

Practice Questions - Marking Scheme
Session 2022-23
Class XII
Mathematics (Code – 041)

SECTION A - Multiple Choice Questions - 1 Mark each		
Q.No.	Answer/Solution	Marks
Q.1	C. $\sec^{-1} x$	1
Q.2	B. P and Q must be square matrices of the same order.	1
Q.3	D. all - i), ii) and iii)	1
Q.4	A. -48	1
Q.5	C. $\frac{1}{4}$	1
Q.6	B. $-\tan \frac{1}{x} - B$, where B is a constant.	1
Q.7	D. 4	1
Q.8	C. 9 sq units	1
Q.9	B. $\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} = 0$	1
Q.10	C. $\frac{4}{3} e^{3x} + 1$	1
Q.11	A. only ii)	1
Q.12	C. $\frac{9}{2} \hat{i} + \frac{3}{2} \hat{j} + \frac{1}{2} \hat{k}$	1
Q.13	D. 0	1
Q.14	B. 60°	1
Q.15	D. 8	1
Q.16	B. It has a unique solution.	1
Q.17	D. 0.08	1
Q.18	A. Minimise $Z = x + y$	1
Q.19	C. (A) is true but (R) is false.	1
Q.20	C. (A) is true but (R) is false.	1
SECTION B - VSA questions of 2 marks each		
Q.21	<p>Solves the RHS to obtain $\frac{2\pi}{3}$ as follows:</p> $\cos^{-1}(-1) - \operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ $= \pi - \frac{\pi}{3}$ $= \frac{2\pi}{3}$ <p>Equates the LHS to obtain $x = -\frac{1}{\sqrt{3}}$ as follows:</p>	0.5
		1

$$\cot^{-1}(x) = \frac{2\pi}{3}$$

$$x = \cot\left(\frac{2\pi}{3}\right)$$

$$x = -\left(\frac{1}{\sqrt{3}}\right)$$

Finds $\tan^{-1}\left(\frac{1}{x}\right)$ as $-\frac{\pi}{3}$ as follows:

$$\tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$$

OR

i) Finds the domain as $(-\infty, \frac{2}{5}] \cup [\frac{4}{5}, \infty)$ as follows:

$$5x - 3 \leq -1 \text{ or } 5x - 3 \geq 1$$

$$\therefore x \leq \frac{2}{5} \text{ or } x \geq \frac{4}{5}$$

ii) Finds the range as $[-2, 3\pi - 2]$ as follows:

$$0 \leq \cos^{-1}\left(\frac{1}{2x-1}\right) \leq \pi$$

$$3(0) - 2 \leq 3 \cos^{-1}\left(\frac{1}{2x-1}\right) - 2 \leq 3(\pi) - 2$$

$$3(0) - 2 \leq y \leq 3(\pi) - 2$$

$$-2 \leq y \leq 3\pi - 2$$

0.5

1

1

Q.22

Writes the expression for C as $\frac{1}{2}(A - A')$.

Finds A' as:

$$A' = \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}$$

0.5

0.5

	<p>Finds C as:</p> $C = \frac{1}{2} \left(\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix} \right)$ $= \begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & -2 \\ 2 & 2 & 0 \end{bmatrix}$	1
Q.23	<p>Differentiates y with respect to x using chain rule as:</p> $\frac{dy}{dx} = 4(e^{\sec x} + x)^3 \frac{d}{dx}(e^{\sec x} + x)$ <p>Simplifies the above differential as:</p> $\frac{dy}{dx} = 4(e^{\sec x} + x)^3 (e^{\sec x} \sec x \tan x + 1)$	1 1
Q.24	<p>Substitutes $\vec{v} = k(\hat{q} + \hat{r})$ in $\hat{p} \cdot \vec{v} = \hat{q} \cdot \vec{v}$ to get:</p> $\hat{p} \cdot \hat{q} + \hat{p} \cdot \hat{r} = \hat{q} \cdot \hat{q} + \hat{q} \cdot \hat{r}$ $\hat{p} \cdot \hat{q} + \hat{p} \cdot \hat{r} = 1 + \hat{q} \cdot \hat{r}$ <p>Rearranges the terms of the above equation as:</p> $1 - \hat{p} \cdot \hat{q} + \hat{q} \cdot \hat{r} - \hat{p} \cdot \hat{r} = 0$ <p>Simplifies the above equation as:</p> $\implies \hat{p}(\hat{p} - \hat{q}) - \hat{r}(\hat{p} - \hat{q}) = 0$ $\implies (\hat{p} - \hat{q}) \cdot (\hat{p} - \hat{r}) = 0$ <p style="text-align: center;">OR</p> <p>Uses the cross-product of vectors and writes:</p> $\text{Area of QRST} = \vec{RQ} \times \vec{RS} = \vec{a} \times \vec{b} $ <p>Uses the cross-product of vectors and writes:</p> $\text{Area of QRTP} = \vec{RQ} \times \vec{RT} $	0.5 0.5 1 0.5 0.5

	<p>Simplifies RHS of the above equation as:</p> $\begin{aligned} &= \vec{RQ} \times (\vec{RQ} + \vec{QT}) \\ &= \vec{a} \times (\vec{a} + \vec{b}) \\ &= (\vec{a} \times \vec{a}) + (\vec{a} \times \vec{b}) \\ &= 0 + \vec{a} \times \vec{b} = \vec{a} \times \vec{b} \end{aligned}$ <p>Concludes that the area of parallelogram QRST is equal to the area of parallelogram QRTP.</p>	1
Q.25	<p>i) Expands the vector form to get the following:</p> $x\hat{i} + y\hat{j} + z\hat{k} = (x_1 + \lambda x_1)\hat{i} + (y_1 + 2\lambda y_1)\hat{j} + (z_1 + 3\lambda z_1)\hat{k}$	0.5
	<p>Eliminates λ by equating the like coefficients of the position vectors of the x, y and z axes to get the cartesian equation as follows:</p> $\frac{x - x_1}{x_1} = \frac{y - y_1}{2y_1} = \frac{z - z_1}{3z_1}$	0.5
	<p>ii) Assumes the coordinates of B as (x_2, y_2, z_2) and compares the cartesian form of the equation from step 2 with the regular form of the cartesian equation to find:</p> $x_2 = 2x_1, y_2 = 3y_1 \text{ and } z_2 = 4z_1$	0.5
	<p>Substitutes values $x_1 = (-2)$, $y_1 = 5$ and $z_1 = (-3)$ in the equations from step 3 to get coordinates of B as $(-4, 15, -12)$.</p>	0.5

SECTION C - Short Answer Questions of 3 Marks each

Q.26

Finds $\frac{du}{d\theta}$ as:

$$\frac{du}{d\theta} = e^{\sin^{-1}\theta} \times \frac{1}{\sqrt{1-\theta^2}}$$

Finds $\frac{dv}{d\theta}$ as:

$$\frac{dv}{d\theta} = e^{-\cos^{-1}\theta} \times \frac{1}{\sqrt{1-\theta^2}}$$

Uses parametric differentiation and finds $\frac{du}{dv}$ as:

$$\frac{du}{dv} = \frac{e^{\sin^{-1}\theta}}{e^{-\cos^{-1}\theta}} = e^{\sin^{-1}\theta + \cos^{-1}\theta} = e^{\frac{\pi}{2}}$$

Concludes that the given statement is true as $e^{\frac{\pi}{2}}$ is a constant.

OR

Rewrites the given equation by taking logarithm on both sides as:

$$m(\log x) - n(\log y) = (m - n)(\log x + \log y)$$

Differentiates the above equation as:

$$\frac{m}{x} dx - \frac{n}{y} dy = (m - n) \left(\frac{1}{x} dx + \frac{1}{y} dy \right)$$

Rearranges the above equation to get:

$$\frac{n}{x} dx = \frac{m}{y} dy$$

Finds $\frac{dy}{dx}$ to be $\frac{ny}{mx}$.

Q.27

Interprets the question statement and writes it as:

$$(3x - 1)f(x) = \frac{d}{dx} \left(3x^4 - \frac{13}{3}x^3 + \frac{3}{2}x^2 + C \right)$$

Finds the derivative in the above step as:

$$(3x - 1)f(x) = 12x^3 - 13x^2 + 3x$$

	<p>Factorises the above cubic polynomial as:</p> $(3x - 1)f(x) = (3x - 1)(4x^2 - 3x)$ <p>and determines the value of $f(x)$ as $(4x^2 - 3x)$.</p> <p>Substitutes $x = 6$ in $f(x)$ and evaluates $f(6)$ as 126.</p>	<p>1</p> <p>0.5</p>
<p>Q.28</p>	<p>Rewrites the integral using the identity $\operatorname{cosec}^2 x = 1 + \cot^2 x$ as:</p> $\int \cot^2 x (1 + \cot^2 x)^2 \operatorname{cosec}^2 x \, dx$ <p>Substitutes $\cot x = u$ and hence $\operatorname{cosec}^2 x \, dx = -du$ in the above step and rewrites the integral as:</p> $-\int u^2 (1 + u^2)^2 \, du$ <p>Integrates the above expression as:</p> $-\left[\frac{u^3}{3} + \frac{u^7}{7} + \frac{2u^5}{5} \right]$ <p>Substitutes $\cot x$ in place of u in the above expression to get:</p> $-\left[\frac{\cot^3 x}{3} + \frac{\cot^7 x}{7} + \frac{2\cot^5 x}{5} \right]$ <p>Substitutes the limits in the above expression to get $\frac{92}{105}$.</p>	<p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>1</p>
<p>Q.29</p>	<p>Rearranges the given differential equation as:</p> $\frac{dy}{dx} + \frac{y}{\sqrt{1-x^2}} = \frac{e^{-\sin^{-1}x}}{\sqrt{1-x^2}}$ <p>Finds the integrating factor as follows as the equation obtained in the above step is of the form $\frac{dy}{dx} + y P(x) = Q(x)$.</p> <p>Integrating factor = $e^{\int P(x) \, dx}$</p> $= e^{\int \frac{1}{\sqrt{1-x^2}} \, dx} = e^{\sin^{-1}x}$	<p>0.5</p> <p>0.5</p>

Finds the solution as:

$$ye^{\int P(x) dx} = \int Q(x) \times e^{\int P(x) dx} dx + C$$

$$\Rightarrow ye^{\sin^{-1}x} = \int \frac{e^{-\sin^{-1}x}}{\sqrt{1-x^2}} \times e^{-\sin^{-1}x} dx + C$$

$$\Rightarrow ye^{\sin^{-1}x} = \sin^{-1}x + C$$

Where C is the constant of integration.

Substitutes $x = y = 0$ in the above equation and finds the value of C as 0.

Writes the particular solution as:

$$ye^{\sin^{-1}x} = \sin^{-1}x$$

OR

Rearranges the given equation in terms of $\frac{y}{x}$ as:

$$\frac{dy}{dx} = \frac{y}{x} + \frac{y}{x^2} \sqrt{y^2 - x^2}$$
$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \frac{y}{x} \sqrt{\frac{y^2}{x^2} - 1}$$

Considers $y = vx$ and finds $\frac{dy}{dx}$ in terms of v as:

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

Equates the RHS obtained in steps 1 and 2 to get:

$$x \frac{dv}{dx} = v \sqrt{v^2 - 1}$$

Rearranges the terms using the variable separable method as:

$$\frac{dv}{v\sqrt{v^2-1}} = \frac{dx}{x}$$

Integrates on both sides to find the general solution as:

$$\sec^{-1}v = \log|x| + C$$

or

$$\sec^{-1}\frac{y}{x} = \log|x| + C, \text{ where } C \text{ is the constant of integration.}$$

1.5

0.5

0.5

0.5

0.5

0.5

1

Q.30

Takes the number of hens and cows to be x and y respectively and formulates the linear programming problem as follows:

$$\text{Maximise } Z = 12x + 40y$$

subject to constraints,

$$25x + 75y \leq 900$$

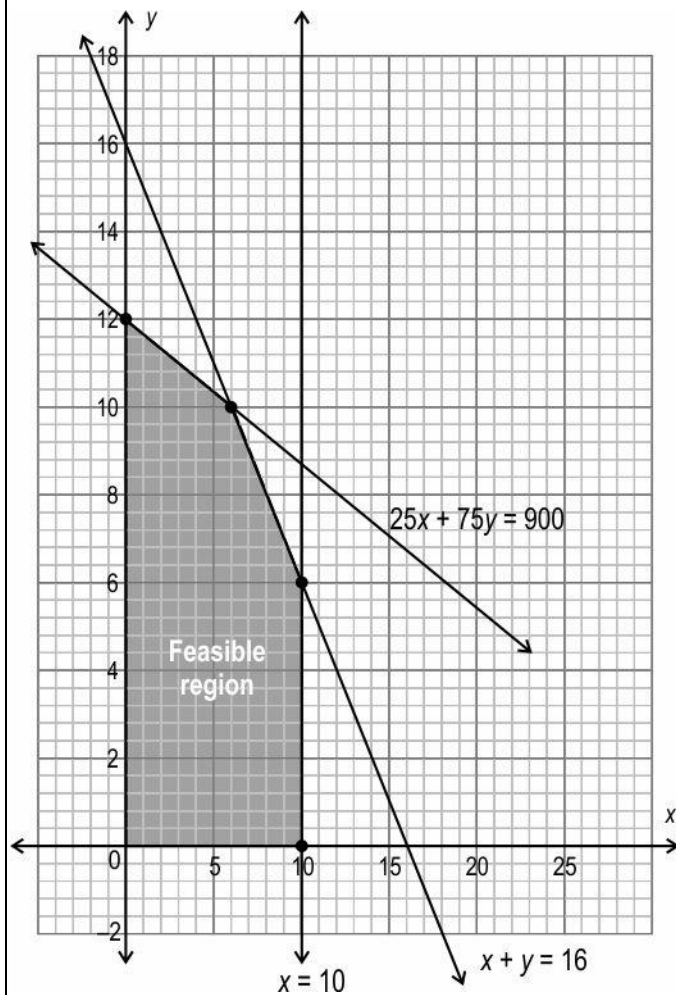
$$x + y \leq 16$$

$$x \leq 10$$

$$x \geq 0$$

$$y \geq 0$$

Graphs the constraints and marks the feasible region as:



1.5

1.5

Q.31

Takes E, F and G to be the events of taking out green marbles in the first, second and third draws respectively, and writes:

$$P(E) = P(\text{green marble in first draw}) = \frac{8}{14}$$

Finds the probability that the second marble taken out is green provided first is also green as:

$$P(F|E) = P(\text{green marble in the second draw}) = \frac{7}{13}$$

Finds the probability that the third marble taken out is green provided first two are also green as:

$$P(G|EF) = P(\text{green marble in third draw}) = \frac{6}{12}$$

Finds the probability that all three marbles taken out are green in colour as:

$$P(E) \times P(F|E) \times P(G|EF)$$

$$= \frac{8}{14} \times \frac{7}{13} \times \frac{6}{12}$$

$$= \frac{10}{91}$$

OR

Assumes the number of students as a random variable X and writes that it can take values of 0, 1 and 2.

Finds $P(X = 0)$ as:

$P(\text{non-student and non-student})$

$$= \frac{10}{18} \times \frac{9}{17}$$

$$= \frac{90}{306}$$

Finds $P(X = 1)$ as:

$P(\text{student and non-student})$ or $P(\text{non-student and student})$

$$= \frac{8}{18} \times \frac{10}{17} \times \frac{10}{18} \times \frac{8}{17}$$

0.5

0.5

0.5

1.5

0.5

1.5

$$= \frac{160}{306}$$

Finds $P(X = 2)$ as:

$P(\text{student and student})$

$$= \frac{8}{18} \times \frac{7}{17}$$

$$= \frac{56}{306}$$

Writes the required probability distribution as:

X	0	1	2
P(X)	$\frac{90}{306}$	$\frac{160}{306}$	$\frac{56}{306}$

0.5

0.5

SECTION D - Long answer type questions (LA) of 5 marks each

Q.32

Assumes the number of litres of orange juice, beetroot juice and kiwi juice as x , y and z , respectively to frame equations as follows:

$$500x + 20y + 800z = 1860$$

$$2x + 5y + 3z = 22$$

$$100x + 120y + 200z = 760$$

Writes the above system of equations in the matrix form using $AX = B$ as

$$\begin{bmatrix} 500 & 20 & 800 \\ 2 & 5 & 3 \\ 100 & 120 & 200 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1860 \\ 22 \\ 760 \end{bmatrix}$$

Finds $|A| = 110000 \neq 0$ and hence writes that A is non-singular and has a unique solution.

Finds $adj A$ as:

$$\begin{bmatrix} 640 & 92000 & -3940 \\ -100 & 20000 & 100 \\ -260 & -58000 & 2460 \end{bmatrix}$$

0.5

0.5

0.5

1

	<p>Finds A^{-1} using A and $adj A$ as:</p> $A^{-1} = \frac{1}{ A } \times adj A$ $= \frac{1}{5500} \begin{bmatrix} 32 & 4600 & -197 \\ -5 & 1000 & 5 \\ -13 & -2900 & 123 \end{bmatrix}$ <p>Writes that $X = A^{-1}B$ and finds X as</p> $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ <p>Concludes that 2 litres of orange juice, 3 litres of beetroot juice and 1 litre of kiwi juice should go into the mixture.</p>	<p>1</p> <p>1</p> <p>0.5</p>
<p>Q.33</p>	<p>Writes the endpoints of the ellipse as $(-9, 0)$, $(9, 0)$, $(0, 6)$ and $(0, -6)$ respectively.</p> <p>Expresses y in terms of x as:</p> $y = \pm \frac{6}{9} \sqrt{9^2 - x^2}$ <p>Sets up the equation for the area of the shaded region as:</p> $\text{Shaded Area} = \left \int_{-9}^0 \frac{6}{9} \sqrt{9^2 - x^2} dx \right + \text{Area of 2 triangles} + \int_0^9 \frac{6}{9} \sqrt{9^2 - x^2} dx$ $= 2 \int_0^9 \frac{6}{9} \sqrt{9^2 - x^2} dx + \text{Area of 2 triangles}$ <p>Evaluates the 1st part of the above equation as:</p> $2 \int_0^9 \frac{6}{9} \sqrt{9^2 - x^2} dx$ $= 2 \times \frac{6}{9} \left[\frac{x}{2} \sqrt{81 - x^2} + \frac{81}{9} \sin^{-1} \frac{x}{9} \right]_0^9$	<p>1</p> <p>1</p> <p>1</p>

	<p>Applies the upper and the lower limit and finds the value of the integral as:</p> $= 2 \times \frac{6}{9} \left\{ \left[\frac{9}{2} \sqrt{81 - 81} + \frac{81}{2} \sin^{-1} \frac{9}{9} \right] - \left[\frac{0}{2} \sqrt{81 - 0} + \frac{81}{2} \sin^{-1} \frac{0}{9} \right] \right\}$ $= 2 \times \frac{6}{9} \times \frac{81}{2} \times \frac{\pi}{2} = 27\pi$ <p>Evaluates the 2nd part of the equation from step 2 as:</p> <p>Area of 2 triangles = $2 \times \frac{1}{2} \times 9 \times 6 = 54$ sq units</p> <p>Adds the area obtained in step 3 and 4 to find the area of the shaded region in terms of π as:</p> <p>($27\pi + 54$) sq units or $27(\pi + 2)$ sq units.</p>	<p>1</p> <p>0.5</p> <p>0.5</p>
Q.34	<p>Compares $\vec{r}_1 = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r}_2 = \vec{a}_2 + \mu \vec{b}_2$ with the given equations to get $\vec{a}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$, $\vec{a}_2 = 2\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b}_2 = -\hat{i} + \hat{j} + 2\hat{k}$.</p> <p>Notes that the lines are skewed and writes the formula to find the shortest distance between the lines (d) as follows:</p> $d = \left \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{ \vec{b}_1 \times \vec{b}_2 } \right $ <p>Solves $(\vec{b}_1 \times \vec{b}_2)$ as $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ -1 & 1 & 2 \end{vmatrix} = 7\hat{i} + \hat{j} + 3\hat{k}$.</p> <p>Finds $\vec{b}_1 \times \vec{b}_2 = \sqrt{49 + 1 + 9} = \sqrt{59}$, and $(\vec{a}_2 - \vec{a}_1) = \hat{i} + \hat{k}$.</p> <p>Finds $(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)$ as $7 + 3 = 10$.</p> <p>Substitutes values from above steps to find distance as $\frac{10}{\sqrt{59}}$ units.</p> <p style="text-align: center;">OR</p> <p>Writes that $\frac{x-4}{1} = \frac{y-2}{3} = \frac{z-1}{2} = \lambda$.</p> <p>Assumes P ($x, y, z$) to be the point of intersection of the two lines. Finds $x = \lambda + 4, y = 3\lambda + 2$ and $z = 2\lambda + 1$.</p> <p>Takes Tara's position as T(2, -2, 1) to find the direction ratios of TP as $(\lambda + 2), (3\lambda + 4)$ and (2λ).</p>	<p>1</p> <p>1</p> <p>1.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>1</p>

	<p>Notes that the dot product of the direction ratios of the given line and TP will be 0, since they are perpendicular, and $\cos 90^\circ = 0$.</p> <p>Writes that $1(\lambda + 2) + 3(3\lambda + 4) + 2(2\lambda) = 0$.</p> <p>Solves the above equation to find $\lambda = (-1)$.</p> <p>Substitutes the value of λ to find $x = 3, y = 7$ and $z = 2$.</p> <p>Finds the length of TP as $\sqrt{83}$ units, using the following formula:</p> $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$	<p>1.5</p> <p>0.5</p> <p>1</p>
<p>Q.35</p>	<p>Rewrites the given integral as:</p> $\int x^4 x^3 \sin(2x^4) dx$ <p>Substitutes $x^4 = u$ and hence $x^3 dx = \frac{1}{4} du$ in the above integral to get:</p> $\frac{1}{4} \int u \sin(2u) du$ <p>Uses integration by parts to integrate the above expression as:</p> $\frac{1}{4} [u \int \sin(2u) du - \int (\int \sin(2u) du) du]$ <p>Integrates the above expression to get:</p> $-\frac{1}{8} u \cos(2u) + \frac{1}{16} \sin(2u) + C$ <p>Substitutes x^4 in place of u in the above expression to get:</p> $-\frac{1}{8} x^4 \cos(2x^4) + \frac{1}{16} \sin(2x^4) + C$ <p style="text-align: center;">OR</p> <p>Expands the denominator using the identity $(a^3 - b^3)$ as:</p> $\int \frac{1}{(2-x)(x^2+2x+4)} dx$ <p>Rewrites the integral as a sum of two integrals using partial fractions as:</p> $\frac{1}{12} \int \frac{1}{(2-x)} dx + \frac{1}{12} \int \frac{x+4}{x^2+2x+4} dx$	<p>0.5</p> <p>1</p> <p>1</p> <p>2</p> <p>0.5</p> <p>0.5</p> <p>1</p>

	<p>Solves the first integral as:</p> $-\frac{1}{12} \log 2 - x $ <p>Rewrites the second integral as:</p> $\frac{1}{24} \int \frac{2x+2}{x^2+2x+4} dx + \frac{1}{4} \int \frac{1}{(x+1)^2 + (\sqrt{3})^2} dx$ <p>Solves the above integral as:</p> $\frac{1}{24} \log x^2+2x+4 + \frac{1}{4\sqrt{3}} \tan^{-1} \left(\frac{x+1}{\sqrt{3}} \right)$ <p>Concludes the final answer as:</p> $-\frac{1}{12} \log 2 - x + \frac{1}{24} \log x^2+2x+4 + \frac{1}{4\sqrt{3}} \tan^{-1} \left(\frac{x+1}{\sqrt{3}} \right) + C$	<p>0.5</p> <p>1</p> <p>1.5</p> <p>0.5</p>
SECTION E - Case Studies/Passage based questions of 4 Marks each		
Q.36i)	<p>Lists all the elements of R as:</p> $R = \{(C, PB), (PB, C), (V, PB), (PB, V), (PB, SwD), (SwD, PB), (PB, ShD), (ShD, PB), (SwD, ShD), (ShD, SwD)\}$	1
Q.36ii)	<p>Writes that the relation R is symmetric.</p> <p>Gives a reason. For example, for every $(x_1, x_2) \in R$, $(x_2, x_1) \in R$ as every direct ship/direct ferry runs in both the directions.</p>	<p>0.5</p> <p>0.5</p>
Q.36iii)	<p>Writes that R is not transitive.</p> <p>Gives a reason. For example,</p> <p>$(C, PB) \in R$ as there is a direct ship from Chennai to Port Blair.</p> <p>$(PB, SwD) \in R$ as there is a direct ferry from Port Blair to Swaraj Dweep.</p> <p>But $(C, SwD) \notin R$ as there is no direct ship/ferry from Chennai to Swaraj Dweep.</p> <p style="text-align: center;">OR</p>	<p>0.5</p> <p>1.5</p>

	Writes that the function f is one-one.	0.5
	Gives a reason. For example, no two elements of set Y are mapped to a common element in set X .	0.5
	Writes that the function f is not onto.	0.5
	Gives a reason. For example, $C \in X$ (co-domain of f) but it has no pre-image in Y .	0.5
Q.37i)	Finds the rate at which the amount of drug is changing in the blood stream 5 hours after the drug has been administered as: $C'(t) = -3t^2 + 9t + 54$ $\Rightarrow C'(5) = 24 \text{ mg/hr}$	1
Q.37ii)	Equates the derivative $C'(t)$ to 0 and factorises $C'(t)$ as $3(3 + t)(6 - t)$. Writes that for $t \in (3, 4)$, $3 > 0,$ $(3 + t) > 0$ and $(6 - t) > 0$ Therefore, $C'(t) > 0$. Concludes that $C(t)$ is strictly increasing in the interval $(3, 4)$. <p style="text-align: center;">OR</p> Equates the derivative $C'(t) = -3t^2 + 9t + 54$ to 0 and finds the critical points as $t = 6$ hours and $t = (-3)$ hours. Differentiate $C'(t)$ to get $C''(t)$ as: $C''(t) = -6t + 9$ Finds $C''(6)$ as (-27) and writes that $C(t)$ attains its maximum at $t = 6$ hours, as $C''(6) = (-27) < 0$. Concludes that 6 hours after the drug is administered, C_{\max} is attained.	0.5 1.5 0.5 0.5 0.5 0.5
Q.37iii)	Finds the value of $C(t)$ at $t = 6$ hours as: $C(6) = -(6)^3 + 4.5(6)^2 + 54(6)$ $\Rightarrow C(6) = 270$	1

	Writes the amount of drug in the bloodstream when the effect of the drug is maximum as 270 mg.	
Q.38i)	<p>Takes $P(S)$, $P(C)$ and $P(T)$ as the probabilities that a person selected randomly from the staff prefers sugar, coffee and tea respectively.</p> <p>Finds $P(T) = P(C') = 1 - 0.6 = 0.4$.</p> <p>Finds $P(S T) = 1 - 0.2 = 0.8$.</p> <p>Uses theorem on total probability and finds the probability that a randomly selected staff prefers a beverage with sugar as:</p> $P(S) = P(C) \times P(S C) + P(T) \times P(S T)$ $= 0.6 \times 0.9 + 0.4 \times 0.8 = 0.86 \text{ or } \frac{86}{100} \text{ or } \frac{43}{50}$	<p>1</p> <p>1</p>
Q.38ii)	<p>Uses the sum of probabilities = 1 and finds the following probabilities:</p> <ul style="list-style-type: none"> ◆ $P(\text{without sugar} \text{coffee}) = 1 - 0.9 = 0.1$ ◆ $P(\text{tea}) = 1 - 0.6 = 0.4$ <p>Uses Bayes' theorem to find the probability that a staff selected at random prefers coffee given that it is without sugar, $P(\text{coffee} \text{without sugar})$ as:</p> $\frac{P(\text{coffee}) \times P(\text{without sugar} \text{coffee})}{P(\text{coffee}) \times P(\text{without sugar} \text{coffee}) + P(\text{tea}) \times P(\text{without sugar} \text{tea})}$ $= \frac{0.6 \times 0.1}{0.6 \times 0.1 + 0.4 \times 0.2}$ <p>(Award 0.5 marks if only the formula for Bayes' theorem is written correctly.)</p> <p>Simplifies the above expression and finds the required probability as $\frac{6}{14}$ or $\frac{3}{7}$.</p>	<p>0.5</p> <p>1</p> <p>0.5</p>