Supplementary Textual Material

in

Physics

for

Class XI

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SUPPLEMENTARY TEXTUAL MATERIAL IN PHYSICS
CLASS XI

ERRORS IN MEASUREMENT

UNIT - I

The term ‘Error’ in Physics, has a meaning quite different from ‘mistake’. An experimentalist can commit a mistake due to carelessness or ‘casualness’ on his/her part. We do not expect such a behavior from any serious experimentalist. ‘Mistakes’, therefore, do not get any consideration, or quantitative classification, in the observations of a ‘well planned’ and ‘well executed’ experiment.

No matter how well planned, and carefully executed our experiment is, we still cannot avoid **‘errors’** in our observations. This is because no measurement is ever perfect. Errors can arise from causes like: a built-in fault in the ‘design’ or ‘graduations’ of a measuring instrument, **or** a faulty way of carrying out the measurement on the part of the observer. The more important errors, in Physics, however, are what are known as (i) random errors; and (ii) the errors caused by the limitations of the measuring devices used in a given situation. The ‘limitation’, of any measuring instrument, is quantitatively specified through its **‘least count’** – the minimum magnitude of the relevant quantity that it can measure.

Random errors are errors which cannot be associated with any systematic or constant cause or with any definite ‘law of action’. These errors, are usually assumed to follow the well known **‘Gaussian law of Normal Distribution.**

In simple words, this law implies that the probability of an error $(+\Delta x)$ in a measurement is the same as the probability of an error $(-\Delta x)$ in that very measurement. Also, in a carefully carried out experiment, small magnitudes of ‘error’ are more likely than larger magnitudes of ‘error’. The ‘Normal Distribution law’ is graphically represented by a curve of the type shown here (Figure (i))

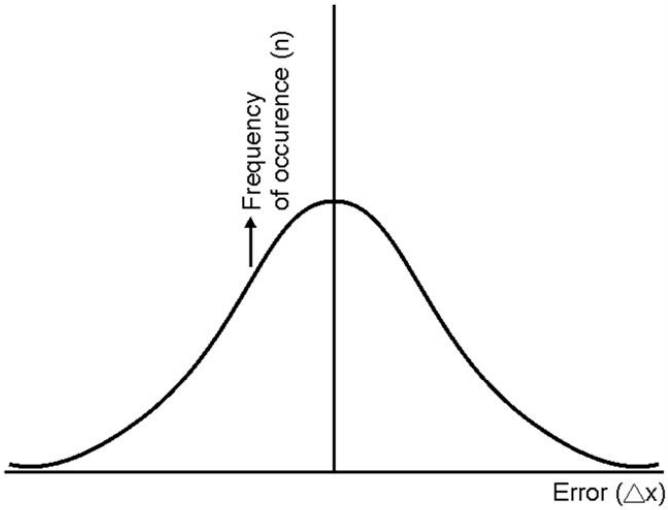


Fig. (i) **Illustrating Gaussian law of Normal Distribution**

A very significant 'conclusion', from this law, is that the 'arithmetic mean' of a large number of observations, is likely to be much closer to its 'true value' than any of the individual observations.

It is for this reason that we are always advised to take a large number of observations, and use their arithmetic mean, for doing, our 'calculations' **or** for drawing our 'conclusions' or 'inferences'.

The 'least counts', of the measuring instruments, used in an experiment, play a very significant role in the 'precision' associated with that experiment. Scientists are, therefore, constantly striving to design instruments, and measuring techniques, that have better (smaller) values for their 'least count'!

We can appreciate the importance of 'least count' by taking the simple example of the 'measurement of a length'. When we use a meter scale for this purpose, we can rely on this measurement only up to a 'mm'. This is because the least count of a meter scale is 1mm only. However, the use of a (simple) vernier caliper, pushes up this reliability to $\left(\frac{1}{10}\right)^{th}$ of a mm or 0.1mm while the use of (usual) 'screw gauge' would take this reliability to $\left(\frac{1}{10}\right)^{th}$ of a mm or 0.01mm.

Most of our experiments require us to use the measured values, of a number of different physical quantities , and put then in the appropriate 'formula' , to calculate the required quantity. We then calculate the 'percentage reliability , or 'maximum error', in our final result:

1. by associating a 'relative error'-equal to the ratio of the least count (of the measuring instrument used) to the measured value, with each of the quantities involved in our formula.
2. by using the standard 'rules' for finding the error in a 'sum or difference' , 'product or quotient', or 'power' , of different quantities, involved in a given formula.

We illustrate these ideas – for calculating the maximum error--through a few examples.

Example 1 : Suppose we use a physical balance to measure the mass of an object and find the mean value of our observations to be 156.28g.

Since we are somewhat uncertain, about the measurement, due to our instrument's imperfections, we need to express this in our result. Let the least count of physical balance be 0.1g: This implies that the uncertainly, of any measurement made with this instrument, is $\pm$0.1g. Therefore, we would report the mass of this object to be (156.3g $\pm$ 0.1g). This implies that we can only say that the mass of the object is somewhere between 156.2g and 156.4g.

Example 2 : It is required to find the volume of a rectangular block. A vernier caliper is used to measure the length, width and height of the block. The measured values are found to be 1.37cm, 4.11cm, and 2.56cm, respectively.

Solution : The measured (nominal) volume of the block is, therefore,

$V = \ell \times w \times h$

$= (1.37 \times 4.11 \times 2.56)cm^3.$

$= \quad 14.41cm^3$

However, each of these measurements has an uncertainty of $\pm$ 0.01cm, the least count of the vernier caliper. We can say that the values of length, width, and height should be written as

$\ell = (1.37cm \pm 0.01cm)$

$w = (4.11cm \pm 0.01cm)$

$h = (2.56cm \pm 0.01cm)$

We thus find that the lower limit, of the volume of the block, is given by

V_{min} = 1.36cm x 4.10cm x 2.55cm

= 14.22 cm^3

This is 0.19cm^3 **lower** than the (nominal) measured value.

The upper limit can also be calculated:

V_{max} = 1.38cm x 4.12cm x 2.57cm

= 14.61cm^3

This is 0.20cm^3 **higher** than the measured value.

As a practical rule, we choose the higher of these two deviations (from the measured value) as the uncertainty, in our result. We, therefore, should report the volume of the block as (14.41cm^3 ± 0.20cm^3).

Example 3 : In an experiment, on determining the density of a rectangular block, the dimensions of the block are measured with a vernier caliper (with a least count of 0.01cm) and its mass is measured with a beam balance of least count 0.1g. How do we report our result for the density of the block?

Solution : Let the measured values be:

Mass of block (m) = 39.3g

Length of block (ℓ) = 5.12cm

Breadth of block (b) = 2.56cm

Thickness of block (t) = 0.37cm

The density of the block is given by

$$\boldsymbol{\rho} = \frac{\text{mass}}{\text{volume}} = \frac{\text{m}}{\ell \times \text{b} \times \text{t}}$$

$$= = \frac{39.3\text{g}}{5.12\text{cm} \times 2.56\text{cm} \times 0.37\text{cm}} = 8.1037\text{g cm}^{-3}$$

Now uncertainty in m = $\pm$ 0.01g

uncertainty in ℓ = $\pm$ 0.01cm

uncertainty in b = $\pm$ 0.01cm

uncertainty in t = $\pm$ 0.01cm

Maximum relative error, in the density value is, therefore, given by

$$\frac{\Delta\rho}{\rho} = \frac{\Delta\ell}{\ell} + \frac{\Delta b}{b} + \frac{\Delta t}{t} + \frac{\Delta m}{m}$$

$$= \frac{0.01}{5.12} + \frac{0.01}{2.56} + \frac{0.01}{0.37} + \frac{0.1}{39.3}$$

= 0.0019 + 0.0039 + 0.027 + 0.0024

= 0.0358

Hence $\Delta\rho$ = 0.0358 x 8.1037g cm^{-3} = $\simeq$ 0.3g cm^{-3}

We cannot, therefore, report the calculated value of ρ (= 8.1037 gm^{-3}) up to the fourth decimal place. Since $\Delta\rho$ = 0.3g cm^{-3} the value of ρ can be regarded as accurate up to the first decimal place only. Hence the value of ρ must be rounded off as 8.1 g cm^{-3} and the result of measurements should be reported as

ρ = (8.1 $\pm$ 0.3) g cm^{-3}.

A careful look, at the calculations done above, the main contribution to this (large) error in the measurement of ρ, is contributed by the (large) relative error (0.027) in the measurement of t, the smallest of the quantities measured. Hence the precision of the reported value of ρ could be increased by measuring t with an instrument having a least count smaller than 0.01cm. Thus if a micrometer screw gauge (least count = 0.001cm), (rather) than a vernier caliper were to be used, for measuring t, we would be reporting our result for ρ with a considerably lower degree of uncertainty. Experimentalists keep such facts in mind while designing their 'plan' for carrying out different measurements in a given experiment.

EXERCISES

1. The radius of a sphere is measured as (2.1$\pm$ 0.5)cm

   Calculate its surface area with error limits.

   [Ans. {(55.4 $\pm$ 26.4)cm^2}]

2. The voltage across a lamp is (6.0 $\pm$ 0.1) volt and the current passing through it is (4.0 $\pm$ 0.2) ampere. Find the power consumed by the lamp.

   [Ans. {(24.0 $\pm$ 1.6) watt}]

3. The length and breadth of a rectangular block are 25.2cm and 16.8cm, which have both been measured to an accuracy of 0.1cm. Find the area of the rectangular block.

   [Ans. {(423.4 $\pm$ 4.2) cm^2}]

4. A force of (2500 + 5) N is applied over an area of (0.32 $\pm$ 0.02) m^2. Calculate the pressure exerted over the area.

   [Ans. {(7812.5 $\pm$ 503.9) N/m^2}]

5. To find the value of 'g', by using a simple pendulum, the following observations were made :

   Length of the thread ℓ = (100 $\pm$ 0.1) cm

   Time period of oscillation T = (2 $\pm$ 0.1) s

   Calculate the maximum permissible error in measurement of 'g'. Which quantity should be measured more accurately & why?

6. For a glass prism, of refracting angle 60°, the minimum angle of deviation, D_m, is found to be 36°, with a maximum error of 1.05°, when a beam of parallel light is incident on the prism. Find the range of experimental value of refractive index

$'\mu'$ It is known that the refractive index $'\mu'$ of the material of the prism is given by:

$$\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin(A/2)}$$

[Ans. ($1.46 \leq \mu \leq 1.51$, with a mean value of 1.49)]

7. The radius of curvature of a concave mirror, measured by a spherometer, is given by

$$R = \frac{\ell^2}{6h} + \frac{h}{2}$$

The value of ℓ and h are 4.0cm and 0.065cm respectively, where ℓ is measured by a meter scale and h by a spherometer. Find the relative error in the measurement of R.

[Ans. (0.08)]

8. In Searle's experiment, the diameter of the wire, as measured by a screw gauge, of least count 0.001cm, is 0.500cm. The length, measured by a scale of least count 0.1cm, is 110.0cm. When a weight of 40N is suspended from the wire, its extension is measured to be 0.125cm by a micrometer of least count 0.001cm. Find the Young's modulus of the material of the wire from this data.

[Ans. {($2.2 \times 10^{11} \pm 10.758 \times 10^{9}$) N/m^2)]

9. A small error in the measurement of the quantity having the highest power (in a given formula), will contribute maximum percentage error in the value of the physical quantity to whom it is related. Explain why?

10. The two specific heat capacities of a gas are measured as $C_p = (12.28 \pm 0.2)$ units and $C_v = (3.97 + 0.3)$ units. Find the value of the gas constant R.

[Ans. {(8.31 ± 0.5) units}].

MOTION IN A VERTICAL CIRCLE

UNIT – IV

Consider a particle P suspended in a vertical plane, by a massless, inextensible string from a fixed point O. In equilibrium, the string is vertical with P vertically below the point of suspension O, as shown in Figure (i) (a).

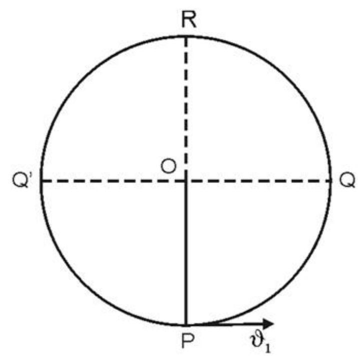


Figure (i) (a) **Figure (i) (b)**

Let the particle P be imparted an initial velocity $\overline{\vartheta_1}$ in a horizontal direction, as shown in Figure (i) (b). Under the tension in the string, the particle starts moving along a vertical circular path of radius equal to the length of the string. The point of suspension O is the center of this circle. It turns out that the initial velocity , $\overline{\vartheta_1}$ has to be more than a certain minimum critical value so that the particle may describe a vertical circular motion around point O.

The motion of a particle in a vertical circle, differs from that in a horizontal circle. In a horizontal circular motion, the force of gravity plays no role in the motion of the particle. However, in a vertical circular motion, gravity plays a very important role. It is easy to realize that a vertical circular motion has to be a **non-uniform circular motion**. In this case, the **velocity of the particle** varies both in **magnitude** and **direction**. In other words, in a vertical circular motion even the speed of the particle does not remain constant. As the particle moves up the circle, from its lowest position P, its speed continuously decreases till it reaches the highest point of its circular path. This is due to the work done against the force of gravity. When the particle moves down the circle, i.e., from R $\rightarrow$ Q' $\rightarrow$ P, its speed would keep on increasing. This is because of the work done, by the force of gravity, on the particle.

To obtain the basic characteristics of a vertical circular motion, consider an instantaneous position of particle, say at L. In this position, let the string make an angle θ with the vertical line OP, as shown in figure (ii)

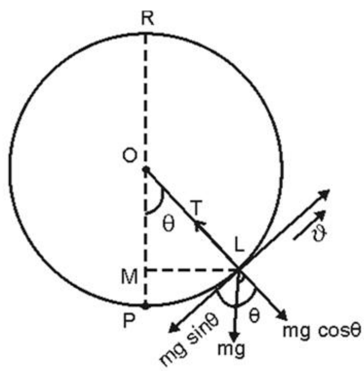


Figure (ii)

The forces acting on the particle (of mass m) at this position, L, are

(i) its weight = mg ; acting vertically downwards
(ii) the tension; T; in the string acting along LO.

The instantaneous velocity $\vec{\vartheta}$ of the particle is along the direction of the tangent to the circle at L. The corresponding instantaneous centripetal force, force, on the particle, equals $\frac{m\vartheta^2}{r}$ where r (= length of string l) is the radius of the particle's circular path. This force must act along $\overline{LO}$. We must, therefore, have

$$\frac{m\vartheta^2}{r} = T - mg\cos\theta$$

$$\therefore T = \frac{m\vartheta^2}{r} + mg\cos\theta \qquad \text{(i)}$$

We can take the horizontal direction, at the lowest point P, as the position of zero gravitational potential energy. Now, as per the law of conservation of energy;

Total energy at P = Total energy at L

$$\therefore \frac{1}{2} m\vartheta_1^2 + 0 = \frac{1}{2} m\vartheta^2 + mgh \qquad \text{(ii)}$$

where MP = h, is the vertical height by which particle has risen above P. From right angled triangle OML,

$$OM = OL\cos\theta = r\cos\theta$$
$$\therefore MP = h = OP - OM$$
$$= r - r\cos\theta$$

$$= r\,(1 - \cos\theta) \qquad \text{(iii)}$$

From Eqns. (ii) and (iii), we get

$$\vartheta_1^2 = \vartheta^2 + 2gr\,(1 - \cos\theta) \qquad \text{(iv)}$$

We now substitute the value of ϑ^2, from Eqn (iv), in Eqn (i). Hence

$$T = \frac{m}{r}\left[\vartheta_1^2 - 2gr\,(1 - \cos\theta)\right] + mg\cos\theta$$
$$= \frac{m\vartheta_1^2}{r} - 2mg\,(1 - \cos\theta) + mg\cos\theta$$
$$= \frac{m\vartheta_1^2}{r} - 2mg + 3mg\cos\theta \qquad \text{(v)}$$

This relation gives the tension T, in the string as a function of θ. We now use this relation to see the details of the particle when it is at the **(i)** lowest **(ii)** mid – way (horizontal) and **(iii)** highest position of its circular path.

When the particle is at the lowest point P of its vertical circular path, we have $\theta = 0^\circ$. The tension T_p in the string in this position, from Eqn (V) ,is

$$T_p = \frac{m\vartheta_1^2}{r} - 2mg + 3mg\cos 0^\circ$$
$$= \frac{m\vartheta_1^2}{r} + mg \qquad \text{(vi)}$$

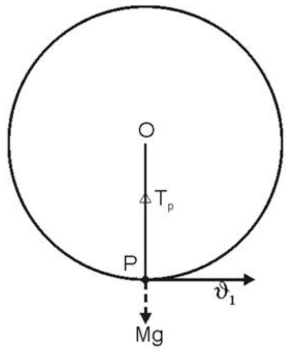


Figure (iii)

Consider next the case, when the particle is in position Q, where the string is momentarily) in its horizontal position. Clearly $\theta = \pi/2$ here. Let $\vec{\vartheta_2}$ be the instantaneous velocity of the particle here. Let T_Q be the instantaneous tension in the string here. Using Eqn (v), we have

$$T_Q = \frac{m\vartheta_1^2}{r} - 2mg + 3mg\cos\left(\pi/2\right)$$

$$= \frac{m\vartheta_1^2}{r} - 2mg \qquad \text{(vii)}$$

The change in the tension, as the particle moves from P to Q, equals $(T_P - T_Q)$. We have $(T_P - T_Q)$

$$= \left(\frac{m\vartheta_1^2}{r} + mg\right) - \left(\frac{m\vartheta_1^2}{r} + 2mg\right) = 3mg$$

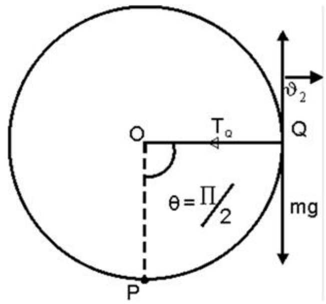


Figure (iv)

We next consider the particle P to be at the highest point R of its circular path. Let $\vec{\vartheta_3}$ be the instantaneous velocity of the particle here. In this position, $\theta = \pi$. If T_R denotes the tension in string in this position, we have, from eqn(v),

$$T_R = \frac{m\vartheta_1^2}{r} - 2mg + 3mg\cos(\pi)$$

$$= \frac{m\vartheta_1^2}{r} - 5mg \qquad \text{(viii)}$$

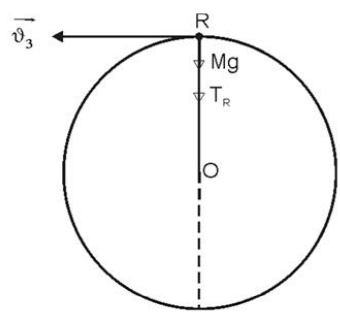


Figure (V)

Hence the change in the tension in the string, as the particle moves from P to R, along the vertical circle, is $(T_P - T_R) = \left(\frac{m\vartheta_1^2}{r} + mg\right) - \left(\frac{m\vartheta_1^2}{r} + 5mg\right) = 6mg$

It is thus seen that, the tension in the string is **maximum** when the particle is at **lowest point** P and is **minimum** at the **highest point** R of its vertical circular path. This is so because at the highest point, a part of the centeripetal force, needed to keep the particle moving in its circular path, is provided by the weight (mg) of the particle.

From Eqn (viii), it is easy to realize that T_R can be **(a)** positive; **(b)** negative or **(c)** zero depending on the value of ϑ_1 If T_R becomes a negative number, the **string would get slackened,** and the particle will be unable to continue moving along its vertical circular path. It will fall down before it is able to complete its circular path. Hence for completing the vertical circle, the minimum value of T_R has to be zero, We, therefore, have

$$(T_R)_{min} = \frac{m\,(\vartheta_1)^2_{min}}{r} - 5mg = o$$

$$\therefore\ (\vartheta_1)_{min} = \sqrt{5gr} \qquad \text{(ix)}$$

Using Eqn (iv) the minimum speed, which the particle must have at the highest point R, so that it is able to complete the vertical circle, is given by

$$(\vartheta_1)^2_{min} = (\vartheta_3)^2_{min} + 2gr\,(1 - \cos \Pi\ )$$

$$\therefore\ 5gr = (\vartheta_3)^2_{min} + 4gr$$

$$\text{or } (\vartheta_3)_{min} = \sqrt{gr} \qquad \text{(x)}$$

When the particle completes its motion along the vertical circle it is referred to as **"looping the loop".** For this to be possible, **the minimum speed at the lowest point** , must be $\sqrt{5gr}$.

The values of the tension in the string, when the particle is just able to do 'looping the loop', correspond to $\vartheta_1 = (\vartheta_1)_{min} = \sqrt{5gr}$

Hence , in this case,

T_P = 6mg [From Eqn (vi)]

and T_R = mg [From Eqn (viii)]

The results obtained above (for 'looping the loop') are put to many practical applications. We list below some of these applications.

(i) The pilot of an air craft can successfully loop a vertical circle (of radius r), if the velocity of the air-craft, at the lowest point of its vertical circle, is more than $\sqrt{5gr}$

(ii) Consider a bucket full of water being rotated in a vertical circle. The water, in the bucket, would not spill-over (even when the bucket is at its highest position along the vertical circle, i.e. when it is upside down), if the starting speed of the bucket, at the lowest point of its path, is **more** than $\sqrt{5gr}$. Under these conditions, the **centerifugal force,** on the water, inside the bucket, is more than the weight mg of the water. Hence the water does not spill over. If, however, the starting speed, at the lowest point of the vertical circular path, is **less** than $\sqrt{5gr}$, water will spillover when the bucket is upside down, i.e. at the highest point of its circular path. It is thus obvious that the **"trick"** really lies in whirling the bucket **"fast enough".**

(iii) A circus acrobat, performing in the **"circle of death",** speeds up his motor cycle, inside the circular cage, before going into a **vertical loop.** When he acquire a speed **more** than $\sqrt{5gr}$, at the lowest point of the intended vertical circular path, he would not fall down, even when he is "upside down" (i.e. at the highest point of his circular path). This is again because, in such a case, the centerifugal force on the motor cyclist, when he is "up-side-down, is more than his weight.

Example 1 : A small stone, of mass 200g, is tied to one end of a string of length 80cm. Holding the other end in hand, the stone is whirled into a vertical circle. What is the minimum speed, that needs to be imparted, at the lowest point of the circular path, so that the stone is just able to complete the vertical circle? What would be the tension in the string at the lowest point of circular path? (Take g $\simeq$10ms^{-2})

Solution : We know that

ϑ_{min} = minimum speed needed at the lowest point, so that particle is just able to complete the vertical circle = $\sqrt{5g\ell}$

Hence $\vartheta_{min} = \sqrt{5 \times 10 \times 0.8}\ \ ms^{-1}$

$\simeq 6.32\ ms^{-1}$

Also T_1 = Tension in the string at the lowest point of its circular path.

$$= \frac{m\vartheta^2{}_{min}}{\ell} + mg = 6mg = 6 \times 0.2 \times 10\,N = 12N$$

Example 2 : A massless string, of length 1.2m, has a breaking strength of 2kgwt. A stone of mass 0.4kg, tied to one end of the string, is made to move in a vertical circle, by holding the other end in the hand. Can the particle describe the vertical circle? (Take g $\simeq$10ms^{-2})

Solution : We are given that

T_{max} = maximum tension in the string so that it does not break

= 2 kgwt = 2x10N = 20N

Let T_1 be the tension in the string when the stone is in its lowest position of its circular path. We know that $T_1 = \frac{m\vartheta_1^2}{r} + mg$.

T_1 would have its minimum value when ϑ_1 equals its minimum value $\left(= \sqrt{5g\ell}\right)$, needed by the stone, to complete its vertical circular path.

Hence $(T_1)_{min} = \frac{m\vartheta_{min}^2}{\ell} + mg = 6mg$

$= 6 \times 0.4 \times 10$

$= 24N$

We thus see that $(T_1)_{min}$ is more than the breaking strength of the string. Hence the particle cannot describe the vertical circle.

Example 3 : A small stone, of mass 0.2kg, tied to a massless, inextensible string, is rotated in a vertical circle of radius 2m. if the particle is just able to complete the vertical circle, what is its speed at the highest point of its circular path? How would this speed get effected if the mass of the stone is increased by 50%? (Take g $\simeq$ 10ms^{-2})

Solution :

Let ϑ_1 be the speed of the stone at the lowest point of its vertical circle. Since the stone is just able to complete the vertical circle, we have

$\vartheta_1 = \sqrt{5gr}$
$= \sqrt{5 \times 10 \times 2}\ ms^{-1} = 10ms^{-1}$

Let ϑ_2 be the speed of the stone, at the highest point, on its circular path. Then

$$\vartheta_2^2 = \vartheta_1^2 - 4gr$$
$$= (10)^2 - 4 \times 10 \times 2$$
$$= 100 - 80 = 20$$
$$\therefore \vartheta_2 = \sqrt{20}\ ms^{-1} \simeq 4.47ms^{-1}$$

It is thus seen that the value of ϑ_2 , **does not depend** on the **mass** (m) of **the stone**. Hence ϑ_2 would **remains the same** when the mass of the stone increases by 50%.

Example 4 : A particle, of , mass 150g, is attached to one end of a massless, inextensible string. It is made to describe a vertical circle of radius 1m. When the string is making an angle of 48.2° with the vertical, its instantaneous speed is $2ms^{-1}$. What is the tension in the string in this position? Would this particle be able to complete its circular path?

(Take g $\simeq$ 10 ms^{-2})

Solution :

The tension T, in the string, when it makes an angle θ, with the vertical, is given by

$$T = \frac{m\vartheta^2}{\ell} + mg\cos\theta$$

where ϑ is the instantaneous speed of the particle.

Here $\vartheta = 2ms^{-1}$, ℓ = 1m, m = 0.15kg, and θ = 48.2°

$$\therefore T = \frac{0.15 \times (2)^2}{1} + (0.15 \times 10 \times \cos 48.2^\circ)$$

= (0.6 + 1.5 x 0.67) N

$\simeq$ 1.6N

Let ϑ_1 be the speed of the particle at the lowest point of its circular path. Then

$$\vartheta_1^2 = \vartheta^2 + 2gr\,(1 - \cos\theta)$$

$$= (2)^2 + 2 \times 10 \times 1 \times (1 - \cos 48.2^\circ)$$

$= (4 + 20 \times (1 - 0.67)$

$\simeq (4 + 6.6) = 10.6$

$\therefore\ \vartheta_1 = \sqrt{10.6}\ \mathrm{m\,s^{-1}} \simeq 3.25\ \mathrm{m\,s^{-1}}$

The minimum value of ϑ_1, so that the particle is able to complete its vertical circle, is $\sqrt{5gr}$.

$\therefore\ (\vartheta_1)_{min} = \sqrt{5 \times 10 \times 1}\ \mathrm{m\,s^{-1}} \simeq 7.07\ \mathrm{ms^{-1}}$

The value of ϑ_1, obtained above, is **less** than this minimum speed. The particle, in the given case, would not be able to complete its vertical circular path.

Example 5 : A bucket, containg 4kg of water, is tied to a rope of length 2.5m and rotated in a vertical circle in such a way that the water in it just does not spill over when the bucket is in its 'upside down' position. What is the speed of bucket at the

(a) highest and (b) lowest point of its circular path? (Take $g \simeq 10\mathrm{ms^{-2}}$)

Solution :

Let ϑ_1 be the speed of bucket at the lowest point of its circular path. Then

$$\vartheta_1 = \sqrt{5gr}$$
$$= \sqrt{5 \times 10 \times 2.5}\ \mathrm{ms^{-1}}$$
$$= \sqrt{125}\ \mathrm{ms^{-1}} \simeq 11.18\ \mathrm{ms^{-1}}$$

Let ϑ_2 be the speed of the bucket at the highest point of its circular path. Then

$$\vartheta_2 = \sqrt{gr} \in$$
$$= \sqrt{10 \times 2.5}\ \mathrm{ms^{-1}}$$
$$= 5\mathrm{ms^{-1}}$$

Example 6 : The figure here shows a smooth 'looping-the-loop' track. A particle, of mass **m**, is released from point A, as shown. If H=3r, would the particle 'loop the loop'?

What is the force on the circular track when the particle is at point (i) B (ii) C ?

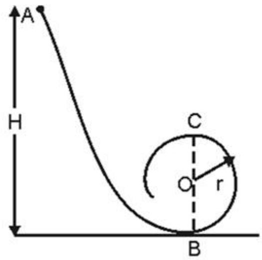


Figure (vi)

Solution :

Let ϑ_B be the speed acquired by the particle at the (lowest) point B. From law of conservation of energy, we have

Total energy at A = Total energy at B

$$\therefore (0 + mgH) = \frac{1}{2} m\vartheta_B^2 + 0$$

$$\therefore \vartheta_B = \sqrt{2gh} = \sqrt{2g \times 3r} = \sqrt{6gr}$$

The minimum, speed, needed by the particle at B, so that it can 'loop the loop' is $\sqrt{5gr}$. Since ϑ_B is more than $\sqrt{5gr}$; the particle would 'loop the loop'.

The forces, acting on particle at B, are as shown here, Let N_1 be the force exerted on the particle by the track. According to Newton's third law, the force exerted by the particle on the track, is equal and opposite to N_1. Now

$$N_1 = mg + \frac{m\vartheta_B^2}{r}$$

$$= mg + \frac{m \times 6gr}{r} = 7mg$$

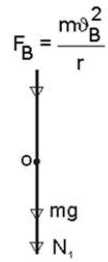


Figure (vi) (a)

Hence the force, exerted, by the particle, on the track, equals 7mg, directed vertically downwards.

The forces, acting on the particle in position C, are as shown in the figure here. The speed, ϑ_c, of the particle, at C, is given by $\vartheta_c^2 = \vartheta_B^2 - 4gr = 6gr - 4gr = 2gr$

$$\therefore\ N_2 = \frac{m\vartheta_c^2}{r} - mg$$

$= 2mg – mg = mg$

Hence the particle exerts a force mg, directed radially outwards, on the track, at point C.

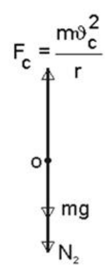


Figure (vi) (b)

EXERCISES

1. A stone of mass 0.2kg, is tied to one end of a string of length 80cm. Holding the other end, the stone is whirled into a vertical circle. What is the minimum speed of the stone at the lowest point so that it just completes the circle. What is the tension in string at the lowest point of the circular path? ($g=10ms^{-2}$).

[Ans: $6.32ms^{-1}$; 12N]

2. A particle, of mass 100g, is moving in a vertical circle of radius 2m. The particle is just 'looping the loop'. What is the speed of particle and the tension in string at the highest point of the circular path? ($g=10ms^{-2}$).

[Ans. $4.47ms^{-1}$, zero]

3. A particle, of mass 0.2kg, attached to a massless string is moving in a vertical circle of radius 1.2m. It is imparted a speed of $8ms^{-1}$ at the lowest point of its circular path. Does the particle complete the vertical circle? What is the change in tension in the string when the particle moves from the position, where the string is vertical, to the position where the string is horizontal?

[Ans. Yes, 6N]

4. A particle, of mass 200g, is whirled into a vertical circle of radius 80cm using a massless string. The speed of particle, when the string makes an angle of 60° with the vertical line, is $1.5ms^{-1}$. What is the tension in the string in this position?

[Ans. 1.56N]

ELASTIC AND INELASTIC COLLISIONS IN TWO DIMENSIONS

UNIT – IV

Consider two particles A and B moving in a plane. If these two particles collide, and still continue moving in the same plane, the collision is referred to as a **two dimensional (or an oblique) collision.** A collision, between two billiard balls is an example of such a two dimensional collision.

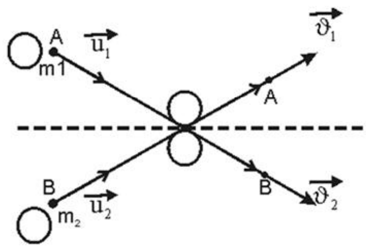
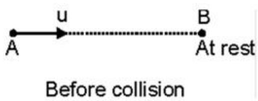


Figure (i)

To analyze the basic details of a two dimensional collision, consider a system of two particles A and B, moving, as shown, before and after the collision.

Since the forces, they exert on each other (during collision) are **internal forces**, and there are no other forces, (i.e.there is no external force), the **linear momentum , of the system is conserved.** When the collision is elastic, the kinetic energy of the system is also conserved. (The collision is inelastic if **kinetic energy of system is not conserved).**

To illustrate, how calculations can be carried out, we now, consider a simple two dimensional elastic collision, taking place , as shown below.



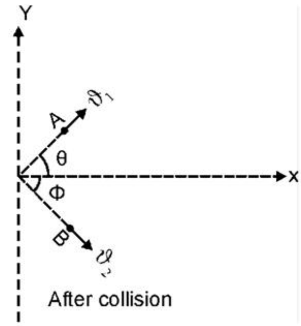


Figure (ii) (a) **Figure (ii) (b)**

Figure ((ii) (a)) shows a particle A, of mass m_1, moving along x-axis, in the x-y plane, with an initial speed u. Particle B, of mass m_2, is initially at rest. When particle A collides with B, the two particles move with speeds ϑ_1 and ϑ_2, in the x-y plane, after the collision (Figure (ii) (b))

After the collision, let particle A move in a direction inclined at an angle θ , with its initial direction of motion. Angle θ is known as the **angle of scattering.**

After the collision, let particle B move along a direction, making an angle ϕ, with the initial direction of motion of A. Angle Φ is known as the **angle of recoil.**

Knowing m_1, m_2, and u , we have to do the needed calculations. The law of conservation of linear momentum, used for the x and y components separately, gives us the two equations:

$$m_1u + 0 = m_1\vartheta_1 \cos\theta + m_2\,\vartheta_2\,\cos\phi \qquad \text{(i)}$$

and $$0 = m_1\vartheta_1 \sin\theta - m_2\,\vartheta_2\,\sin\phi \qquad \text{(ii)}$$

We have assumed the collision to be **perfectly elastic**. Hence total K.E. before collision = total K.E. after collision

$$\therefore \frac{1}{2}m_1u^2 + 0 = \frac{1}{2}m_1\vartheta_1^2 + \frac{1}{2}m_2\,\vartheta_2^2 \qquad \text{(iii)}$$
or $$m_1u^2 = m_1\vartheta_1^2 + m_2\,\vartheta_2^2$$

We thus have **three equations** in all but need to find four unknown parameters. A **complete solution,** is therefore, **NOT POSSIBLE.** However if the value of any one of the four unknowns, i.e, $\vartheta_1, \vartheta_2, \theta$ or ϕ, is **given,** the remaining three can be calculated, using Eqns (i), (ii) and (iii).

[For an **inelastic** two dimensional collision, we only have two equations i.e Eqn (i) and (ii). Hence, in this case, if two of the four unknowns, say θ and ϕ are given, we can calculate the remaining two unknowns (i e ϑ_1, and ϑ_2) using Eqns (i) and (ii)].

Special case : We now consider the special case of two dimensional collision of two particles of equal mass. Eqns (i), (ii) and (iii), in this case, reduce to

$$u = \vartheta_1 \cos\theta + \vartheta_2 \cos\phi \qquad \text{(iv)}$$

$$o = \vartheta_1 \sin\theta - \vartheta_2 \sin\phi \qquad \text{(v)}$$

$$u^2 = \vartheta_1^2 + \vartheta_2^2 \qquad \text{(vi)}$$

From Eqns (iv) and (vi)

$$\left(\vartheta_1 \cos\theta + \vartheta_2 \cos\phi\right)^2 = \vartheta_1^2 + \vartheta_2^2$$

$$\therefore\ \vartheta_1^2 \cos^2\theta + \vartheta_2^2 \cos^2\phi + 2\vartheta_1\vartheta_2 \cos\theta\cos\phi = \vartheta_1^2 + \vartheta_2^2$$

$$\begin{aligned} \text{or } 2\vartheta_1\vartheta_2 \cos\theta\cos\phi &= \vartheta_1^2(1-\cos^2\theta) + \vartheta_2^2(1-\cos^2\phi) \\ &= \vartheta_1^2 \sin^2\theta + \vartheta_2^2 \sin^2\phi \end{aligned} \qquad \text{(vii)}$$

Using Eqn (v), we can rewrite Eqn (vii) as

$$2\vartheta_1\vartheta_2 \cos\theta\cos\phi = 2\vartheta_1^2 \sin^2\theta$$

$$\text{or } \cos\theta = \left(\frac{\vartheta_1}{\vartheta_2}\right)\frac{\sin^2\theta}{\cos\phi} \qquad \text{(viii)}$$

Now $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

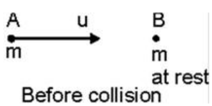
$$= \left(\frac{\vartheta_1}{\vartheta_2}\right)\frac{\sin^2\theta}{\cos\phi}\cos\phi - \frac{\vartheta_1}{\vartheta_2}\sin^2\theta \quad (\text{from Eqn (V) and (viii)})$$

$$= \frac{\vartheta_1}{\vartheta_2}\sin^2\theta - \frac{\vartheta_1}{\vartheta_2}\sin^2\theta$$

$$= 0$$

$$\therefore \theta + \phi = \pi/2$$

We thus see that in the special case, of a perfectly elastic (two-dimensional) collision, between two particles, of the same mass, the two particles move along mutually perpendicular directions after the collision. This is illustrated in the figure below.



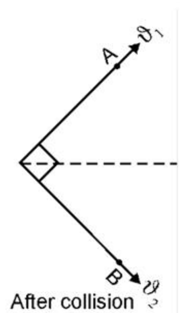
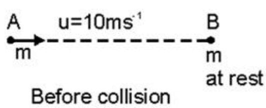


Figure (iii)(a)

Figure (iii)(b)

Example 1: A and B are two particles having the same, mass **m**. A is moving along x-axis with a speed of $10m^{-1}$, and B is at rest. After undergoing a perfectly elastic collision, with B, particle A gets scattered through an angle of 30^o. What is the direction of motion of B, and the speeds of A and B, after this collision?

Solution:



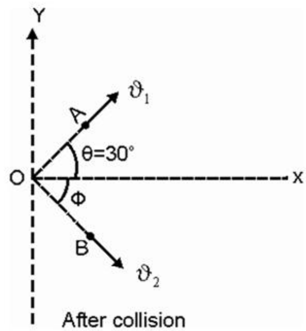


Figure (iv)(a) **Figure (iv)(b)**

Figure (iv), (a) and (b), show the particles A and B, before and after the collision. Since A and B have the same mass, and the collision is perfectly elastic, we would have

$$\theta + \phi = 90^\circ$$
$$\therefore \phi = 90^\circ - 30^\circ = 60^\circ \qquad \text{(i)}$$

Using law of conservation of linear momentum, we get

(i) for the x-components,

$$u = 10 = \vartheta_1 \cos 30^\circ + \vartheta_2 \cos 60^\circ$$

$$\text{or} \quad 10 = \frac{\sqrt{3}}{2}\vartheta_1 + \frac{\vartheta_2}{2}$$

$$\therefore 20 = \sqrt{3}\,\vartheta_1 + \vartheta_2 \qquad \text{(ii)}$$

and (ii) for the y – components,

$$o = \vartheta_1 \sin 30^\circ - \vartheta_2 \sin 60^\circ$$

$$\therefore \vartheta_1 \frac{1}{2} = \frac{\sqrt{3}}{2}\vartheta_2 \text{ or } \vartheta_1 = \sqrt{3}\,\vartheta_2 \qquad \text{(iii)}$$

From Eqn (ii) and (iii), we get

$$20 = 3\vartheta_2 + \vartheta_2 \text{ or } \vartheta_2 = 5\text{ms}^{-1}$$

$$\text{and } \vartheta_1 = \sqrt{3}\,\vartheta_2$$

$$= 1.732 \times 5\text{ms}^{-1} = 8.66\text{ms}^{-1}$$

Example 2: Two particles, A and B, of masses **m** and **2m**, are moving along the X and Y- axis, respectively, with the same speed ϑ.

They collide, at the origin, and coalesce into one body, after the collision. What is the velocity of this coalesced mass? What is the loss of energy during this collision?

Solution :

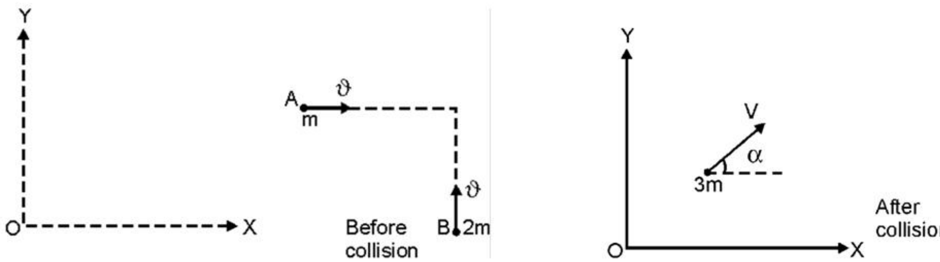


Figure (v) (a) Figure (v) (b)

Figures (v) (a), and (v) (b), show the two particles before and after the collision. Let V be the speed of the combined mass and let the direction of $\vec{V}$ be making an angle α with the positive x-axis, after the collision. Using law of conservation of linear momentum, we have

For the x-components,

$$m\vartheta = 3m\,V\cos\alpha \qquad \text{(i)}$$

and for the y-components

$$2m\vartheta = 3m\,V\sin\alpha \qquad \text{(ii)}$$

From Eqns (i) and (ii), we get

$$\tan\alpha = \frac{2m\vartheta}{m\vartheta} = 2$$

$\therefore \alpha = \tan^{-1}(2) \simeq 63.4^{\circ}$ (iii)

Also $V^2 = \left(\frac{\vartheta}{3}\right)^2 + \left(\frac{2\vartheta}{3}\right)^2 = \frac{5}{9}\vartheta^2$

$\therefore \quad V = \left(\frac{\sqrt{5}}{3}\right)\vartheta$ (iv)

Now K_i = Total K.E. before collision = $\frac{1}{2}m\vartheta^2 + \frac{1}{2}(2m)\vartheta^2 = \frac{3m\vartheta^2}{2}$

and K_f = Total K.E. after collision = $\frac{1}{2}(3m)V^2 = \left(\frac{3}{2}\right)\left(\frac{5}{9}\right)m\vartheta^2$

$$= \frac{5}{6}m\vartheta^2$$

Hence ΔK = Loss of kinetic energy during the collision = $K_i - K_f$

$$= \left(\frac{3}{2} - \frac{5}{6}\right)m\vartheta^2 = \frac{2}{3}m\vartheta^2$$

EXERCISES

1. A billiard ball A moving with an initial speed of $1 ms^{-1}$, undergoes a perfectly elastic collision with another identical ball B at rest. A is scattered through an angle of 30^{o}. What is the angle of recoil of B? What is the speed of ball A after the collision?

$\left[\text{Ans. } 60^{o};\ \frac{\sqrt{3}}{2} ms^{-1}\right]$

2. Two identical balls, A and B, undergo a perfectly elastic two dimensional collision. Initially A is moving with a speed of $10 ms^{-1}$ and B is at rest. Due to collision, A is scattered through an angle of 30^{o}. What are the speeds of A and B after the collision?

$\left[\text{Ans.}\vartheta_A = 5\sqrt{3}\ \text{ms}^{-1}, \vartheta_B = 5\ \text{ms}^{-1}\right]$

3. A and B are two identical balls. A, moving with a speed of $6 ms^{-1}$, along the positive x-axis, undergoes a collision with B, initially at rest. After collision, each ball moves along directions making angles of $\pm 30^\circ$ with the x-axis. What are the speeds of A and B after the collision? Is this collision perfectly elastic?

$\left[\text{Ans.}\vartheta_A = \vartheta_B = 2\sqrt{3}\ \text{ms}^{-1}, \text{No}\right]$

NON – CONSERVATIVE FORCES

UNIT – IV

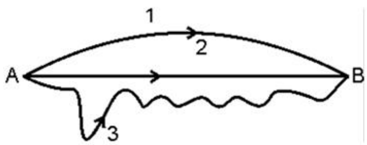


Figure (i)(a)

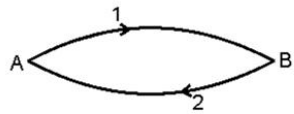


Figure (i)(b)

A force is non-conservative; if **work done** by, or against the force, in moving from one point A, in space, to another point B **DEPENDS ON THE PATH FOLLOWED IN MOVING FROM A to B**. In Figure (i) (a), let W_1, W_2 and W_3 denote the works done in moving a body, from A to B, along three different paths 1, 2 and 3 respectively. For a non-conservative force $W_1 \neq W_2 \neq W_3$

Figure (i) (b) shows a particle moving along a **closed path** $A \to 1 \to B \to 2 \to A$. Let W_1 be the work done along the path $A \to 1 \to B$ and let W_2 be the work done along the path $B \to 2 \to A$. For a non-conservative force $|W_1| \neq |W_2|$. **Therefore, in such a case, the net work done, along the closed path is not-zero.** Expressed mathematically.

$\oint \vec{F}.d\vec{s} \neq o$, i.e. for a non-conservative force, the work done along a closed path is not zero.

Two of the common examples, of non-conservative forces, are

(i) **Force of friction**
(ii) **Viscous force**

Non- Conservative forces are usually velocity **dependent.** Consider a particle moving from A to B, on a horizontal, rough surface.

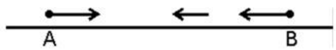


Figure (ii)

The force of friction coming into play, is a non-conservative force. Let W be the work done against the force of friction in moving from A to B. When the body moves from B to A, W would again be the work done against the force of friction. The net work done, against friction, in the round trip is therefore, 2W. If this work is being done at the cost of kinetic energy, the loss of Kinetic energy, in the round trip, is numerically equal to 2W.

Consider next, a body having both a kinetic energy (K), and a potential energy (U), moving in a non-conservative force field. The total energy E (=K+U), of the body, does not remain constant. Let E_i and E_f represent the total values of the initial and final energy. If W is the work done, against the non-conservative force, we would have

$$E_i - E_f = W$$

The work done, by non-conservative forces, appears in some other form of energy like heat, sound, light etc. When we take into account all forms of energy, the general **law of conservation of energy still holds good both for conservative as well as non-conservative forces.**

Example 1 : A particle, of mass 0.1kg, has an initial speed of $4ms^{-1}$ at a point A on a rough horizontal road. The coefficient of friction, between the object and the road is 0.15. The particle moves to a point B, at a distance of 2m from A. What is the speed of the particle at B?

(Take $g \simeq 10ms^{-2}$)

Solution:

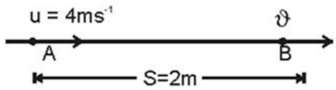


Figure (iii) (a)

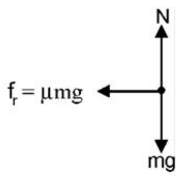


Figure (iii) (b)

Figure (iii) (a) here shows the initial position A and the final position B of the object. Let the speed of particle at B be ϑ. The particle does work against force of friction which is a non-conservative force.

$\therefore K_i - K_f = W$

where W is the work done against the force of friction. Figure (iii) (b) shows all the forces acting on the object

$\therefore W = f_r \times S = \mu mg\ S = (0.15 \times 0.1 \times 10 \times 2)\ J = 0.3J$

Hence $\frac{1}{2} \times 0.1\ [4^2 - \vartheta^2] = 0.3$

$$\text{Or } 16 - \vartheta^2 = 6$$

$$\therefore \vartheta = \sqrt{10}\ ms^{-1} \simeq 3.16 ms^{-1}$$

Example 2: A particle, of mass 0.2kg, has an initial speed of $5ms^{-1}$ at the bottom of a rough inclined plane, of inclination 30^o and vertical height 0.5m. What is the speed of the particle as it reaches the top of the inclined plane?

$\left(\text{Take } \mu = 1/\sqrt{3}; g \simeq 10ms^{-2}\right)$

Solution:

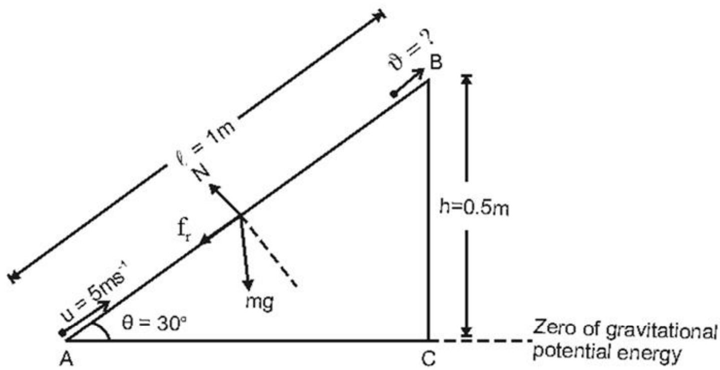


Figure (iv)

The figure here shows the initial position A and the final position B of the particle. Take the base AC, of the inclined plane, as the zero of gravitational potential energy. The forces acting on the particle are as shown. f_r = force of friction between the particle and the inclined plane = $\mu N = \mu mg\cos\theta = \left(\frac{1}{\sqrt{3}} \times 0.2 \times 10 \times \frac{\sqrt{3}}{2}\right) N = 1N$

Also; $\frac{h}{\ell} = \sin 30° = 0.5 \therefore \ell = \left(\frac{0.5}{0.5}\right) m = 1m$

Hence W, the work done by the particle, against the force of friction, in moving from A to B = $(\mu mg\cos\theta) \times \ell$ = 1N x 1m = 1J

Now $\quad E_i - E_f = W$

$$\therefore \frac{1}{2} \times 0.2 \times (5)^2 - \left[\frac{1}{2} \times 0.2\,\vartheta^2 + 0.2 \times 10 \times 0.5\right] = 1$$

or $2.5 - 0.1\vartheta^2 - 1 = 1$

$\therefore 0.1\vartheta^2 = 0.5$

$\therefore \vartheta = \sqrt{5} ms^{-1} \simeq 2.24 ms^{-1}$

POISSON'S RATIO

UNIT – VII

Careful experiments show that when a wire is having a longitudinal strain, there is not only an increase in its length but also a slight decrease in its diameter. The length of the wire **increases** along the direction of the applied deforming forces-, its diameter **decrease** occurs along the perpendicular, (or lateral) direction. This fact is not only true for wires but for all other bodies under strain. The ratio, of the change, ΔD, in diameter, to the original diameter, D, is called lateral strain i.e. strain at right angles to the deforming forces. Thus,

$$\text{Lateral strain} = \frac{\text{Change in diameter}}{\text{Original diameter}} = \frac{\Delta D}{D} \qquad \text{(i)}$$

$$\text{Also longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta \ell}{\ell} \qquad \text{(ii)}$$

Poisson pointed out that, within elastic limits, the lateral strain is directly proportional to the longitudinal strain. The ratio, of the lateral strain, to the longitudinal strain, is therefore, a constant for a given material and is known as its **Poisson's ratio.** It is denoted by the Greek letter σ.

$$\text{Hence, } \sigma = \left(\frac{\text{- lateral strain}}{\text{longitudinal strain}}\right)$$

$$\text{or} \quad \sigma = \left(\frac{-\Delta D}{D}\right) \Big/ (\Delta \ell / \ell)$$

$$\text{i.e. } \sigma = -\left(\frac{\ell}{D}\frac{\Delta D}{\Delta \ell}\right) \qquad \text{(iii)}$$

The negative sign is put in because the change in diameter is in opposite sense to the change in length of the wire.

Poisson's ratio, being a ratio of two types of strains, is a dimensionless quantity i.e. σ is a pure number.

The value of Poisson's ratio depends only upon the nature of the material and for most of the substances it lies between 0.2 to 0.4 . When a body under tension suffers no change in volume, i.e. the body is perfectly incompressible, the value of the Poisson's ratio is maximum, and equals 0.5.

Example: A 10kg mass is attached to one end of a copper wire, 3m long and 1mm in diameter. Calculate the lateral compression produced in it. (Poisson's ratio is 0.25 and Young's modulus, of the material of the wire, is 12.5×10^{10} N/m^2.)

Solution:

Let $\Delta\ell$ be the increase in the length of the wire.

Now $\Delta\ell = \dfrac{F.\ell}{\pi r^2 Y}$ $\left(\because Y = \dfrac{\left(F/\pi r^2\right)}{\left(\Delta\ell/\ell\right)}\right)$

$$\therefore \Delta\ell \simeq \frac{10 \times 9.8 \times 3}{3.14 \times \left(0.5 \times 10^{-3}\right)^2 \times 12.5 \times 10^{10}}$$

$$= 0.2993 \times 10^{-2} m$$

Now Poisson's ratio, σ $= = \dfrac{-\Delta D}{D} \dfrac{\ell}{\Delta\ell}$

$$\therefore \Delta D = - \frac{\Delta\ell\, D\, \sigma}{\ell}$$

$$= \frac{-0.2993 \times 10^{-2} \times 10^{-3} \times 0.25}{3}$$

$$\simeq - 2.5 \times 10^{-7} m$$

$$= -0.25 \mu m$$

Thus the lateral compression, produced in the wire, equals (nearly) $0.25 \mu m$.

EXERCISES

1. One end of a nylon rope, of length 4.5m and diameter 6mm, is fixed to a free limb. A monkey, weighing 100N, jumps to catch the free end and stays there. Find the elongation of the rope and the corresponding change in the diameter. Given Young's modulus of nylon = $4.8 \times 10^{11} N/M^2$ and Poisson's ratio of nylon = 0.2.

   [Ans. (8.8×10^{-9}m)]

2. Determine the Poisson's ratio of the material of a wire whose volume remains constant under an external normal stress.

3. A material has Poisson's ratio 0.5. If a uniform rod, of it, undergoes a longitudinal strain of 2×10^{-3}, what is the percentage increases in its volume?

   [Ans. (zero %)]

ELASTIC ENERGY

UNIT – VII

The force 'F' needed to stretch a wire of length L by an amount ℓ is given by-

$F = \frac{YA\ell}{L}$, where A is the cross-sectional area of the wire and Y its Young's modulus. (i)

The work done, by this force, in stretching the wire, gets stored in it as its elastic potential energy. This work can be calculated as follows:

dW = work done for a small increase $d\ell$ in the length of the wire, = $Fd\ell$

Hence the total work done, for an increase ℓ in the length of the wire, is

$$W = \int_0^{\ell} F d\ell \qquad \text{(ii)}$$

$$= \frac{YA}{L} \int_0^{\ell} \ell d\ell \qquad (\text{ from (i)})$$

$$= \left(\frac{YA}{L}\right)\frac{\ell^2}{2} = \frac{1}{2}\left(\frac{YA\ell}{L}\right).\ell$$

$$= \frac{1}{2}F\ell \qquad (\text{from (i)})$$

This work done equals the elastic potential energy, U. Hence

$$U = \frac{1}{2}F\ell = \frac{1}{2}\text{Force x extension} \qquad \text{(iii)}$$

Now, volume of the wire = AL.Therefore, the potential energy per unit volume

$$= \frac{1}{2}\frac{F\ell}{AL} \qquad = \frac{1}{2}\left(\frac{YA\ell}{L}\right)\frac{\ell}{AL}$$

$$= \frac{1}{2}\left(\frac{Y\ell}{L}\right).\frac{\ell}{L}$$

$$= \frac{1}{2}\left(\frac{F}{A}\right).\left(\frac{\ell}{L}\right)$$

$$= \frac{1}{2}\text{ stress x strain} \qquad \text{(iv)}$$

Thus the **elastic potential energy, per unit volume of the wire (energy density), equals half the product of its stress and strain.**

EXERCISES

1. A metallic wire is stretched by suspending a weight from it. If ℓ is the longitudinal strain and Y is the Young's Modulus; show that elastic potential energy per unit volume is given by $\frac{1}{2}Y\ell^2$.

2. When the load on a wire is increased from 3kgwt to 5kgwt, the elongation, increases from 0.61mm to 1.02mm. How much work is done during this extension of the wire?

   [Ans. (16.023×10^{-3} J)]

3. A steel wire, of length 4m, is stretched through 2mm. The cross-sectional area of the wire is $2.0mm^2$. If Young's Modulus of steel is 2.0×10^{11} N/m^2, find (i) the energy density of the wire and (ii) the elastic potential energy stored in the wire.

   [Ans. (2.5×10^4 J/m^3, 0.2 J)]

4. A load of 31.4kg is suspended from a wire of radius 10^{-3}m and density 9×10^3 kg/m^3. Calculate the change in temperature of the wire if 75% of the work done is converted into heat. The Young's modulus and the specific heat capacity of the material of the wire are 9.8×10^{10} N/m^2, and 490 J/kg/K, respectively.

   $\left[\text{Ans.}\left(\frac{1}{120}\text{ K}\right)\text{or } 0.0083^{\circ}\text{C}\right]$

CRITICAL VELOCITY

UNIT – VII

The flow of a liquid, as we know, can be **(i)** stream lined or **(ii)** turbulent . Stream lined, or laminar flow, corresponds to an orderly, steady and systematic flow in which all particles, of the flowing liquid, crossing a particular point, have the same velocity, as their predecessors. When this velocity of flow keeps on changing in a random, haphazard or abrupt manner, the flow of the liquid is referred to as a turbulent flow.

The transition, from a stream lined flow to a turbulent flow, for a given liquid, flowing in a given 'container', takes place when its velocity of flow exceeds a certain limiting or critical value. This critical (transitional) velocity is referred to as its **critical velocity** (ϑ_c)

It was, in 1883, that Osborne Reynolds showed experimentally that the value of this critical velocity depends upon

(i) the coefficient of viscosity (η) of the flowing liquid,
(ii) the density, ρ, of the flowing, liquid, and
(iii) the diameter, d, of the pipe through which the liquid is flowing.

Hence $\vartheta_c = f(\eta, \rho, d)$

A simple use, of the methods of dimensional analysis, leads us to the result

$$\vartheta_c = R\frac{\eta}{\rho d}$$

The dimensionless constant R = $\dfrac{\rho d\,\vartheta_c}{\eta}$ has come to be known as the **Reynolds's number.**

The above result shows that the value of ϑ_c increases with **(i)** an increase in the value of η and **(ii)** a decrease in the values of ρ and d. Thus the flow of a more viscous, but less dense liquid, through a narrow pipe (preferably a capillary tube) is more likely to be stream lined. A lower value of the coefficient of viscosity, a higher density and a broader size of the 'container', all contribute towards making the flow turbulent at relatively low values of the velocity of flow. It is hardly surprising, therefore, that wherever experimental investigations, (under conditions of a streamlined flow, of a given liquid), have to be carried out, we always use a 'capillary tube' as the 'container' through which the liquid is made to flow.

EXERCISES

1. What should be the maximum average velocity of water in a tube of diameter 0.5cm, so that the flow is laminar? The viscosity of water is 0.00125 Nsm^{-2}.

   [Ans. (0.5 m/s)]

2. What should be the average velocity of a water in a tube of radius 0.005 m, so that the flow is just turbulent? The viscosity of water is 0.001 Pas.

   [Ans. (0.2 m/s)]

3. Water flows at a speed of 6cm/s through a tube of radius 1cm. Coefficient of viscosity of water at room temperature is 0.01poise. What is the nature of the flow?

   [Ans. {R<2000, (R = 1200) flow is likely to be laminar}.]

RELATION BETWEEN C_p AND C_v

UNIT VII

We know that the specific heat capacity, of a gas, can have any value, ranging from zero to infinity, depending on the conditions under which it is being heated. It is, therefore, meaningless to speak about the specific heat capacity of a gas without specifying the conditions under which it is being heated. Of the very many specific heat capacities that can be associated with a gas, there are two such capacities of a gas, that are of special significance. These are: **the specific heat capacity at constant Volume** (C_v) and that at **constant pressure** (C_p). They are defined as follows :

(i) The molar specific heat capacity, of a gas, at constant volume, (C_v), is the amount of heat energy required to raise the temperature of 1 mole of the gas, through 1K, when its volume has been kept constant.

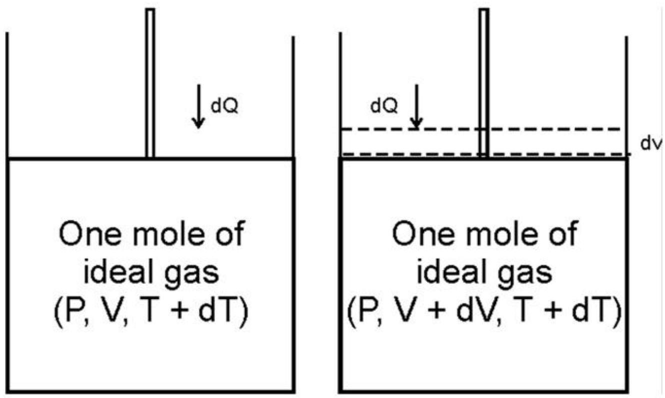
(ii) The molar specific heat, capacity of a gas, at constant pressure (C_p), is the amount of heat energy required to raise the temperature of 1mole of the gas, through 1K, when its pressure has been kept constant.

It is easy to realize that the specific heat capacity, of a gas at constant pressure, is greater than its specific heat capacity at constant volume. In other words, C_p is greater than C_v. This is because when heat is supplied, to a gas, keeping its volume constant, the gas would not do any work against external pressure. Hence all the energy, supplied to it, is used up in raising the temperature of the gas. However, when the gas is heated at constant pressure, its volume increases, The heat energy supplied in this case, is therefore, required not only for increasing the temperature but also for doing work against the external pressure. The gas, therefore, would need, more heat energy to heat it through 1K.

It is clear, therefore, that **C_p has to be greater than C_v.**

The difference, between the two, is the thermal equivalent of the work done in expanding the gas against the external pressure.

We can deduce the relation between the two by using the first law of thermodynamics. Consider one mole of an ideal gas enclosed in a cylinder fitted with a frictionless piston.



Heated at constant Volume **Heated at constant pressure**

Figure (i) (a) **Figure (i) (b)**

Let P, V and T denote the (initial) pressure, volume and (absolute) temperature of the gas. Let the gas be heated at constant volume so that its temperature goes up to T + dT. If the amount of heat energy, supplied, is denoted by dQ, we would have

$dQ = 1 x C_v \times dT = C_v\, dT$ (i)

From the first law of thermodynamics, we have

$dQ = dU + dW$ (ii)

Here dW (=PdV) = zero as there is no change in the volume of the gas. Hence

$dQ = C_v\, dT = dU$ (iii)

Now when the same gas is heated under constant pressure conditions, the heat energy, required to change its temperature by same amount dT, would be given by

$dQ = 1 \times C_p\, dT = C_p\, dT$ (iv)

Again applying the first law of thermodynamics, to this process, we now get

$C_p\, dT = dU + PdV$ (v)

Substituting the value of dU, from Eq (iii) in Eq (v)

$C_p\, dT = C_v\, dT + PdV$ (vi)

For one mole of an ideal gas, we have

$PV = RT$ (vii)

Differentiating this equation, (when P is constant), we get

$PdV = RdT$ (viii)

Hence $C_p\, dT = C_v\, dT + RdT$

$\therefore C_p = C_v + R$

or $C_p - C_v = R$ (ix)

Here C_p, C_v and R, all need to be measured in the same units, i.e, J mol^{-1} K^{-1}

Example: Find the value of C_v and C_p for nitrogen. (Given R = 8.3 J mol^{-1} K^{-1}; also, for a diatomic gas, $C_v = \left(\frac{5}{2}\right) R$.

Solution:

Nitrogen is known to exist as a diatomic molecule.

Hence C_v (for nitrogen) $= \frac{5R}{2} = \frac{5}{2} \times 8.3$ J mol^{-1} K^{-1}

$= 20.75$ J mol^{-1} K^{-1}

Further, $C_p = C_v + R$

$\therefore C_p$ (for nitrogen) = (20.75 + 8.3) J mol-1 K^{-1}

$= 29.05$ K mol^{-1} K^{-1}

EXERCISES

1. Calculate the specific heat capacity at constant volume for a gas. Given specific heat capacity at constant pressure is 6.85 cal mol^{-1} K^{-1}, R = 8.31 J mol^{-1} K^{-1} and J = 4.18 J cal^{-1}

   [Ans. (4.862 cal mol^{-1} K^{-1})]

2. The difference, between the two specific heat capacities (at constant pressure and volume) of a gas is 5000 J kg^{-1} K^{-1} and the ratio of these specific heat capacities is 1.6. Find the two specific heat capacities, i.e., C_v and C_p

   [Ans. (C_v = 8333. 33 J kg^{-1} K^{-1}) (C_p = 13333. 33 J kg^{-1} K^{-1})

3. Specific heat capacity, of argon, at constant pressure is 0.125 cal/g/K and at constant volume is 0.075 cal/g/K. Calculate the density of argon at STP. Given J = 4.18 J/cal and normal pressure = 1.01 x 10^5 Nm^{-2}.

   [Ans. (1.77 x 10^3 kg/m^3)]

BLACK BODY RADIATION

UNIT VII

The term radiation refers to the continual emission of energy from the surfaces of all bodies. This energy is referred as radiant energy and usually corresponds to the infrared region of the electromagnetic spectrum.

All bodies emit radiations at all temperatures. However, the wavelengths emitted, are characteristic of the body's temperature. The earth, at an ideal radiation temperature of 255K, radiates energy mainly in the far end of the infra red (or thermal radiation) region of the electromagnetic spectrum. Thermal radiations (or infra red rays, with wavelengths ranging from about 800nm to about 400000nm) produce heat when they are absorbed by a body. It is this characteristic that distinguishes thermal radiation from the rest of the electrometric spectrum.

Emissive power and absorptive power:

The amount of heat energy, radiated by a body per second, depends upon **(i)** the area of its surface **(ii)** its surface temperature and **(iii)** the nature of its surface. For a given surface, experimental results show that, at a particular temperature, and for a particular wavelength range [say λ to $(\lambda + d\lambda)$], the amount of radiant energy emitted, per unit area of a surface per unit time, and per unit wavelength range, is a constant. We call this constant as the **emissive power** (e_λ) of the given surface at the given temperature and wavelength. This implies that for a (given) surface, of emissive power e_λ, the amount of radiant energy emitted in the wavelength range [λ to $(\lambda + d\lambda)$], per second per unit area = $e_\lambda \, d\lambda$. The S.I unit of emissive power is J s^{-1} m^{-2} or Wm^{-2}.

When radiation is incident over a surface, a part of it gets reflected, a part of it get refracted and the rest is absorbed We know, from our (daily life) experience, that a bright polished surface reflects most of the radiation incident upon it, whereas a rough black surface absorbs most of radiation falling on it. Thus the degrees of reflection, transmission and absorption are different, for different surfaces.

To account for the energy absorbed, we define a term called the **absorptive power** of the surface.

The **absorptive power**, a_λ, of a surface, at a given temperature, and for a wavelength (λ), is the ratio of the amount of heat energy absorbed by the surface in a certain time, to the total energy incident on it, in the same time, within a narrow wavelength range, around the wavelength λ. This implies that if a total radiant energy say 'dQ', in the wavelength range λ to $(\lambda + d\lambda)$, is incident, per unit time, on a surface of absorptive power a_λ, the amount of radiant energy absorbed by this surface, in unit time equals $a_\lambda\, dQ$.

a_λ is clearly a dimensionless quantity, as it is defined as the ratio of two similar physical quantities.

Black Body – Suppose a beam of isotropic (i.e., uniform in all directions) thermal radiation is incident on a surface. Let 'a' be the fraction of the total thermal energy of all wavelengths absorbed by this surface. The remaining fraction is either reflected (r) or transmitted (t). Then, from law of conservation of energy,

$$a + t + r = 1$$

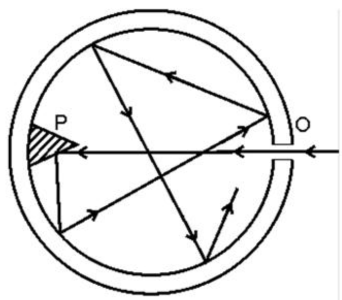
An (ideal) object, whose absorptivity is unity (a = 1), is knows as a **perfect black body.** Hence, a **perfect black body** is one which **neither reflects nor transmits,** but absorbs all the thermal radiations incident on it, irrespective of their wavelengths. Such an object would, therefore appear black to the human eye. No object, in actual practice, can be a perfect absorber, i.e., a perfect black body. Lamp black, or platinum black, is the nearest approach to such a body. It absorbs as much as 95% to 97% of the radiations incident on it. However, we can design a body which is almost 100% black. **Ferry's black body** is one such body.

BLACK BODY RADIATION – ITS SPECTRUM

In1859, Kirchoff suggested an excellent idea to design a near perfect black body. He suggested that a large enclosure, with a small hole, on one of its sides, would be an excellent absorber. This is because any radiation, going inside the enclosure, through the hole, would get repeatedly reflected around inside it with a part of it getting absorbed on each reflection. The in going radiation would, therefore, have hardly any chance of coming, out. It would, therefore, be getting (almost) completely absorbed inside the enclosure. **Ferry** designed a **'black body'** using this idea, at a later date.

Ferry's black body consists of a hollow enclosure, with a narrow opening very small in size compared to the size of the enclosure. A conical projection was located inside it just opposite to the hole.

The 'system' acts as an almost perfect absorber. Radiation, entering the hole, suffers innumerable diffuse reflections and absorptions at the inner walls and has a negligible chance of coming out. It thus (eventually) gets absorbed (almost) fully, irrespective of the material of the walls of the enclosure.



Fery's black body

Figure (i)

When such an enclosure is heated in a suitable bath (that can maintain its temperature), the radiation, coming out from the small hole, (the cavity radiations) are akin to **'black body radiations'.** The most significant feature, of these black body radiations, is that the wavelength range, of the emitted radiation, is independent of the size, shape or material of the enclosure and depends only on the temperature of the 'black body'.

Having thus succeeded in designing a (near) perfect black body, experiments were carried out to study the distribution of energy, among the different wavelengths, emitted by the black body. This study was carried out for different temperatures of the black body.

The general shape of these experimental curves, for radiation energy, emitted by a black body over different wavelengths, (for different temperatures of the black body), is as shown here.

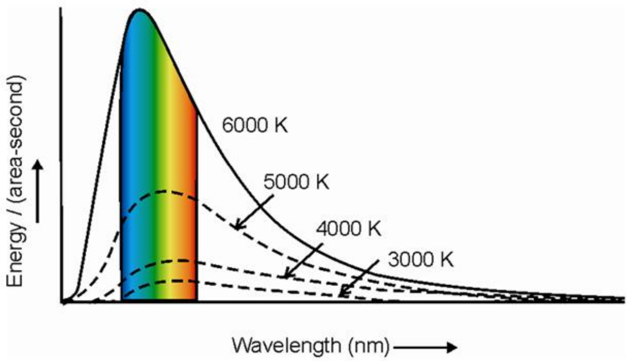


Figure (ii)

A study of these curves reveals the following interesting features:

1. At a given temperature, the (emitted) energy is not uniformly distributed, i.e., the energy, for different wavelengths, is different.
2. At a given temperature, the energy of emission has a maximum value for a particular wavelength, λ_m. With increase in temperature, this wavelength, of maximum emission, keeps on decreasing. In other words, 'maximum', of each curve, shifts towards shorter wavelengths as the temperature is raised.
3. The total area, under the curve, shows a (large) increase even for a small increase in temperature. This area, under each curve, would represent the total energy emitted (over all the wavelengths) at a particular temperature. It follows, therefore, that the total energy emitted, increases quite rapidly with an increase in temperature.

WEIN'S DISPLACEMENT LAW:- We have already noticed that as the temperature increases, the peak (maximum), of the radiant energy, emitted by the black body, moves towards shorter wavelengths. In 1893, Wien analysed the problem theoretically Applying the principles of thermo dynamics, he found that the product of the peak wavelength (λ_m) and the Kelvin temperature (T) of the black body should remain constant. Thus

$$\lambda_m T = b \qquad \text{(vii)}$$

where b is a constant. This result is known as **Wien's displacement Law.** According to this law : The wavelength (λ_m), for which the radiant energy emitted by a blackbody is a maximum, is inversely proportional to the Kelvin temperature (T) of the black body. Thus

$$\lambda_m \ \alpha \frac{1}{T}$$

or $\lambda_m T = b$

The constant ,b, here, has come to be known as Wien's constant. Its experimentally obtained value is (nearly) 2.898×10^{-3} m K.

Applications of Wien's law:

Wien's law helps us to understood why the colour , of a piece of iron, heated over a flame, first becomes dull red, then reddish yellow and finally white hot. As the temperature of iron increases, the wavelength of maximum emission, becomes smaller and smaller. This leads to the observed changes in colour.

This law is also used for estimating the surface temperatures of celestial bodies such as the stars and the moon.

Example 1 : Light, from the moon, is found to have a peak (or wavelength of maximum emission) at $\lambda = 14\ \mu$m. Given that the Wien's constant (b) equals 2.898×10^{-3} mK, estimate the temperature of the moon.

Solution : Using Wien's law ($\lambda_m T=b$), we get

$$T = \frac{2.898\text{mk} \times 10^{-3}}{14 \times 10^{-6}} \simeq 207\,\text{K}$$

Thus the surface temperature of moon is about 207 K.

STEFAN'S LAW:

We have seen above that the experimental observations, on black body spectra, show that the total thermal energy emitted, (by the given black body), increases (quite fast) with an increase in its temperature.

On the basis of experimental measurements, it was found that the thermal radiation energy emitted per second (i.e., power radiated), from the surface of a body, is proportional to its surface area A and to the fourth power of its absolute temperature T.

For a perfect radiator (or a perfect black body), the energy emitted per unit time (H), can, therefore, be expressed as

$H = A\,\sigma\,T^4$, where σ is a universal constant (viii)

This relation, referred to as **Stefan's law,** was obtained experimentally by Stefan. It was later proved theoretically by Boltzmann and is now known as the **Stefan – Boltzmann's Law.** The constant σ is called Stefan-Boltzmann constant. Its value, in S.I. units is $5.67 \times 10^{-8}\ Wm^{-2}\ K^{-4}$

For any object, which is not perfectly black, the energy emitted (at a given temperature, T) is **less** than that given by Eqn (viii). For such objects, we define a dimensionless quantity ε, known as its, emission coefficient or degree of blackness. In general, value of ε would lie between 0 and 1. ε =1 for a perfectly black body. Hence the energy emitted, per unit time, by an object, of emission coefficient ε, would be given by

$H = A\,\varepsilon\,\sigma\,T^4$ ____ (9)

Example 2 : Determine the surface area of the filament of a 100W incandescent lamp radiating out its labelled power at 3000K. Given $\sigma = 5.7 \times 10^{-8}\ Wm^{-2}\ K^{-4}$, and emissivity '$\varepsilon$' of the material of the filament = 0.3.

Solution : We have $H = A\,\varepsilon\,\sigma\,T^4$

$$\therefore A = \frac{H}{\varepsilon\sigma T^4} = \frac{(100W)}{0.3 \times \left(5.7 \times 10^{-8}\ W\,m^{-2}\,K^{-4}\right) \times (3000K)^4}$$

$= 7.25 \times 10^{-5}\ m^2$

APPROXIMATE FORM OF STEFAN - BOLTZMANN LAW

Consider an object, at an absolute temperature T, kept in surroundings, at a temperature, T_o ($T > T_o$).Regarding both the object and surroundings as perfectly black, we have

H_1 = rate of emission of energy by the body

$= A\ \sigma\ T^4$

H_2 = rate of energy absorbed by the body from the surroundings

= rate of emission of energy by the surroundings

$= A\ \sigma\ T_0^4$

H = Net rate of loss of energy of the body

$= H_1 - H_2 = A\ \sigma\ (T^4 - T_0^4)$

For small values of (T-T_o); i.e. small differences of temperature, between the body and the surroundings, it can be easily seen that $H\ \alpha\ (T\text{-}T_o)$. This result corresponds to the well known **Newton's law of cooling**.

Example 3 : A man, the surface area of whose skin is $2m^2$, is sitting in a room where air temperature is 20^oC. If his skin temperature is 28^oC and emissity of his skin equals 0.97, find the rate at which his body loses heat. $\left(\text{Given } \sigma = 5.67 \times 10^{-8}\ Wm^{-2}\ K^{-4}\right)$

Solution : We have

$\mathbf{H} = \sigma\ \varepsilon\ A\ (T^4 - T_0^4)$

Here $T = (28 + 273)K = 310K$

$T_o = (20 + 273)K = 293K$

$\therefore H = (5.67 \times 10^{-8}\ Wm^{-2}\ K^{-4}) \times 0.97 \times 2m^2\ [(301\ K)^4 - (293K)^4]$

$\simeq$ 9 2.2 W

Here, since $(T - T_o)$ is small, let us also calculate H by using the approximate form of Stefan - Boltzmann law. We have

$H \simeq \sigma \; \varepsilon \, A \, (T - T_0)$

$= 5.67 \times 10^{-8} \times 0.97 \times 2 \times 17$ W

~ 88 W

The approximate value is thus (nearly) 4.3% lower than its more exact value.

<u>"THE SOLAR CONSTANT" AND SURFACE TEMPERATURE OF THE SUN</u>

The radiations emitted by the sun are absorbed by the earth. The energy absorbed per unit area, per unit time on earth varies with time. However, the average energy absorbed per unit area, per minute, by the earth is a constant. This is known as the solar constant (S). ($S = 8.135$ J m^{-2} min^{-1}). Knowing the value of S, we can estimate the surface temperature of the Sun.

Let R denote the radius of sun. We consider the earth to be moving in a circular orbit of radius r, around the sun.

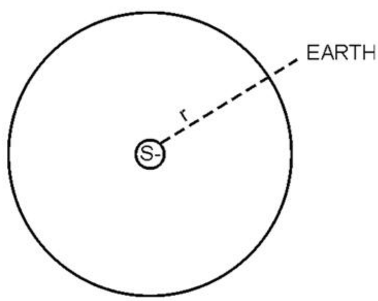


Figure (iii)

Regarding sun as a perfectly black body, the energy radiated per second by the sun = H = $\left(4\pi R^2\right)\sigma\, T^4$, where T is the surface temperature of the sun. This energy is radiated equally in all directions. The earth lies on the surface of a sphere of radius r with the sun as its centre. The total energy, crossing the entire surface of this sphere, per second, is H. Hence the energy received, per unit area, per second by the earth

$$= \frac{H}{4\pi r^2} = \frac{(4\pi R^2)\ \sigma T^4}{4\pi r^2} = \left(\frac{R}{r}\right)^2 \sigma T^4$$

Hence the Solar constant

$$S = \left(\frac{R}{r}\right)^2 \sigma T^4 \times 60$$

Knowing R, r, σ and S, T can be calculated from this equation. Taking R = 7×10^8m, r = 1.5×10^{11}m

S = 8.135×10^4 Jm^2 min^{-1}, σ = 5.77×10^{-8} Wm^{-2} K^{-1},` we get

T $\approx$ 5700 K

The actual temperature of the sun would be more than this estimated value because the sun is not a perfectly black body.

GREEN HOUSE EFFECT:

The greenhouse effect is a naturally occurring process that aids in heating the earth's surface and atmosphere. It results from the fact that certain atmospheric gases, such as carbon dioxide, water vapour and methane, (collectively known as green house gases), are able to change the energy balance of the earth by absorbing long wave radiations emitted from the earth's surface.

The whole of solar radiations do not reach the earth; nearly 25% of them are reflected back into space by the clouds and the gases. About 25% more are absorbed by the atmospheric gases. Hence only about 50% of the (incoming) solar radiations are able to penetrate the earth's atmosphere. The majority of this energy is then used to heat the earth's surface. This (heated up) surface becomes a radiator of energy in the long wave (infrared radiation) band. Depending upon their concentration, the green house gases, trap these long wave radiations. A part of this energy is re-radiated back to the surface of earth to be returned back to the atmosphere for re-radiation. The cycle continues until no more long wave radiations are available for absorption. The amount of heat energy, added to the atmosphere, by the green house effect, is determined by the concentration of the green house gases in the earth's surface.

The downward flux, of long wave radiations by the green house gases, is called green house flux. This plays a very significant role in keeping the earth warm, at an average or mean temperature, around (nearly) 15° C. In the absence of this flux, the earths mean temperature would drop to -18°C. At such a low temperature , water will frezze and most of life on earth would not survive.

The above phenomenon, of keeping the earth warm, due to the presence of certain active gases (green house gases) in the atmosphere, is called the green house effect. The name is based on the existence of a similar warmer interior in a glass enclosed 'green house'. In such 'a green house', glass panes, carbon dioxide and water vapour, allow the short wavelength radiations to enter but 'prevent the escape' of long wavelength (infra / heat) radiations. (Glass is effectively opaque to heat). Such green houses are used for growing tropical plants.

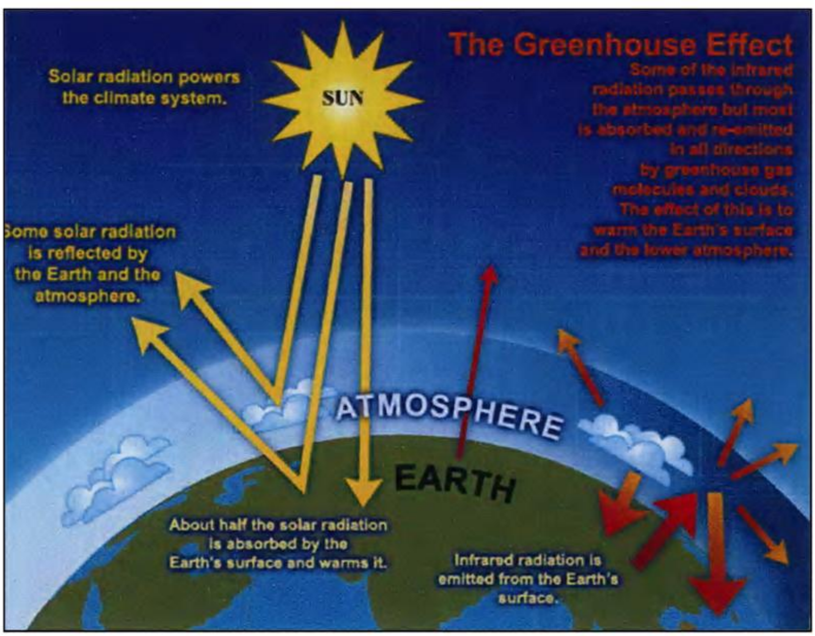


Figure (iv)

We are now in a position to appreciate the concern of 'environmentalists' about 'Global warming.

This concern is primarily due to 'human being-caused activities' which are resulting in an increase, in the percentage of carbon dioxide in the atmosphere. It is being feared that this increase in the concentration, of this most significant of the greenhouse gases, would push up the average temperature of the earth. This may cause the polar ice-caps to melt which can result in vast spread flooding and devastation. Scientists and governments, around the world, are, therefore, thinking of ways and means of minimizing, and avoiding, this possible danger to mankind and earth!

EXERCISES

1. Sketch the variation of total power radiated by a black body with respect to the absolute temperature of the black body.
2. A black body, at 2000K, emits maximum energy at a wavelength of 1.56 μm. At what temperature will it emit maximum energy at a wavelength of 1.8 μm?

   [Ans. 1670K]

3. A spherical black body with a radius of 12cm radiates 450 W power at 500K. What would be the power of radiation if radius were to be halved and the temperature doubled?

   [Ans.1800W]

4. The spectrum of a black body at two temperatures 27°C and 327°C is shown in the figure. Let A_1 and A_2 be the respective areas under the two curves. Estimate the ratio A_2 / A_1

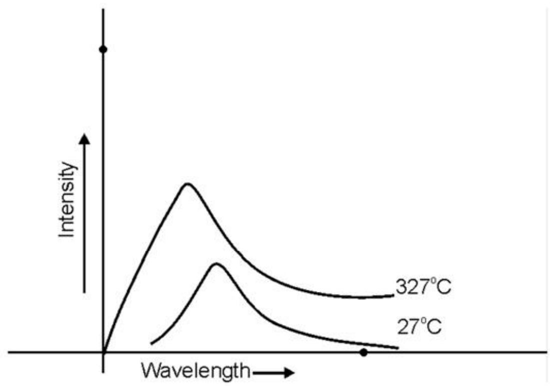


Figure (v)

[Ans. 16:1]

5. If each square meter, of sun's surface, radiates energy at the rate of 6.3 x $10^7 J/m^2/s$ and the Stefan's constant is 5.669 x 10^{-8} $W/m^2/K^4$, calculate the temperature of the sun's surface, assuming Stefan's law applies to the sun's radiation.

[Ans. 5.773K]

6. How much faster does a cup of tea cool by 1°C, when at 373K than when at 303K. Consider the tea as a black body. Take the room temperature as 293K and Stefan's constant as 5.7 x 10^{-8} $W/m^2/K^4$

[Ans. 11.3 times]

7. Two bodies, A and B, have thermal emisivities of 0.01 and 0.81 respectively. The outer surface area as of both the bodies are same. The two emit the same total radiated power. The wavelength λ_B, corresponding to the maximum intensity in radiations from B, is shifted from the wavelength λ_A, corresponding to the maximum intensity in radiations from A, by 1 μm. If the temperature of body A is 5802K, find the temperature of body B and the wavelength λ_B.

   [Ans. 1934 K, 1.5μm]

8. The tungsten filament, of an electric lamp, has a length of 0.25m and a diameter of 6 x 10^{-5}m. The power rating of the lamp is 100W. If the emissivity of the filament is 0.8, estimate the steady temperature of the filament. Stefan's constant = 5.67 x 10^{-8} W/m^2/K^4.

   [Ans. 2616 K]

9. "Good reflectors are poor emitters of thermal radiation". Explain.

10. If the earth did not have an atmosphere, it would become intolerably cold. Why?

SPEED OF A WAVE MOTION

UNIT - X

The speed of a mechanical (elastic) wave, is clearly expected to depend upon the properties of the medium through which the wave is travelling. The medium could be a solid, liquid or a gas. All media are characterized by their physical properties, like mass, density, elasticity, and temperature. Any alternation, in the properties of the medium, would cause a change in the speed of propagation of a wave through it. For example, an increase, in the tension of a spring, increases the wave speed through it. Hence one can say that the speed, of a (given type of) mechanical wave, in a medium, depends upon the properties of the medium.

For given type of waves (in a given medium), this speed, however does not change with a change in the characteristics (amplitude, wave length or frequency) of the wave. For example, the speeds of different types of sound waves, infrasonic, audible or ultrasonic, in a given medium, are all given by

$$\vartheta = \sqrt{\frac{E}{\rho}}$$

where E is the (appropriate) elastic constant and ρ is the density of the medium. **The speed is thus dependent only on the properties of the medium.**

It is interesting to note that the speed of propagation of electromagnetic waves is also determined by the 'characteristics' of the medium through which they are propagating. E. M. waves, as we know, are a 'combination' of the oscillations of electric and magnetic fields in mutually perpendicular directions. The relevant 'properties' of the medium, determining their speed of propagation, are the 'permittivity' and the 'permeability ' of the medium. In vacuum, the speed of propagation (c) of all types of e.m. waves, is given by

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

Here μ_o = permeability of vacuum and ε_o = permittivity of vacuum.

In any other material medium, the speed of propagation (ϑ) of e.m. waves, is given by $\vartheta = \frac{1}{\sqrt{\mu \varepsilon}}$

where μ = permeability of the medium and ε = permittivity of the medium. It however, turns out that the values of μ and ε, of a material medium, are not quite the same for e.m. waves of different frequencies. The speed of propagation of these different waves, is, therefore, not quite the same and this leads to the well known 'dispersion' phenomenon – resulting in the formation of 'spectra'.