<table>
<thead>
<tr>
<th>Question No</th>
<th>Answer</th>
<th>Hints/Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(c)</td>
<td>In a skew-symmetric matrix, the (i, j)th element is negative of the (j, i)th element. Hence, the (i, i)th element = 0.</td>
</tr>
<tr>
<td>2.</td>
<td>(a)</td>
<td>$</td>
</tr>
<tr>
<td>3.</td>
<td>(b)</td>
<td>The area of the parallelogram with adjacent sides $AB$ and $AC =</td>
</tr>
<tr>
<td>4.</td>
<td>(c)</td>
<td>The function $f$ is continuous at $x = 0$ if $\lim_{x \to 0} f(x) = f(0)$. We have $f(0) = k$ and $\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(1 - \cos \frac{1}{8x^2}\right) = \lim_{x \to 0} \left(\frac{2\sin^2 \frac{2x}{8x^2}}{4x^2}\right) = \lim_{x \to 0} \left(\frac{\sin \frac{2x}{2x}}{2x}\right)^2 = 1$. Hence, $k = 1$.</td>
</tr>
<tr>
<td>5.</td>
<td>(b)</td>
<td>$\frac{x^2}{2} + \log</td>
</tr>
<tr>
<td>6.</td>
<td>(c)</td>
<td>The given differential equation is $4\left(\frac{dy}{dx}\right)^3 \frac{d^2 y}{dx^2} = 0$. Here, $m = 2$ and $n = 1$. Hence, $m + n = 3$.</td>
</tr>
<tr>
<td>7.</td>
<td>(b)</td>
<td>The strict inequality represents an open half plane and it contains the origin as $(0, 0)$ satisfies it.</td>
</tr>
<tr>
<td>8.</td>
<td>(a)</td>
<td>Scalar Projection of $3\hat{i} - \hat{j} - 2\hat{k}$ on vector $\hat{i} + 2\hat{j} - 3\hat{k} = \frac{(3\hat{i} - \hat{j} - 2\hat{k}) \cdot (i + 2j - 3k)}{</td>
</tr>
<tr>
<td>9.</td>
<td>(c)</td>
<td>$\int_2^3 \frac{x}{x^2 + 1} = \frac{1}{2} \left[\log(x^2 + 1)\right]_2^3 = \frac{1}{2} \left(\log 10 - \log 5\right) = \frac{1}{2} \log\left(\frac{10}{5}\right) = \frac{1}{2} \log 2$.</td>
</tr>
<tr>
<td>10.</td>
<td>(c)</td>
<td>$(AB^{-1})^{-1} = (B^{-1})^{-1}A^{-1} = BA^{-1}$.</td>
</tr>
<tr>
<td>11.</td>
<td>(d)</td>
<td>The minimum value of the objective function occurs at two adjacent corner points $(0.6, 1.6)$ and $(3, 0)$ and there is no point in the half plane $4x + 6y &lt; 12$ in common with the feasible region. So, the minimum value occurs at every point of the line-segment joining the two points.</td>
</tr>
<tr>
<td>12.</td>
<td>(d)</td>
<td>$2 - 20 = 2x^2 - 24 \implies 2x^2 = 6 \implies x^2 = 3 \implies x = \pm \sqrt{3}$.</td>
</tr>
<tr>
<td>13.</td>
<td>(b)</td>
<td>$</td>
</tr>
<tr>
<td>14.</td>
<td>(c)</td>
<td>$P(A' \cap B') = P(A') \times P(B')$ (As $A$ and $B$ are independent, $A'$ and $B'$ are also independent.) $= 0.7 \times 0.4 = 0.28$.</td>
</tr>
<tr>
<td>15.</td>
<td>(c)</td>
<td>$ydx - xdy = 0 \implies ydx - xdy = 0 \implies \frac{dy}{y} = \frac{dx}{x} \implies \int \frac{dy}{y} = \int \frac{dx}{x} + \log K, K &gt; 0 \implies \log</td>
</tr>
</tbody>
</table>
16. (a) \[ y = \sin^{-1}x \]
\[
\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2} \frac{dy}{dx} = 1
\]
Again, differentiating both sides w. r. to \( x \), we get
\[
\sqrt{1-x^2} \frac{d^2y}{dx^2} + \frac{dy}{dx} \left( \frac{-2x}{2\sqrt{1-x^2}} \right) = 0
\]
Simplifying, we get \((1-x^2)y_2 = xy_1\)

17. (b) \[ |\vec{a} - 2\vec{b}|^2 = (\vec{a} - 2\vec{b}) \cdot (\vec{a} - 2\vec{b}) \]
\[
|\vec{a} - 2\vec{b}|^2 = \vec{a} \cdot \vec{a} - 4\vec{a} \cdot \vec{b} + 4\vec{b} \cdot \vec{b}
\]
\[= |\vec{a}|^2 - 4\vec{a} \cdot \vec{b} + 4|\vec{b}|^2\]
\[= 4 - 16 + 36 = 24\]
\[|\vec{a} - 2\vec{b}|^2 = 24 \implies |\vec{a} - 2\vec{b}| = 2\sqrt{6}\]

18. (b) The line through the points (0, 5, -2) and (3, -1, 2) is
\[
\frac{x - 3}{3 - 0} = \frac{y - 5}{-1 - 5} = \frac{z + 2}{2 + 2}
\]
\[\text{or,} \quad \frac{x}{3} = \frac{y - 5}{-6} = \frac{z + 2}{4}\]
Any point on the line is \((3k, -6k + 5, 4k - 2)\), where \( k \) is an arbitrary scalar.
\[3k = 6 \implies k = 2\]
The z-coordinate of the point \( P \) will be \( 4 \times 2 - 2 = 6 \)

19. (c) \( \sec^{-1}x \) is defined if \( x \leq -1 \) or \( x \geq 1 \). Hence, \( \sec^{-1}2x \) will be defined if \( x \leq -\frac{1}{2} \) or \( x \geq \frac{1}{2} \).
Hence, A is true.
The range of the function \( \sec^{-1}x \) is \( [0, \pi] - \{\frac{\pi}{2}\}\)
R is false.

20. (a) The equation of the x-axis may be written as \( \vec{r} = t\hat{i} \). Hence, the acute angle \( \theta \) between the given line and the x-axis is given by
\[\cos\theta = \frac{|1 \times 1 + (-1) \times 0 + 0 \times 0|}{\sqrt{1^2 + (-1)^2 + 0^2} \times \sqrt{1^2 + 0^2 + 0^2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}\]

SECTION B (VSA questions of 2 marks each)

21. \[
sin^{-1}[\sin\left(\frac{13\pi}{7}\right)] = \sin^{-1}[\sin\left(\frac{2\pi - \frac{\pi}{7}}{\frac{\pi}{7}}\right)]\]
\[= \sin^{-1}\left[\sin\left(-\frac{\pi}{7}\right)\right] = -\frac{\pi}{7}\]
\[\text{OR}\]
Let \( y \in N \) (codomain). Then \( \exists \) \( 2y \in N \) (domain) such that
\[
f(2y) = \frac{2y}{2} = y. \text{ Hence, } f \text{ is surjective.}\]
\[1, 2 \in N \text{ (domain) such that } f(1) = 1 = f(2)\]
Hence, \( f \) is not injective.

22. Let \( AB \) represent the height of the street light from the ground. At any time \( t \) seconds, let the man represented as \( ED \) of height 1.6 m be at a distance of \( x \) m from \( AB \) and the length of his shadow \( EC \) be \( y \) m.
Using similarity of triangles, we have
\[
\frac{4}{1.6} = \frac{x+y}{y} \Rightarrow 3y = 2x\]
\[\frac{1}{2}\]
Differentiating both sides w.r.to, we get \( \frac{dy}{dt} = 2 \frac{dx}{dt} \)
\[
\frac{dy}{dt} = 2 \times 0.3 \Rightarrow \frac{dy}{dt} = 0.2
\]
At any time \( t \) seconds, the tip of his shadow is at a distance of \((x + y)\) m from \( AB \).
The rate at which the tip of his shadow moving
\[
\frac{dx}{dt} + \frac{dy}{dt} m/s = 0.5 m/s
\]
\[
\frac{dy}{dt} m/s = 0.2 m/s
\]
The rate at which his shadow is lengthening
\[
\frac{dx}{dt} m/s = 0.5 m/s
\]

23. \( \vec{a} = \hat{i} - \hat{j} + 7\hat{k} \) and \( \vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k} \)
Hence \( \vec{a} + \vec{b} = 6\hat{i} - 2\hat{j} + (7 + \lambda)\hat{k} \) and \( \vec{a} - \vec{b} = -4\hat{i} + (7 - \lambda)\hat{k} \)
\( \vec{a} + \vec{b} \) and \( \vec{a} - \vec{b} \) will be orthogonal if, \((\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0\)
i.e., if, \(-24 + (49 - \lambda^2) = 0 \Rightarrow \lambda^2 = 25\)
i.e., if, \( \lambda = \pm 5\)

OR
The equations of the line are \( 6x - 12 = 3y + 9 = 2z - 2 \), which, when written in standard symmetric form, will be
\[
\frac{x-2}{6} = \frac{y-3}{3} = \frac{z-1}{2}
\]
Since, lines are parallel, we have \( \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \)
Hence, the required direction ratios are \( \left( \frac{1}{6}, \frac{1}{3}, \frac{1}{2} \right) \) or \((1,2,3)\)
and the required direction cosines are \( \left( \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right) \)

24. \( y\sqrt{1 - x^2} + x\sqrt{1 - y^2} = 1 \)
Let \( \sin^{-1}x = A \) and \( \sin^{-1}y = B \). Then \( x = \sin A \) and \( y = \sin B \)
\( y\sqrt{1 - x^2} + x\sqrt{1 - y^2} = 1 \Rightarrow \sin B\cos A + \sin A\cos B = 1 \)
\( \Rightarrow \sin(A + B) = 1 \Rightarrow A + B = \sin^{-1}1 = \frac{\pi}{2} \)
\( \Rightarrow \sin^{-1}x + \sin^{-1}y = \frac{\pi}{2} \)
Differentiating w.r.to \( x \), we obtain \( \frac{dy}{dx} = -\frac{\sqrt{1 - y^2}}{\sqrt{1 - x^2}} \)

25. Since \( \vec{a} \) is a unit vector, \( \therefore |\vec{a}| = 1 \)
\[
\begin{align*}
(x - \hat{a}) \cdot (x + \hat{a}) &= 12. \\
\Rightarrow x \cdot \hat{x} + \hat{x} \cdot \hat{a} - \hat{a} \cdot \hat{x} - \hat{a} \cdot \hat{a} &= 12 \quad \frac{1}{2} \\
\Rightarrow |\hat{x}|^2 - |\hat{a}|^2 &= 12. \\
\Rightarrow |\hat{x}|^2 - 1 &= 12 \\
\Rightarrow |\hat{x}|^2 &= 13 \Rightarrow |\hat{x}| = \sqrt{13} \\
\end{align*}
\]

SECTION C
(Short Answer Questions of 3 Marks each)

26. \[
\int \frac{dx}{\sqrt{3 - 2x - x^2}} \\
= \int \frac{dx}{\sqrt{-x^2 + 2x - 3}} = \int \frac{dx}{\sqrt{4 - (x+1)^2}} \quad 2 \\
= \sin^{-1}\left(\frac{x+1}{2}\right) + C \left[ \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \right] \quad 1
\]

27. \[
P(\text{not obtaining an odd person in a single round}) = P(\text{All three of них throw tails or All three of them throw heads}) \\
= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{1}{4} \quad \frac{1+1/2}{2} \\
P(\text{obtaining an odd person in a single round}) \\
= 1 - P(\text{not obtaining an odd person in a single round}) = \frac{3}{4} \quad \frac{1}{2} \\
The required probability \\
= P(\text{‘In first round there is no odd person’ and ‘In second round there is no odd person’ and ‘In third round there is an odd person’}) \\
= \frac{1}{4} \times \frac{1}{2} \times \frac{3}{4} = \frac{3}{64} \quad 1
\]

OR

Let X denote the Random Variable defined by the number of defective items.

\[
P(X=0) = \frac{4}{6} \times \frac{3}{5} = \frac{2}{5} \\
P(X=1) = 2 \times \left(\frac{2}{6} \times \frac{4}{5}\right) = \frac{8}{15} \quad 2 \\
P(X=2) = \frac{2}{6} \times \frac{1}{5} = \frac{1}{15} \quad 2
\]

<table>
<thead>
<tr>
<th>(x_i)</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p_i)</td>
<td>(\frac{2}{5})</td>
<td>(\frac{8}{15})</td>
<td>(\frac{1}{15})</td>
</tr>
<tr>
<td>(p_ix_i)</td>
<td>0</td>
<td>(\frac{8}{15})</td>
<td>(\frac{2}{15})</td>
</tr>
</tbody>
</table>

Mean = \(\sum p_ix_i = \frac{10}{15} = \frac{2}{3}\) \quad \frac{1}{2}

28. \[
\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sin x + \sqrt{\cos x}} \ dx \quad \text{...(i)}
\]
Using \( \int_a^b f(x) \, dx = \int_a^b f(a + b - x) \, dx \)

\[
1 = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{6} + \frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{6} + \frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{6} + \frac{\pi}{2} - x\right)}} \, dx
\]

\[
1 = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x + \sqrt{\sin x}}} \, dx \quad \text{(ii)}.
\]

Adding (i) and (ii), we get

\[
2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x + \sqrt{\cos x}}} \, dx + \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x + \sqrt{\sin x}}} \, dx
\]

\[I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x + \sqrt{\cos x}}} \, dx \quad \text{OR} \]

\[
\int_0^4 |x - 1| \, dx = \int_0^1 (1-x) \, dx + \int_1^4 (x-1) \, dx
\]

\[= \left[ x - \frac{x^2}{2} \right]_0^1 + \left[ \frac{x^2}{2} - x \right]_1^4 = (1 - \frac{1}{2}) + (8 - 4) - \left( \frac{1}{2} - 1 \right) = 5
\]

29. \( y \, dx + (x - y^2) \, dy = 0 \)

Reducing the given differential equation to the form \( \frac{dx}{dy} + P \, x = Q \) we get, \( \frac{dx}{dy} + \frac{x}{y} = y \)

\[\text{IF} = e^{\int P \, dy} = e^{\int \frac{1}{y} \, dy} = e^{\log y} = y
\]

The general solution is given by

\[
x \times \text{IF} = \int Q \times \text{IF} \, dy \Rightarrow xy = \int y^2 \, dy
\]

\[\Rightarrow xy = \frac{y^3}{3} + C, \text{ which is the required general solution}
\]

OR

\( x \, dy - y \, dx = \sqrt{x^2 + y^2} \, dx \)

It is a Homogeneous Equation as

\[\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x} = \sqrt{1 + \left( \frac{y}{x} \right)^2 + \frac{y}{x}} = f \left( \frac{y}{x} \right).
\]

Put \( y = vx \)

\[
\frac{dy}{dx} = v + x \frac{dv}{dx}
\]
\[ v + x \frac{dv}{dx} = \sqrt{1 + v^2} + v \]

Separating variables, we get
\[ \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x} \]

Integrating, we get
\[ \log|v + \sqrt{1 + v^2}| = \log|x| + \log K, \quad K > 0 \]

\[ \log \left| y + \sqrt{x^2 + y^2} \right| = \log x^2 K \]

\[ \Rightarrow y + \sqrt{x^2 + y^2} = \pm Kx^2 \]

\[ \Rightarrow y + \sqrt{x^2 + y^2} = Cx^2, \text{ which is the required general solution} \]

30. We have \( Z = 400x + 300y \) subject to \( x + y \leq 200, x \leq 40, x \geq 20, y \geq 0 \)

The corner points of the feasible region are C(20,0), D(40,0), B(40,160), A(20,180)

<table>
<thead>
<tr>
<th>Corner Point</th>
<th>( Z = 400x + 300y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(20,0)</td>
<td>8000</td>
</tr>
<tr>
<td>D(40,0)</td>
<td>16000</td>
</tr>
<tr>
<td>B(40,160)</td>
<td>64000</td>
</tr>
<tr>
<td>A(20,180)</td>
<td>62000</td>
</tr>
</tbody>
</table>

Maximum profit occurs at \( x = 40, y = 160 \)
and the maximum profit = \( \text{₹} 64,000 \)

31. \[ \int \frac{x^3 + x + 1}{(x^2 - 1)} \ dx = \int \left( x + \frac{2x + 1}{(x-1)(x+1)} \right) \ dx \]

Now resolving \( \frac{2x + 1}{(x-1)(x+1)} \) into partial fractions as

\[ \frac{2x + 1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} \]

We get
\[ \frac{2x + 1}{(x-1)(x+1)} = \frac{3}{2(x-1)} + \frac{1}{2(x+1)} \]
Hence, \(\int \frac{x^3 + x + 1}{x^2 - 1} \, dx = \int \left( x + \frac{2x + 1}{(x-1)(x+1)} \right) \, dx \)
\[= \int \left( x + \frac{3}{2(x-1)} + \frac{1}{2(x+1)} \right) \, dx \]
\[= \frac{x^2}{2} + \frac{3}{2} \log|x - 1| + \frac{1}{2} \log|x + 1| + C \]
\[= \frac{x^2}{2} + \frac{1}{2} \left( \log|x - 1|^3 (x + 1) \right) + C \)

\[\text{SECTION D} \]

\((\text{Long answer type questions (LA) of 5 marks each})\)

32. The points of intersection of the parabola \(y = x^2\) and the line \(y = x\) are \((0, 0)\) and \((1, 1)\).

Required Area = \(\int_0^1 y_{\text{parabola}} \, dx + \int_1^2 y_{\text{line}} \, dx\)

Required Area = \(\int_0^1 x^2 \, dx + \int_2^1 x \, dx \)
\[= \left[ \frac{x^3}{3} \right]_0^1 + \left[ \frac{x^2}{2} \right]_1^2 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6} \)

33. Let \((a, b) \in N \times N\). Then we have \(ab = ba\) (by commutative property of multiplication of natural numbers)

\[\Rightarrow (a, b) R (a, b) \]

Hence, \(R\) is reflexive.

Let \((a, b), (c, d) \in N \times N\) such that \((a, b) R (c, d)\). Then \(ad = bc\)

\[\Rightarrow cb = da \text{ (by commutative property of multiplication of natural numbers)} \]

\[\Rightarrow (c, d) R (a, b) \]

Hence, \(R\) is symmetric.

Let \((a, b), (c, d), (e, f) \in N \times N\) such that
(a, b) R (c, d) and (c, d) R (e, f).
Then \( ad = bc, \ cf = de \)
\[ \Rightarrow adcf = bcde \]
\[ \Rightarrow af = be \]
\[ \Rightarrow (a, b)R(e, f) \]
Hence, R is transitive.
Since, R is reflexive, symmetric and transitive, R is an equivalence relation on \( N \times N \).

**OR**

Let \( A \in P(X) \). Then \( A \subset A \)
\[ \Rightarrow (A, A) \in R \]
Hence, R is reflexive.
Let \( A, B, C \in P(X) \) such that
\( (A, B), (B, C) \in R \)
\[ \Rightarrow A \subset B, B \subset C \]
\[ \Rightarrow A \subset C \]
\[ \Rightarrow (A, C) \in R \]
Hence, R is transitive.
\( \emptyset, X \in P(X) \) such that \( \emptyset \subset X \).
Hence, \( (\emptyset, X) \in R \).
But, \( X \not\subset \emptyset \), which implies that \( (X, \emptyset) \not\in R \).
Thus, R is not symmetric.

34. The given lines are non-parallel lines. There is a unique line-segment PQ (P lying on one and Q on the other, which is at right angles to both the lines. PQ is the shortest distance between the lines. Hence, the shortest possible distance between the insects = PQ

The position vector of P lying on the line
\[ \vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}) \]
is \( (6 + \lambda)\hat{i} + (2 - 2\lambda)\hat{j} + (2 + 2\lambda)\hat{k} \) for some \( \lambda \)
The position vector of Q lying on the line
\[ \vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}) \]
is \( (-4 + 3\mu)\hat{i} + (-2\mu)\hat{j} + (-1 - 2\mu)\hat{k} \) for some \( \mu \)
\[ \vec{PQ} = (-10 + 3\mu - \lambda)\hat{i} + (-2\mu - 2 + 2\lambda)\hat{j} + (-3 - 2\mu - 2\lambda)\hat{k} \]
Since, PQ is perpendicular to both the lines
\[ (-10 + 3\mu - \lambda) + (-2\mu - 2 + 2\lambda)(-2) + (-3 - 2\mu - 2\lambda)2 = 0, \]
\[ i.e., \mu - 3\lambda = 4 \quad \text{...(i)} \]
and \( (-10 + 3\mu - \lambda)3 + (-2\mu - 2 + 2\lambda)(-2) + (-3 - 2\mu - 2\lambda)(-2) = 0, \)
\[ i.e., 17\mu - 3\lambda = 20 \quad \text{...(ii)} \]
solving (i) and (ii) for \( \lambda \) and \( \mu \), we get \( \mu = 1, \lambda = -1. \)
The position vector of the points, at which they should be so that the distance between them is the shortest, are
\[ 5\hat{i} + 4\hat{j} \text{ and } -\hat{i} - 2\hat{j} - 3\hat{k} \]
\[ \vec{PQ} = -6\hat{i} - 6\hat{j} - 3\hat{k} \]
The shortest distance = \[ |\vec{PQ}| = \sqrt{6^2 + 6^2 + 3^2} = 9 \]

OR
Eliminating $t$ between the equations, we obtain the equation of the path $\frac{x}{2} = \frac{y}{-4} = \frac{z}{4}$, which are the equations of the line passing through the origin having direction ratios $<2, -4, 4>$. This line is the path of the rocket.

When $t = 10$ seconds, the rocket will be at the point $(20, -40, 40)$. Hence, the required distance from the origin at 10 seconds =

$$\sqrt{20^2 + 40^2 + 40^2} km = 20 \times 3 \text{ km} = 60 \text{ km}$$

The distance of the point $(20, -40, 40)$ from the given line $= \frac{|\overrightarrow{a} \times \overrightarrow{b}|}{|\overrightarrow{b}|} = \frac{|-30x(10t-20)+10k|}{|10t-20+10k|} km = \frac{|-300t+300k|}{|10t-20+10k|} km = 300 \sqrt{3} km$.

**SECTION E (Case Studies/Passage based questions of 4 Marks each)**

36. (i) $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function, is differentiable everywhere, hence, differentiable in $(0, 12)$

(ii) $f'(x) = -0.2x + m$

Since, 6 is the critical point,

$f'(6) = 0 \Rightarrow m = 1.2$

(iii) $f(x) = -0.1x^2 + 1.2x + 98.6$

$f'(x) = -0.2x + 1.2 = -0.2(x - 6)$

<table>
<thead>
<tr>
<th>In the Interval</th>
<th>$f'(x)$</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0, 6)$</td>
<td>+ve</td>
<td>$f$ is strictly increasing in $[0, 6]$</td>
</tr>
<tr>
<td>$(6, 12)$</td>
<td>-ve</td>
<td>$f$ is strictly decreasing in $[6, 12]$</td>
</tr>
</tbody>
</table>
OR

(iii) \( f(x) = -0.1x^2 + 1.2x + 98.6, \)
\( f'(x) = -0.2x + 1.2, f'(6) = 0, \)
\( f''(x) = -0.2 \)
\( f''(6) = -0.2 < 0 \)
Hence, by second derivative test 6 is a point of local maximum. The local maximum value = \( f(6) = -0.1 \times 6^2 + 1.2 \times 6 + 98.6 = 102.2 \)
We have \( f(0) = 98.6, f(6) = 102.2, f(12) = 98.6 \)
6 is the point of absolute maximum and the absolute maximum value of the function = 102.2.
0 and 12 both are the points of absolute minimum and the absolute minimum value of the function = 98.6.

37. (i)
Let \( (x, y) = \left( x, \frac{b}{a} \sqrt{a^2 - x^2} \right) \) be the upper right vertex of the rectangle.
The area function \( A = 2x \times 2 \frac{b}{a} \sqrt{a^2 - x^2} \)
\( = \frac{4b}{a} x \sqrt{a^2 - x^2}, x \in (0, a). \)

(ii) \( \frac{dA}{dx} = \frac{4b}{a} \left[ x \times \frac{-x}{\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2} \right] \)
\( = \frac{4b}{a} \left\{ \frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}} \right\} = - \frac{4b}{a} \times \frac{2 \left( x + \frac{a}{\sqrt{2}} \right) \left( x - \frac{a}{\sqrt{2}} \right)}{\sqrt{a^2 - x^2}} \)
\( \frac{dA}{dx} = 0 \Rightarrow x = \frac{a}{\sqrt{2}} \)
x = \frac{a}{\sqrt{2}} is the critical point.

(iii) For the values of x less than \( \frac{a}{\sqrt{2}} \) and close to \( \frac{a}{\sqrt{2}} \), \( \frac{dA}{dx} > 0 \)
and for the values of x greater than \( \frac{a}{\sqrt{2}} \) and close to \( \frac{a}{\sqrt{2}} \), \( \frac{dA}{dx} < 0 \).
Hence, by the first derivative test, there is a local maximum at the critical point \( x = \frac{a}{\sqrt{2}} \). Since there is only one critical point, therefore, the area of the soccer field is maximum at this critical point \( x = \frac{a}{\sqrt{2}} \).
Thus, for maximum area of the soccer field, its length should be \( a\sqrt{2} \) and its width should be \( b\sqrt{2} \).

OR
(iii) \( A = 2x \times 2 \frac{\beta}{a} \sqrt{a^2 - x^2}, x \in (0, a) \).

Squaring both sides, we get

\[
Z = A^2 = \frac{16\beta^2}{a^2} x^2(a^2-x^2) = \frac{16\beta^2}{a^2} (x^2 a^2 - x^4), x \in (0, a).
\]

A is maximum when \( Z \) is maximum.

\[
dZ = \frac{16\beta^2}{a^2} (2xa^2 - 4x^3) = \frac{32\beta^2}{a^2} x(a + \sqrt{2}x)(a - \sqrt{2}x)
\]

\[
\frac{dZ}{dx} = \frac{32\beta^2}{a^2} (a^2 - 6x^2)
\]

\[
\frac{d^2Z}{dx^2} = \frac{32\beta^2}{a^2} (a^2 - 3a^2) = -64\beta^2 < 0
\]

Hence, by the second derivative test, there is a local maximum value of \( Z \) at the critical point \( x = \frac{a}{\sqrt{2}} \). Since there is only one critical point, therefore, \( Z \) is maximum at \( x = \frac{a}{\sqrt{2}} \), hence, \( A \) is maximum at \( x = \frac{a}{\sqrt{2}} \).

Thus, for maximum area of the soccer field, its length should be \( a\sqrt{2} \) and its width should be \( b\sqrt{2} \).

38. (i) Let \( P \) be the event that the shell fired from A hits the plane and \( Q \) be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

\( E_1 = PQ, E_2 = \bar{P}Q, E_3 = \bar{P}Q, E_4 = P \bar{Q} \)

Let \( E \) = The shell fired from exactly one of them hits the plane.

\[
P(E_1) = 0.3 \times 0.2 = 0.06, \ P(E_2) = 0.7 \times 0.8 = 0.56, \ P(E_3) = 0.7 \times 0.2 = 0.14, \ P(E_4) = 0.3 \times 0.8 = 0.24
\]

\[
P \left( \frac{E}{E_1} \right) = 0, \ P \left( \frac{E}{E_2} \right) = 0, \ P \left( \frac{E}{E_3} \right) = 1, \ P \left( \frac{E}{E_4} \right) = 1
\]

\[
P(E) = P(E_1)P \left( \frac{E}{E_1} \right) + P(E_2)P \left( \frac{E}{E_2} \right) + P(E_3)P \left( \frac{E}{E_3} \right) + P(E_4)P \left( \frac{E}{E_4} \right)
\]

\[
= 0.14 + 0.24 = 0.38
\]

(ii) By Bayes’ Theorem,

\[
P \left( \frac{E_3}{E} \right) = \frac{P(E_3)P \left( \frac{E}{E_3} \right)}{P(E_1)P \left( \frac{E}{E_1} \right) + P(E_2)P \left( \frac{E}{E_2} \right) + P(E_3)P \left( \frac{E}{E_3} \right) + P(E_4)P \left( \frac{E}{E_4} \right)}
\]

\[
= \frac{0.14 \times 7}{0.38 \times 19}
\]

NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses \( E_1 \) and \( E_2 \) are actually eliminated as \( P \left( \frac{E}{E_1} \right) = P \left( \frac{E}{E_2} \right) = 0 \)

**Alternative way of writing the solution:**

(i) \( P(\text{Shell fired from exactly one of them hits the plane}) \)

\[
= P(\text{Shell from A hits the plane and Shell from B does not hit the plane}) \lor (\text{Shell from A does not hit the plane and Shell from B hits the plane})
\]

\[
= 0.3 \times 0.8 + 0.7 \times 0.2 = 0.38
\]

(ii) \( P(\text{Shell fired from B hit the plane/Exactly one of them hit the plane}) \)

\[
= P(\text{Shell fired from B hit the plane } \cap \text{ Exactly one of them hit the plane})
\]

\[
= P(\text{Exactly one of them hit the plane})
\]
\[
\begin{array}{c}
P(\text{Shell from only 8 hit the plane}) \\
P(\text{Exactly one of them hit the plane}) \\
\begin{align*}
0.14 &= \frac{7}{0.38} = \frac{19}{19} \\
0.14 &= \frac{7}{0.38} = \frac{19}{19}
\end{align*}
\end{array}
\]