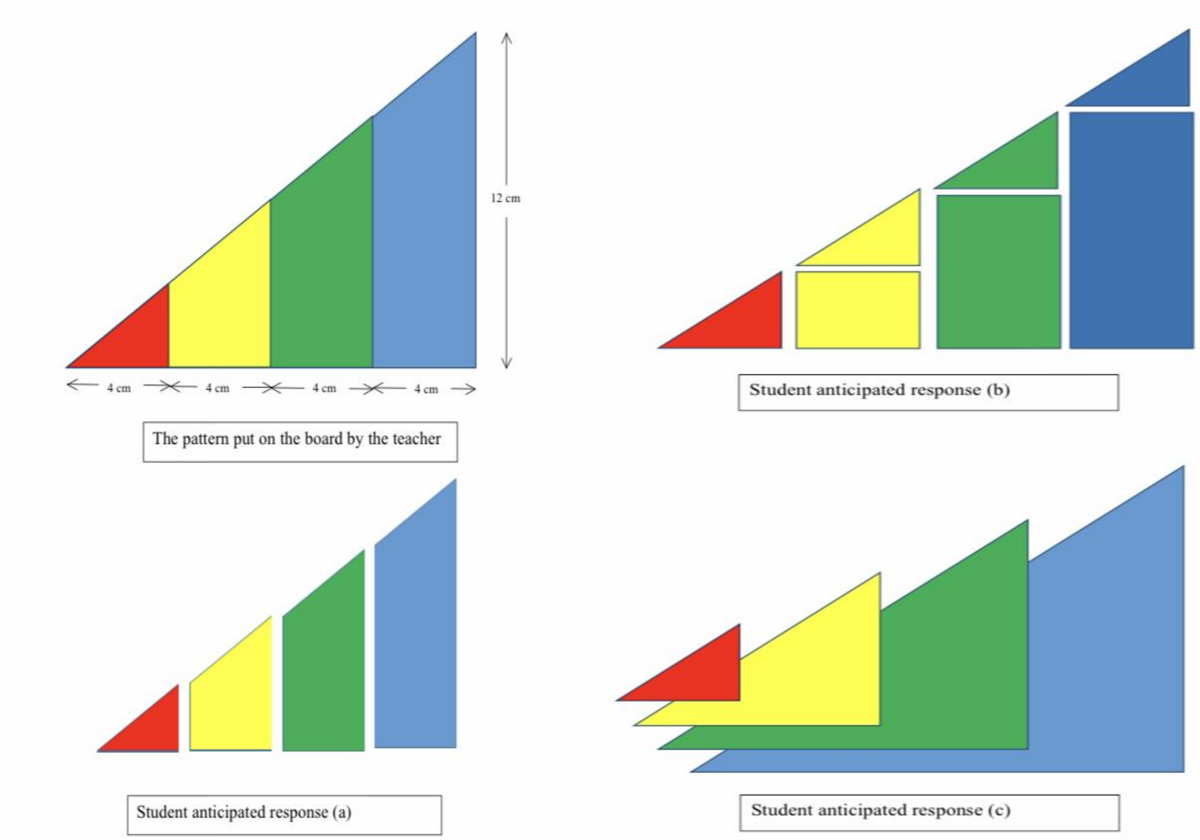
- Ask the class: Do you believe that having all three angles of a triangle congruent is another way to prove triangle congruence? Is AAA a triangle congruence postulate? Why or why not? o Remind them that a general definition of congruence states that figures must be the same size and the same shape in order to be considered congruent. Refer to the symbol for congruence (≅)
- Compare this to the symbol for equality and discuss that this refers to the equality of values whereas congruence refers to figures having the same size and shape.
- Allow the students to discuss. Call on a few students to share their thoughts. Encourage them to draw pictures to support their claims.
- Explain to the students that the measures of the angles of a triangle do not dictate the lengths of the sides. Show several examples of pairs of triangles that have the same shape (where corresponding angles are congruent) but that are not the same size. Try to reach a consensus that the triangles have the same shape but not the same size.
- Ask the class: Can you think a word that you could use to describe these triangles that look very much alike but that we have all agreed, are not congruent? (Hopefully, they will come up with the word similar!) to Introduce the symbol for similarity (~) and compare it to the previous symbols for congruence and equality. Explain that this symbol, when written alone, can be used to denote that two figures have the same shape but not necessarily the same size.
- Ask the class: Do you think that, in the same way congruent triangles have interesting properties; similar triangles will have interesting properties? Today, we will find out what those properties are!

Now the teacher will ask the students to:

	● List all Triangle Congruence Postulates that you know. There are five! Draw a picture of congruent triangles with the corresponding parts indicated for each postulate ● . Do you believe that having all three angles of a triangle congruent is another way to prove triangle congruence? Is AAA a triangle congruence postulate? Why or why not? ● Draw several examples of pairs of triangles that have the same shape (corresponding angles are congruent) but that are not the same size.
Source	https://www.wccusd.net/cms/lib/CA01001466/Centricity/domain/60/lessons/geometry%20lessons/InvestigatingSimilarTrianglesV5.pdf

2 Activity

The teacher will ask the students to
Recreate the triangular pattern that is on the board with cut outs and different coloured cards. Feel free to use as many shapes as you like, stick the shapes together any way you like or overlap shapes. And As shown in the board plan, we anticipate the following responses from the students:
a. Students will recreate the pattern using strips rather than triangles.
b. Use rectangles and triangles.
c. Students will use similar triangles.
And then, the teacher can ask students to pair up and classify the triangles they used in the categories of similar triangles.

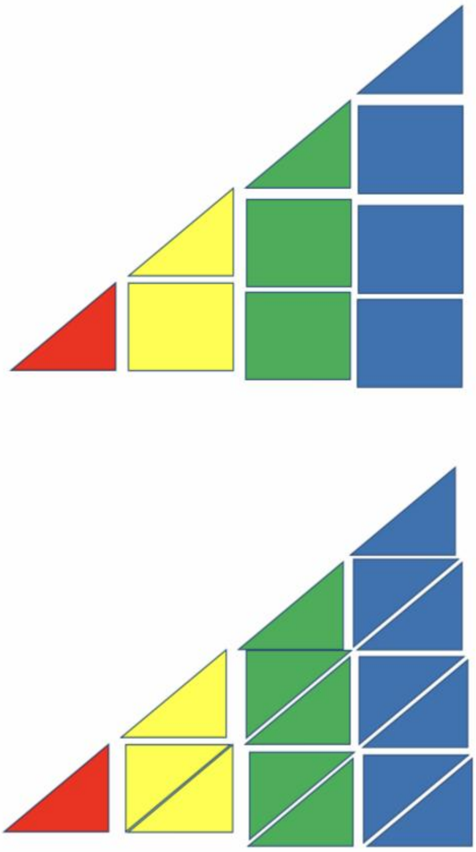


The pattern put on the board by the teacher

Student anticipated response (b)

Student anticipated response (a)

Student anticipated response (c)



Source: https://www.projectmaths.ie/documents/MathsCounts2016/SimilarTriangles.pdf

7. COORDINATE GEOMETRY

QR Code:

V7Z4A1

Learning outcome and Learning Objectives:

<table>
<tr><th>Content area / Concepts</th><th>Learning Objectives</th><th>Learning Outcome</th></tr>
<tr><td>Introduction</td><td>Identify x and y coordinate in order to plot points on the graph</td><td rowspan="6">Derives formulae to establish relations for geometrical shapes in the context of a coordinate plane, such as finding the distance between two given points, in order to determine coordinates of a point between any two given points, to find area of a triangle etc.</td></tr>
<tr><td rowspan="2">Distance Formula</td><td>Apply and derive distance formula in order to determine the distance between two coordinates on the graph</td></tr>
<tr><td>Apply distance formula in order to solve various mathematical and real-life situations graphically</td></tr>
<tr><td rowspan="2">Section Formula</td><td>Apply and derive section formula in order to divide the line segment in a given ratio</td></tr>
<tr><td>Apply distance and section formula in order to determine the vertices/diagonals/mid points of given geometrical shapes</td></tr>
<tr><td>Area of a Triangle</td><td>Apply and derive the formula of area of triangle geometrically in order to determine the area of quadrilateral/triangle</td></tr>
</table>

Test items

LOB: Identify x and y coordinate in order to plot points on the graph

1. Sheena was asked to plot a point 10 unit on the left of the origin and other point 4 units directly above the origin. Which of the following are the two points?
   Option 1:(10,0) and (0,-4)
   Option 2:(-10,0) and (4,0)
   Option 3:(10,0) and (0,4)
   Option 4:(-10, 0) and (0, 4)

Correct Answer: Option 4

2. Three points lie on a vertical line. Which of the following could be those points?
   Option 1:(0, 4), (4, 0), (0, 0)
   Option 2:(4, 3), (5, 3), (-12, 3)
   Option 3:(-8, 7), (-8,-8), (-8, -100)
   Option 4:(-8,3), (-8, 8), (8,7)

Correct Answer: Option 3

LOB: Apply and derive distance formula in order to determine the distance between two coordinates on the graph

1. On a graph, two-line segments, AB and CD of equal length are drawn. Which of these could be the coordinates of the points, A, B, C and D?
   Option 1: A(-3,4) B(-1,2) and C(3,4) D(1,2)
   Option 2: A(-3,-4) B(-1,2) and C(3,4) D(1,2)
   Option 3: A(-3,4) B(-1,-2) and C(3,4) D(1,2)
   Option 4: A(3,4) B(-1,2) and C(3,4) D(1,2)

Correct Answer: Option 1

2. The distance between two points, M and N, on a graph is given as $\sqrt{10^2 + 7^2}$. The coordinates of point M are (–4. 3). Given that the point N lies in the first quadrant, which of the following is true about the all possible x coordinates of point N?
   Option 1: They are multiple of 2.
   Option 2: They are multiples of 3.
   Option 3: They are multiples of 5.
   Option 4: They are multiples of 6.

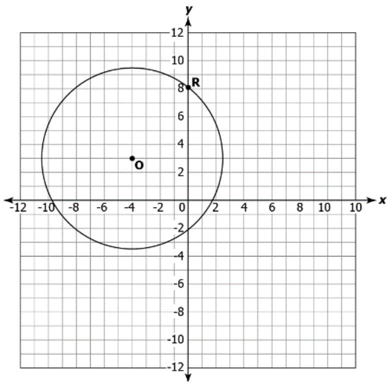
Correct Answer: Option 2

LOB: Apply distance formula in order to solve various mathematical and real-life situations graphically

1. On a coordinate grid, the location of a bank is (–4, 8) and the location of a post office is (2, 0). The scale used is 1 unit = 50 m. What is the shortest possible distance between the bank and the post office?
   Option 1: 200 m
   Option 2: 500 m
   Option 3: 700 m
   Option 4: 800 m

Correct Answer: Option 2

2. The graph of a circle with centre O with point R on its circumference is shown.



What is the side length of the square that circumscribes the circle?

Option 1: $2\sqrt{41}$Units

Option 2:$\sqrt{41}$ Units

Option 3: $3\sqrt{17}$ Units

Option 4: $6\sqrt{17}$ Units

Correct Answer: Option 1

LOB: Apply and derive section formula in order to divide the line segment in a given ratio

1. A point G divides a line segment in the ratio 3:7. The segment starts at the origin and ends at a point K having 20 as its abscissa and 40 as its ordinate. Given that G is closer to the origin than to point K, which of the following are the coordinates of point G?

   Option 1: (14, 28)

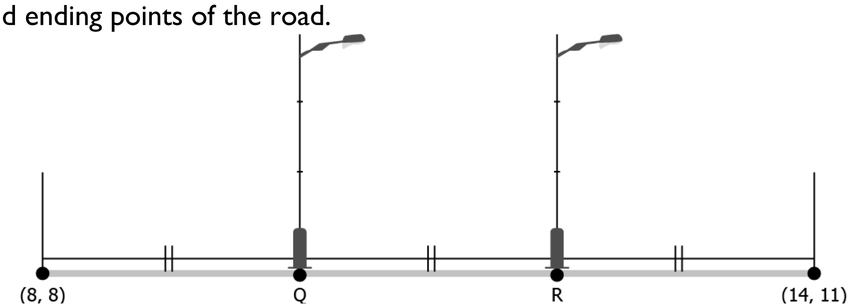
   Option 2: (28, 14)

   Option 3: (12, 6)

   Option 4: (6, 12)

Correct Answer: Option 4

2. Two poles are to be installed on an elevated road as shown in the diagram. The diagram also shows the starting and ending points of the road.



Which of the following are the coordinates of the poles?

Option 1: Q (10,9) and R(12,8)

Option 2: Q(10,8) and R (12,11)

Option 3: Q (10,9) and R(12,10)

Option 4: Q(-10, 9) and R(0, 11)

Correct Answer: Option 3

LOB: Apply distance and section formula in order to determine the vertices/ diagonals/ mid points of given geometrical shapes

1. Which of the following are the coordinates of the intersection points of the diagonals of the rectangle ABCD with vertices A(0,3), B(3,0), C(1,-2) and D(-2,1)?

   Option 1: (1.5, 1.5)

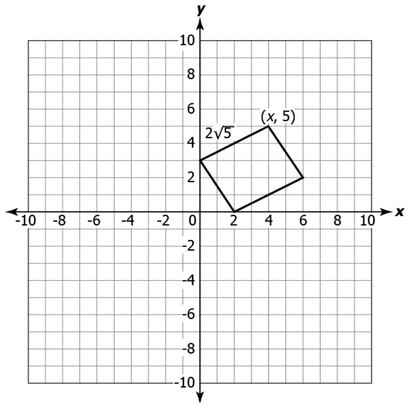
   Option 2:$(\frac{1}{2}, \frac{1}{2})$

   Option 3: ($-\frac{1}{2}, -\frac{1}{2}$)

   Option 4: (2, -1)

Correct Answer: Option 2

2. The figure shows a parallelogram with one of its vertices intersecting the *y*-axis at 3 and another vertex intersecting the *x*-axis at 2.



If (*m*, *n*) is the intersection point of the diagonals of the parallelogram, which relation is correct?

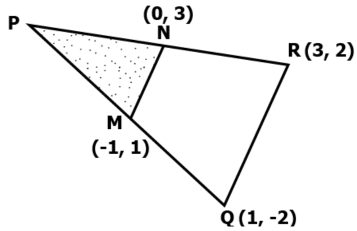
Option 1: $m = 0.5 + n$
Option 2: $m = n - 0.5$
Option 3: $m = 1.50 + n$
Option 4: $m = n - 1.50$
Correct Answer: Option 1

LOB: Apply and derive the formula of area of triangle geometrically in order to determine the area of quadrilateral/triangle

1. A triangle is drawn on a graph. Two of the vertices of the triangle intersect the *y*-axis at -3 and *x*-axis at 5. The third vertex is at (2, 4). What is the area of the triangle?

Option 1:16 square units
Option 2: 14.5 square units
Option 3: 8 square units
Option 4: 6.5 square units
Correct Answer: Option 2

2. Observe the triangles *PMN* and *PQR* shown below. The area of the triangle *PQR* is 14 square units.

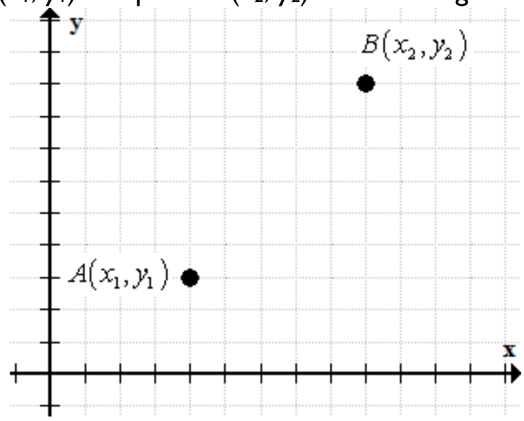
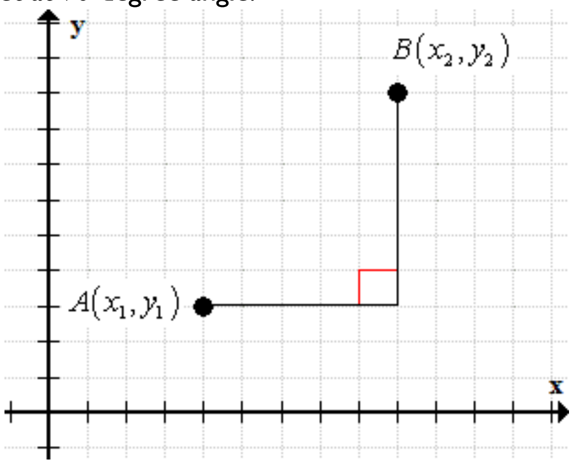


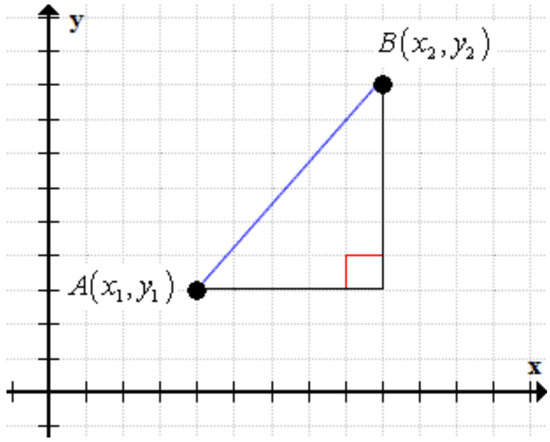
What is the area of the triangle *PMN*?

Option 1: 3.5 square units
Option 2: 7 square units
Option 3: 2.5 square units
Option 4: 1 square unit
Correct Answer: Option 1

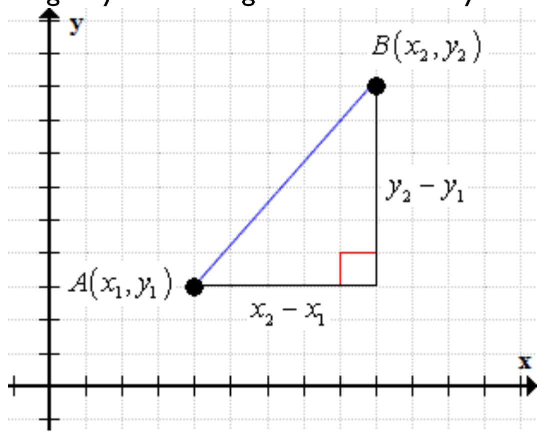
Suggested Teacher Resources

1 Lesson Plan

Objective	Apply and derive distance formula in order to determine the distance between two coordinates on the graph
Prerequisite	Pythagoras theorem, solving for root, solving equations, basic of coordinate system, solving by putting values in variables
Vocabulary words	Hypotenuse, cartesian plane
Materials required	Chart, graph paper, trace paper, geometry box
Procedure	Opening: Students would be doing this activity with their peers. Teacher will draw a cartesian plane on the chart for students to see and ask students to find distance between point A (x_1, y_1) and point B (x_2, y_2). Shown in figure.  1. First, construct the vertical and horizontal line segments passing through each of the given points such that they meet at 90-degree angle.  2. Next, connect points A and B to reveal a right triangle.

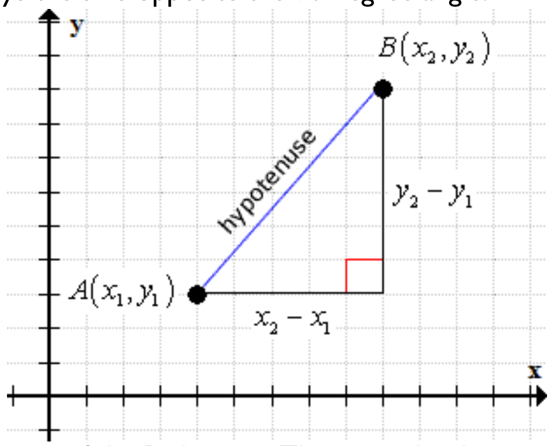


3. Find the legs of a right triangle by subtracting the x-values and y-values accordingly.



4. The distance between the points A and B is just the hypotenuse of right triangle.

Ask students to discuss the method to calculate the distance AB
Note: Hypotenuse is always the side opposite the 90-degree angle.



5. Finally, applying the concept of the Pythagoras Theorem, the distance formula is calculated as follows:

$$\textbf{hypotenuse} = \textbf{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Questions of Area and Perimeter in the Coordinate Plane

1. Create a full-page coordinate plane in portrait orientation. Label the x-axis and y-axis. Use a scale of 1 unit per gridline.

2. Create the following figures:
   a. Triangle ABC with *A* (-7, 19), *B* (-15, 7), and *C* (-2, 7)
   b. Rectangle *DEFG* with *D* (2, 12), *E* (12, 18), *F* (15, 13), and G (5, 7)
   c. Square *HJKL* with *H* (-9, -7), *J* (3, -2), *K* (8, -14), and *L* (-4, -19)

<table>
<tr><td></td><td>3. Use coordinate geometry to find the perimeter and area of each figure (In other words, do not use a ruler or any other measuring device.) Assume that each increment on the grid paper is 1 cm
Use the following table to note down the observations:
<table><tr><td>Perimeter</td><td>Area</td></tr><tr><td></td><td></td></tr></table></td></tr>
<tr><td>source</td><td>https://betterlesson.com/lesson/489378/distances-in-the-coordinate-plane</td></tr>
</table>

2 Activity

Setup: Euler's Village Learning Task

Question to students:
You would like to build a house close to the village of Euler. There is a beautiful town centre in the village, and the road you would like to build your house on begins right at the town square. The road follows an approximately north east direction as you leave town and continues for 90 km. It passes right by a large shade tree located approximately 2 km east and 3 km north of the town square. There is a stretch of the road, between 3 km and 12 km to the east of town, which currently has no houses. This stretch of road is where you would like to locate your house. Building restrictions require all houses sit parallel to the road. All water supplies are linked to town wells and the closest well to this part of the road is 5 km east and 12 km north of the town centre.

1. How far from the well would it be if the house was located on the road 3 km east of town? 5 km east of town? 10 km east of town? 12 km east of town? (For the sake of calculations, do not consider how far the house is from the road, just use the road to make calculations)
2. The cost of the piping leading from the well to the house is a major concern. Where should you locate your house in order to have the shortest distance to the well? Justify your answer mathematically.
3. If the cost of laying pipes is Rs 250 per linear km, how much will it cost to connect your house to the well?
4. The builder of your house is impressed by your calculations and wants to use the same method for placing other houses. Describe the method you used. Would you want him to place the other houses in the same manner?
5. Describe what the house, the road, and the shortest distance represent mathematically. Describe your method mathematically.
6. Write a formula that the builder could use to find the cost of laying pipes to any house along this road. How would you have to change your formula for another road?

Reference:

1. https://www.chilimath.com/lessons/intermediate-algebra/derivation-of-distance-formula/
2. http://jwilson.coe.uga.edu/EMAT6680Fa05/Hardy/Unit%206%20-%20Coordinate%20Geometry.pdf

8. INTRODUCTION TO TRIGONOMETRY

QR Code:

Q2W7N7

Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Trigonometric Ratios	Describe trigonometry in order to study the relationship between side and angle of a triangle	Determines all trigonometric ratios with respect to a given acute angle (of a right triangle) in order to use them in solving problems in daily life contexts like finding heights of different structures or distances from them
	Define and distinguish various trigonometric ratios in order to describe and verify sine, cosine, tangent, cosecant, secant, cotangent of an angle	
	Use given trigonometric ratio(s) in order to find and verify other trigonometric ratios/angles of the triangle	
Trigonometric Ratios of Some Specific Angles	Compute the trigonometric ratio of 0^0, 30^0, 45^0, 60^0, 90^0 in order to know and apply the value of specific angles	
Trigonometric Ratios of Complementary Angles	Compute the trigonometric ratio of complimentary angles in order to apply the values in mathematical problems	
Trigonometric Identities	Compute and apply trigonometric identities in order to simplify and solve mathematical problems	

Test items

LOB: Describe trigonometry in order to study the relationship between side and angle of a triangle

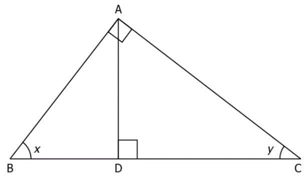
1. Which of the following completes the statement below?

The adjacent and _______ of right triangle depends on the $\angle x$ being referred to in that triangle and for the complementary angle $(90-x)^0$ of that triangle, the adjacent and opposite are ____________.

Option 1: opposite side, reversed
Option 2: opposite side, same
Option 3: hypotenuse, reversed
Option 4: hypotenuse, same

Correct Answer: Option 1

2. Considering the diagram below.



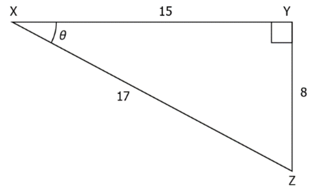
Which of the following statements is true?

Option 1: Side AC is adjacent to $\angle y$ in triangle ADC and triangle ABC
Option 2: Side AC is adjacent to $\angle y$ in triangle ADC and side BC is adjacent to $\angle y$ in triangle ABC
Option 3: Side DC is adjacent to $\angle y$ in triangle ADC and side AC is adjacent to $\angle y$ in triangle ABC
Option 4: Side AC is adjacent to $\angle y$ in triangle ADC and side DC is adjacent to $\angle y$ in triangle ABC

Correct Answer: Option 3

LOB: Define and distinguish various trigonometric ratios in order to describe and verify sine, cosine, tangent, cosecant, secant, cotangent of an angle

1. Consider the triangle shown below.

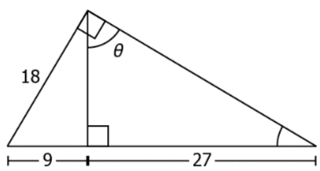


What are the values of tan Θ, cosec Θ and sec Θ?

Option 1: $\tan\theta = \frac{8}{15}$,$\text{cosec}\theta = \frac{17}{15}$, $\sec\theta = \frac{17}{8}$
Option 2: $\tan\theta = \frac{8}{15}$, $\text{cosec}\theta = \frac{17}{8}$,$\sec\theta = \frac{17}{15}$
Option 3: $\tan\theta = \frac{17}{15}$,$\text{cosec}\theta = \frac{8}{15}$, $\sec\theta = \frac{17}{8}$
Option 4: $\tan\theta = \frac{8}{15}$,$\text{cosec}\theta = \frac{17}{15}$, $\sec\theta = \frac{8}{17}$

Correct Answer: Option 2

2. Observe the figure shown.



Which of these is the value of cos θ?

Option 1: $\frac{1}{2}$
Option 2: $\frac{2}{1}$

Option 3: $\frac{2\sqrt{3}}{3}$

Option 4: $\frac{3\sqrt{3}}{2}$

Correct Answer: Option 1

LOB: Use given trigonometric ratio(s) in order to find and verify other trigonometric ratios/angles of the triangle

1. If sin Θ = $\frac{7}{\sqrt{85}}$, what are the values of tan Θ, cos Θ and cosec Θ?

Option 1: tan Θ= $\frac{7}{6}$, cos Θ= $\frac{6}{\sqrt{85}}$ and cosec Θ= $\frac{\sqrt{85}}{7}$

Option 2: tan Θ= $\frac{6}{7}$, cos Θ= $\frac{7}{\sqrt{85}}$ and cosec Θ= $\frac{\sqrt{85}}{7}$

Option 3: tan Θ= $\frac{7}{6}$, cos Θ= $\frac{7}{\sqrt{85}}$ and cosec Θ= $\frac{\sqrt{85}}{7}$

Option 4: tan Θ= $\frac{6}{7}$, cos Θ= $\frac{6}{\sqrt{85}}$ and Cosec Θ= $\frac{\sqrt{85}}{6}$

Correct Answer: Option 1

2. The two legs AB and BC of right triangle ABC are in a ratio 1:3. What will be the value of sin C?

Option 1: $\frac{1}{\sqrt{10}}$

Option 2: $\frac{3}{\sqrt{10}}$

Option 3: $\frac{1}{3}$

Option 4: $\frac{1}{2}$

Correct Answer: Option 1

LOB: Compute the trigonometric ratio of 0,30,45,60,90 in order to know and apply the value of specific angles

1. What is the value of $\frac{3-\sin^2 60°}{\tan 30° \tan 60°}$?

Option 1: $1\frac{1}{4}$

Option 2: $2\frac{1}{4}$

Option 3: $2\frac{1}{2}$

Option 4: $2\frac{3}{4}$

Correct Answer: Option 2

2. The value of $\frac{4-sin^2 45^0}{cot\ k\ tan\ 60^0}$ is 3.5.
What is the value of *k*?

Option 1: 30°
Option 2: 60°
Option 3: 45°
Option 4: 90°

Correct Answer: Option 2

LOB: Compute the trigonometric ratio of complimentary angles in order to apply the values in mathematical problems

1. What is the value of $\frac{5 \text{ cosec } 55°}{\sec\ 35°}$?

Option 1: $\frac{5}{2}$
Option 2: 5
Option 3: $\frac{15}{2}$
Option 4: 15

Correct Answer: Option 2

2. If sec 2*x* = $\frac{1}{\sin{(x-36)^0}}$, where 2*x* is an acute angle, what is the value of *x*?

Option 1: 36
Option 2: 42
Option 3: 48
Option 4: 52
Correct Answer: Option 2

LOB: Compute and apply trigonometric identities in order to simplify and solve mathematical problems

1. Which of these is equivalent to $\frac{2\tan x(\sec^2 x-1)}{\cos^3 x}$?
   Option 1: $2\tan^3 x \operatorname{cosec} x$
   Option 2: $2\tan^3 x \sec^3 x$
   Option 3: $2\tan^3 x \operatorname{cosec}^3 x$
   Option 4: $2\cot^3 x \sec^3 x$
   Correct Answer: Option 2

2. Which of the following option makes the statement below true?

$$\frac{\frac{1}{\sec x}+\sec x}{\cos^2 x-1-\tan^2 x}=?$$

   Option 1: $-\operatorname{cosec} x \cot x$
   Option 2: $-\sec x \cot x$
   Option 3: $-\operatorname{cosec} x \tan x$
   Option 4: $-\sec x \tan x$
   Correct Answer: Option 1

Suggested Teacher Resources

1 **Lesson Plan**

Objectives	Students will be able to-: • Correctly identify the hypotenuse in a right-angled triangle, and the opposite and adjacent sides of a given angle • Find a pattern linking the ratio of sides of a triangle with the angles and hence understand the concepts of sine, cosine and tangent ratios of angles • Apply trigonometry to solve problems including those involving angles of elevation and depression.
Prerequisite Knowledge	Triangle properties, Ratios
Material Required	Calculators, measuring tapes graph paper cm rulers
Procedure	Teacher will ask students to find the angle of elevation of the sun. • Fill in Student Activity 1A. • Inside Classroom Fill in Student Activity 1C. From your scaled diagram work out the angle of elevation of the sun. • Now ask students using the angle of elevation (from Student Activity 1C) and the length of the shadow of an object whose height you cannot physically measure (from Student Activity 1B), again use a scaled diagram to determine the height of the object. • Alternatively, Teacher will use ratios in similar triangles. • Fill in Student Activity 1E. • Note: Later on, having learned about the trigonometric ratios of sin, cos and tan studentscan use these trigonometric ratios to determine the angle of elevation of the sun and hence the height of for example a tall tree or a goalpost. • Why is the study of triangles? • important? – show students the images in Appendix B • What is the hypotenuse of a right-angled triangle? • Ask them to Collaborate in pairs as you fill in Student Activity 2. • Mark the hypotenuse on the triangles in Student Activity 2 of right-angled triangles. How many other angles are in the triangle? • How many degrees do they add up to? What are these angles called? Mark either one of the two complementary angles on the triangle with an arc. What is the name of one of the arms of that angle? • The other side, which is beside the marked angle, is given the name “adjacent”. Label it. • Label the third side of the triangle as the opposite side. • Describe the opposite side. • Repeat this process of marking angles and labelling sides for all the triangles on Student Activity 1. • If you were to work out the ratio of any two of the sides in a right-angled triangle how would you do this? Would the order matter? Explain your answer. • How many possible ratios could be worked out for a right-angled triangle? When giving the ratios use the names for the sides.

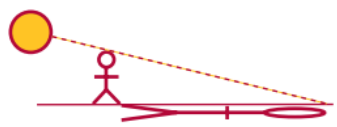
	• What is the relationship between the first pair? If I knew the answer to the first ratio was ½, what would be the answer to the second one?
	• Student Activities 3, 4, 5, 6, 7 and 8: For each triangle of the five triangles mark the 90° angle and one other given angle (given on the sheet). Label the sides of the right-angled triangles as hyp (hypotenuse), adj (adjacent), and opp (opposite). Measure each side correct to the nearest mm and calculate the ratios opp/hyp, adj/hyp, opp/adj. One student is to measure, one to calculate ratios and one to double check changing roles on each triangle.
	• Ask students to Write down what you have observed from your answers.
	• Of the 3 ratios which of them can never be bigger than 1? Explain.
	• Is it possible for any of the ratios to be bigger than 1? If so, which one or which » ones and why?
	• The sides and angles of triangles are related to each other. This is called TRIGONOMETRY – Trigon meaning triangle and metria meaning measurement.
	• Go back to Student Activities 3 – 8 and fill in the name of each ratio, for example for the sheet with angles of 30° fill in opp/hyp = sin 30° etc.
	• On the master table, Student Activity 9 fill in the mean value you have calculated for sin, cos and tan of the angle from your own Student Activity 3, 4, and 5,6,7and 8 Tell the rest of the class the values you have calculated.
	• There are other units for measuring angles such as radians, which you will meet later on so you must be sure your calculator is in degree mode if you are using degrees
	• Using the calculator ask them to check the values of sin, cos and tan of the angles, which you have calculated through measurement. Check the measurements of the rest of the class also.
	• Using the answers on Student Activity 9, answer the questions on Student Activity 10.
	• If students knew the ratios how would you find out the angle? Given that the sin of an angle is 0.5 how do you find the angle?
	• Given
	  ○ Sin¯'A=0.7 find A
	  ○ Cos¯'=0.2 find B
	  ○ tan¯'=1.75 find C.
	• Ask them to Reflection List what you have learned today.
	• Write down anything you found difficult. Write down any questions you may have.

Source	https://www.projectmaths.ie/documents/T&L/IntroductionToTrigonometry.pdf

2 **Activity**

Activity: me and my shadow

When the sun is high, your shadow is short



When the sun is low, your shadow is long

Student Activity 1A

• Show the angle of elevation of the sun on the above diagram. Call it A.

• Describe the angle of elevation of the sun in terms of the two arms of the angle.____

____

• Measure the height of one of the students in your group and the length of their shadow.

• Height of the student: ____________cm. Length of the shadow _________________________cm.

• Draw a rough sketch of a right-angled triangle to model the situation and write in the measurements.

Student Activity 1B

Measure the length of the shadow of some tall object e.g. flagpole or goalpost. Length of the shadow of a tall object which you cannot physically measure e.g. goalpost ________________________________cm.

Student Activity 1C

Back in class – Measuring the angle of elevation of the sun

- Decide what scale to use.
- Draw an accurate diagram on graph paper.

Question-:

Measure the angle of elevation of the sun from Diagram 1 above using a protractor.

- Angle of elevation of the sun at __________ (time) on ________ (date) was _________.
- Check your answer with other students in the class.
- If you were to measure the angle of elevation of the sun at 10 a. m and another class measured the angle at 11 a.m. what would be the difference in the measurements?________________________________.

Student Activity 1D

Knowing the angle of elevation of the sun, measure the height of a tall object using the length of its shadow as previously measured.

- Decide what scale to use. Scale: ________________________
- Draw an accurate diagram on graph paper using the length of the shadow, the angle of elevation of the sun and forming right-angled triangle (ASA).

Question-:

Measure the height of the goalpost from Diagram 2 above and using the scale factor convert to its actual height.

- Check the answer with other students in the class.

Conclusion for part 2:

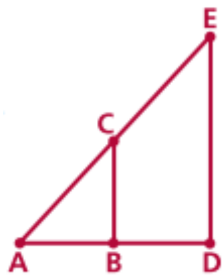
The height of the goalpost is ____________________cm approximately. Would you expect the same answer if you took the measurements at different times of the day? Explain your answer.____

____

____

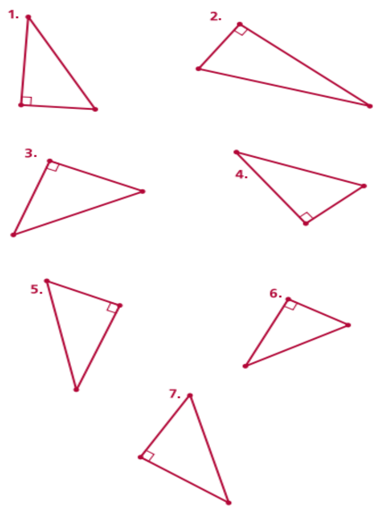
Student Activity 1E-:

- Using graph paper draw the above 2 diagrams overlapping, with the angles of elevation of the sun superimposed as shown by example in the diagram on the right. Label the diagram on the graph paper as in the diagram on the right.



Student Activity 2-:

Labelling Sides in Right Angled Triangles



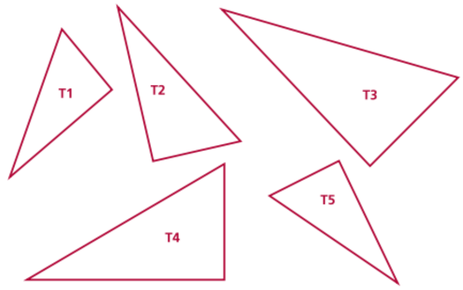
Student Activity 3

Calculating ratios for similar right-angled triangles with angles of 30°

• Measure and label the 90° and the 30° angles in the following triangles. What is the measure of the third angle?

• Label the hypotenuse as "hyp". With respect to the 30° angle, label the other sides as "adj" for adjacent and "opp" for

• Complete the table below.



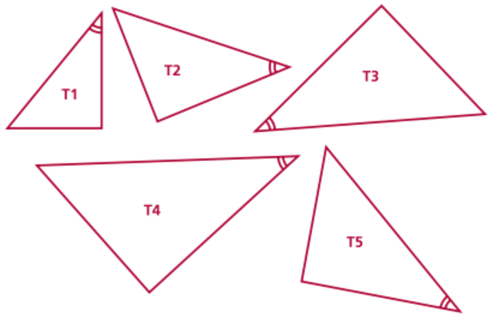
Student Activity 4

Calculating ratios for similar right-angled triangles with angles of 40°

• Measure and label the 90° and the 40° angles in the following triangles. What is the measure of the third angle?

• Label the hypotenuse as "hyp". With respect to the 40° angle, label the other sides as "adj" for adjacent and "opp" for opposite.

• Complete the table below.



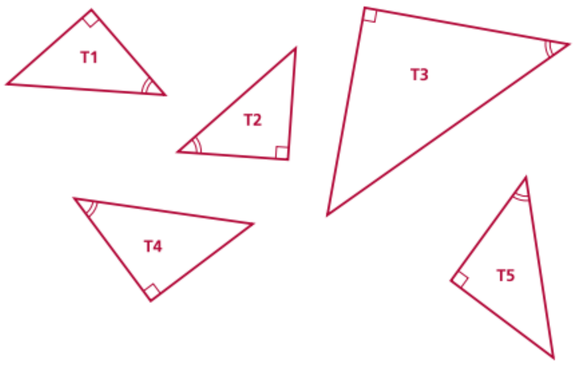
Student Activity 5

Calculating ratios for similar right-angled triangles with angles of 45°

• Measure and label the 90° and the 45° angles in the following triangles. What types of right angled triangle are these triangles?____

• Label the hypotenuse as "hyp". With respect to the 45° angle, label the other sides as "adj" for adjacent and "opp" for opposite.

• Complete the table below.



Student Activity 6

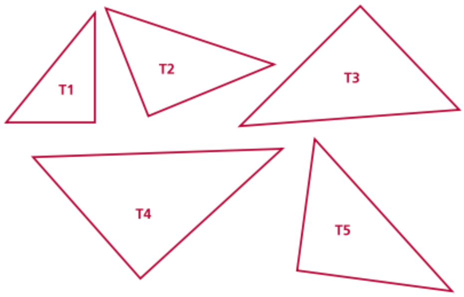
Calculating ratios for similar right-angled triangles with angles of 50°.

• Label the 90° and the 50° angles in the following triangles. What is the measure of the third angle?

____

• Label the hypotenuse as "hyp". With respect to the 50° angle, label the other sides as "adj" for adjacent and "opp" for opposite.

• Complete the table below.



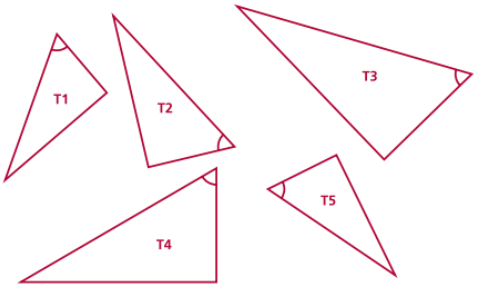
Student Activity 7

Calculating ratios for similar right-angled triangles with angles of 60°

• Measure and label the 90° and the 60° angles in the following triangles. What is the measure of the third angle?____

• Label the hypotenuse as "hyp". With respect to the 60° angle, label the other sides as "adj" for adjacent and "opp" for opposite.

• Complete the table below.



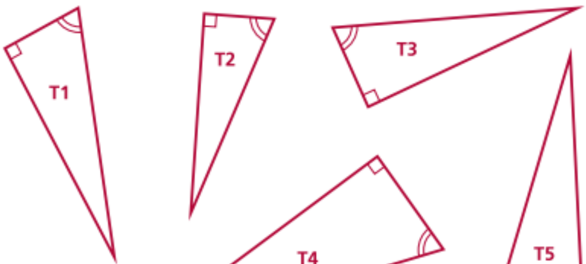
Student Activity 8

Calculating ratios for similar right-angled triangles with angles of 70°.

• Measure and label the 90° and the 70° angles in the following triangles. What is the measure of the third angle?____

• Label the hypotenuse as "hyp". With respect to the 70° angle, label the other sides as "adj" for adjacent and "opp" for opposite.

• Complete the table below



Student Activity 9

Master table of class results for ratios of sides in right angled triangles.

Angle/°	Opp/hyp	Check	Adj/hyp	Check	Opp/adj	Check
30°						
40°						
45°						
50°						
60°						
70°						

Student Activity 10

Using the master table of class results answer the following questions

1. What do you notice about sin 30° and cos 60°?

2. What do you notice about cos 30° and sin 60°?

3. Can you explain what you have noticed using diagrams?
4. How would you describe angles 30° and 60°?

5. Can you find similar examples in the master table?

6. For what angle in a right-angled triangle is the opposite side one half of the hypotenuse?

Draw a diagram to illustrate your answer.

7. For what angle in a right-angled triangle are the opposite and adjacent sides equal?

8. Calculate SinA/CosA for each angle A. Compare this to the value of Tan A. What do you notice? Can you justify the answer?

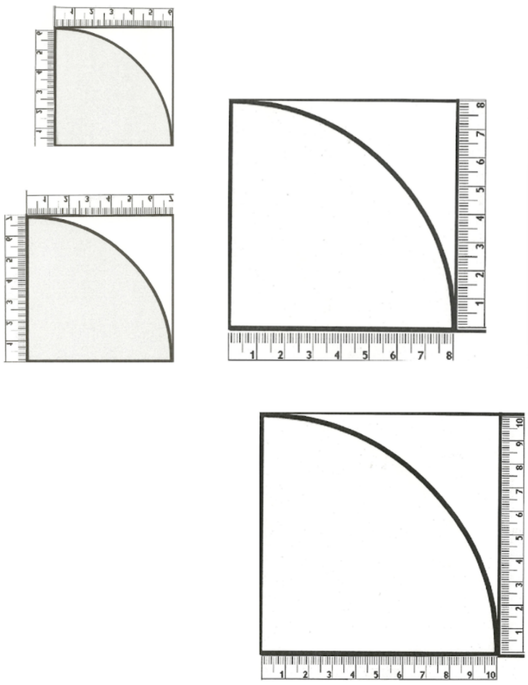
3

Activity

Materials required: quarter circle, geometry box

Setup:

1. Do not mention anything the term "trigonometry" at the start of this exercise. Give a brief revision of Pythagoras theorem.
2. Divide the students into groups of three (if the number of students does not divide evenly into groups of 3, a group or two of 2 students works fine). Give each member of a group the two handouts (the same sized [quarter circles](), and a [table]()), a protractor, and an additional straightedge (this seemed to help when measuring the angles).



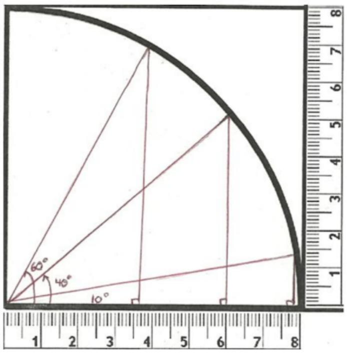
Quarter circles

Degrees	opp	adj	hyp	opp	adj	opp

				hyp	hyp	adj
0						
30						
45						
60						
90						

Table

3. The table contains nine different angle measures. Ask the group to split these angle measures up between the group members, so that each student is responsible for three angle measures. Explain that they are to use the protractor to mark off their angle measures on their quarter circle. Once they have marked off their angle measures, they draw three right triangles, as shown in the example. Provide an example of this on the board, just to make sure that everyone gets off to a good start.



4. Next start the measurement phase. Briefly discuss with the students the meaning of the words "opposite", "adjacent", and "hypotenuse" with respect to right triangles. When all seem clear on these concepts, I explain that each student is responsible for measuring and recording the lengths of the opposite and adjacent sides of all three of his or her triangles, accurate to one decimal place. The adjacent side of the triangles is measured easily using the horizontal ruler on the diagram; using their straightedge or protractor, the students can extend the opposite side of the triangle horizontally, connecting to the vertical ruler on the diagram. Also discuss the length of the hypotenuses, which will always be equal to the radius of their quarter-circle.
5. Once each member of the group has recorded their own data in their table, they share their data with the other members of the group, so that each group member ends the measurement phase with *opp*, *adj*, and *hyp* filled in for all nine angle measurements.
6. At this point, with *opp*, *adj*, and *hyp* filled in for each angle, students are ready to find and fill in the ratios.
7. When everyone's tables are complete, end the data gathering stage, and are ready for discussion.

Reference: https://betterlesson.com/lesson/448228/introduction-to-trigonometry

9. APPLICATIONS OF TRIGONOMETRY

QR Code:

T7A3T4

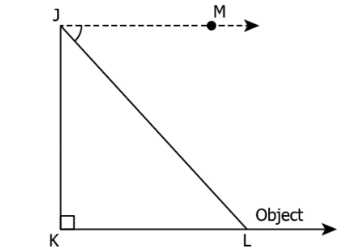
Learning outcome and Learning Objectives:

<table>
<tr><th>Content area / Concepts</th><th>Learning Objectives</th><th>Learning Outcome</th></tr>
<tr><td rowspan="2">Heights and Distances</td><td>Identify line of sight in order to determine angle of elevation and angle of depression</td><td rowspan="2">Determines all trigonometric ratios with respect to a given acute angle (of a right triangle) in order to use them in solving problems in daily life contexts like finding heights of different structures or distances from them.</td></tr>
<tr><td>Apply trigonometric ratios (of specific angles) in order to determine heights and distances of the objects</td></tr>
</table>

Test items

LOB: Identify line of sight in order to determine angle of elevation and angle of depression

1. Consider the figure shown.

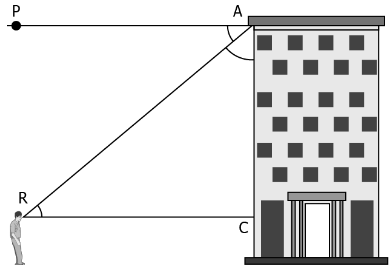


If $\angle LJM$ is the angle of depression, what is the line of sight in the figure shown?

Option 1: JM
Option 2: JL
Option 3: JK
Option 4: KL

Correct Answer: Option 2

2. Observe the figure below.



If AR is line of sight, which of these angles represents the angle of elevation?

Option 1: $\angle$ARC
Option 2: $\angle$RAP
Option 3: $\angle$ACR
Option 4: $\angle$RAC

Correct Answer: Option 1

LOB: Apply trigonometric ratios (of specific angles) in order to determine heights and distances of the objects

1. The angles of elevation of two cars, from the car to the top of a building are 45° and 30°. If the cars are on the same side of the building and are 50 m apart, what is the height of the building?

Option 1: $25(\sqrt{3}-1)$ m
Option 2: $25(\sqrt{3}+1)$ m
Option 3: $50(\sqrt{3}-1)$m
Option 4: $50(\sqrt{3}+1)$ m

Correct Answer: Option 2

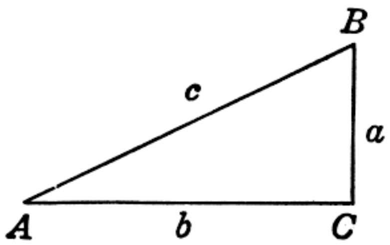
2. From the top of a 42 m high light house, the angles of depression of two ships that are in a straight line are 60° and 45° respectively. What is the distance between the two ships? (Use$\sqrt{3} = 1.73$)

Option 1: 17.8 m
Option 2: 24.2 m
Option 3: 32.39 m
Option 4: 66.28 m

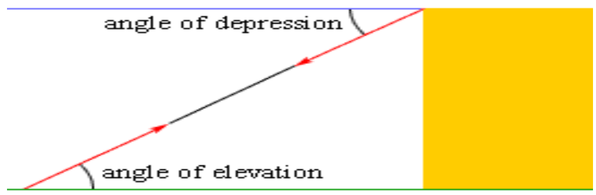
Correct Answer: Option 1

Suggested Teacher Resources

I Lesson Plan

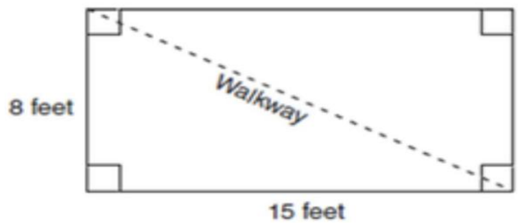
<table>
<tr><td>Objective</td><td>To apply trigonometric ratios (of specific angles) in order to determine heights and distances of the objects</td></tr>
<tr><td>Material required</td><td>None</td></tr>
<tr><td>Prerequisite Knowledge</td><td>Trigonometric signs</td></tr>
<tr><td>Procedure</td><td>The teacher will start by brushing up previous concepts to see how much students have understood till now.


Identify the sine, cosine and tangent of <A
Identify the sine, cosine and tangent of <B
Describe the relationship between the sines and cosines of <A and <B
Explain when to use trigonometric ratios instead of Pythagoras theorem.

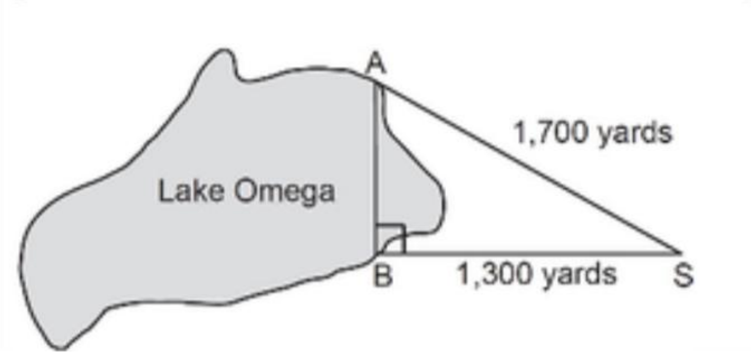
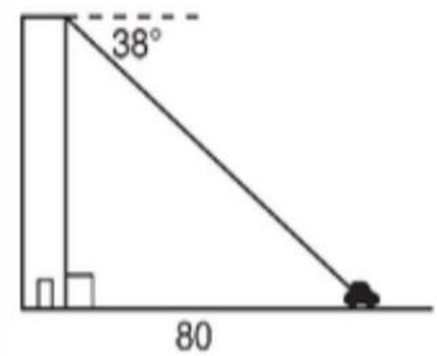
Next, the teacher draws the following diagram on the board:


And asks what is the relationship between these two angles.
Next, the teacher assigns pairs of students one or two questions from the activity sheet, which consists of real-world problems involving right triangles. The students work together to solve the problem. Both students record their responses on their own sheet, but one student writes the work on a separate paper, as well. The recorder must ensure that his or her partner understands the work shown on the paper enough for the partner to be able to present the solution to the class.

Nancy's rectangular garden is represented in the diagram below. If a diagonal walkway crosses her garden, what will be the length of this walkway in feet?



</td></tr>
</table>

	• Campsite A and campsite B are located directly opposite to each other on shores on Lake omega. The two campsites form a right triangle with Suraj's position, S. The distance from Campsite B to Suraj's position is 1300 yards and Campsite A is 1700 yards. What is the distance between Campsite A and Campsite B?  • From the top of an apartment building, the angle of depression from the car parked on the street is 38 degrees. The car is parked 80feet away from the base of the building. Find the height of the building.  • A 28 feet ladder is leaning against a house. The bottom of the ladder is 6 feet from the base of the house. Find the measure of the angle formed by the ladder and the ground.
Source	https://betterlesson.com/lesson/resource/2854340/mixed-trigonometry-practice

2

Activity

Materials List: Each group should have:

- A4 size paper
- protractor
- Pencil
- Scale, long thread for measuring

Preparation: Worksheet for students: Draw a river as shown in figure 1 with width of river exactly 15 cm apart for all worksheets that are given to the students.

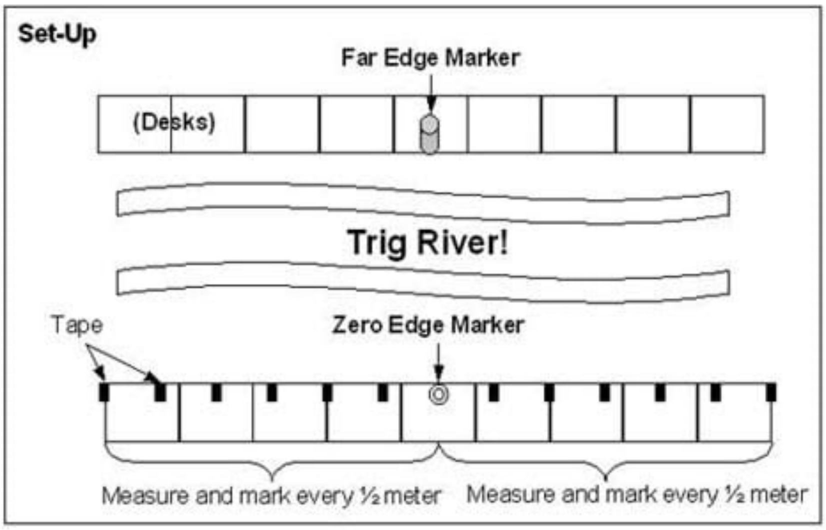


Figure 1

Introduction/Motivation

Teacher will ask "Is it possible to determine the width of a river without crossing it? "
(Answer: It is possible to come very close to determining the width of a river of any size, using triangles.) The same principle used to determine the width of a river can be applied to other situations, including determining the height of a hill, a tree or a building. The simple geometric shape that makes this all possible is the *triangle*. During this activity, you will learn how to use triangles to determine the width of a river.

Setup:

Define a "river" for the students. For example, if working inside, rearrange desks to form the two "banks" of the river (with space for students to work on each "shore"). If working outside, choose a spot with two widely spaced (2-5 meters) and roughly parallel lines to define the "river" banks. For example, a wide sidewalk or a strip of grass with straight edges. If a small "river" is being measured, have the students measure in centimetres, if a larger "river" is being used, have the students measure in meters. Because these are ratios of distance, the result at the end should have the same unit (meters, centimetres, etc.) used to make the initial measurement.

Students will form 5-6 groups and work together to answer the questions

1. Give each student the worksheet.
2. On one side of the river (as close to the middle of that side as possible, set an object that will be the Far Edge Marker. Normally this represents a tree right at the edge of the opposite side of the river.
3. Directly across from the marker, place a Zero Edge Marker (see Figure 1). All the students should be on this side of the river.
4. Lay the measuring tape along the "zero edge bank" with one end at the Zero Edge Marker and place a piece of tape every ½ meter on the river edge of the desks. Repeat this in the other direction (see Figure 1).
5. Each student should make an estimate of how wide the river is and record it on the back of their worksheet (anywhere on the paper is acceptable).
6. Each group will work from a different tape mark. When both students of a group are at their designated mark and have written on their worksheet the distance their tape is from the Zero Edge Marker, give each group a protractor.

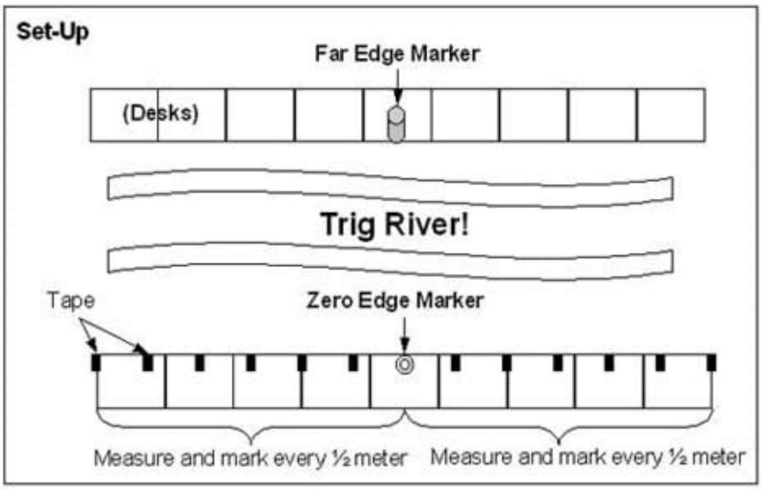


Figure 1. Set up for angle measurement.

7. Lay the protractor with the centre point on the middle of the tape and the zero-angle pointing toward the Zero Edge Marker (see Figure 2)

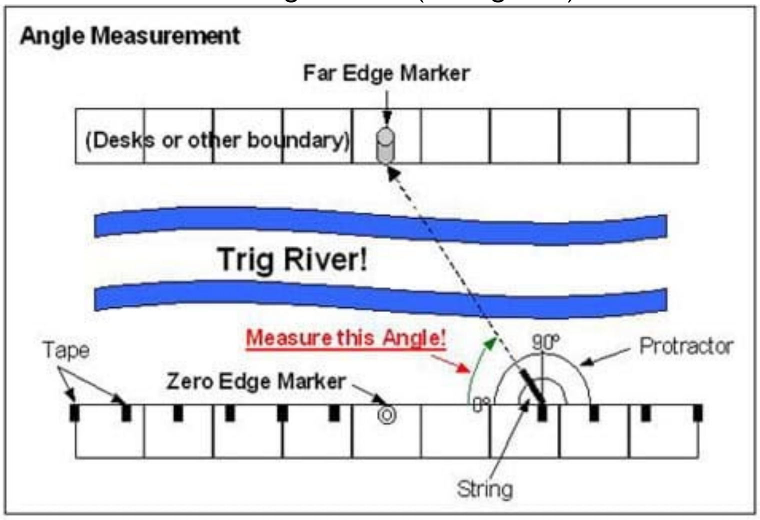


Figure 2. Angle Measurement

8. One student will hold the protractor in place while the other places one end of the string on the centre point of the protractor and aims the other end at the Far Edge Marker. Read the angle the string passes over on the protractor (counting up from zero; this should not be more than 90 degrees), and record it on the worksheet. While the students do this, the teacher can measure the actual distance between the two markers; do not reveal the distance yet.

9. Students alternate between measuring the values through protector.

10. Students will finalize the method and make it presentable.

11. Have students compare their estimate of the river width to the actual measurement. How close was their estimate?

Question and Answer #1: Were students who were closer to the zero marker more or less accurate than those further away?

Answer: Students close to the zero marker should be less accurate because the values of the tangents of angles close to 90° become large quickly and a small error in the angle measurement results in a large distance error. Note that the same problem would be seen as the measured angle approaches zero degrees, but a student would have to be infinitely far away for that.

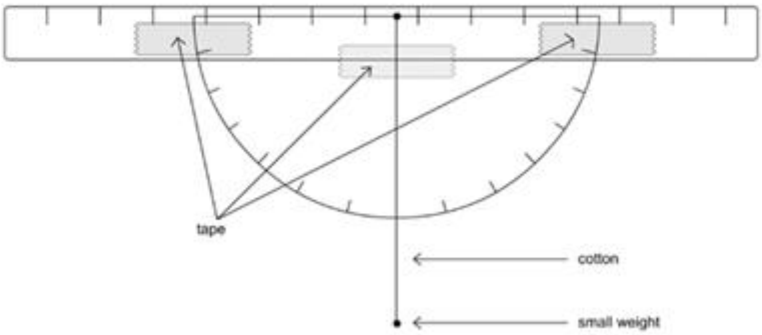
Question and Answer #2: Could this measuring method be used in the wilderness if you did not have a calculator or Trig Tables?

Answer: It is not easy to memorize the tangent values for all angles but one value is very easy to remember: tan (45). Have the students find this value and then explain why they get a simple answer.

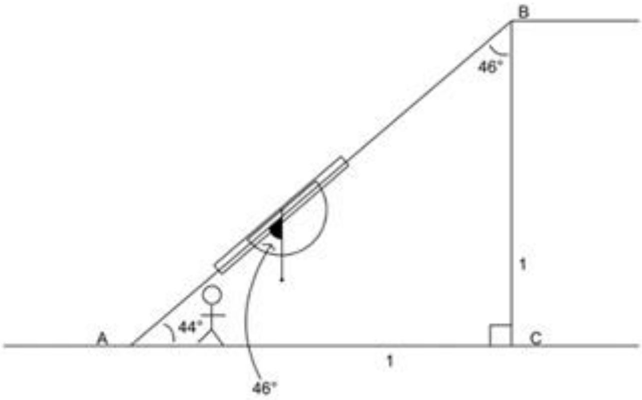
3 Activity

In this session the students are grouped in threes. Each group constructs a clinometer, uses it to measure angles, and prepares for a practical application of trigonometry to be undertaken in Session 2.

1. Issue each group with a clear plastic ruler, a clear plastic protractor, clear tape, cotton and a small weight. Before the session it is suggested that the teacher drill a small hole in the centre of the protractors large enough to pass the cotton through. Each group makes a clinometer. See diagram below.



2. The next task is to gain some facility in using it to measure angles. Explain how it is done by using the diagram below.

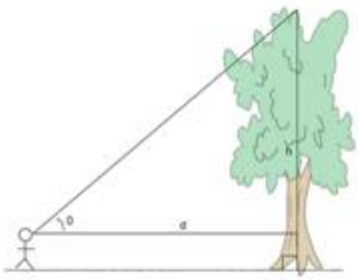


3. The clinometer allows the students to measure the angle ABC. They are then able to calculate the angle BAC. The students can then be sent to a few places around the school to practise measuring such angles. Usually it is best for one person to hold the clinometer in such a way that his or her eye looks along the ruler to the top of the object concerned, and a team member reads off the angle. Students should take turns doing this. The aim, here, is to become proficient at taking angles so that this is not an issue during Session 2. For each measurement, the students should sketch a diagram like Diagram 1.2 and calculate the angle BAC.
4. The above may be completed before the session has concluded. If this happens, the time could be spent usefully revising some of the material from trigonometry units studied earlier.

Session 2

In this session the students learn how to measure the height of one or more suitable objects, carry out the activity, and write up a poster explaining how this procedure is carried out.

1. Each student will need to measure the distance between his or her feet and eyes. Imagine the tall object to be measured is a tree. The diagram below indicates how this is done.



2. Students measure the distance d from the base of the tree. They use the clinometer to find the angle θ. Using tan, they can find h (because h = d x tan(θ)). And, of course, they need to add on the distance between their feet and eyes at the end.
3. Before you send the groups out, go through two practice examples using imaginary figures. Once you are sure the groups understand the process, get each group to take the necessary measurements and calculate the heights concerned. Each group should make tree measurements of the angle concerned (each at a different distance); one for each member of the group.
4. On their return to class, lest the activity be quickly forgotten, get each student to make a small poster that fully describes how a clinometer is made and how it was used for making the relevant measurements just completed.

10. CIRCLES

QR Code:

G8K6Q2

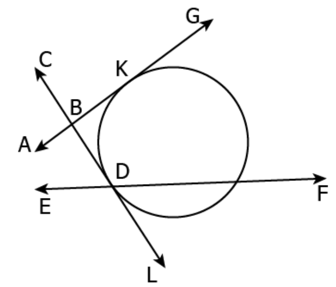
Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Tangent to a Circle	Draw, identify and differentiate between secant and tangent of a circle in order to prove and apply various theorems related to circles	-
Number of Tangents from a Point on a Circle	Prove and apply theorems related to tangent of a circle in order to determine number of tangents from the given point(s)	
	Prove and apply theorem related to tangent of a circle in order to determine length of the tangent	

Test items

LOB: Draw, identify and differentiate between secant and tangent of a circle in order to prove and apply various theorems related to circles

1. A figure is shown below.



Which of the following is true?

Option 1: Lines AG and CL are the tangents and line EF is a secant to the circle.
Option 2: Lines AG and CL are the secants and line EF is a tangent to the circle.
Option 3: Line AG is a tangent and lines EF and CL are the secants to the circle.
Option 4: Line AG is a secant and lines EF and CL are the tangents to the circle.

Correct Answer: Option 1

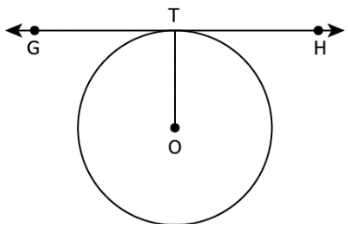
2. A circle passes through point P. How many tangents and secants to the circle are possible that pass through P?

Option 1: Tangent: 1; Secant: 1
Option 2: Tangent: Infinite; Secant: 1
Option 3: Tangent: 1; Secant: Infinite
Option 4: Tangent: Infinite; Secant: Infinite

Correct Answer: Option 3

LOB: Prove and apply theorems related to tangent of a circle in order to determine number of tangents from the given point(s)

1. A circle with centre O is shown below.

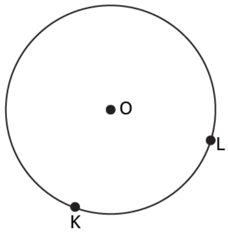


Which of the following statements is true?

Option 1: There can be only one line passing through point T such that it is parallel to OT.
Option 2: There can be only one line passing through point T such that it is perpendicular to OT.
Option 3: There can be any number of lines passing through point T such that they are parallel to OT.
Option 4: There can be any number of lines passing through point T such that they are perpendicular to OT.

Correct Answer: Option 2

2. A circle is shown below.



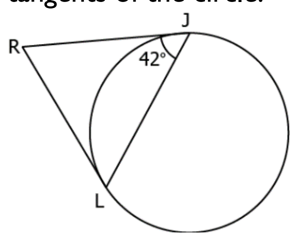
Statement I: There is only one line passing through point K which makes an angle of $90°$ with OK.
Statement II: The shortest distance of a tangent passing through point L from the centre O is equal to the radius of the circle, OL.
Statement III: One tangent can pass through two points K and L of a circle.
Which statement(s) is/are correct?

Option 1: Statement I and II
Option 2: Statement I and III
Option 3: Statement II and III
Option 4: Statement I, II and III

Correct Answer: Option 1

LOB: Prove and apply theorem related to tangent of a circle in order to determine length of the tangent

1. In the figure shown, RJ and RL are tangents of the circle.

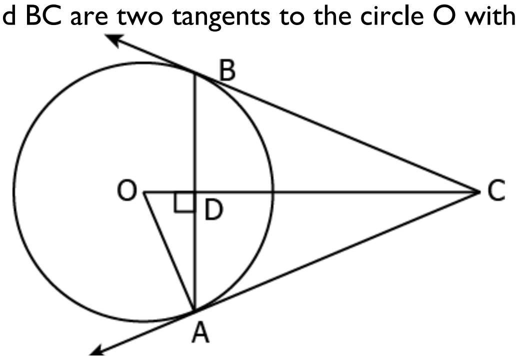


What is the measure of $\angle JRL$?

Option 1: $90° - 42°$
Option 2: $90° - 84°$
Option 3: $180° - 42°$
Option 4: $180° - 84°$

Correct Answer: Option 4

2. In figure is shown, AC and BC are two tangents to the circle O with radius 5 cm.



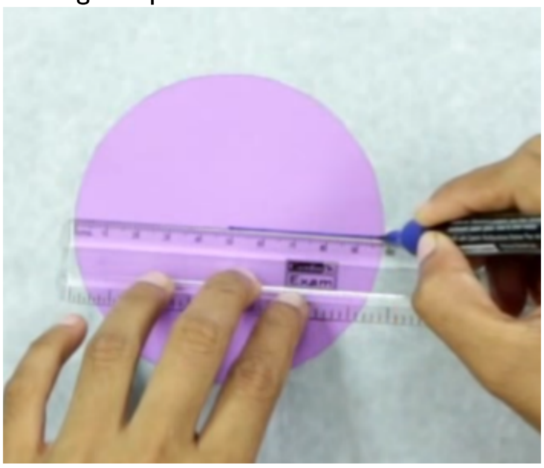
If OC = 13 cm and OD: DC = 3: 10, then what is the perimeter of triangle ABC?

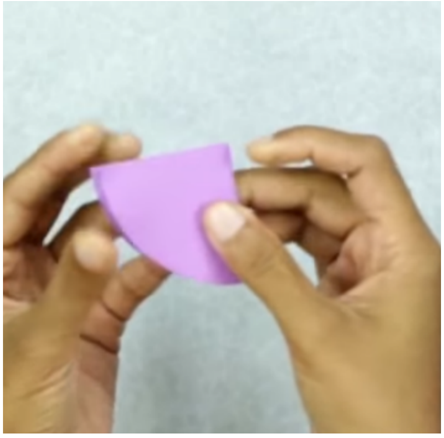
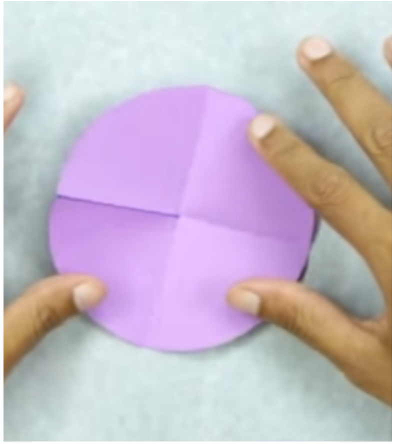
Option 1: 24 cm
Option 2: 30 cm
Option 3: 32 cm
Option 4: 34 cm

Correct Answer: Option 3

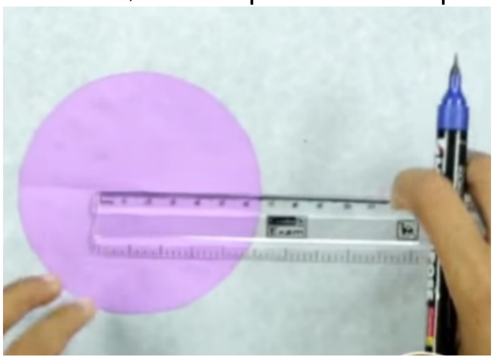
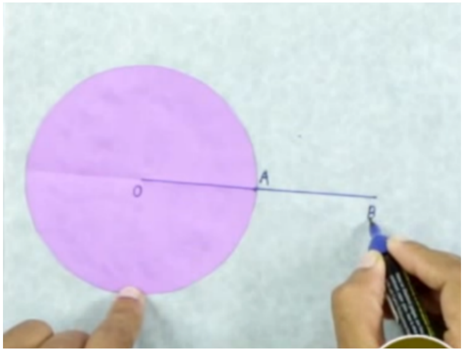
Suggested Teacher Resources

I Lesson Plan

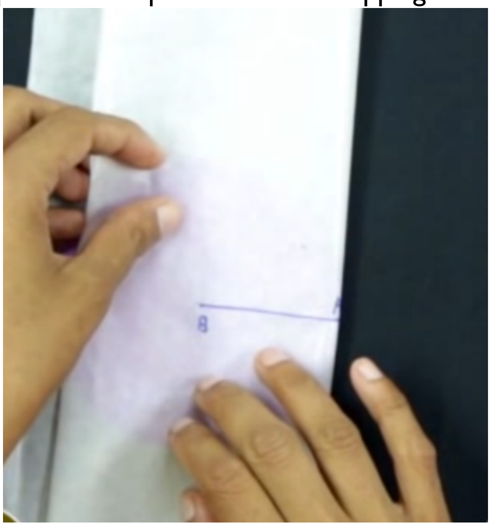
Objectives	Prove and apply theorem related to tangent of a circle in order to determine length of the tangent
Prerequisite Knowledge	• Definition of tangents and radius
Material Required	• Multicoloured chalks
Procedure	Do Now: Divide students into groups of 3. Students will be doing recall of concepts that will come handy in the lesson. Properties of chords: 1. If two angles in a circle are congruent, then they determine two central angles that are _______ 2. If two chords in a circle are congruent, then their _____ are congruent. 3. The perpendicular from the centre of a circle to a chord is the ____ of the chord. 4. Two congruent chords in a circle are ________ from the centre of the circle. 5. A tangent to a circle _____ to the radius drawn to the point of contact. Note for teacher: Throughout the whole-class discussion, focus on the use of congruent triangles, CPCTC, and properties of isosceles triangles as ways to justify how students know. Activity: Draw Tangent to the circle Setup: Divide coloured paper among students and they will be performing the activity with their peers. Instructions: 1. Cut a circle of any radius using compass and scissor. Paste it in the copies.  2. Fold the circle twice from the middle as shown.

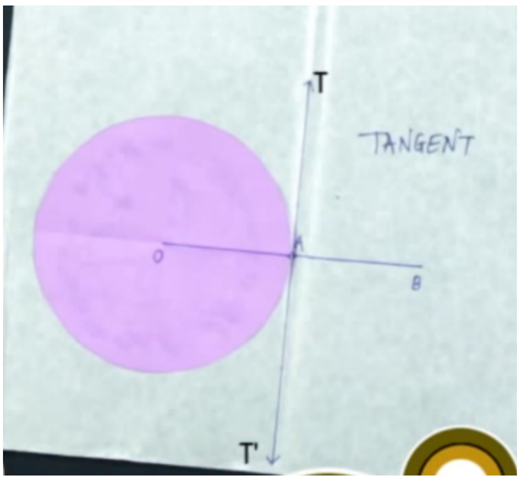
3. Join the centre of the circle to any point on the outside of the circle using pen and scale. Mark the centre O, external point B and the point where circle cut the line at A.

4. Fold the paper such that point O and point B are overlapping.



5. Mark the crease with pen and draw a straight line. Name it TT'.

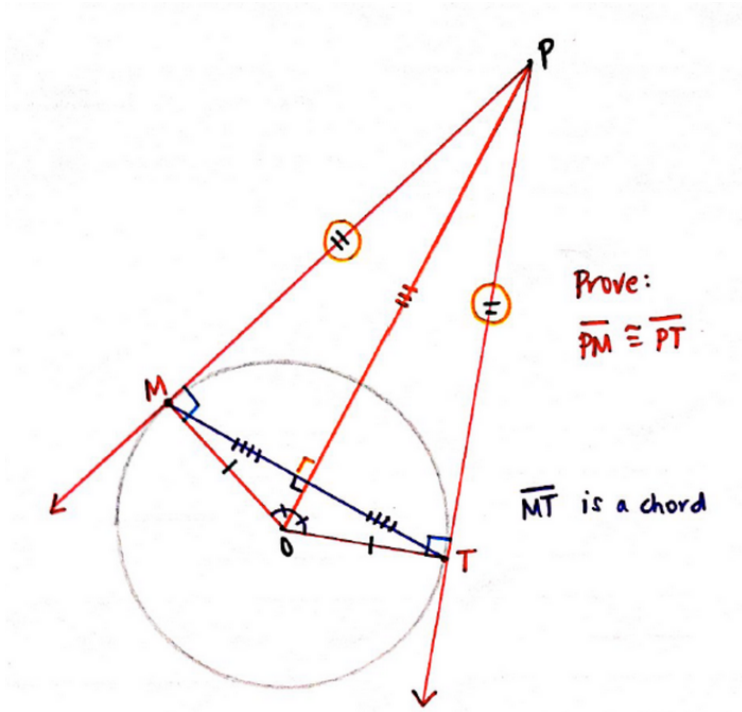


TT' is a tangent.

<table>
<tr><td></td><td>6. Take any point on tangent and fold the paper with crease just touching the circumference of the circle. This crease will give another tangent to the circle.
7. Note down the observation such as lengths of the tangents drawn, angle between them.

Observation: The length of the tangents drawn from external point to the circle are equal in length.

Students will work in pairs to prove that tangents from an external point to the circle are equal.

Proof:


On proving congruence between triangle PMO and triangle PTO by RHS, PM and PT will be equal using C.P.C.T.

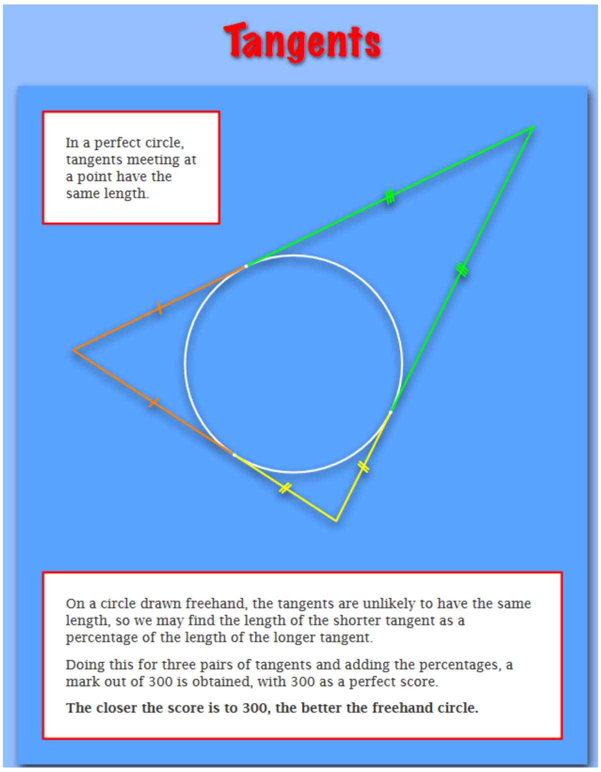
Teacher can push students to find alternative proof.

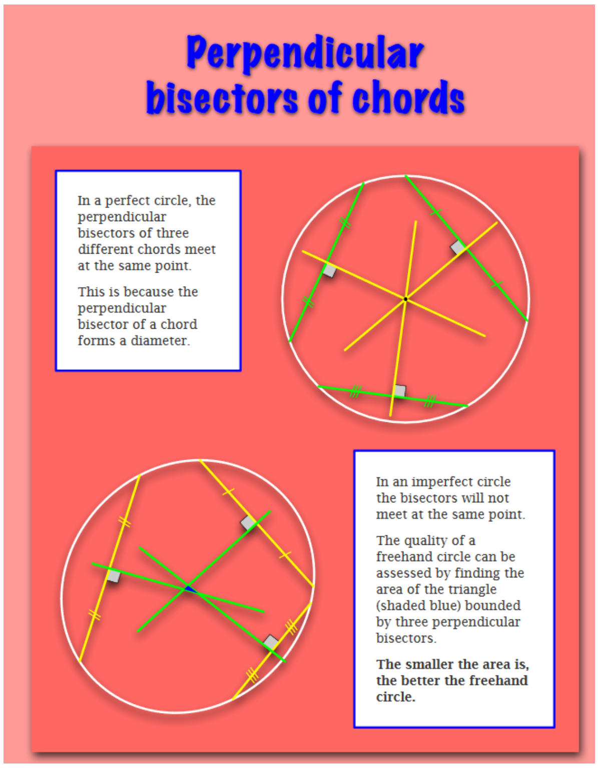
Alternative Proof:
In Triangle TOM
MO = TO (Radii)
So, Angle OMT = Angle OTM (Isosceles triangle property)
So, 90° - Angle OMT = 90° - Angle OTM
Angle PMT = Angle PTM
Triangle PTM is an isosceles triangle.
Hence, PM = PT</td></tr>
<tr><td>Source</td><td>https://betterlesson.com/lesson/resource/2732864/notes-chords</td></tr>
</table>

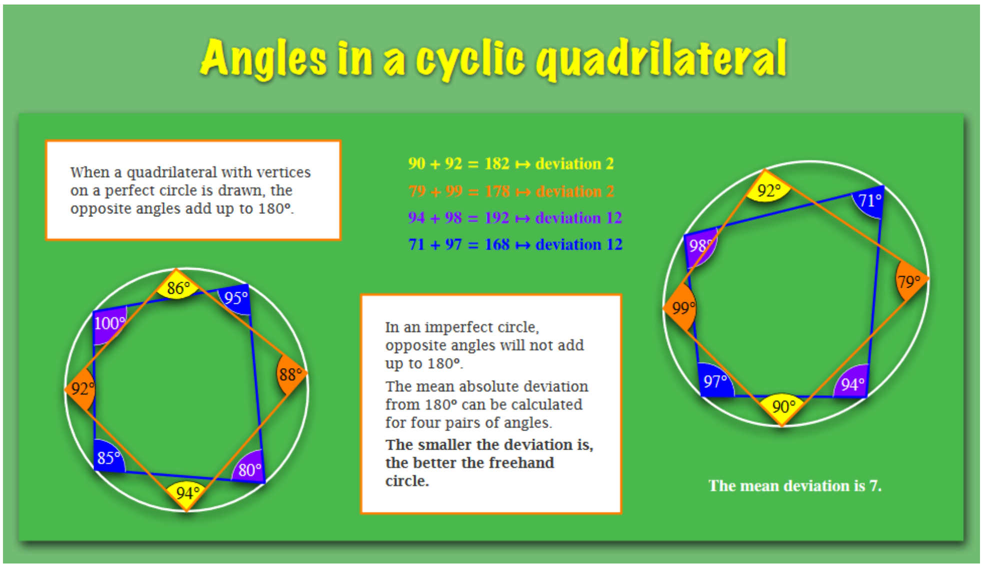
2

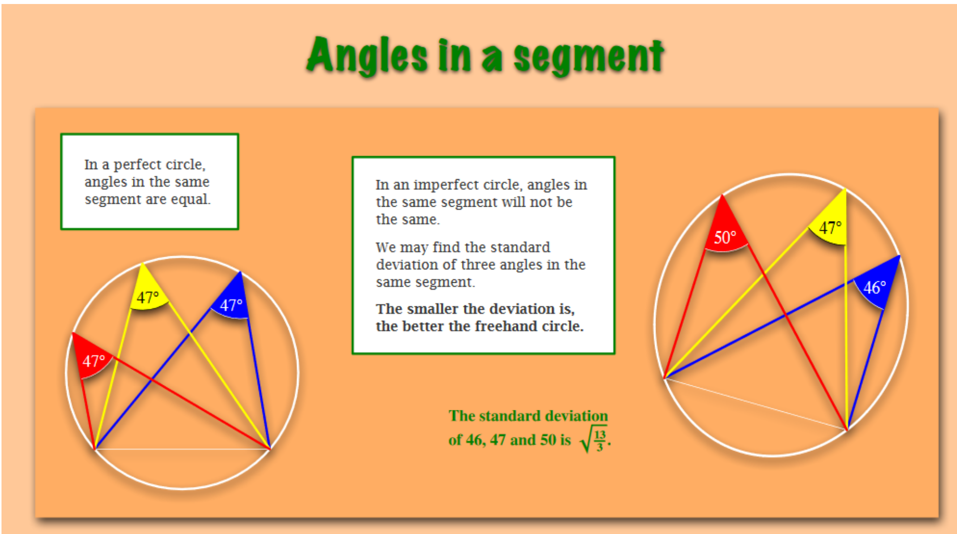
Activity

Objective: Use theorems of circles to argue which hand drawn figure is closer to a circle.
Setup: It's a competition between pairs to see who can draw the best hand drawn circle using just a pencil. The partner will use theorems of circles to identify if the circle drawn is closer to an actual circle or not. The most theorems students can remember to test the more points they will earn.









Source: https://www.projectmaths.ie/workshops/seminar2014-2015/DrawingAFreeCircleByHand.pdf

11. CONSTRUCTIONS

QR Code:

B1E7Q2

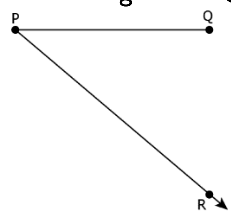
Learning outcome and Learning Objectives:

<table>
<tr><th>Content area / Concepts</th><th>Learning Objectives</th><th>Learning Outcome</th></tr>
<tr><td>Division of a Line Segment</td><td>List and execute steps of construction in order to divide a line segment in a given ratio</td><td rowspan="3">Examines each step and reasons out each step, in order to:
A) Construct a triangle similar to a given triangle as per a given scale factor.
B) Construct a pair of tangents from an external point to a circle and justify procedures</td></tr>
<tr><td>Construction of a similar triangle</td><td>List and execute steps of construction in order to construct a similar triangle as per a given scale factor</td></tr>
<tr><td>Construction of Tangents to a Circle</td><td>List and execute steps of construction in order to construct tangent(s) to a given circle</td></tr>
</table>

Test items

LOB: List and execute steps of construction in order to divide a line segment in a given ratio

1. A part of the construction to divide the line segment PQ in the ratio 3: 4 is shown below.



Which of these is the next step to divide the line segment PQ in the ratio 3: 4?

Option 1: Locate 4 points, P_1, P_2, P_3, and P_4 on the ray PR, such that $PP_1 = P_1P_2 = P_2P_3 = P_3P_4$.

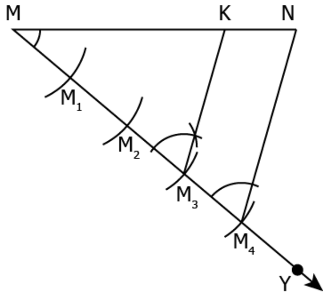
Option 2: Locate 4 points, P_1, P_2, P_3, and P_4 on the ray PR, such that $PP_1 = 2P_1P_2 = 3P_2P_3 = 4P_3P_4$.

Option 3: Locate 7 points, P_1, P_2, P_3, P_4, P_5, P_6, and P_7 on the ray PR, such that $PP_1 = P_1P_2 = P_2P_3 = P_3P_4 = P_4P_5 = P_5P_6 = P_6P_7$.

Option 4: Locate 7 points, P_1, P_2, P_3, P_4, P_5, P_6, and P_7 on the ray PR, such that $PP_1 = 2P_1P_2 = 3P_2P_3 = 4P_3P_4 = 5P_4P_5 = 6P_5P_6 = 7P_6P_7$.

Correct Answer: Option 3

2. Divya divided a line segment MN in the ratio 3: 1 as shown below, where $MM_1 = M_1M_2 = M_2M_3 = M_3M_4$.



Which of these options shows that MK: KN = 3: 1?

Option 1: As M_3K is parallel to M_4N, therefore $\frac{MM_3}{M_3M_4} = \frac{MK}{KN}$ by BPT. Also, as $\frac{MM_3}{M_3M_4} = \frac{3}{1}$, so $\frac{MK}{KN} = \frac{3}{1}$.

Option 2: As M_3K is parallel to M_4N, therefore $\frac{MM_3}{M_3M_4} = \frac{KN}{MK}$ by BPT. Also, as $\frac{MM_3}{M_3M_4} = \frac{3}{1}$, so $\frac{MK}{KN} = \frac{3}{1}$.

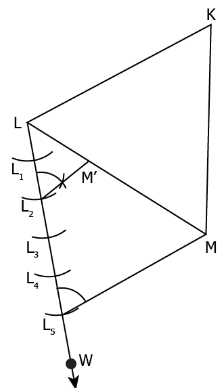
Option 3: As, M_3K is parallel to M_4N, therefore $\frac{MM_3}{M_3M_4} = \frac{MK}{KN}$ by BPT. Also, as $\frac{MM_3}{M_3M_4} = \frac{3}{1}$, so $\frac{MK}{KN} = \frac{1}{3}$.

Option 4: As, M_3K is parallel to M_4N, therefore $\frac{MM_3}{M_3M_4} = \frac{KN}{MK}$ by BPT. Also, as $\frac{MM_3}{M_3M_4} = \frac{3}{1}$, so $\frac{MK}{KN} = \frac{1}{3}$.

Correct Answer: Option 1

LOB: List and execute steps of construction in order to construct a similar triangle as per a given scale factor

1. Peter is asked to construct a similar triangle with sides $\frac{2}{5}$ of the corresponding sides of triangle KLM. Part of his construction is shown below.



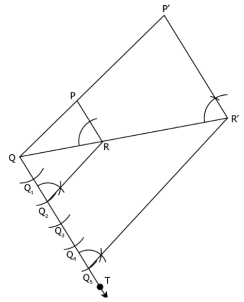
Which of these is the next step to construct the similar triangle?

Option 1: Draw a line through M' parallel to KM to intersect KL at K'.

Option 2: Draw a line through L_2 parallel to KM to intersect KL at K'.
Option 3: Draw a line through M' parallel to LW to intersect KL at K'.
Option 4: Draw a line through L_2 parallel to KM to intersect LM at K'.

Correct Answer: Option 1

2. Dilip constructed a triangle P'QR' similar to triangle PQR with sides $2\frac{1}{2}$ times the corresponding sides of triangle PQR as shown below.



Which of these options shows that P'Q: PQ = 5: 2?

Option 1: As $\Delta PQR \sim \Delta P'QR'$ by AA similarity criteria and as $\frac{QR'}{QR} = \frac{QQ_5}{QQ_2} = \frac{5}{2}$, therefore $\frac{P'Q}{PQ} = \frac{QR'}{QR} = \frac{5}{2}$.

Option 2: As $\Delta PQR \sim \Delta P'QR'$ by SSS similarity criteria and as $\frac{QR'}{QR} = \frac{QQ_5}{QQ_2} = \frac{5}{2}$, therefore $\frac{P'Q}{PQ} = \frac{QR'}{QR} = \frac{5}{2}$

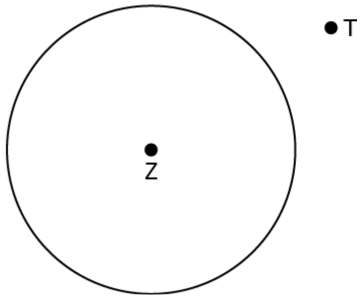
Option 3: As $\Delta PQR \sim \Delta P'QR'$ by AA similarity criteria and as $\frac{QR'}{QR} = \frac{QQ_5}{QQ_2} = \frac{2}{5}$, therefore $\frac{P'Q}{PQ} = \frac{QR'}{QR} = \frac{2}{5}$

Option 4: As $\Delta PQR \sim \Delta P'QR'$ by SSS similarity criteria and as $\frac{QR'}{QR} = \frac{QQ_5}{QQ_2} = \frac{2}{5}$, therefore $\frac{P'Q}{PQ} = \frac{QR'}{QR} = \frac{2}{5}$

Correct Answer: Option 1

LOB: List and execute steps of construction in order to construct tangent(s) to a given circle

1. A circle with centre Z and a point T outside the circle is shown below.



Which options describes the first two steps to construct two tangents from point T to the circle?

Option 1:
- Join ZT and bisect it. Let N be the midpoint of ZT.
- With N as centre and NZ as radius, draw a circle.

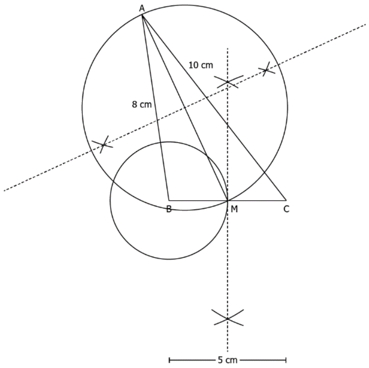
Option 2:
- Join ZT and trisect it. Let N be the midpoint of ZT.
- With N as centre and NZ as radius, draw a circle.

Option 3:
- Join ZT and bisect it. Let N be the midpoint of ZT.
- With N as centre and ZT as radius, draw a circle.

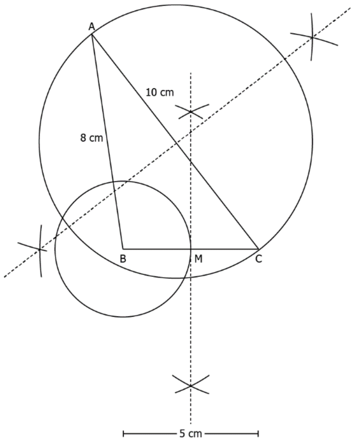
Option 4:
- Join ZT and trisect it. Let N be the midpoint of ZT.
- With N as centre and ZT as radius, draw a circle.

Correct Answer: Option 1

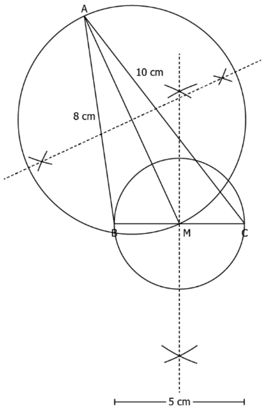
2. In a triangle ABC, AB = 8 cm, BC = 5 cm and AC = 10. Which of the following construction is used in constructing tangents from A to a circle with diameter BC?



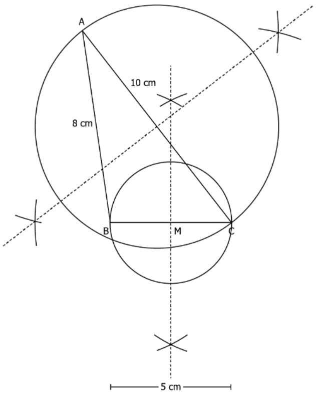
Option 1:



Option 2:



Option 3:



Option 4:

Correct Answer: Option 3

Suggested Teacher Resources

1 Lesson Plan

Objectives	Students will be able to: • Construct an angle and its bisector using a straightedge and compass • State the steps used to construct an angle and its bisector when using a straightedge and compass • Verify that the construction of their angle bisector is correct.
Prerequisite Knowledge	• Drawing a parallel line using compass. • Knowledge of Similarity of Triangles.
Material Required	• Pen/Pencil and notebook. • Compass and scale.
Procedure	Start the class with the following activity- Part-1 • Draw two-line segments of equal measurements on the board. • The lengths of the line segments should be in decimals. For example, you may draw two-line segments, each measuring 15.7 cm. • Then, select two students and ask them to divide the given line segments using only a ruler. • Ask one of the students to divide the line segment in the ratio 5:7 and the other learner to divide the other line segment in the ratio 2:3. • Thereafter, ask the students to measure the divisions to check if the line segments are divided according to the given ratios. Now ask the students to share the difficulties they faced while dividing the line segment. Part 2 • After the students have answered, demonstrate the division of a line segment in a given ratio using a compass. • Thereafter, mathematically prove the construction. Finally, ask the students if they found this method simpler than the earlier method. Part 3 • After the demonstration, divide the class into pairs. • Provide each pair with the length of a line segment and a ratio. • The students need to draw the line segment in their books and then divide it in a given ratio using a compass (by using the methods covered in Part 2). • Also, ask each pair to mathematically prove the constructions. Thereafter, randomly select students and ask them to demonstrate the construction and proof to the class. End the class with the following manner – Hold a quiz pertaining to the following topics learnt previously: • Similarity of triangles • Criteria for similarity of triangles

	Note - You may also draw triangles (with the measurements of lengths and angles) on the board. Then, ask the students to identify whether the triangles are similar and the criterion they used to come to the conclusion. (This quiz will act as a pre-requisite for the upcoming lesson i.e. constructing a similar triangle to a given triangle)
Reflection Questions	• What were some mathematical concepts that you learned in class today? • What was easy and difficult? • Where do you think we divide a given length in a specific ratio? Can you see something around you that is divided in a ratio?
Expected Outcome	• Students will understand the importance and requirement of construction in real life. • Students will be able to divide a line segment in a given ratio.

2 Activity

Time- 20 min

- The class needs to be split into two groups in order to do stations. Station 1 is teacher facilitated and Station 2 is student facilitated. Students will spend 10 minutes at each station. You should already have your desks split into two sections with groups of two within those sections (Students will be working in pairs. See "Additional Info/Instructions" on how to pair your students). For both stations, students will need to have their own compass, straightedge, and a notebook they use to keep their notes. Each station will need its own individual materials but those will be provided at the station (see below for more details on each station). Place a start and end time on the board so the students are able to keep track of their remaining time.
- **Station 1** (Guided practice with the teacher): In this station, you will need patty paper, chart paper (this is like over-sized sticky-note paper, you can also use poster boards or any type of large paper approximately 30" x 25"), and glue or tape. Keep these items in a box or container and take them to the station when it is time for the Guided Practice. Before starting the station, place one glue/tape for every pair of desks and a sheet of patty paper on each desk. The same steps that were done in the Teaching Phase will be completed again. This time the students in this station will follow along with the teacher. The teacher will model the construction on chart paper while the students do it on patty paper and glue/tape it in the note's parts of their notebook for future reference. If patty paper is not accessible, the students can complete the construction in the notebook. By using the patty paper, the students are able to fold along the ray that bisects the angle and verify their construction. Instruct them to also measure the angles using a protractor in order to verify their construction. Then refer back to the guiding question.
- **Station 2** (In groups of two, without teacher): The students will work in groups of two. Near Station 2 place the copies of the "Guided Practice Station 2" worksheet (See "Guided Practice Station 2 – Sample" for a guide on completing the worksheet). Instruct the students to pick up a worksheet on their way to Station 2. Designate one student to help run the PowerPoint slide show. You can present the slide show so that it changes automatically (See Further Recommendations for instructions to presenting the slide show). Even if you have the slide show on automatic, still designate a student to help with the PowerPoint in case the video needs to be replayed or a slide needs to be viewed a second time. Students will watch the "Constructing Angle Bisector" PowerPoint and complete the "Guided Practice Station 2" worksheet. As they watch the PowerPoint, they will copy each step in the "Image" column, write what they did in the "My Description" column, and write the actual step in the "Steps" column. The actual step will be written out in the PowerPoint so they can check their work. They are allowed to work with their partner to discuss the steps and construction, but each student should have their own worksheet. The worksheet should be inserted in the notes part of their notebook for future reference.
- After ten minutes has elapsed, the students in Station 1 will go to Station 2 and the students in Station 2 will go to Station 1.

Source-: https://www.cpalms.org/Public/PreviewResourceLesson/Preview/130859

12. AREAS RELATED TO CIRCLES

QR Code:

V5M5U7

Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Perimeter and Area of a Circle — A Review	Describe the relationship between circumference and diameter of a circle in order to define	-
	Apply the concepts of circumference and area of in order to solve in for various circular objects in real life	
Areas of Sector and Segment of a Circle	Describe sector and segment of a circle in order to differentiate between the two	
	Describe minor and major sector of a circle in order to differentiate between the two	
	Describe minor and major segment of a circle in order to differentiate between the two	
	Apply the formula of area of sector and segment of a circle in order to compute the area of a specified region	
	Calculate the length of an arc of a circle in order to comment whether it is the major arc or minor arc	
Areas of Combinations of Plane Figures	Calculate the area of various combinations of plane figures in order to apply the concepts of circles, quadrilaterals and triangles	

Test items

LOB: Describe the relationship between circumference and diameter of a circle in order to define π

1. Which of these is equivalent to π?

Option 1: $\frac{\text{Circumference}}{\text{radius}}$

Option 2: $\text{Circumference} \times \text{diameter}$

Option 3: $\frac{\text{Circumference}}{\text{diameter}}$

Option 4: $\text{Circumference} \times \text{radius}$

Correct Answer: Option 3

2. Raman draws a circle with diameter 6 units. He draws another circle by increasing the radius of the previously drawn circle by 4 units. What would be the quotient if he divides the circumference of the newly formed circle by its diameter?

Option 1: π

Option 2: 2π

Option 3: 8

Option 4: 12

Correct Answer: Option 1

LOB: Apply the concepts of circumference and area of in order to solve in for various circular objects in real life

1. A circular garden, of circumference 88 m is surrounded by a pathway of width 3.5 m. Ajay wants to put fence around the pathway. What is the cost of fencing the pathway at the rate of ₹70 per metre? (Use $\pi = \frac{22}{7}$)

Option 1: ₹3,080

Option 2: ₹3,850

Option 3: ₹6,160

Option 4: ₹7,700

Correct Answer: Option 4

2. A fountain is enclosed by a circular fence of circumference 11 m and is surrounded by a circular path. The circumference of the outer boundary of the path is 16 m. A gardener increased the width of the pathway by decreasing the area enclosed by the fence such that the length of the fence is decreased by 3 m. The path is to be covered by the bricks which cost ₹125 per m^2. What will be the total cost, to the nearest whole number, required to cover the area by the bricks? (Use $\frac{22}{7}$ for π)

Option 1: ₹79,545

Option 2: ₹39,772

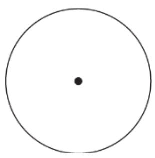
Option 3: ₹9878

Option 4: ₹1,910

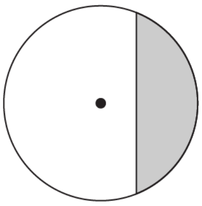
Correct Answer: Option 4

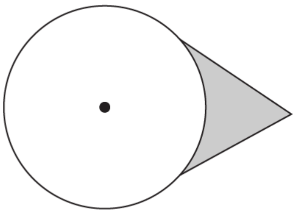
LOB: Describe sector and segment of a circle in order to differentiate between the two

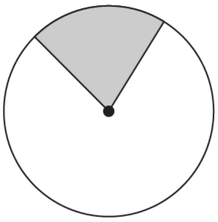
1. Consider a circle below.

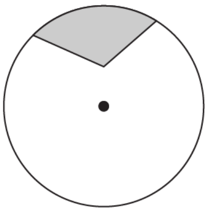


Aman shades a part in the circle which is enclosed by two radii and its corresponding arc. Which of these could he have drawn?

Option 1: 

Option 2: 

Option 3: 

Option 4: 

Correct Answer: Option 3

2. Consider the statements below.

Statement A: A quarter circle represents a sector of the circle.

Statement B: A semicircle represents both sector and segment of the circle.

Which of these statements is correct?

Option 1: Only Statement A

Option 2: Only Statement B

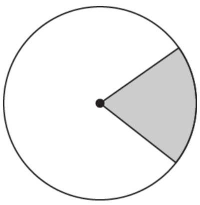
Option 3: Both the statements A and B

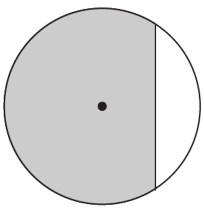
Option 4: Neither statement A nor B

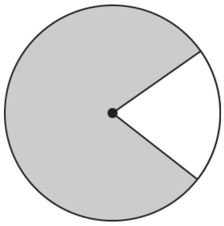
Correct Answer: Option 3

LOB: Describe minor and major sector of a circle in order to differentiate between the two

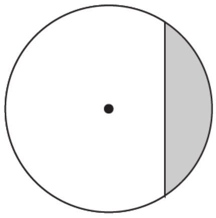
1. Which of the following options represents the shaded region as the major sector and unshaded region as the minor sector?

Option 1: 

Option 2: 



Option 3:



Option 4:

Correct Answer: Option 3

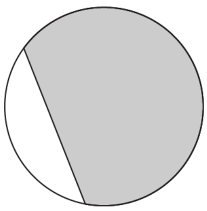
2. To form a circle of radius *r*, four minor sectors of equal measure are joined.
Which of these options completes the sentence below?
The sum of the area of the four minor sectors is equal to the _.

Option 1: circumference of the circle of radius *r*.
Option 2: area of the semicircle of diameter 2*r*.
Option 3: area of the circle of diameter 2*r*.
Option 4: circumference of the circle of diameter *r*.

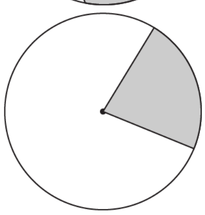
Correct Answer: Option 3

LOB: Describe minor and major segment of a circle in order to differentiate between the two

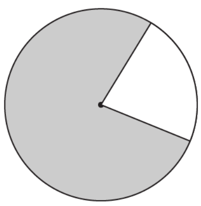
1. To show the minor segment of a circle, a student shades the region enclosed between a chord and the minor arc.
Which of these shows the region the student could have shaded?



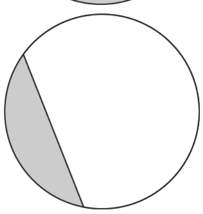
Option 1:



Option 2:



Option 3:



Option 4:

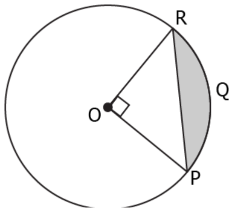
Correct Answer: Option 4

2. Which of these is equivalent to the sum of the lengths of arc corresponding to the minor and major segment of a circle of radius 12 cm?

Option 1: 12π cm
Option 2: 24π cm
Option 3: 36π cm
Option 4: 144π cm
Correct Answer: Option 2

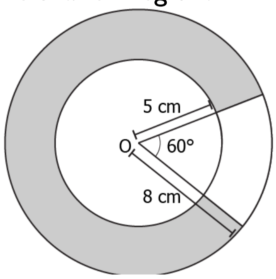
LOB: Apply the formula of area of sector and segment of a circle in order to compute the area of a specified region

1. Observe the figure below:



What is the area of the segment PQR, if the radius of the circle is 7 cm? (Use $\pi = \frac{22}{7}$)
Option 1: 11 cm^2
Option 2: 14 cm^2
Option 3: 17.3 cm^2
Option 4: 91 cm^2
Correct Answer: Option 2

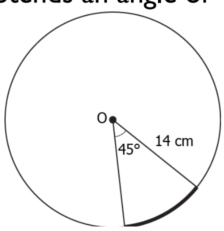
2. Two concentric circles of radius 8 cm and 5 cm are shown below, and a sector forms an angle of 60° at the centre O. What is the area of the shaded region?



Option 1: 38π cm^2
Option 2: $\frac{77\pi}{2}$ cm^2
Option 3: $\frac{11\pi}{2}$ cm^2
Option 4: $\frac{195\pi}{6}$ cm^2
Correct Answer: Option 4

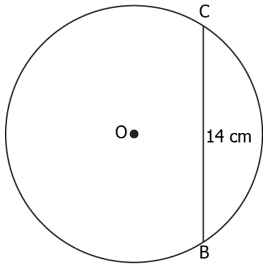
LOB: Calculate the length of an arc of a circle in order to comment whether it is the major arc or minor arc

1. An arc of a circle of radius 14 cm, subtends an angle of 45° at the centre as shown:



Which of these options is correct?
Option 1: The arc shown is a major arc and its length is 38.5 cm.
Option 2: The arc shown is a minor arc and its length is 11 cm.
Option 3: The arc shown is a minor arc and its length is 5.5 cm.
Option 4: The arc shown is a major arc and its length is 77 cm.
Correct Answer: Option 2

2. A circle with centre O of diameter 28 cm and a chord BC of length 14 cm is shown below:



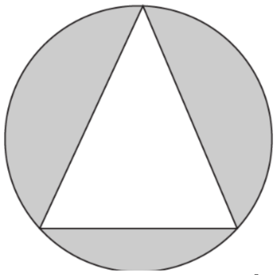
What is the length of the major arc of the circle, to the nearest tenth?

Option 1: 7.3 cm
Option 2: 14.7 cm
Option 3: 73.3 cm
Option 4: 146.7 cm

Correct Answer: Option 3

LOB: Calculate the area of various combinations of plane figures in order to apply the concepts of circles, quadrilaterals and triangles

1. In equilateral triangle of side 28 cm is inscribed in a circle of diameter 32 cm, as shown below:

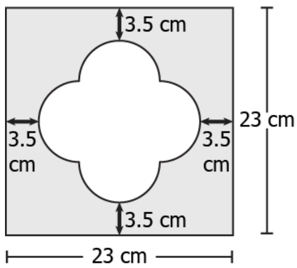


What is the area of the shaded region? (Use $\pi = 3.14$ and $\sqrt{3}$=1.73)

Option 1: 125.68 cm^2
Option 2: 411.84 cm^2
Option 3: 464.76 cm^2
Option 4: 2876.28 cm^2

Correct Answer: Option 3

2. Ankita shaded a square cardboard, as shown below:



Which of these is closest to the area of the shaded region?

Option 1: 263.9 cm^2
Option 2: 364.4 cm^2
Option 3: 439.9 cm^2
Option 4: 492.4 cm^2

Correct Answer: Option 2

Suggested Teacher Resources

1 Lesson Plan

Objective	To construct a tangent to a circle at a point on it when the centre of the circle is known and calculate the area of the figures thus formed.
Material Required	Butter paper, coloured sheet of paper, compass, scale, scissors
Prerequisite Knowledge	To draw a line perpendicular to a given line at a point on it
Procedure	The teacher starts by discussing the various aspects of a circle namely sector, segment, arc, radius, diameter, circumference etc. Then, the teacher asks the students to follow the following steps for the activity; ● Draw a circle of a given radius with centre O on a coloured sheet and paste it on the butter paper. ● Let A be the given point on the circle, Join OA and extend it to B which will lie outside the circle giving you a segment OAB. ● Fold the Paper along OB and press the two parts to form a crease along the line OB. Now fold the paper in such a way that BA falls along OA and it passes through A. press the two parts to form a crease. Unfold the paper and mark the crease by a line T'AT. ● The line segment T'AT is the required Tangent on the point A. Additional: ● Then the teacher asks the students to join the points T' and T to O to get two triangles. ● And asks students what can they say about these two triangles? ● Are these triangles congruent? ● By which congruence rule? ● Next, the teacher would ask students to calculate the area of the circle and the two triangles.
Source	https://www.youtube.com/watch?v=86g36f5cBoo

2 Activity

Material Required: Circular cardboard cut-outs or paper plates that represent the whole pizza

- Sector cardboard cut-outs or paper plates representing the slices of the pizza. (Use scissors to cut slices (sectors) out of the plates)
- Rulers for measuring the radius
- Protractors for measuring the central angles

Students should be placed in groups of 3-5 students according to the teacher's preference.
Each group should receive two whole pizzas, two slice cut-outs, two rulers, and two protractors. The teacher will cut these circles to specific sizes and create a quick reference sheet with measurements and answers.

The teacher will:

- Model how to compare the slices to the pizza.
- Ask students how many of their slices would make a whole pizza.
- Allow the students to discuss their findings.
- Ask, "If there are *x* slices in your pizza, what fraction does your slice represent?"

- Explain the definition of sectors and display the definition on the interactive white board for students to take notes.

The teacher continues probing by asking the following questions:

- "How many degrees are in an entire circle?"
- "How many degrees does your slice represent?"

If students are having difficulty, the teacher can pose guiding questions:

- How many total slices are in your pizza?
- What fraction does one slice represent? (1 out of 6)
- If the total number of degrees in a circle is 360, what is 1/6 of 360?
- So, how many degrees is 1/6 of 360?

The teacher explains that the slice of the pizza represents a fraction of the total pizza, or a sector of the circle.

The teacher asks:

- "How do we find the area of a circle?"
- "How would we find the area of one half of a circle?"
- "How many degrees are there in one half of a circle?"
- "How did you come to this conclusion?" (wait for response)
- "How would we find the area of one fourth of a circle?"
- "How many degrees are there in one fourth of a circle?"
- "How did you come to this conclusion?" (wait for response)
- "How would you find the area of a sector with a central angle of 350 degrees?"
- "Who remembers what the central angle is?"
- "What are some properties of the central angle?"
- "How can we use the central angle to calculate the area of a fraction of a circle?"
- Now that we have the fraction of the circle we need; how can we use it to find the area of the sector? (Allow students to discuss their findings)

After the class comes to a consensus, the teacher would go to the board to display the formula for the area of a sector.

Students then:

- measure and record the length of the radius of the pizza in their notebooks
- measure and record the measure of the angle of the sector of the pizza in their notebooks
- using the radius that they just found on their pizzas, find the area of a pizza slice
- compare their answers within their groups

The teacher would:

- circulate and observe student work

13. SURFACE AREAS AND VOLUMES

QR Code:

T2U2Z4

Learning outcome and Learning Objectives:

<table>
<tr><th>Content area / Concepts</th><th>Learning Objectives</th><th>Learning Outcome</th></tr>
<tr><td>Surface Area of a Combination of Solids</td><td>Apply formulas of surface area of 3D solids in order to derive the area of a new solid</td><td rowspan="6">Visualizes objects in surrounding as a combination of different solids like cylinder and a cone, cylinder and a hemisphere, combination of different cubes etc.in order to find their surface areas and volumes</td></tr>
<tr><td>Volume of a Combination of Solids</td><td>Apply formulas of volume of 3D solids in order to derive the volume of a new solid</td></tr>
<tr><td>Conversion of Solid from One Shape to Another</td><td>Apply formulas of volumes of 3d solids in order to derive the volume of the new converted solid</td></tr>
<tr><td rowspan="2">Frustum of a Cone</td><td>Apply the formula of surface area of a cone in order to derive the area of the frustrum</td></tr>
<tr><td>Apply the formula of volume of a cone in order to derive the volume of the frustrum</td></tr>
<tr><td>Applications of surface areas and volumes</td><td>Use concepts of surface areas and volumes for a variety of 3-D objects in order to apply into real life situations</td></tr>
</table>

Test items

LOB: Apply formulas of surface area of 3d solids in order to derive the area of a new solid

1. A medicine-capsule is in the shape of a cylinder of radius 0.25 cm with two hemispheres stuck to each of its ends. The length of the entire capsule is 2 cm. What is the total surface area of the capsule? (Take π as 3.14)
   Option 1: 3.14 cm^2
   Option 2: 2.7475 cm^2
   Option 3: 0.98125 cm^2
   Option 4: 0.785 cm^2

Correct Answer: Option 1

2. Arman wants to polish the object, which is composed of a cylinder surmounted by a hemisphere. If the whole length of the solid is 5 m and the diameter of the hemisphere is 7 m, what is the cost of polishing the surface area of the solid at the rate of 50 paise per sq. m.? (Use π as $\frac{22}{7}$)
   Option 1: `1100
   Option 2: `550
   Option 3: `110
   Option 4: `55

Correct Answer: Option 4

LOB: Apply formulas of volume of 3d solids in order to derive the volume of a new solid

1. Two identical solid cubes of side k units are joined end to end. What is the volume, in cubic units, of the resulting cuboid?
   Option 1: k^3
   Option 2: $2k^3$
   Option 3: $3k^3$
   Option 4: $6k^3$

Correct Answer: Option 2

2. A vendor sells glasses of juice, which is in the form of a cylinder mounted by a hollow hemisphere. The diameter of the hemisphere and the cylinder is 8.4 cm. If the total height of the glass is 15 cm, which of these is closest to the volume, in cubic centimeters, of 8 such glasses? (Use π as $\frac{22}{7}$)
   Option 1: 599 cm^3
   Option 2: 754 cm^3
   Option 3: 6032 cm^3
   Option 4: 29083 cm^3

Correct Answer: Option 3

LOB: Apply formulas of volumes of 3d solids in order to derive the volume of the new converted solid

1. Four solids spheres of the same size are made by melting a solid metallic cylinder of base diameter 4 cm and height 36 cm. What is the diameter of each sphere?
   Option 1: 3 cm
   Option 2: 6 cm
   Option 3: 7 cm
   Option 4: 14 cm

Correct Answer: Option 2

2. A cube of side length 66 cm is filled with spherical metallic balls of diameter 0.6 cm and it is assumed that $\frac{5}{8}$ space of the cube remains unfilled. What are number of balls that can be filled in the cube? (Use π as $\frac{22}{7}$)

Option 1: 952875
Option 2: 959777
Option 3: 1165812
Option 4: 2264031
Correct Answer: Option 1

LOB: Apply the formula of surface area of a cone in order to derive the area of the frustum

1. A circle of radius 5 units is cut into two pieces of 30% and 70% of the total area respectively. The smaller section is curled until the two straight edges meet, and the bottom is made for the cone that is formed by the section. What is the surface area of the cone?
   Option 1: 25π
   Option 2: $\frac{39}{2}\pi$
   Option 3: $\frac{39}{4}\pi$
   Option 4: 3π
Correct Answer: Option 3

2. A solid cone is divided into a frustum and a smaller cone. The radii of the ends of a frustum are 14 cm and 56 cm, and the height is 30 cm. Which of these is closest to the total surface area of the frustum? (Use π as $\frac{22}{7}$)
   Option 1: 13173 cm^2
   Option 2: 21009 cm^2
   Option 3: 21827 cm^2
   Option 4: 42656 cm^2
Correct Answer: Option 3

LOB: Apply the formula of volume of a cone in order to derive the area of the frustum

1. A student is drinking water in a glass which is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. What is the capacity of the glass? (Use π as $\frac{22}{7}$)
   Option 1: $\frac{217}{3}$ cm^3
   Option 2: $\frac{216}{7}$ cm^3
   Option 3: $\frac{318}{3}$ cm^3
   Option 4: $\frac{308}{3}$ cm^3
Correct Answer: Option 4

2. From the top of a solid cone of height 12 cm and base 6 cm of a cone of height 4 cm is removed by a plane parallel to the base. Which of these is closest to the volume of the frustum of the cone? (Use π as $\frac{22}{7}$)
   Option 1: 235 cm^3
   Option 2: 364 cm^3
   Option 3: 436 cm^3
   Option 4: 564 cm^3
Correct Answer: Option 3

LOB: Use concepts of surface areas and volumes for a variety of 3-D objects in order to apply into real life situations

1. Abhinav made a model for his school project in the shape of a cylinder of radius 7 cm and height 21 cm, with hemisphere surmounted on one end. He wants to cover the entire model with decorative paper. What is the area, in centimeter square, of paper that is required to cover the model? (Use π as $\frac{22}{7}$)
   Option 1: 1386 cm^2
   Option 2: 1232 cm^2
   Option 3: 1012 cm^2

Option 4: 976 cm^2

Correct Answer: Option 2

2. Asif is melting three solid metal spheres of radii 3 cm, 4 cm, and 5 cm respectively to form a new sphere. How much more is the surface area of the new sphere as compared to the combined surface area of the three spheres?

    Option 1: 4π

    Option 2: 16π

    Option 3: 36π

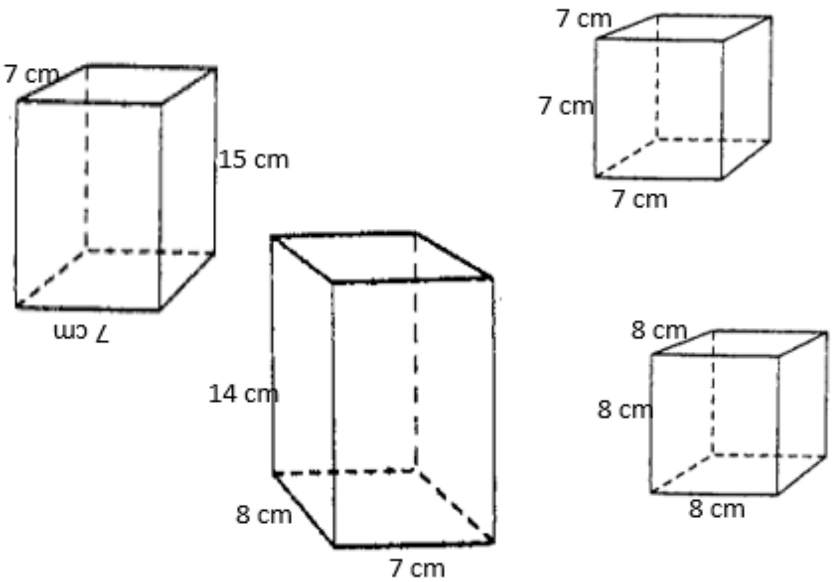
    Option 4: 64π

Correct Answer: Option 4

Suggested Teacher Resources

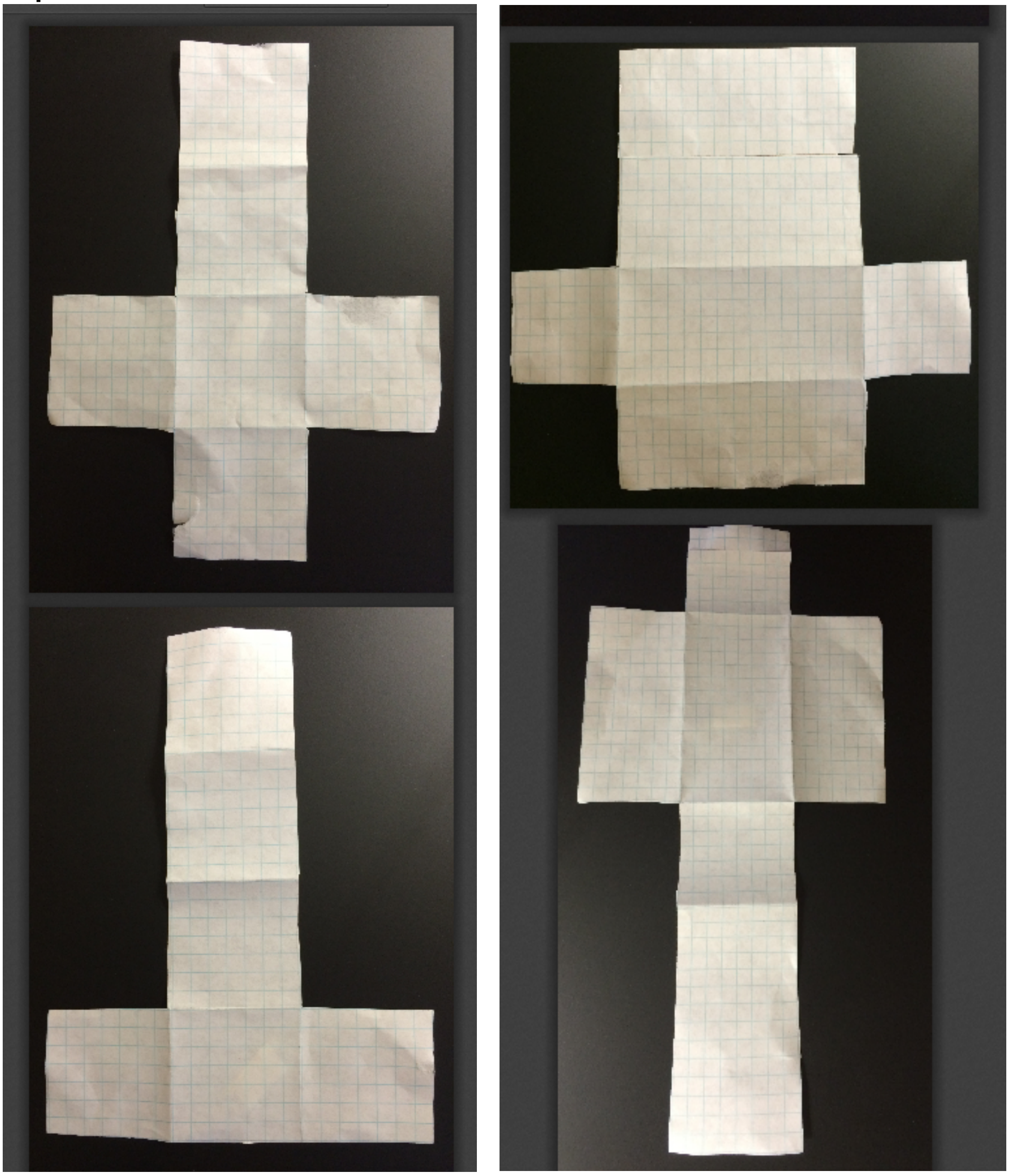
I

Lesson Plan

Objectives	Apply formulas of volume of 3D solids in order to derive the volume of a new solid
Prerequisite Knowledge	How to calculate volume of a given solid or a combination of solid
Material Required	Paper-Sheets/graph paper, Scissor, Scale, glue, A4 size blank sheets, Pack of biscuits.
Procedure	Activity: Lunchbox **Mr. Anurag is purchasing a new lunch box but wants to make sure he buys one large enough to hold his favourite menu. He loves to have one juice box, one box of biscuits, one box of fruits and one box of raisins. He decided that if he found the volume of each of these favourite menu items, then compared it with the volume of different lunch boxes, he would know which one to buy.** What would be the volume of the smallest lunchbox that he should buy? Show your work and then justify your answer. By adding together, the volume of all the boxes we know the total volume. The lunch box would have to hold at least this amount. What other ideas should Mr. Weaver consider before buying his new lunch box? He needs to think whether the dimensions of the lunchbox would allow the items to fit. If the lunch box is only 5 cm high, it could not hold any of the items. 

Setup: Students will solve the worksheet in group of 5 and present their answer in front of class once everyone agrees upon the answer.

Sample Prism:



1. Ask students to make rectangular prisms (if you feel your students need them) to the students, poster paper, and present the problem sheet. Explain to students that these are just models and ask them what models are useful for. (responses/So we don't use the real food items/so we don't make a mess/to allow us to manipulate the shapes) (suggested measurement using graph paper with ¼ inch grid squares (7x7x7; 7x7x15; 7x8x14; 8x8x8 = 2374 units cubed) sample rectangular prisms

2. Allow students to work in partners to solve the problem. Give time to struggle with the problem. (20-25 min)
3. Circulate around the room to listen to conversations checking for understanding (**responses**: We can combine volumes, we need to count the units on each shape, we can find the area of the objects and add them together, place the shapes together). If frustration is evident begin with probing questions.
4. Continue to circulate the room. At this point you may see three types of students, those who understand and have completed the Group Problem, those who are using the manipulative to solve the problem and are doing well, and those students who are still using the manipulative and are still frustrated.

5. For those students who have completed the task hand, you might give them a different problem.

Problem: How many boxes?

Any box with a volume of 36 cubic cm will ship for only Rs 10.00. Salma is very excited about this low price because she is shipping birthday presents to her cousins. She started thinking about all the different size boxes she could use. Help her out by making an organized list of the dimensions of the boxes that would have a volume of 36 cubic cm. Please write equations for each box to show that your answers are correct.

Answer:
6 x 6 x 1 = 36 cubic inches
3 x 6 x 2 = 36 cubic inches
3 x 3 x 4 = 36 cubic inches
2 x 3 x 6 = 36 cubic inches
2 x 2 x 4 = 36 cubic inches
2 x 1 x 18 = 36 cubic inches

6. For those who are still working allow them time to work.

7. Once everyone has finished allow time for sharing (Several models can be used however; like a **gallery walk** for this instance since students wrote and explained their solutions.)

Gallery Walk-have students pretend they are in an art gallery. Their hands are behind their back or in their pockets. Students will then circulate around the room and quietly comment on what they see in other solutions. Once complete the students must return to their area and write down one different solution that they saw for solving the problem in their math copies.)

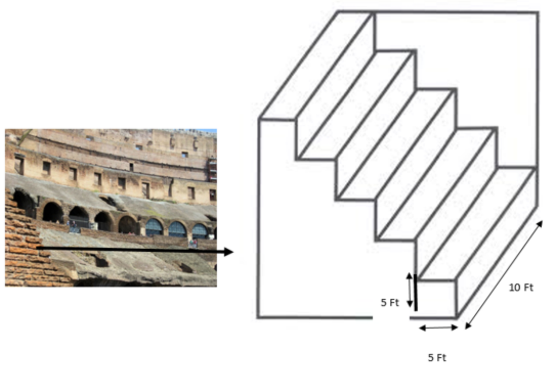
Closure: Once each group has shared display the posters around the room.

Ask the question:

What can be done when you have multiple right rectangular prisms when trying to find the volume of all?

Responses may include: (After finding the volume of each prism add them together/You can combine the prisms together to find the total volume of the prisms)

Q) While Saket was walking up the stairs to the Roman Coliseum he wondered what the total volume of the space underneath the stairs was. Each stair is the same width and length. The height of each step is the same. He sketched out the staircase and measured the first stair to get one right rectangular prism. Saket also sent a picture of the staircase to use as reference. The drawing below represents Bob's sketch. Determine the total volume of the space under the staircase for the Coliseum.



Answer: The volume underneath the first step 5x5x10 = 250 cubic feet
Since the step height between steps is the same, there are 15 groups of 250 cubic feet.
15 x 250 = 3,750 cubic feet

Source	https://www.cpalms.org/Public/PreviewResourceLesson/Preview/73388

2 Activity

Pack of Biscuits Challenge

Objective – Design a pack of biscuits using paper sheets and find its surface area and volume.

Materials Required - Paper-Sheets, Scissor, Scale, glue, A4 size blank sheets.

Introduction – You work as a Design Engineer at a biscuit factory and your company have just created a new biscuit for which you need to design a box that is efficient and attractive!

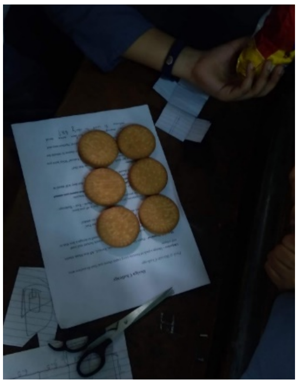
There would we two winners – The most efficient one and the most attractive one!

Give them examples of some common arrangements of biscuits and some shapes in which the design is possible and ask them to be as creative as possible!

Constraints –
1) Pack of biscuits should be in 'One Part'.
2) No gaps in it for the air intake!

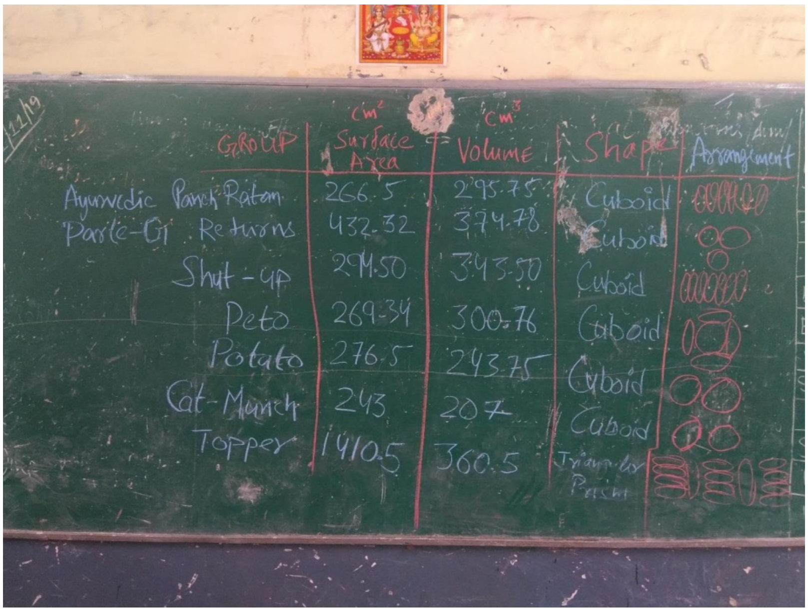
Instructions-
1. Use the materials provided judiciously, that is all you have.
2. Follow the process- Brainstorm – Design – Build – Test – Redesign – Share.
3. Design a pack of biscuits capable of holding 12 biscuits.
4. Design your pack of biscuits keeping in mind that they will travel in trucks and should not break in transportation.
5. Calculate the surface area and volume of your design.

Student work examples–



Expected Outcomes –

Display the result in this manner to announce the winner and to ask the reflective questions –



Reflect and Share-

1. Did you get stuck at any point? What was that like?
2. What did you change along the way?
3. What was the hardest part of the challenge? Easiest?
4. What would you do differently if you had the chance to rebuild the design?
5. What has changed in your understanding about 'Surface area and Volume'? What have you learned?
6. Now, can you give some examples where surface area and volume come into action in daily life?

Targeted Learnings –

1. Surface area and volume are dependent on each other.
2. Surface area and volume are also dependent on the arrangement of the things inside the shape.
3. E.g. Potato Chips and French Fries, Nail, Water cleaning by cloth

Give post-work to find other areas where surface area and volume comes into action.

14. STATISTICS

QR Code:

U1A9P6

Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Mean of Grouped Data	Apply direct method in order to calculate the mean of the grouped data	Calculates mean, median and mode in order to apply them to real life contexts
	Apply assumed mean method in order to calculate the mean for a grouped data	
Mode of Grouped Data	Compute the mean and mode of the given data in order to interpret the two measures of central tendency	
Median of Grouped Data	Apply formula for the median of a given grouped data in order to calculate missing values of Frequency	
Mean, median and mode	Differentiate between mean, median and mode with examples in order to understand most effective measure of central tendency in various cases	
Graphical Representation of Cumulative Frequency Distribution	Derive the co-ordinates to plot on the graph in order to represent the two ogives	
	Graph both ogives for the data obtained in order to determine the median of the given grouped data	

Test items

LOB: Apply direct method in order to calculate the mean of the grouped data

1. The table below shows the time taken by a group of students to complete 100 m dash.

Time taken (in sec)	Number of students
18 – 20	3
20 – 22	18
22 – 24	26
24 – 26	19
26 – 28	9
28 – 30	5

Which of these is the mean time taken, in sec, by the group of students to complete the 100 m dash when calculated using direct method?

Option 1: 18.16
Option 2: 18.96
Option 3: 22.7
Option 4: 23.7

Correct Answer: Option 4

2. When calculated using direct method, the mean of the data set shown in the table below is 31.

Class Interval	Frequency
0 – 10	22
10 – 20	24
20 – 30	35
30 – 40	30
40 – 50	27
50 – 60	m
60 – 70	6
70 – 80	4

What is the frequency for the class interval 50-60?

Option 1: 10
Option 2: 12
Option 3: 20
Option 4: 24

Correct Answer: Option 2

LOB: Apply assumed mean method in order to calculate the mean for a grouped data

1. The table below summarizes the data about the heights of students in a class.

Height (in cm)	Number of students
130-140	6
140-150	14
150-160	20
160-170	12
170-180	8

When calculated using assumed mean method what is the mean height of students in the class?

Option 1: 154.33 cm
Option 2: 154.67 cm
Option 3: 155.33 cm
Option 4: 155.67 cm

Correct Answer: Option 3

2. Amith grows cucumbers in his farm. He collects some of them and measures their lengths and represents his data as shown below.

Length (in mm)	Number of cucumbers
110 – 120	10
120 – 130	18
130 – 140	12
140 – 150	n
150 – 160	26
160 – 170	11
170 – 180	19

When calculated using assumed mean method, Amith gets the mean length of the cucumber as 147.25 cm. Which of the following statement is true?

Option 1: There are a smaller number of cucumbers of length (140 – 150) mm than of length (120 – 130) mm.

Option 2: There are a smaller number of cucumbers of length (140 – 150) mm than of length (170 – 180) mm.

Option 3: There are a greater number of cucumbers of length (140 – 150) mm than of length (150 – 160) mm.

Option 4: There are a greater number of cucumbers of length (140 – 150) mm than of length (130 – 140) mm.

Correct Answer: Option 4

LOB: Compute the mean and mode of the given data in order to interpret the two measures of central tendency

1. A data set is shown.

Class interval	Frequency
4 – 6	2
6 – 8	6
8 – 10	20
10 – 12	28
12 – 14	12
14 – 16	10
16 – 18	2

What is the mean and mode of the data shown?

Option 1: Mean: 10; Mode: 10.33

Option 2: Mean: 10; Mode: 10.67

Option 3: Mean: 11; Mode: 10.33

Option 4: Mean: 11; Mode: 10.67

Correct Answer: Option 4

2. The table below shows the time taken by a group of 40 students to solve a work sheet.

Number of minutes	Number of students
20 – 24	1
24 – 28	2
28 – 32	4
32 – 36	k
36 – 40	10
40 – 44	8
44 – 48	3

Which of the following statements is true?

Option 1: The mean time taken by the students is more than the sum of time taken by all the students to solve the worksheet.

Option 2: The time taken by the maximum number of students is less than the mean time taken by the students to solve the worksheet.

Option 3: The time taken by the maximum number of students is more than the mean time taken by the students to solve the worksheet.

Option 4: The time taken by the maximum number of students is more than the sum of time taken by all the students to solve the worksheet.

Correct Answer: Option 2

LOB: Apply formula for the median of a given grouped data in order to calculate missing values of Frequency

1. A grouped data is shown below

Class Interval	Frequency	Cumulative frequency
0 – 10	4	4
10 – 20	a	$4 + a$
20 – 30	4	$8 + a$
30 – 40	b	$8 + a + b$
40 – 50	5	$13 + a + b$

If the median of the grouped data is 22.50 and the total frequency is 20, then what is the value of a?

Option 1: 1
Option 2: 2
Option 3: 4
Option 4: 5

Correct Answer: Option 4

2. The table below shows the time spent by 100 students on exercising every day.

Time (in minutes)	Number of students
0 – 10	x
10 – 20	24
20 – 30	32
a30 – 40	21
40 – 50	y
50 – 60	5

If the median time spent by the students on exercising is 26.25 minutes which statement correctly compares the frequencies of the class intervals 0-10 and 40-50?

Option 1: There are twice as many students who exercise 0- 10 minutes each day as the number of students who exercise 40-50 minutes each day.
Option 2: There are twice as many students who exercise 40- 50 minutes each day as the number of students who exercise 0-10 minutes each day.
Option 3: There are thrice as many students who exercise 0- 10 minutes each day as the number of students who exercise 40-50 minutes each day.
Option 4: There are thrice as many students who exercise 40- 50 minutes each day as the number of students who exercise 0-10 minutes each day.

Correct Answer: Option 2

LOB: Differentiate between mean, median and mode with examples in order to understand most effective measure of central tendency in various cases

1. A grouped data is shown below:

Class Interval	Frequency
0 – 15	2
15 – 30	26
30 – 45	32
45 – 60	42
60 – 75	28
75 – 90	30

Which of the following is the most effective measure of central tendency?

Option 1: Mean because the data has extreme data points.
Option 2: Median because the data has extreme data points.
Option 3: Mean because the data has no extreme data points.
Option 4: Median because the data has no extreme data points.

Correct Answer: Option 2

2. The table below shows the age of people attending a musical concert.

Age (in years)	Number of people
0 – 10	2
10 – 20	19
20 – 30	32
30 – 40	23
40 – 50	19

If five more people of age 21 yrs., 32 yrs., 35 yrs., 44 yrs., and 11 yrs. attend the concert, which statement describes the central tendency of the new data?

Option 1: The central tendency of the data increases by 0.1 as the mean increases by 0.1.
Option 2: The central tendency of the data increases by 0.1 as the median increases by 0.1.
Option 3: The central tendency of the data increases by 0.2 as the mean increases by 0.2.
Option 4: The central tendency of the data increases by 0.2 as the median increases by 0.2.

Correct Answer: Option 4

LOB: Derive the co-ordinates to plot on the graph in order to represent the two ogives

1. The table below shows the heights of plants in Pranay's backyard.

Height (in cm)	**Number of Plants**
0 – 50	10
50 – 100	25
100 – 150	30
150 – 200	28
200 – 250	12
250 – 300	14

Which of the following are the coordinates of less than ogive curve?

Option 1: $(0, 10), (50, 35), (100, 65), (150, 93), (200, 105), (250, 119)$
Option 2: $(10, 0), (35, 50), (65, 100), (93, 150), (105, 200), (119, 250)$
Option 3: $(10, 50), (35, 100), (65, 150), (93, 200), (105, 250), (119, 300)$
Option 4: $(50, 10), (100, 35), (150, 65), (200, 93), (250, 105), (300, 119)$

Correct Answer: Option 4

2. The table below shows the age group of 50 women members of a club.

Age (in years)	**Number of Women**
20 – 25	2
25 – 30	y
30 – 35	16
35 – 40	z
40 – 45	7
45 – 50	4

Given that $y - z = 1$, Aruna and Anjali wrote the coordinates of less than ogive curve and more than ogive curve respectively as shown below.

Aruna:$(20, 2), (25, 13), (30, 29), (35, 39), (40, 46), (45, 50)$
Anjali:$(20, 50), (25, 48), (30, 38), (35, 22), (40, 11), (45, 4)$

Who is/are correct?

Option 1: Only Aruna
Option 2: Only Anjali
Option 3: Both of them
Option 4: Neither of them

Correct Answer: Option 4

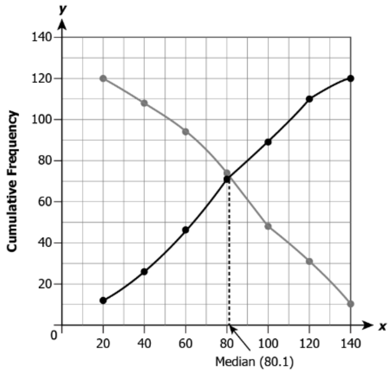
LOB: Graph both ogives for the data obtained in order to determine the median of the given grouped data for

1. A grouped data is shown below.

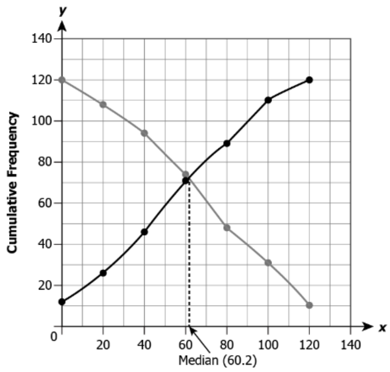
Class Interval	Frequency
0 – 20	12
20 – 40	14
40 – 60	20
60 – 80	25
80 – 100	18
100 – 120	21
120 – 140	10

Which graph shows the median of the data?

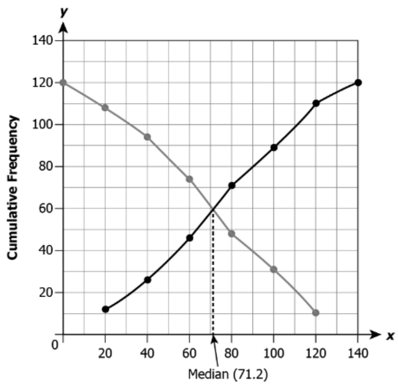
Option 1:



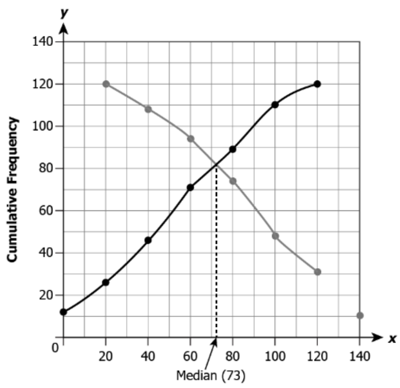
Option 2:



Option 3:



Option 4:



Correct Answer: Option 3

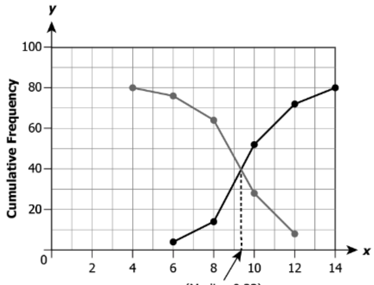
2. The table below shows the number of hours a group of 80 students sleep every day.

Number of hours	Number of Students
4 – 6	4
6 – 8	a

8 – 10	36
10 – 12	b
12 – 14	8

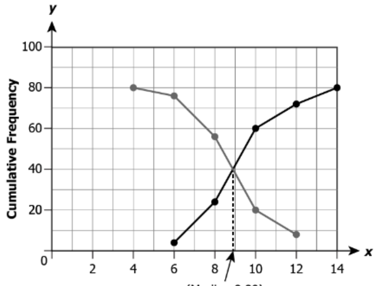
Given that $b - a = 8$, which of the following represents the median number of hours the group of students sleep?

Option 1:



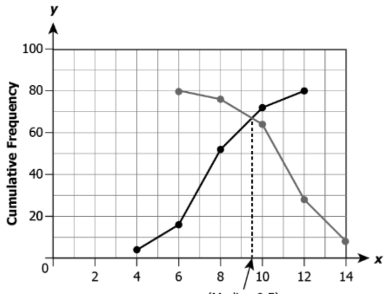
(Median 9.33)

Option 2:



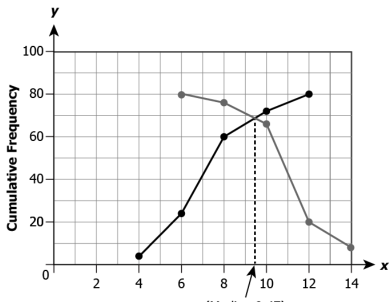
(Median 8.89)

Option 3:



(Median 9.5)

Option 4:



(Median 9.47)

Correct Answer: Option 1

Suggested Teacher Resources

1

Lesson Plan

Objectives	Students will be able to- 1. construct a histogram from data with 70% accuracy. 2. calculate mean, median, mode, and range from data with 70% accuracy. 3. construct a circle graph from the data in a histogram with 70% accuracy.
Prerequisite Knowledge	• Forming grouped frequency distribution table of the given data. • Finding mean, mode, and median of the given data.
Material Required	1.Empty Water Bottles 2.Cereal Boxes to build crafts 3. Tape 4.Scissors 5. Yarn 6.Copy of design worksheet for each student 7.Copy of notes for direct instruction 8.Copy of directions and rubric for each student. 9. Fishing line for zip line 10.Dolls for crafts 11.Paper Towel Rolls
Procedure	• Start the class with engineering design challenge where they will work in groups to complete the challenge. • Students will work in groups of four to create a craft to send down a zipline in the library (or anywhere there is room to use the zipline). There will be three ziplines already set up and ready to go for the students to use. A craft will be considered successful if it can travel the entire zipline without losing the doll sitting in the craft. • Groups can only use the materials provided to their group. Each group will get the same materials but will need to use them differently. Each group will get one fishing line, one doll, one paper towel roll, two empty water bottles, one cereal box, tape, scissors, a yard of yarn, and copies of design worksheets, notes, directions, and rubrics to follow along. • Students must complete the worksheet as they work. This will help facilitate them going through the engineering design process. • Students will have one hour to work on their prototype and build a craft. They can test their craft on each zipline as many times as needed so that they can have a successful run on the zipline with a fastest time. The group with the fastest time will be the "winners" • After one hour, each group will present their craft to the class and send it down the zipline for their official run. Since they practiced before, one last run is their final run to show the class. One student will be a timekeeper and one student will record the times for all students in a google spreadsheet. • Using the data from the zipline challenge, complete the notes for direct instruction calculating measures of central tendency and creating histograms and circle graphs. • Students will generate their own question/topic to research. Students will need a minimum of 10 data points. From this data, they will calculate measures of central tendency and create a histogram and circle graph. Students will have a choice on how they present the information. They will present the information the next day

	and each for 10 minutes. This project will replace a traditional multiple-choice test for this unit.
Source	https://orise.orau.gov/stem/documents/k-12/lesson-plans/statistics.pdf

2 Activity

Problem Statement:
Barbie needs to travel from 1 end of the zipline to the other end using ___________________ materials.

STEP 1: Individually SKETCH a radical solution that uses the material at your station. (5 minutes)

STEP 2: Group Share and Feedback SHARE
● Each person in your group will take 1 minute to share their idea. While they share you need to fill out Chart:

1 thing you liked about the design	1 thing that could be changed to improve	1 Aspect you would like to include in group design

FEEDBACK
● Then EACH teammate will share their feedback (what they wrote in the chart) to that person.

STEP 3: GROUP DESIGN and Testing (35 minutes)
● Take everything put in their last column and brainstorm on a design that uses those aspects to create a NEW design that represents the groups ideas
● Once you are done, you can test it once.

STEP 4: Test in front of Class to Input your Time

STEP 5: Reflection. How did your carrier do? What did you like about your design? What would you change about your design? How could you make your carrier better?

Source-:https://orise.orau.gov/stem/documents/k-12/lesson-plans/statistics.pdf

15. PROBABILITY

QR Code:

E3F5F5

Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Probability — A Theoretical Approach	Differentiate between Empirical probability and theoretical probability in order to find the two for a variety of cases	Calculates in order to determine the probability of a given event
	Calculate the probability of given events in an experiment in order to comment whether they are complementary events/Sure event/impossible event	
	Represent using organized lists, tables, or tree diagrams in order to List the sample space for compound events	
	Calculate the probability of various events in order to rank them from most to least probable	

Test items

LOB: Differentiate between Empirical probability and theoretical probability in order to find the two for a variety of cases

1. Tanvi has a box containing four cards labelled A, B, C and D. She randomly picks a card from the box, records the label on the card and put it back in the box. She repeats this experiment 80 times and records her observation in the table shown below.

Card A	11
Card B	16
Card C	25
Card D	28

Which of the following shows the empirical probability and theoretical probability of picking Card C the next time?

Option 1: Empirical probability = $\frac{5}{16}$; Theoritical probability = $\frac{1}{4}$
Option 2: Empirical probability = $\frac{5}{16}$; Theoritical probability = $\frac{1}{2}$
Option 3: Empirical probability = $\frac{5}{11}$; Theoritical probability = $\frac{1}{4}$
Option 4: Empirical probability = $\frac{5}{11}$; Theoritical probability = $\frac{1}{2}$

Correct Answer: Option 1

2. Mandy has a bag containing 1 red, 1 green, 1 yellow, 1 black and 1 blue ball. She randomly picks the ball from the bag notes it colour and keeps it back in the bag. She repeats this 40 times. The table shows the number of times each colour ball she gets. The number of times the black ball is picked is missing in the table.

Red ball	Green ball	Yellow ball	Black ball	Blue ball
10	6	5	?	10

She then repeats the experiment 10 more times and gets red ball twice, green ball once, yellow ball thrice, black ball once and blue ball thrice.
Which of these is a valid conclusion as the number of trials of the experiment increases?

Option 1: The empirical probability of picking yellow ball gets closer to its theoretical probability.
Option 2: The empirical probability of picking red ball does not get closer to its theoretical probability.
Option 3: The empirical probability of picking yellow ball gets further away from its theoretical probability.
Option 4: The empirical probability of picking red ball becomes equal to its theoretical probability.

Correct Answer: Option 1

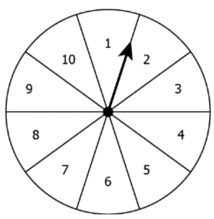
LOB: Calculate the probability of given events in an experiment in order to comment whether they are complementary events/Sure event/impossible event

1. If a card is drawn from a deck of cards, what is the probability of a card drawn to be a red or a black card and what can we say about that event?

Option 1: 1 and it is a sure event.
Option 2: 0 and it is a sure event.
Option 3: 1 and it is an impossible event.
Option 4: 0 and it is an impossible event.

Correct Answer: Option 1

2. A spinner is shown below.



Some of the events are listed below, when the spinner is spinned.
Event A: The spinner lands on a multiple of 11.
Event B: The spinner lands on a number less than 11.
Event C: The spinner lands on a number more than 10.
Which of the following statement is true about the three events?

Option 1: Probability of Event A is 1, so A is a sure event while the probabilities of Events B and C are 0, so they are impossible events.

Option 2: Probability of Event A is 1, so A is an impossible event while the probabilities of Events B and C are 0, so they are sure events.

Option 3: Probability of Event B is 1, so B is a sure event while the probabilities of Events A and C are 0, so they are impossible events.

Option 4: Probability of Event B is 1, so B is an impossible event while the probabilities of Events A and C are 0, so they are sure events.

Correct Answer: Option 3

LOB: Represent using organized lists, tables, or tree diagrams in order to List the sample space for compound events

1. hen four coins are tossed simultaneously, which of the following represents the sample space?

Option 1:

HHHH	HHHT	HHTH	HTHH
THHH	HHTT	TTHH	HTTT
THTT	TTHT	TTTH	TTTT

Option 2:

HHHH	HHHT	HHTH	HTHH
THHH	THHT	HTTH	HTTT
THTT	TTHT	TTTH	TTTT

Option 3:

HHHH	HHHT	HHTH	HTHH
THHH	HHTT	HTHT	HTTH
THHT	THTH	TTHH	HTTT
THTT	TTHT	TTTH	TTTT

Option 4:

HHHH	HHHT	HHTH	HTHH
THHH	HHTT	HTHT	HTTH
HTTH	HTHT	TTHH	HTTT
THTT	TTHT	TTTH	TTTT

Correct Answer: Option 3

2. To win a price in a game, you need to first choose one of the 4 doors, 1, 2, 3, 4 and then need to choose one of the three boxes A, B, C and then need to choose between two colours red and green.

How many of the possible outcomes of this game include selecting Box A and red colour?

Option 1: 4
Option 2: 8
Option 3: 12
Option 4: 24

Correct Answer: Option 1

LOB: Calculate the probability of various events in order to rank them from most to least probable

1. A box has 10 equal size cards. Of the 10 cards, 4 are blue, 3 are green, 2 are yellow and 1 is red. If a card is randomly drawn from the box, which is the colour that the card is most likely to have?

Option 1: Red
Option 2: Blue

Option 3: Green
Option 4: Yellow
Correct Answer: Option 2

2. Of 50 students in a class, 16 prefer cricket, 8 prefer football, 7 prefer basketball and rest of the students prefer either tennis or hockey. There are twice as many students who prefer tennis as the number of students who prefer hockey. A student is randomly selected from the class. Which statement is correct?
   Option 1: The probability of selecting a student who prefer tennis is more than that of selecting a student who prefer cricket.
   Option 2: The probability of selecting a student who prefer hockey is more than that of selecting a student who prefer cricket.
   Option 3: The probability of selecting a student who prefer hockey is more than that of selecting a student who prefer tennis.
   Option 4: The probability of selecting a student who prefer basketball is more than that of selecting a student who prefer cricket.

   Correct Answer: Option 1

Suggested Teacher Resources

I Lesson Plan

Objective	To calculate the probability of given events in an experiment in order to comment whether they are complementary events/Sure event/impossible event
Material Required	A deck of Tarot cards
Prerequisite Knowledge	Probability
Procedure	The teacher will start the class by giving the introduction: When you throw a dice, you don't know on which side it will fall. You've got "one chance in 6 to get a 2", "one chance in 6 to get a 1" or "3 chances in 6 to get an odd number". In the probability language, to throw a dice is a **random experiment**. You can't foresee the result. Tomorrow's weather is a random experiment too because nobody, even the weather forecast, can really say what the weather will be tomorrow. The different results of a random experiment are called **elementary events**. An **event** is a sum of elementary events. The **universal set** is the set of elementary events associated with a random experiment. We denote it by Ω. Probability theory is very closely associated to set theory, then when you throw a dice, the results {1}, {2}, {3}, {4}, {5}, {6} are each one elementary events, $\{1\} \cup \{3\} \cup \{5\} = \{1;3;5\}$ is an event and the universal set is the set $\Omega = \{1,2,3,4,5,6,\}$. Then, the teacher will explain: If all results have the same chance to happen, the elementary events are equally likely outcomes. It's the case when you throw a dice. In that case, if A is an event, then the probability of the event A is a number between 0 and 1 which is: $$P(A) = \frac{number\ of\ basic\ events\ in\ A}{number\ of\ basic\ events\ in\ \Omega}$$ For example, if A is the event 'to get a number greater or equal to 3', then $$P(A) = \frac{4}{6} = \frac{2}{3}$$ The elementary events are not always equally likely. If today is a sunny day, the probability that tomorrow will be sunny is not the same as the probability that it will rain tomorrow. Lucky, Mayank, Bhanu and Suraj are playing tarot. In a tarot deck there are 78 cards, 22 of these are trumps and there are 14 cards of each colour. Lucky picks up a card in the deck. What is the probability for the card to be a 5 but not the 5 of trump? Thanks to the last formula, this probability is $$P = \frac{4}{78}$$ Mayank picks up a card. What is the probability for the card to be smaller than 5? There are 4 of each colour, then $$P = \frac{16}{78}$$

The same way, the probability for Bhanu to pick up a trump is $\frac{22}{78}$.

Then Suraj picks up a card. What is the probability for the card to be a 2 or a heart? The answer is not all the hearts added to all the 2's because the 2 of hearts would count twice! The solution is

$$P = \frac{4 + 14 - 1}{78}$$

Because we count twice the 2 of hearts, we subtract it once. To get 3 different probabilities, we can also write:

$$P = \frac{4}{78} + \frac{14}{78} - \frac{1}{78}$$

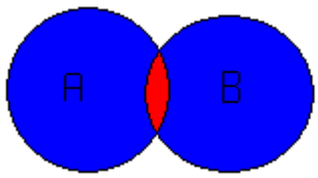
Usually, if A and B are events, we denote by $A \cap B$ (this reads 'A inter B' or 'A cap B') the sets of elementary events that are both in A and B, and we denote by $A \cup B$ ('A union B' or 'A cup B') the set of elementary events that are in either in A or in B (or both). You can translate Suraj's experiment by:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If the events A and B don't contain any common elementary event, we say that they are independent (for example with the dice if A = {1; 2} and B = {5; 6}). Then $A \cap B = \varnothing$, thus $P(A \cap B) = 0$ (the probability of an impossible event is null). The formula below is:

$P(A \cup B) = P(A) + P(B)$

To understand this concept, you can also represent the events with circles.



If two circles intersect then the area of the whole surface is equal to the area of the first circle added to the area of the second circle minus the area of their intersection. If the two circles don't intersect then you just have to add the areas.

Source	http://www.fmaths.com/probability1/lesson.php

2 Activity

Mathematical Objective: Students will notice that the experimental and theoretical probabilities differ over smaller samples but converge with larger samples

Materials/Resources:

- Coins
- Coloured pencils for graphing

Assumption of Prior Knowledge

- Students should know how to calculate basic probability
- Students should know how to plug in numbers into a formula
- Students should know how to collect and graph data \

Introduction: Setting Up the Mathematical Task

- Students will perform an experiment in which you explore the nature of probability in relation to the number of trials. They will perform the experiment 5 times using 5 different number of trials that are between 1 and 100 using a Coin. After you complete the 5 experiments, you will summarize your findings and discuss with the class. After the class discussion, you will draw your conclusion about the probabilities based on your experiment and the discussion.

- In groups of 4, students will conduct experiments and discover what happens to experimental probability.
- The teacher will review how to find the probability of an event with class and review the term "Theoretical" and "Experimental" probability in the context of an event.
- Students will work within their group and then each group will share results at the end.
- The teacher will float around and listen for groups making interjections only when needed and asking only facilitating questions.

Student Exploration:
Small Group Work (Groups of 4 where possible) Whole Class Sharing/Discussion (For Closure)
Student/Teacher Actions:

- The teacher should have available several ideas and questions for reminding students on how to graph data in different ways.
- The teacher will facilitate a discussion at the end of class, allowing students to discuss what happened as the number of trials increases.
- The class will complete activity with each person rolling 10 trials of a Coin and combining all of the data together to test what happens with very large number of trials.
- Students will complete closure slip as individuals. Monitoring Student Responses
- Students Should:

O Respectfully communicates with their group
O Listen to each member's contribution
O Take their role seriously and change roles with each experiment
O Communicate their thinking while answering questions as a group

- Closure Activity (either as exit slip or hw)

O Question 1: What is the difference between experimental and theoretical probability?
O Question 2: What happened to the experimental probability as the number of trials increased?
Task:
Teacher will draw the following table on board which students will make in their copies

Theoretical Probability: List each outcome of the sample space in the table below and calculate the corresponding theoretical probability of each.

Sample space	head	tail
Probability		

Experimental Probability: Each group will perform the experiment 5 times using 5 different number of trials between 1 and 100. The group will draw conclusions about the probabilities in relation to the theoretical probability and discuss with the class at the end of the period.
Conduct Each Experiment with a chosen number of trials between 1 and 100 and record your data for the trials, graph the frequency of your data outcomes, calculate probabilities, and then calculate the experimental error of the first sample after each section.

Calculate the Experimental Error for the first sample outcome versus the theoretical probability.

Trails	Number of heads	Number of tails	$\frac{\text{Cumulative number of heads}}{\text{Total number of times the coin is tossed}}$	$\frac{\text{Cumulative number of tails}}{\text{Total number of times the coin is tossed}}$
3	2	1	⅔	⅓
5	3	2	3/5	2/5

Summary: Summarize your results about the theoretical and experimental probabilities in relation to the number of trials used for each experiment. Be prepared to speak about your summary during the class discussion.

Reference: https://www.radford.edu/rumath-smpdc/Performance/src/Christina%20Perdue%20%20AFDA%20Probability%20&%20Law%20of%20Large%20Numbers.pdf