MATHEMATICS (041)
MARKING SCHEME
CLASS XII (2026-27)

SECTION: A (Solution of MCQs of 1 Mark each, no part marks)

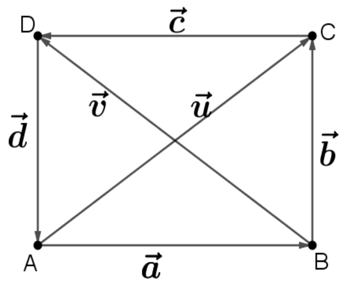
Q. No.	ANS	HINTS/SOLUTION	Marks
1.	**(D)**	$\pi + \cos^{-1} x$	**1**
	(D)	**For Visually Impaired:** $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{\pi\}$	**1**
2.	**(D)**	$\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$	**1**
3.	**(A)**	$\cot^{-1}\left(2\cos\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right) = \cot^{-1}\left(2\cos\left(\frac{1}{2}\times\frac{\pi}{3}\right)\right)$ $= \cot^{-1}\left(2\cos\left(\frac{\pi}{6}\right)\right) = \cot^{-1}\left(2\times\frac{\sqrt{3}}{2}\right) = \cot^{-1}(\sqrt{3}) = \frac{\pi}{6}$	**1**
4.	**(B)**	$3a = 1, b = -2, 2c = 4 \Rightarrow \sqrt{6a + 2b + 3c} = \sqrt{2 - 4 + 6} = \sqrt{4} = 2$	**1**
5.	**(C)**	$\lvert adj(PQ)\rvert = \lvert PQ\rvert = \lvert P\rvert\lvert Q\rvert = 2 \times 3 = 6$ ($\because$ order of P and $Q = 2 \times 2$)	**1**
6.	**(D)**	$[1 \quad x]\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}\begin{bmatrix}4\\-1\end{bmatrix} = O \Rightarrow [1 \quad 2x]\begin{bmatrix}4\\-1\end{bmatrix} = O \Rightarrow 4 - 2x = 0 \Rightarrow x = 2$	**1**
7.	**(A)**	Let $y = \cos^{-1}(2x^2 - 1)$ and $u = \cos^{-1}(x)$ $\Rightarrow y = \cos^{-1}(2cos^2u - 1)$ as $\cos u = x$ $\Rightarrow y = \cos^{-1}(\cos(2u)) = 2u$ $\Rightarrow \frac{dy}{du} = 2$	**1**
8.	**(D)**	$f(x)$ is differentiable $\Rightarrow f(x)$ is continuous $\Rightarrow c = 0$ and $a = 2b$ $\Rightarrow (a, b, c)$ becomes $(2b, b, 0)$ which is satisfied by option (D) $(3, \frac{3}{2}, 0)$ only	**1**
9.	**(D)**	y is decreasing $\Rightarrow \frac{dy}{dx} < 0 \Rightarrow a\{1 + 2\cos\left(\frac{x}{a}\right)\} < 0 \Rightarrow \cos\left(\frac{x}{a}\right) < -\frac{1}{2}$ $\Rightarrow \frac{x}{a} \in \left(\frac{2\pi}{3}, \pi\right] \Rightarrow x \in \left(\frac{2\pi a}{3}, \pi a\right]$	**1**
10.	**(A)**	$\frac{dy}{dx} > y \Rightarrow 4x^3 > x^4 \Rightarrow x^3(4 - x) > 0 \Rightarrow x(x - 4) < 0 \Rightarrow 0 < x < 4$	**1**
11.	**(B)**	The order is 2 and the term containing $\frac{d^2y}{dx^2}$ has the maximum power 2 in the equation, so the degree is 2	**1**
12.	**(D)**	$\int \frac{1}{1+e^{2x}}dx = \int\left(\frac{1+e^{2x}}{1+e^{2x}} - \frac{e^{2x}}{1+e^{2x}}\right)dx = x - \frac{1}{2}log\lvert 1 + e^{2x}\rvert + C$ So $P = \frac{1}{2}$	**1**
13.	**(C)**	Magnitude of $2\hat{\imath} - u\hat{\jmath} + 3\hat{k}$ = 4 $\Rightarrow \sqrt{4 + u^2 + 9} = 4$ $\Rightarrow u^2 = 16 - 13 = 3 \Rightarrow u = \pm\sqrt{3}$	**1**
14.	**(B)**	Direction cosines of the vector $\hat{\imath} + \hat{\jmath} + \sqrt{2}\,\hat{k}$ are $\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$ Let angle = γ. Then $\cos\gamma = \frac{1}{\sqrt{2}} \Rightarrow \gamma = \frac{\pi}{4}$	**1**
15.	**(B)**	Both the vectors $\hat{\imath} + \hat{\jmath}$ and $\hat{\imath} - \hat{\jmath}$ are in XY-plane So a vector perpendicular to both is $3\hat{k}$	**1**

16.	**(D)**	The probability that it rains on a particular day is $\frac{1}{5}$. Probability that it rains on three consecutive days = $\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}=\frac{1}{125}$	**1**
17.	**(D)**	Let the events be A: sum of the numbers appeared is a multiple of 5 B: at least one number is 4 $A=\{(1,4),(2,3),(3,2),(4,1),(4,6),(5,5),(6,4)\}$ $A\cap B=\{(1,4),(4,1),(4,6),(6,4)\}$ So required probability = $P(B\|A)=\frac{4}{7}$	**1**
18.	**(A)**	Probability of occurrence of any one of them is $\frac{1}{3}$ $\Rightarrow P(A'\cap B)+P(A\cap B')=\frac{1}{3}\Rightarrow P(A')P(B)+P(A)P(B')=\frac{1}{3}$ $\Rightarrow \left(1-\frac{1}{6}\right)k+\frac{1}{6}(1-k)=\frac{1}{3}\ \Rightarrow k\left(\frac{5}{6}-\frac{1}{6}\right)=\frac{1}{3}-\frac{1}{6}\Rightarrow k=\frac{1}{6}\times\frac{6}{4}=\frac{1}{4}$	**1**
19.	**(B)**	Both assertion and reason are true, but reason is not the correct explanation of the assertion $\int_{-1}^{1}\|x-2\|\,dx=\ \int_{-1}^{1}(-x+2)\,dx=4$	**1**
20.	**(A)**	Both assertion and reason are true, and reason is the correct explanation of the assertion.	**1**

Section –B

[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

21(A).	$I=\int\frac{1}{x\sqrt{x^6-1}}dx=\frac{1}{6}\int\frac{6x^5}{x^6\sqrt{x^6-1}}dx$ Let $x^6-1=t^2$ $\Rightarrow 6x^5dx=2tdt$ $I=\frac{1}{3}\int\frac{1}{(t^2+1)}dt=\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)+C$	 **½** **1+½**
	OR	
21(B).	Let $I=\int\frac{cos\,x-\sin\,x}{\sqrt{\sin(2x)}}\,dx=\int\frac{cos\,x-\sin\,x}{\sqrt{1+\sin(2x)-1}}\,dx=\int\frac{cos\,x-\sin\,x}{\sqrt{(sinx+cosx)^2-1}}\,dx$ Let $sinx+cosx=t\quad\Rightarrow(\cos x-\sin\,x)dx=dt$ $I=\int\frac{1}{\sqrt{t^2-1}}\,dt=log\left\|t+\sqrt{t^2-1}\right\|+C$ $=log\left\|sinx+cosx+\sqrt{(sinx+cosx)^2-1}\right\|+C$ $=log\left\|sinx+cosx+\sqrt{\sin(2x)}\right\|+C$	**½** **½** **½** **½**
22.	$y=\sin(x+y)\Rightarrow\sin^{-1}y=x+y$ $\Rightarrow\frac{1}{\sqrt{1-y^2}}=\frac{dx}{dy}+1\ \Rightarrow\ \frac{dx}{dy}=\frac{1-\sqrt{1-y^2}}{\sqrt{1-y^2}}$ $\Rightarrow\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{1-\sqrt{1-y^2}}$ So, $\frac{dy}{dx}=0\ \ \Rightarrow y=\pm1$	**1** **1**

	Alternative way: $y = \sin(x+y) \Rightarrow \frac{dy}{dx} = \cos(x+y)\left(1+\frac{dy}{dx}\right)$ $\Rightarrow \frac{dy}{dx}(1-\cos(x+y)) = \cos(x+y)$ $\Rightarrow \cos(x+y) = 0$ as $\frac{dy}{dx} = 0$ $\Rightarrow \sin(x+y) = \pm 1 \Rightarrow \mathrm{y} = \pm 1$	**1** **1**
23(A).	Given points are $A(2,6,5), B(3,p,7), C(-4,3,4)$ and $D(-1,0,q)$ Direction ratios of the line $\overleftrightarrow{AB}$ are $\langle 1,\ p-6,\ 2\rangle$ and Direction ratios of the line $\overleftrightarrow{CD}$ are $\langle 3,-3,q-4\rangle$ As the lines are parallel, so we have $\frac{1}{3} = \frac{p-6}{-3} = \frac{2}{q-4}$ $\Rightarrow -1 = p-6\ and\ q-4 = 3\times 2$ $\Rightarrow p = -1+6 = 5,\ q = 6+4 = 10$ Thus, $\frac{q}{p} = \frac{10}{5} = 2$	**1** **½** **½**
	OR	
23 (B).	 Let vectors representing the four sides of the rectangle are as follows $\overrightarrow{AB} = \vec{a}, \overrightarrow{BC} = \vec{b}, \overrightarrow{CD} = \vec{c}, and\ \overrightarrow{DA} = \vec{d}$ Then $\overrightarrow{AC} = \vec{u} = \vec{a}+\vec{b}$ …(i)and $\overrightarrow{BD} = \vec{v} = \vec{b}+\vec{c} = \vec{b}-\vec{a}$ …(ii) Solving (i) and (ii), $\vec{a} = \frac{1}{2}(\vec{u}-\vec{v})\ and\ \vec{b} = \frac{1}{2}(\vec{u}+\vec{v})$ Also, $\vec{c} = -\vec{a} = \frac{1}{2}(\vec{v}-\vec{u})$ and $\vec{d} = -\vec{b} = -\frac{1}{2}(\vec{u}+\vec{v})$	**½** **1** **½**
24.	Given $\|\vec{a}\| = \sqrt{6}$, $\vec{a}\cdot\vec{b} = 1$ and $\vec{b}\times\vec{a} = 3\hat{\imath}+4\hat{\jmath}-2\hat{k}$ So $\left\|\vec{a}\times\vec{b}\right\| = \left\|\vec{b}\times\vec{a}\right\| = \sqrt{29}$ We know, $\left\|\vec{a}\times\vec{b}\right\|^2 + \left\|\vec{a}\cdot\vec{b}\right\|^2 = \|\vec{a}\|^2\left\|\vec{b}\right\|^2$ $\Rightarrow 29+1 = 6\left\|\vec{b}\right\|^2 \Rightarrow \left\|\vec{b}\right\|^2 = \frac{30}{6} = 5 \Rightarrow \left\|\vec{b}\right\| = \sqrt{5}$	**½** **½** **1**
25.	Constraints are $3x+2y \geq 10, x+2y \leq 6,$ $y \geq 0$	**1½** **½**

	For Visually Impaired: As maximum value exists at (p, q) and $\left(2p, \frac{q}{2}\right)$ So $Z\ at\ (p, q) = Z\ at\left(2p, \frac{q}{2}\right)$ $\Rightarrow 3p + 4q = 3 \times 2p + 4 \times \frac{q}{2}$ $\Rightarrow 3p + 4q = 6p + 2q$ $\Rightarrow 6p - 3p = 4q - 2q$ $\Rightarrow 3p = 2q$	**1** **1**

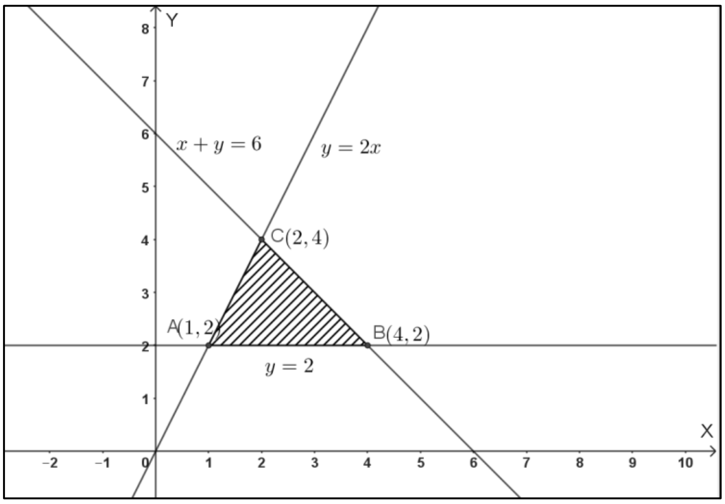
Section –C

[This section comprises of solution of short answer type questions (SA) of 3 marks each]

26(A)	Let $A = PQ$ and $B = QP$. Then $A = PQ = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$ $B = QP = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 4 \\ 2 & 0 & 0 \end{bmatrix}$ Here $\|A\| = 4 \neq 0$ So, A is a non-singular matrix, hence its inverse exists. Now $A^{-1} = \frac{1}{\|A\|} adj\ A = \frac{1}{4}\begin{bmatrix} 2 & 0 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ Here $\|B\| = 0$ So, B is a singular matrix, hence its inverse does not exist.	**1** **1** **½** **½**
27.	Let A be the area of the circular region covered by the drone and let r be its radius. Let h be the height of the drone at any time t. Given $\frac{dA}{dh} = 44\ m^2\ per\ metre$ and $\frac{dr}{dt} = 6\ m/s$ To find $\frac{dh}{dt}$ when r = 14 m. $A = \pi r^2 \quad \Rightarrow \frac{dA}{dt} = 2\pi r\frac{dr}{dt} \qquad \Rightarrow \frac{dA}{dh}.\frac{dh}{dt} = 2\pi r\frac{dr}{dt}$ $44\ \frac{dh}{dt} = 2(\frac{22}{7})6r \quad \Rightarrow \left.\frac{dh}{dt}\right]_{r=14m}$ = 12 m/s The height of the drone increases at the rate of 12 m/s at that moment.	**½** **1 ½** **1**
28(A).	$\frac{dx}{dt} = 3sin^2t\ cos\ t$ and $\frac{dy}{dt} = -3\ cos^2t\ sin\ t$ $\frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt} = -cot\ t$ $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right).\frac{dt}{dx} = cosec^2t\left(\frac{1}{3sin^2t\cos t}\right) = \frac{1}{3}(sec\ t.cosec^4t)$ At the point $\left(\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$, $sin^3t = \frac{1}{2\sqrt{2}} \Rightarrow t = \frac{\pi}{4}$ $\left.\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}} = \frac{4\sqrt{2}}{3}$	**1** **1** **1**

	OR	
28(B).	Let $y = x^{logx} + (logx)^x = u + v$ where $u = x^{logx}$ and $v = (logx)^x$ $log\,u = (log\,x)^2$ and $log\,v = x.log(log\,x)$ $\frac{1}{u}\frac{du}{dx} = 2(\,log\,x).\frac{1}{x}$ and $\frac{1}{v}\frac{dv}{dx} = log(log\,x) + \frac{1}{log\,x}$ $\frac{du}{dx} = x^{logx}\left(2(\,log\,x).\frac{1}{x}\right)$ and $\frac{dv}{dx} = (logx)^x\left(log(log\,x) + \frac{1}{log\,x}\right)$ $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{2}{x}\,x^{logx}\;log\,x + \;(logx)^x\left(log(log\,x) + \frac{1}{log\,x}\right)$ **Alternative Solution:** Since $x = e^{logx}$ $y = x^{logx} + (logx)^x = e^{\log(x^{logx})} + e^{\log((logx)^x)}$ $= e^{(\log x)^2} + e^{x\log(\log x)}$ $\frac{dy}{dx} = e^{(\log x)^2}\left[\frac{2\log x}{x}\right] + e^{x\log(\log x)}[\log(\log x) + x\left(\frac{1}{\log x}\right)\left(\frac{1}{x}\right)]$ $OR\;\; x^{logx}\left[\frac{2\log x}{x}\right] + (logx)^x[\log(\log x) + \left(\frac{1}{\log x}\right)]$	1 1 1 1 1+1
29(A).	$xdy + (y - x^2e^x)dx = 0$ is a linear differential equation given by $\frac{dy}{dx} + \frac{1}{x}\,y = xe^x$ I.F = $e^{\int\frac{1}{x}dx} = \;e^{log\,x} = x$ Solution is $y.x = \int x^2e^x\,dx$ using by parts $yx = x^2e^x - \int 2xe^x\,dx$ using by parts again $yx = x^2e^x - 2\{xe^x - e^x\} + C$ y (1) = 0 $\Rightarrow$ $C = -e$ Particular solution is $yx = x^2e^x - 2\{xe^x - e^x\} - e = e^x\{x^2 - 2x + 2\} - e$	½ ½ ½ ½ ½ ½
	OR	
29(B).	$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ is a homogeneous differential equation. Let $y = v\,x$ $\Rightarrow \frac{dy}{dx} = v + x\,\frac{dv}{dx}$ So, equation reduces to	

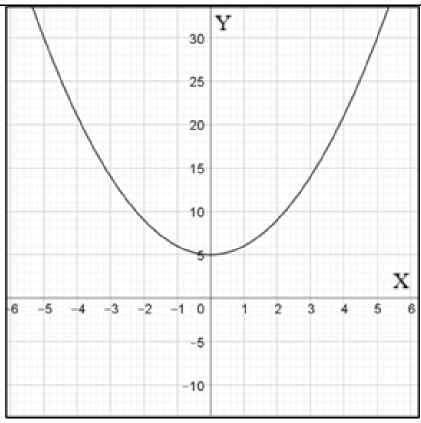
	$v + x\,\frac{dv}{dx} = \frac{v}{1+v^2} \quad \Rightarrow x\,\frac{dv}{dx} = \frac{v-v-v^3}{1+v^2}$ $\int \frac{1+v^2}{v^3}\,dv = -\int \frac{dx}{x}$ $\int (\frac{1}{v^3} + \frac{1}{v})\;\; dv = -log\,\lvert x\rvert + \log C \Rightarrow \frac{-1}{2v^2} + \log\lvert v\rvert = -\log\lvert x\rvert + \log C$ $\log\left\lvert\frac{y}{C}\right\rvert = \frac{x^2}{2y^2}$ or $y = C\,e^{\frac{x^2}{2y^2}}$ $C = 1$ when $x = 0$ and $y = 1$. The particular solution is $y = e^{\frac{x^2}{2y^2}}$	**1** **1** **1**
30(A).	A point on the line $x - 2 = \frac{2y+3}{4} = 2 - z = k\ is\ P\left(k+2, \frac{4k-3}{2}, 2-k\right)$ And direction ratios of the given line are $\langle 1, 2, -1\rangle$. Direction Ratios of the line joining the points $A(2,1,4)$ and this point P are $\langle k+2-2, \frac{4k-3}{2} - 1, 2-k-4\rangle = \langle k, \frac{4k-5}{2}, -k-2\rangle$ As $\overrightarrow{AP}$ is perpendicular to the given line, So $1 \times k + 2 \times \frac{4k-5}{2} + (-1) \times (-k-2) = 0$ $\Rightarrow k + 4k - 5 + k + 2 = 0 \Rightarrow 6k - 3 = 0 \Rightarrow k = \frac{1}{2}$ So, foot of the perpendicular is $P\left(k+2, \frac{4k-3}{2}, 2-k\right) = \left(\frac{1}{2} + 2, \frac{2-3}{2}, 2 - \frac{1}{2}\right) = \left(\frac{5}{2}, \frac{-1}{2}, \frac{3}{2}\right)$ Direction ratios of the line $\overleftrightarrow{AP}$ are $\langle k, \frac{4k-5}{2}, -k-2\rangle = \langle \frac{1}{2}, -\frac{3}{2}, -\frac{5}{2}\rangle$ or $\langle 1, -3, -5\rangle$ And equation of $\overleftrightarrow{AP}$ is $\frac{x-2}{1} = \frac{y-1}{-3} = \frac{z-4}{-5}$ OR $x - 2 = \frac{1-y}{3} = \frac{4-z}{5}$	½ ½ ½ ½ ½ ½
	OR	
30(B).	Given lines are $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$ and $\frac{x-5}{2} = y - \frac{1}{2} = \frac{3-z}{2} \Rightarrow \frac{x-5}{2} = \frac{y-\frac{1}{2}}{1} = \frac{z-3}{-2}$ Let the angle between the lines be θ, then $\cos\theta = \frac{2\times2+2\times1+1\times(-2)}{\sqrt{9}\times\sqrt{9}} = \frac{4}{9} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{9}\right)$ Now required line will be in the direction of the vector $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 2 & 1 \\ 2 & 1 & -2 \end{vmatrix} = -5\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$ So required line is $\frac{x-1}{-5} = \frac{y-1}{6} = \frac{z-1}{-2}$ OR $\frac{x-1}{5} = \frac{y-1}{-6} = \frac{z-1}{2}$	½ 1 1 ½

31.	 Feasible region is bounded with three vertices $A(1,2), B(4,2), C(2,4)$ Now $Z_A = 100, Z_B = 220, Z_C = 200$ So, Maximum Value of Z is 220 at the point $B(4,2)$.	**1 ½** **1** **½**
	For Visually Impaired: The vertices of the feasible region are $A(20,0), B(40,0), C(30,10),\ D(0,20)$ and $E(0,10)$. Now $Z_A = 50 \times 20 - 20 \times 0 = 1000$ $Z_B = 50 \times 40 - 20 \times 0 = 2000$ $Z_C = 50 \times 30 - 20 \times 10 = 1300$ $Z_D = 50 \times 0 - 20 \times 20 = -400$ $Z_E = 50 \times 0 - 20 \times 10 = -200$ Minimum value of Z is -400 at $D(0,20)$ Maximum Value of Z is 2000 at $B(40,0)$	**2** **1**

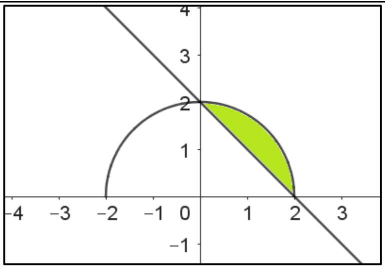
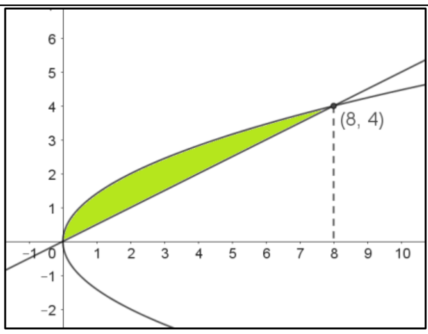
Section –D

[This section comprises of solution of long answer type questions (LA) of 5 marks each]

32(A).	Given relation R is defined on $A = N \times N$ such that $(a,b)R(c,d)\ if\ and\ only\ if\ a^2 + d^2 = b^2 + c^2$ Reflexive: Let $(p,q) \in A = N \times N$ Then as $p,q \in N$, so $p^2 + q^2 = q^2 + p^2$ $\Rightarrow (p,q)R(p,q)\ \forall (p,q) \in N \times N$ $\Rightarrow R$ is a reflexive relation. Symmetric: Let $(p,q)\ and\ (r,s) \in A = N \times N$ such that $(p,q)\ R\ (r,s)$ $\Rightarrow p^2 + s^2 = q^2 + r^2$ $\Rightarrow q^2 + r^2 = p^2 + s^2$ $\Rightarrow r^2 + q^2 = s^2 + p^2$ $\Rightarrow (r,s)\ R\ (p,q)$ Thus, R is a symmetric relation. Transitive:	**1 ½** **1 ½**

	Let $(p,q),(r,s),(u,v) \in A$ such that $(p,q)\ R\ (r,s)\ and\ (r,s)R(u,v)$ $\Rightarrow p^2+s^2=q^2+r^2$ ... (i) and $r^2+v^2=s^2+u^2$ ... (ii) Adding (i) and (ii), $p^2+s^2+r^2+v^2=q^2+r^2+s^2+u^2$ $\Rightarrow p^2+v^2=q^2+u^2$ $\Rightarrow (p,q)\ R\ (u,v)$ Thus, R is a transitive relation So, R is a reflexive, symmetric and transitive relation. Hence R is an equivalence relation	1 ½ ½
	OR	
32(B).	 Yes, R is a function as the graph shows every element of the set $(-\infty,\infty)$ has a unique image in the set $[0,\infty)$. Now, let us denote this function by $f:(-\infty,\infty)\rightarrow[0,\infty)$ as $f(x)=x^2+5$ Injective Function: Here $f(-1)=(-1)^2+5=6$ and $f(1)=1^2+5=6$ So $f(-1)=f(1)=6$ Thus f is a many-one function, so not an injective function. Surjective Function: Range of $f(x)=[5,\infty)\neq$ co-domain$=[0,\infty)$ So, it is not surjective. **Alternate Method:** Correct graph as above Yes, R is a function as the graph shows every element of the set $(-\infty,\infty)$ has a unique image in the set $[0,\infty)$. Now, let us denote this function by $f:(-\infty,\infty)\rightarrow[0,\infty)$ as $f(x)=x^2+5$ Injective Function: Let $a,b\in(-\infty,\infty)$, such that $f(a)=f(b)$ $\Rightarrow a^2+5=b^2+5\Rightarrow a^2=b^2\Rightarrow a=\pm b$ So f is not injective. Surjective Function: For $y=1$ there is no pre-image as if $1=x^2+5\Rightarrow x^2=-4$ not possible. So, it is not surjective. **For Visually Impaired:** For every element a of the set $(-\infty,\infty)$, $a^2\in[0,\infty)$ and therefore $a^2+7\in[7,\infty)$ Hence every element of the set $(-\infty,\infty)$ has a unique image in the set $[0,\infty)$.	**1 for correct graph** **1** **1 ½** **1 ½** **1** **1** **1 ½** **1 ½**

	So, $f:(-\infty,\infty)\to[0,\infty)$, $f(x)=x^2+7$ is a function. Now, let us denote this function by $f:(-\infty,\infty)\to[0,\infty)$ as $f(x)=x^2+7$ Injective Function: Here $f(-1)=(-1)^2+7=8$ and $f(1)=1^2+7=8$ So $f(-1)=f(1)=8$ Thus f is a many-one function, so not an injective function. Surjective Function: Range of $f(x)=[7,\infty)\neq$ co-domain$=[0,\infty)$ So, it is not surjective. Thus $f(x)$ is neither injective nor surjective and hence not bijective	1 1 ½ 1 ½ 1
33.	Let $I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx$ $I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}\times\frac{a-x}{a-x}}\,dx$ $=\int_{-a}^{a}\frac{a-x}{\sqrt{a^2-x^2}}dx=\int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}}dx-\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx=I_1-I_2$ $I_1=a\int_{-a}^{a}\frac{1}{\sqrt{a^2-x^2}}dx$ $=2a\int_{0}^{a}\frac{1}{\sqrt{a^2-x^2}}dx$ using $\int_{-a}^{a}f(x)\,dx=2\int_0^a f(x)\,dx$ if $f(-x)=f(x)$ $I_1=2a\left[\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a=2a\left(\frac{\pi}{2}\right)=\pi a$ and $I_2=\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx=0$ using $\int_{-a}^{a}f(x)\,dx=0$ if $f(-x)=-f(x)$ $I=I_1-I_2=\pi a$ **Alternate Method:** Let $x=a\cos 2\theta \;\Rightarrow dx=-2\,a\sin(2\,\theta)\,d\theta$ When $x=-a,\theta=\frac{\pi}{2}$ and $x=a,\theta=0$ $I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx=-2\,a\int_{\frac{\pi}{2}}^{0}\tan\theta\sin(2\theta)\,d\theta$ $I=2\,a\int_0^{\frac{\pi}{2}}\tan\theta\sin(2\theta)\,d\theta$ ....(i) using $\int_a^b f(x)dx=-\int_b^a f(x)dx$ $I=2a\int_0^{\frac{\pi}{2}}\tan\left(\frac{\pi}{2}-\theta\right)\sin(\pi-2\,\theta)d\theta$ using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ $I=2a\int_0^{\frac{\pi}{2}}\cot(\theta)\sin(2\theta)\,d\theta$ ....(ii) Adding equations (i) and (ii) $2I=2a\int_0^{\frac{\pi}{2}}(\tan\theta+\cot\theta)\sin(2\theta)\,d\theta$ $\Rightarrow I=2a\int_0^{\frac{\pi}{2}}(sin^2\theta+cos^2\theta)\,d\theta$ $\Rightarrow I=2a\int_0^{\frac{\pi}{2}}1\,d\theta=2a\left(\frac{\pi}{2}\right)=\pi a$	 ½ 1 1 1 1 ½ ½ ½ 1 1 1 1

34(A).	 1 mark for correct figure $y = \sqrt{4-x^2}$ is a semi-circle with radius 2 units and centre (0,0) $x = 2 - y$ which is x + y = 2 is a line with intercepts 2 on each axis. Curves meet at (0,2) and (2,0) Area = $\int_0^2 \left(\sqrt{4-x^2} - (2-x)\right) dx$ $= [\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right) - 2x + \frac{x^2}{2}]_0^2$ $= \pi - 4 + 2 = (\pi - 2)$ sq. units	**1** **1** **1** **1** **1**
	OR	
34(B).	 1 mark for correct figure. $y^2 = 2x$ represents a parabola, whose points of intersection with the given line $x = 2y$ are $(0, 0)$, $(8, 4)$ Area = $\int_0^8 \left(\sqrt{2x} - \frac{x}{2}\right) dx$ $= \left[\sqrt{2}\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{4}\right]_0^8$ $= \frac{64}{3} - 16 = \frac{16}{3}$ sq. units	**1** **1** **1** **1** **1**
34(A).	**For Visually Impaired:** $25x^2 + 16y^2 = 400$ is an Ellipse with the standard equation as $\frac{x^2}{16} + \frac{y^2}{25} = 1$ $\Rightarrow y = \frac{5}{4}\sqrt{4^2 - x^2}$ Area = $4\int_0^4 \left(\frac{5}{4}\sqrt{4^2-x^2}\right) dx$ $= 5\int_0^4 \left(\sqrt{4^2-x^2}\right) dx$ $= 5[\frac{x}{2}\sqrt{16-x^2} + 8\sin^{-1}\left(\frac{x}{4}\right)]_0^4$ $= 5[4(\pi)] = 20\pi$ sq. units	**½** **1** **1** **1½** **1**
	OR	

34(B).	$y^2 = 8x$ represents a parabola, and the area of the region above the x-axis, below the parabola, between the lines $x = 2$ and $x = 5$ is given by Area = $\int_2^5 (\sqrt{8x})\ dx$ $= \left[2\sqrt{2}\frac{2}{3}x^{\frac{3}{2}}\right]_2^5$ $= \sqrt{2}\left(\frac{4}{3}\right)[5\sqrt{5} - 2\sqrt{2}] = \frac{4}{3}[5\sqrt{10} - 4]$ sq. units	**½** **2** **1 ½** **1**
35.	Let the events be E_1 : User uses Alpha smartwatch E_2 : User uses Beta smartwatch E_3 : User uses Gamma smartwatch E_4 : User uses Delta smartwatch A : User gets a false heart-rate alert Clearly $\{E_1, E_2, E_3, E_4\}$ is a partition of the sample space. Then $P(E_1)$=0.45, $P(E_2) = 0.3, P(E_3) = 0.15, P(E_4) = 0.1$ Now $P(A\|E_1) = 0.01,\ P(A\|E_2) = 0.02, P(A\|E_3) = 0.05, P(A\|E_4) = 0.01$ (i) By Theorem of Total Probability, The total probability that a randomly selected user gets a false alert $= P(A)$ $= P(E_1)P(A\|E_1) + P(E_2)P(A\|E_2) + P(E_3)P(A\|E_3) + P(E_4)P(A\|E_4)$ $= 0.45 \times 0.01 + 0.3 \times 0.02 + 0.15 \times 0.05 + 0.1 \times 0.01$ $= 0.0045 + 0.006 + 0.0075 + 0.001 = 0.019$ (ii) By Bayes Theorem, The probability that false alert came from a Beta smartwatch $= P(E_2\|A)$ $= \frac{P(E_2)P(A\|E_2)}{P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)+P(E_3)P(A\|E_3)+P(E_4)P(A\|E_4)}$ $= \frac{0.3\times0.02}{0.019} = \frac{0.006}{0.019} = \frac{6}{19}$	 **1** **1** **1 ½** **1 ½**

Section –E

[This section comprises solution of 3 case- study/passage-based questions of 4 marks]

36.	(i) Given lines are $l_1 : \vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} + 3\hat{\jmath} - \hat{k})\ ) \equiv \vec{a} + \lambda\,\vec{b}$ and $l_2 : \vec{r} = 2\hat{\imath} - \hat{\jmath} + \hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + \hat{k}) \equiv \vec{c} + \mu\,\vec{d}$ Here $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k},\ \vec{b} = \hat{\imath} + 3\hat{\jmath} - \hat{k},\ \vec{c} = 2\hat{\imath} - \hat{\jmath} + \hat{k},\ \vec{d} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ For direction vectors $\vec{b}$ and $\vec{d}$, $\vec{b} \neq \lambda\,\vec{d}$, for any $\lambda \in R$ $\Rightarrow \vec{b}$ is not parallel to $\vec{d}$ So, lines are not parallel. (ii) Given lines are $l_1 : \vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} + 3\hat{\jmath} - \hat{k})\ ) \equiv \vec{a} + \lambda\,\vec{b}$ and $l_3 :\ \vec{r} = 3\hat{\imath} - \hat{\jmath} + \hat{k} + \gamma(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{e} + \gamma\,\vec{f}$	 **1** **1**

	Since $\vec{b}=\vec{f}$, the lines l_1 and l_3 are parallel. (iii) (a) $\vec{c}-\vec{a}=2\hat{\imath}-\hat{\jmath}+\hat{k}-(\hat{\imath}+2\hat{\jmath}+3\hat{k})=\hat{\imath}-3\hat{\jmath}-2\hat{k}$ $\vec{b}\times\vec{d}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 3 & -1\\ 2 & 3 & 1\end{vmatrix}=6\hat{\imath}-3\hat{\jmath}-3\hat{k}=3(2\hat{\imath}-\hat{\jmath}-\hat{k})$ $(\vec{c}-\vec{a}).(\vec{b}\times\vec{d})=(\hat{\imath}-3\hat{\jmath}-2\hat{k}).3(2\hat{\imath}-\hat{\jmath}-\hat{k})=3(2+3+2)=21\neq 0$ So, the lines are skew lines. $\lvert\vec{b}\times\vec{d}\rvert=3\sqrt{2^2+(-1)^2+(-1)^2}=3\sqrt{6}$ Thus, the shortest distance between the given lines is $D=\left\lvert\frac{(\vec{c}-\vec{a}).(\vec{b}\times\vec{d})}{\lvert\vec{b}\times\vec{d}\rvert}\right\rvert=\left\lvert\frac{21}{3\sqrt{6}}\right\rvert=\frac{7}{\sqrt{6}}=\frac{7\sqrt{6}}{6}\ units$ **OR** (b) $l_1:\vec{r}=\hat{\imath}+2\hat{\jmath}+3\hat{k}+\lambda(\hat{\imath}+3\hat{\jmath}-\hat{k})\equiv\vec{a}+\lambda\vec{b}$ and $l_3:\vec{r}=3\hat{\imath}-\hat{\jmath}+\hat{k}+\gamma(\hat{\imath}+3\hat{\jmath}-\hat{k})\equiv\vec{e}+\gamma\vec{b}$ Since the lines are parallel, the shortest distance = $\frac{\lvert\vec{b}\times(\vec{e}-\vec{a})\rvert}{\lvert\vec{b}\rvert}$ $=\frac{1}{\sqrt{11}}\left\lVert\begin{matrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 3 & -1\\ 2 & -3 & -2\end{matrix}\right\rVert=\frac{\lvert-9\hat{\imath}-9\hat{k}\rvert}{\sqrt{11}}=\frac{9\sqrt{22}}{11}units$	 **1** **1** **1** **1**
37.	(i) $W(x)=35+1.6x-0.02x^2$ $W'(x)=1.6-0.04x=0\ \Rightarrow x=40$ Therefore, the value of x is 40 crores at the stationary point of $W(x)$. (ii) $W''(x)=-0.04<0\ \Rightarrow\ W(x)$ is maximum at $x=40$ by the second derivative test. Thus, the WQI is maximum when ₹ 40 crores are spent on the revival. Max. WQI = $W(40)=35+64-32=67$ (iii) (a) If the investment is ₹35 crores, then the WQI = $W(35)$ $W(35)=35+(1.6)(35)-(0.02)(35)^2$ $=(2.6)(35)-(0.02)(1225)$ $=91-24.5=66.5$ Also, 67-66.5 = 0.5 So, the WQI is short by 0.5 units of its maximum level. **OR** (b) $W(60)=35+96-72=59$ 59 is less than the maximum value 67 of WQI, even when the investment has been increased from 40 crores to 60 crores. This shows that the returns are diminishing.	 **1** **½** **½** **1** **1** **1** **1**

38.	(i) Taking price of each kg of apples, oranges and pomegranate as ₹x, ₹y, ₹z respectively. Matrix form is $AX = B$, where $A = \begin{pmatrix} 5 & 10 & 5 \\ 10 & 10 & 5 \\ 8 & 15 & 7 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} 2750 \\ 3500 \\ 4100 \end{pmatrix}$ Here $\lvert A \rvert = 25 \neq 0$ So, the system is consistent and has unique solution Adjoint of the matrix A = adjA = $\begin{pmatrix} -5 & 5 & 0 \\ -30 & -5 & 25 \\ 70 & 5 & -50 \end{pmatrix}$ (ii) Unique solution of the system is given by $X = A^{-1}B = \frac{1}{25}\begin{pmatrix} -5 & 5 & 0 \\ -30 & -5 & 25 \\ 70 & 5 & -50 \end{pmatrix}\begin{pmatrix} 2750 \\ 3500 \\ 4100 \end{pmatrix}$ $\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 150 \\ 100 \\ 200 \end{pmatrix} \Rightarrow x = 150, y = 100, z = 200$ So the price of Apples, Oranges and Pomegranate are respectively ₹150 per kg, ₹100 per kg, ₹200 per kg.	**1** **1** **1** **1**