CBSE

Additional Practice Questions-Marking Scheme
Subject: Mathematics (041)
Class: XII 2023-24

Time Allowed: 3 Hours **Maximum Marks: 80**

SECTION A

Multiple Choice Questions of 1 mark each.

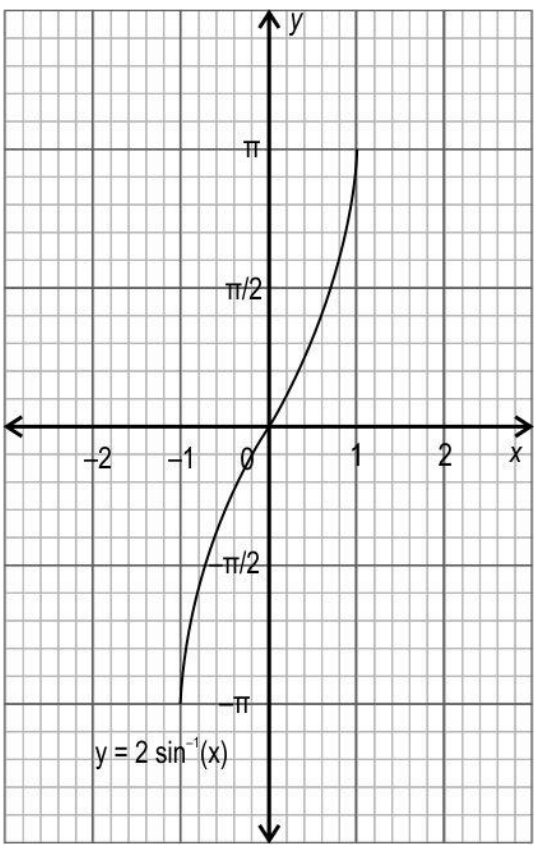
Q No.	Answer/Solution	Marks
1	(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	1
2	(c) $\sqrt{7}$	1
3	(b) $\frac{3}{2}$ sq units	1
4	(d) $\frac{-1}{2\sqrt{x(1-x)}}$	1
5	(b) $f(x)$ is continuous but not differentiable.	1
6	(a) $(-\infty, 0)$	1
7	(a) $\frac{(5e)^x}{\log 5e} + C$	1
8	(a) $6\log(2) - 2$	1
9	(c)	1

	$x^2\left(\frac{dy}{dx}\right)^4 + \sin y - \left(\frac{d^2y}{dx^2}\right)^2 = 0$	
10	(a) only (i)	1
11	(d) only (ii) and (iii)	1
12	(a) $\lambda = \frac{3}{5}, \sigma = 0$	1
13	(b) $(\sqrt{2}, 2, \sqrt{2})$	1
14	(d) $p = 8, q = 4, r = (-3)$	1
15	(a) exactly one	1
16	(d) exists as the inequality $3x + 2y < 6$ does not have any point in common with the feasible region.	1
17	(c) both maximum and minimum	1
18	(c) $\frac{P(M)}{P(M)+P(N)}$	1
19	(d) (A) is false but (R) is true.	1
20	(d) Both (A) and (R) are false.	1

SECTION B

Very short answer questions of 2 marks each.

Q No.	Answer/Solution	Marks

21	Since the domain of inverse of cosine function is [-1, 1], finds the domain of the given function as follows: $-1 \leq x - 1 \leq 1$ So, $0 \leq x \leq 2$ And, $-1 \leq 1 - x \leq 1$ $\Rightarrow 1 \geq x - 1 \geq -1$ So, $2 \geq x \geq 0$ Concludes the domain of $\cos^{-1}(\lvert x - 1 \rvert)$ as [0, 2]. OR	1.5 0.5
	Draws the graphs of $y = 2\sin^{-1}(x)$ as shown below. 	2.0

22	Considers one such matrix as $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$ and frames the following relationship: $\begin{bmatrix} x & y \\ z & w \end{bmatrix} + \begin{bmatrix} x & z \\ y & w \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 4 \end{bmatrix}$ Obtains $x = 3$ and $w = 2$ using the relationship obtained in the previous step as follows: $2x = 6 \Rightarrow x = 3$ $2w = 4 \Rightarrow w = 2$ Writes any value of y and z that satisfies the third relationship obtained in the first step. For example, $y = 0$ and $z = -1$. Writes one such matrix as $\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$. (Award full marks for any other matrix that satisfies the relationship.)	0.5 0.5 0.5 0.5
23	i) Finds $\frac{dx}{dt}$ as $(-cosec^2 t)$. Finds $\frac{dy}{dt}$ as $2cosec\ t\ (-cosec\ t\ cot\ t) = -2cosec^2 t\ cot\ t$. Finds $\frac{dy}{dx}$ as: $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2cot\ t$ *ii*) Finds $\frac{d^2y}{dx^2}$ as: $\frac{d}{dx}(2cot\ t)$ $= \frac{d}{dx}(2x)$ $= 2$ (Award full marks if any alternate method is used.)	0.5 0.5 0.5 0.5

24	Writes that the rate at which the number of active users is increasing or decreasing at a given time is given by $\frac{d}{dt}N(t)$. Finds the derivative of $N(t)$ as: $\frac{d}{dt}N(t) = 1000\ (0.1)\ e^{0.1t}$ Finds the rate of change of active users at 10 minutes past 5 pm as: $\frac{d}{dt}N(10) = 1000(0.1)e^{(0.1)(10)} = 100e$ Concludes that the number of active users are increasing at a rate of $100e$ people per minute at 5:10 pm on that day. OR Finds the derivative of the given function as: $P'(t) = 2000 \times \frac{(-1)}{(1+e^{-0.5t})^2} \times e^{-0.5t} \times (-\frac{1}{2}) = \frac{1000(e^{-0.5t})}{(1+e^{-0.5t})^2}$ Writes that the above quantity is greater than 0 for any value of t. Concludes that the rabbit population is increasing.	0.5 1.0 0.5 1.0 0.5 0.5
25	Substitutes (k - x) by u to get $dx = -du$ and rewrites the given integral as: $I = \int (k - u)(u)^{23}(-du)$ Integrates the expression in above step as: $I = \frac{-ku^{24}}{24} + \frac{u^{25}}{25} + C$ where, C is the constant of integration. Substitutes $u = (k - x)$ back in the above expression and writes: $I = \frac{(-k)(k-x)^{24}}{24} + \frac{(k-x)^{25}}{25} + C$ where, C is the constant of integration.	0.5 1.0 0.5

SECTION C

Short answer questions of 3 marks each.

Q No.	Answer/Solution	Marks
26	Rewrites the numerator of the given integral as: $3x + 5 = A\frac{d}{dx}(x^2 + 4x + 7) + B$ $\Rightarrow 3x + 5 = A(2x + 4) + B$	0.5
	Finds the values of A and B by comparing the coefficients of like terms as: $2A = 3 \Rightarrow A = \frac{3}{2}$ $4A + B = 5 \Rightarrow B = -1$	1.0
	Substitutes the values of A and B in the given integral and integrates the same as: $I = \int \frac{\frac{3}{2}(2x+4)-1}{x^2+4x+7}dx$ $\Rightarrow I = \frac{3}{2}\int \frac{2x+4}{x^2+4x+7}dx - \int \frac{1}{(x+2)^2+(\sqrt{3})^2}dx$ $\Rightarrow I = \frac{3}{2}\log\lvert x^2+4x+7\rvert - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+2}{\sqrt{3}}\right) + C$ where, C is the constant of integration.	1.5
27	Takes $u = (25 + \cos\theta)$. Finds du as: $du = -\sin\theta\, d\theta$	0.5
	Finds the change in limit when $\theta = 0$ and $\theta = \frac{\pi}{2}$ to $u = 26$ and $u = 25$ respectively.	0.5
	Rewrites the given integral using the above substitution and integrates the same as:	

	$\int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{(25+\cos\theta)(26+\cos\theta)}$ $= \int_{26}^{25} \frac{-du}{u(1+u)}$ $= \int_{25}^{26} \frac{du}{u(1+u)}$ $= \int_{25}^{26} \frac{(1+u)-u}{u(1+u)} du$ $= \int_{25}^{26} \frac{1}{u} du - \int_{25}^{26} \frac{1}{1+u} du$ $= [\log u - \log(1+u)]_{25}^{26}$ Applies the limit to find the value of the given definite integral as $\log \frac{26\times 26}{25\times 27}$. (Award full marks if the problem is solved correctly by taking $u = 26 + \cos\theta$.) OR States the property that is going to be used as: If $f(2a - x) = f(x)$, then $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ Takes $2a = \pi$ and $f(x) = h(\sin x)$. Finds $f(2a - x)$ as: $f(2a - x) = f(\pi - x) = h(\sin(\pi - x)) = h(\sin x) = f(x)$ Thus confirms that the property listed in the above step can be applied to the given integral. Hence concludes that: $\int_0^{\pi} h(\sin x)\, dx = 2\int_0^{\frac{\pi}{2}} h(\sin x) dx$	1.5 0.5 1.0 2.0
28	(i) Models the situation and rearranges terms to form a linear differential equation as follows: $0.05\frac{dv}{dt} = -0.4v - 0.5$ $\Rightarrow \frac{dv}{dt} + 8v = -10$	0.5

	(ii) Considering the obtained equation as linear of the form $\frac{dy}{dx}$ + Py = Q with P = 8 and hence takes the integrating factor as: $e^{\int 8dt} = e^{8t}$ Multiplies the differential equation by the integrating factor as follows: $e^{8t}\frac{dv}{dt} + 8ve^{8t} = -10e^{8t}$ $\Rightarrow \frac{d}{dt}(ve^{8t}) = -10e^{8t}$ Integrates both sides to obtain the general solution of the differential equation as follows: $\int \frac{d}{dt}(ve^{8t}) = -10\int e^{8t}$ $\Rightarrow ve^{8t} = \frac{-10}{8}e^{8t} + C$ $\Rightarrow v = -1.25 + Ce^{-8t}$ where C is the constant of integration. Uses the initial condition $v(0) = 10$ m/s to find the value of C as follows: $10 = -1.25 + C$ $\Rightarrow C = 11.25$ Hence, writes the expression for the velocity of the ball as a function of time as follows: $v = -1.25 + 11.25\, e^{-8t}$	0.5 0.5 1.0 0.5
29	Frames the differential equation using the given conditions as follows: $\frac{dy}{dx} = \frac{1}{3} \times \frac{y}{x}$ $\Rightarrow \frac{dy}{dx} = \frac{y}{3x}$ Rearranges the terms to separate the variables as follows: $\frac{dy}{y} = \frac{dx}{3x}$ Integrates both sides to obtain the general solution of the curve. The working may look as follows:	1.0 0.5

	$\int \frac{dy}{y} = \frac{1}{3}\int \frac{dx}{x}$ $\Rightarrow \log y = \frac{1}{3}\log x + \log C_1$ $\Rightarrow y = C_2 x^{\frac{1}{3}}$ where C_1 and C_2 are constants of integration. Substitutes (8, 2) to obtain the equation of the curve as follows: $2 = C_2(8)^{\frac{1}{3}}$ $\Rightarrow C_2 = 1$ $\therefore y = x^{\frac{1}{3}}$	1.0 0.5
	OR (i) Separates the variables and rearranges the terms of the differential equation as follows: $x + (y+1)\frac{dy}{dx} = 2$ $\Rightarrow (y+1)dy = (2-x)dx$ Integrates both sides to obtain the following: $\int (y+1)dy = \int (2-x)dx$ $\Rightarrow \frac{y^2}{2} + y = 2x - \frac{x^2}{2} + C_1$ $\Rightarrow x^2 + y^2 - 4x + 2y + C_2 = 0$ where C_1 and C_2 are constants of integration. Writes that the solution is a general solution of a circle and hence it represents a family of circles. (ii) Substitutes $x = 0$ and $y = 0$ into the general solution to obtain $C_2 = 0$ and writes the particular solution as: $x^2 + y^2 - 4x + 2y = 0$ Rearranges the terms to rewrite the particular solution as $(x - 2)^2 + (y + 1)^2 = 5$ to find the radius as $\sqrt{5}$ units. The working may look as follows: $x^2 + y^2 - 4x + 2y = 0$ Adding 5 to both sides and rearranging terms, $\Rightarrow (x^2 - 4x + 4) + (y^2 + 2y + 1) = 5$ $\Rightarrow (x - 2)^2 + (y + 1)^2 = 5$	0.5 1.0 0.5 0.5 0.5

30	Finds the profit on selling the products as: Profit for each unit of Product A sold = 100 - 60 = Rs 40 Profit for each unit of Product B sold = 150 - 90 = Rs 60. Takes x and y to be the numbers of Product A and Product B to be produced in a day respectively and frames the objective function as: Maximise Z = $40x + 60y$ Writes the constraints of the given linear programming problem as: $60x + 90y \leq 8000$ $x + y \leq 100$ $y \leq 2x$ or $-2x + y \leq 0$ $x, y \geq 0$ OR i) Uses the graph of the feasible region and lists the constraints of the given maximisation problem as: $3x + 2y \leq 12$ $x + 2y \leq 8$ $x, y \geq 0$ ii) Finds the value of the objective function at corner points as:	0.5 1.0 1.5 1.5
	<table><tr><td>Corner point</td><td>$z = 5x + 3y$</td></tr><tr><td>(0, 0)</td><td>0</td></tr><tr><td>(0, 4)</td><td>12</td></tr><tr><td>(2, 3)</td><td>19</td></tr><tr><td>(4, 0)</td><td>20</td></tr></table> Concludes that the objective function attains maximum value at (4, 0) and hence (4, 0) is the optimal solution.	1.0 0.5
31	Applies Bayes' theorem and writes:	

	P(Urgent|Not Important) = $\frac{P(Urgent)\times P(Not\ Important\mid Urgent)}{P(Urgent)\times P(Not\ Important\mid Urgent)+P(Not\ Urgent)\times P(Not\ Important\mid Not\ Urgent)}$ Substitutes respective probabilities in the expression obtained above to find the required probability as follows: P(Urgent|Not Important) $=\frac{\left(\frac{40}{100}\times\frac{1}{2}\right)}{\left(\frac{40}{100}\times\frac{1}{2}\right)+\left(\frac{60}{100}\times\frac{30}{100}\right)}$ Simplifies the above expression to get the probability that a randomly selected task that is not important is urgent as $\frac{10}{19}$ or 52.63%.	1.0 1.5 0.5

SECTION D

Long answer questions of 5 marks each.

Q No.	Answer/Solution	Marks

32	i) Writes that for no $x \in U$, $(x, x) \in R$ as the difference in time between x & x is 0 hours. Concludes that R is not reflexive. Writes that, whenever the difference in time between x and y is 6 hours, the difference in time between y and x is also 6 hours. That is, $(x, y) \in R \Rightarrow (y, x) \in R$. Concludes that R is symmetric. Writes that, if the difference in time between x and y is 6 hours, and the difference in time between y and z is also 6 hours, then the difference in time between x and z could be either 0 hours or 12 hours. That is, $(x, y) \in R$ & $(y, z) \in R$ but $(x, z) \notin R$. Concludes that R is not transitive. ii) From the above steps, concludes that R is not an equivalence relation. OR i) Assumes $f(x) = f(y)$ and evaluates the same as: $\frac{x}{x^2-1} = \frac{y}{y^2-1}$ $\Rightarrow x(y^2-1) = y(x^2-1)$ $\Rightarrow xy^2 - x - yx^2 + y = 0$ $\Rightarrow (y-x)(xy+1) = 0$ Uses the above step to conclude that $x = y$ or $xy = -1$. Takes a pair of numbers x and y such that $xy = -1$ to show that f is not one-one. For example, for $x = \frac{1}{2}$ and $y = -2$, $f(x) = -\frac{2}{3}$ and $f(y) = -\frac{2}{3}$.	 1.5 1.5 1.5 0.5 1.5 0.5 1.0

	ii) Equates $f(x)$ to y and solves the same to express x in terms of y as: $\frac{x}{x^2-1} = y$ $\Rightarrow x = yx^2 - y$ $\Rightarrow yx^2 - x - y = 0$ $\Rightarrow x = \frac{1 \pm \sqrt{1+4y^2}}{2y}$ Since $1 + 4y^2 > 0$, real root exists and also they are not ± 1 $\Rightarrow x = \frac{1 \pm \sqrt{1+4y^2}}{2y} \in R - \{-1, +1\}$	1.5
	Writes that for any $y \in$ R (codomain), there exists $x \in$ R - {-1, 1} (domain) such that $f(x) = y$. Hence concludes that f is onto.	0.5
33	Writes the system of equations as: $100a + 10b + c = 16$ $400a + 20b + c = 22$ $900a + 30b + c = 25$	0.5
	Writes the above system of equations in the form AX = B as: $\begin{pmatrix} 100 & 10 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 16 \\ 22 \\ 25 \end{pmatrix}$	0.5
	Finds |A| as 1(18000 - 12000) - 1(3000 - 9000) + 1(2000 - 4000) = -2000 and writes that A^{-1} exists as $\|A\| \neq 0$.	0.5
	Finds A^{-1} as: $A^{-1} = \begin{pmatrix} \frac{1}{200} & \frac{-1}{100} & \frac{1}{200} \\ \frac{-1}{4} & \frac{2}{5} & \frac{-3}{20} \\ 3 & -3 & 1 \end{pmatrix}$ (Award 1 mark if only all the cofactors are found correctly.)	1.5

	Finds the values of a, b and c as $-\frac{3}{200}, \frac{21}{20}$ and 7 respectively by solving X = A[-1]B as: $$X = A^{-1}B = \begin{pmatrix} \frac{1}{200} & \frac{-1}{100} & \frac{1}{200} \\ \frac{-1}{4} & \frac{2}{5} & \frac{-3}{20} \\ 3 & -3 & 1 \end{pmatrix} \times \begin{pmatrix} 16 \\ 22 \\ 25 \end{pmatrix} = \begin{pmatrix} \frac{-3}{200} \\ \frac{21}{20} \\ 7 \end{pmatrix}$$	1.0
	Finds the equation of the path traversed by the ball as: $y = -\frac{3}{200}x^2 + \frac{21}{20}x + 7$.	0.5
	Writes that when $x = 70$ feet, $y = 7$ feet. So, the ball went by 7 feet above the floor that means 3 feet below the basketball hoop. So, the ball did not go through the hoop.	0.5
34	Finds the equation of the ellipse as: $\frac{x^2}{36} + \frac{y^2}{16} = 1$	0.5
	Expresses y in terms of x as: $y = \pm\frac{4}{6}\sqrt{36 - x^2}$	0.5
	Integrates the above equation with respect to x from limit 0 to 6, that gives the area of one quarter of the ellipse. The working may look as follows: $\int_0^6 \frac{4}{6}\sqrt{36 - x^2}dx$	0.5
	Applies the formula of integration and simplifies as: $\frac{4}{6}\left[\frac{x}{2}\sqrt{6^2 - x^2} + \frac{6^2}{2}\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$	1.0
	Applies the limit and solves further as: $\frac{4}{6}\left[\frac{6}{2} \times 0 + \frac{6^2}{2}\sin^{-1}(1) - 0\right]$	0.5
	Simplifies the above expression to get the area of one-quarter of the base as 6π sq feet.	1.0
	Finds the area of the whole ellipse as $4 \times 6\pi = 24\pi$ sq feet.	0.5
	Finds the volume of water as $24\pi \times 10 = 240\pi$ cubic feet.	0.5

35	i) Takes $\vec{a} = -5\hat{i} + 7\hat{j} - 4\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$	0.5
	Writes the vector equation of the given straight line as: $\vec{r} = (-5\hat{i} + 7\hat{j} - 4\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + \hat{k})$	1.0
	Writes the cartesian equation of the given straight line as: $\frac{x+5}{3} = \frac{y-7}{-2} = \frac{z+4}{1}$	1.0
	ii) Simplifies the vector form obtained in step 2 as: $\vec{r} = (-5 + 3\lambda)\hat{i} + (7 - 2\lambda)\hat{j} + (\lambda - 4)\hat{k}$	1.0
	Writes that at the point where the line crosses *xy*-plane, its *z*-coordinate is zero and equates the *z*-coordinate of the above equation to zero as: $\lambda - 4 = 0$ $\Rightarrow \lambda = 4$	0.5
	Substitutes $\lambda = 4$ in the vector form to get the required point as (7, -1, 0).	1.0
	OR	
	Rewrites the equation of L_1 in cartesian form as: $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$	0.5
	Rewrites the equation of L_2 in cartesian form as: $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$	0.5
	i) Identifies the direction cosines of both the lines as (3, 2, -6) and (2, -12, -3).	0.5
	Finds the cosine of the angle between the two lines as: $\cos\theta = \left\lvert \frac{6-24+18}{\sqrt{49}\sqrt{157}} \right\rvert = 0$	1.5
	(Award 0.5 marks if only the formula of the cosine of the angle between the two lines is written correctly.)	
	Concludes that the angle between the two lines is 90°.	0.5

	ii) Rewrites the equations of L_1 and L_2 in vector form as: $\vec{r}_1 = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 6\hat{k})$, where $\lambda \in R$ $\vec{r}_2 = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(2\hat{i} - 12\hat{j} - 3\hat{k})$, where $\lambda \in R$ Writes that both the lines pass through the origin hence intersect at the origin. (Award full marks if the inference about both lines passing through the origin is drawn without writing the vector forms.) Writes that since both the lines intersect at the origin, the shortest distance between the two lines is 0 units.	1.0 0.5

SECTION E

Case-based questions of 4 marks each.

Q No.	Answer/Solution	Marks
36 i)	Writes the vectors for points P and Q as follows: $\overrightarrow{OP} = -2\hat{i} + \hat{j} + 3\hat{k}$ $\overrightarrow{OQ} = 3\hat{i} + 4\hat{j} - \hat{k}$ Finds the vector representing the flight path of Airplane 1 as: $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ $= (3\hat{i} + 4\hat{j} - \hat{k}) - (-2\hat{i} + \hat{j} + 3\hat{k})$ $= 5\hat{i} + 3\hat{j} - 4\hat{k}$	0.5 0.5
36 ii)	Uses vector subtraction to find the vector representing the flight path from R to Q as: $\overrightarrow{RQ} = \overrightarrow{PQ} - \overrightarrow{PR}$ $= (5\hat{i} + 3\hat{j} - 4\hat{k}) - (5\hat{i} + \hat{j} - 2\hat{k})$ $= 2\hat{j} - 2\hat{k}$	1.0

36 iii)	Finds the cosine of the angle between the vectors representing the flight paths of Airplane 1 and Airplane 2 as: $\cos\theta = \frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{\lvert\overrightarrow{PQ}\rvert\cdot\lvert\overrightarrow{PR}\rvert}$ $= \frac{(5\hat{i}+3\hat{j}-4\hat{k})\cdot(5\hat{i}+\hat{j}-2\hat{k})}{\sqrt{50}\cdot\sqrt{30}}$ $= \frac{18}{5\sqrt{15}}$ Finds the angle between the flight paths as: $\theta = \cos^{-1}\left(\frac{18}{5\sqrt{15}}\right)$ OR Considers a point S which divides PQ internally in the ratio 1:2. Finds the position vector of point S as: $\overrightarrow{OS} = \frac{1(\overrightarrow{OQ})+2(\overrightarrow{OP})}{1+2}$ $= \frac{1(3\hat{i}+4\hat{j}-\hat{k})+2(-2\hat{i}+\hat{j}+3\hat{k})}{3}$ $= -\frac{1}{3}\hat{i} + 2\hat{j} + \frac{5}{3}\hat{k}$ (Award 0.5 marks if only the formula is written correctly.)	1.5 0.5 0.5 1.5
37 i)	Finds the required probability as: $P(5 \text{ from spinner A}) \cap P(8 \text{ from spinner B})$ $= \frac{1}{4} \times \frac{1}{8}$ $= \frac{1}{32}$	1.0
37 ii)	Uses the conditional probability and finds the required probability as follows: $P(\text{Even} \mid \text{Multiple of } 3)$ $= P(\text{Even} \cap \text{Multiple of } 3) \div P(\text{Multiple of } 3)$ $= \frac{\frac{1}{8}}{\frac{2}{8}}$ $= \frac{1}{2}$	1.0

37 iii)	Finds the probability of getting 2 from spinner A and getting 1 from spinner B as: $P_1 = \frac{1}{2} \times \frac{1}{8} = \frac{1}{16}$ Finds the probability of getting 5 from spinner A and getting either 1, 2, 3 or 4 from spinner B as: $P_2 = \frac{1}{4} \times \left[\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right]$ $= \frac{1}{4} \times \frac{4}{8}$ $= \frac{1}{8}$ Writes that P_1 and P_2 are mutually exclusive and hence, finds the probability that she wins a photo frame as: $P_1 + P_2 = \frac{1}{16} + \frac{1}{4}$ $= \frac{5}{16}$ OR Uses the theorem of total probability and writes: P(getting 2) = [P(Spinner A) × P(Getting 2|Spinner A)] + [P(Spinner B) × P(Getting 2|Spinner B)] Finds the required probability by substituting the required probability as: $\left[\frac{65}{100} \times \frac{1}{2}\right] + \left[\frac{35}{100} \times \frac{1}{8}\right]$ $= \frac{59}{160}$	0.5 1.0 0.5 0.5 1.5
38 i)	Identifies that the rod being heated is R_1 and finds the rate of change of temperature at any distance from one end of R_1 as: $\frac{dT}{dx} = \frac{d}{dx}(16 - x)x = \frac{d}{dx}(16x - x^2) = 16 - 2x$ Finds the mid-point of the rod as $x = 8$ m.	1.0 0.5

	Finds the rate of change of temperature at the mid point of R_1 as: $\frac{dT}{dx}_{(\text{at } x=8)} = 16 - 2(8) = 0$	0.5
38 ii)	Identifies that the rod being cooled is R_2 and finds the rate of change of temperature at any distance x m as: $\frac{dT}{dx} = \frac{d}{dx}(x - 12)x = \frac{d}{dx}(x^2 - 12x) = 2x - 12$ Equates $\frac{dT}{dx}$ to 0 to get the critical point as $x = 6$. Finds the second derivative of T as: $\frac{d^2T}{dx^2} = 2$ And concludes that at $x = 6\ m$, the rod has minimum temperature as $\frac{d^2T}{dx^2}_{(\text{at } x=6)} = 2 > 0$. Finds the minimum temperature attained by the rod R_2 as T(6) = (6 - 12)6 = -36 °C.	1.0 0.5 0.5