# Applied Mathematics

## Term - II

**Code-241**

<table>
<thead>
<tr>
<th>Q.N.</th>
<th>Hints/Solutions</th>
<th>Marks</th>
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<tbody>
<tr>
<td><strong>Section – A</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 1 | Given, \( MR = 9 + 2x - 6x^2 \)  
\[ TR = \int (9 + 2x - 6x^2) \, dx \]  
\[ TR = 9x + x^2 - 2x^3 + k \]  
When \( x = 0, TR = 0 \), so \( k = 0 \)  
\[ TR = 9x + x^2 - 2x^3 \]  
\[ ⇒ \]  
\[ px = 9x + x^2 - 2x^3 \]  
\[ ⇒ \]  
\[ p = 9 + x - 2x^2 ] which is the demand function | 1 |

**OR**

\[ TC = \int (50 + \frac{300}{x+1}) \, dx \]  
\[ TC = 50x + 300\log|x + 1| + k \]  
If \( x = 0, TC = ₹2000 \)  
So \( 2000 = 300(\log 1) + k \)  
\[ ⇒ \]  
\[ k = 2000 \]  
So \( TC = 50x + 300\log(x + 1) + 2000 \) | 1 |

2  
\[ R = ₹600 \]  
\[ i = \frac{0.08}{4} = 0.02 \]  
Present value of perpetuity \( P = R \, \frac{i}{i} \)  
\[ ⇒ P = \frac{600}{0.02} = ₹30,000 \] | 1 |

3  
\[ r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1 \]  
\[ = \left(1 + \frac{0.08}{4}\right)^4 - 1 \]  
\[ = (1.02)^4 - 1 = 0.0824 \]  
or 8.24%  
So effective rate is 8.24% compounded annually.  
**OR**  
Present value of ordinary annuity  
\[ = R \left(\frac{1-(1+r)^{-m}}{r}\right) \]  
\[ = 1000 \left(\frac{1-(1.06)^{-5}}{0.06}\right) \]  
\[ = 1000 \left(\frac{1-0.7473}{0.06}\right) = ₹4211.67 \] | 1 |

4  
\[ E(X) = 60\text{kg} \] | 1 |
Standard deviation of $\bar{X} = SE(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{9}{6} = 1.5 \text{ kg}$

<table>
<thead>
<tr>
<th>Year</th>
<th>3 yearly moving total</th>
<th>3 yearly moving average (Trend) (in ₹ lakh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2016</td>
<td>25</td>
<td>---</td>
</tr>
<tr>
<td>2017</td>
<td>30</td>
<td>87</td>
</tr>
<tr>
<td>2018</td>
<td>32</td>
<td>102</td>
</tr>
<tr>
<td>2019</td>
<td>40</td>
<td>117</td>
</tr>
<tr>
<td>2020</td>
<td>45</td>
<td>135</td>
</tr>
<tr>
<td>2021</td>
<td>50</td>
<td>---</td>
</tr>
</tbody>
</table>

1M for 3-yearly moving totals
1M for 3-yearly moving average

Corner Point | Z=3x+2y
-------------|---------
P (2, 2)      | 10      
Q (3, 0)      | 9       

The smallest value of Z is 9. Since the feasible region is unbounded, we draw the graph of $3x + 2y < 9$. The resulting open half plane has points common with feasible region, therefore $Z = 9$ is not the minimum value of Z. Hence the optimal solution does not exist.

Section – B

Substituting, $p_0 = ₹48$ in $p = x^2 + 4x + 3$
We get $x_0 = 5$
$PS = p_0x_0 - \int_{0}^{x_0} g(x) dx$
$= 48 \times 5 - \int_{0}^{5} (x^2 + 4x + 3) dx$
$= 240 - \left[ \frac{x^3}{3} + 2x^2 + 3x \right]_{0}^{5} = ₹133.33$
The trend values are given by 4 quarterly centered moving average.

\[ Y = \text{Year} - 2017 \]

\[ \begin{align*}
X_1 & = 12 \\
X_2 & = 14 \\
X_3 & = 18 \\
X_4 & = 20 \\
X_5 & = 18 \\
X_6 & = 16 \\
X_7 & = 20 \\
X_8 & = 22 \\
X_9 & = 27 \\
X_{10} & = 24 \\
X_{11} & = 30 \\
X_{12} & = 36
\end{align*} \]

\[ \begin{align*}
X^2 & = 9 \\
X^2 & = 4 \\
X^2 & = 1 \\
X^2 & = 0 \\
X^2 & = 1 \\
X^2 & = 4 \\
X^2 & = 9
\end{align*} \]

\[ \begin{align*}
\sum Y & = 532 \\
\sum X^2 & = 28 \\
\sum XY & = 672
\end{align*} \]

\[ a = \frac{\sum Y}{n} = \frac{532}{7} = 76, \quad b = \frac{\sum XY}{\sum X^2} = \frac{672}{28} = 24 \]

\[ Y_c = a + bX, \quad Y_c = 76 + 24X \]

Estimated sales = \( Y_c \) for 2023 = 76 + 24 \times 6 = ₹220 lacs

Define Null hypothesis \( H_0 \) and alternate hypothesis \( H_1 \) as follows:

\[ H_0: \mu = 0.50 \text{ mm} \]

\[ H_1: \mu \neq 0.50 \text{ mm} \]
Thus a two-tailed test is applied under hypothesis $H_0$, we have
\[ t = \frac{\bar{X} - \mu}{s} \sqrt{n - 1} = \frac{0.53 - 0.50}{0.03} \times 3 = 3 \]
Since the calculated value of $t = 3$ does not lie in the internal $-t_{0.025}$ to $t_{0.025}$ i.e., -2.262 to 2.262 for 10-1= 9 degree of freedom so we Reject $H_0$ at 0.05 level. Hence we conclude that machine is not working properly.

We know
\[ \text{CAGR} = \left[ \left( \frac{FV}{IV} \right)^\frac{1}{n} - 1 \right] \times 100, \text{ where, IV= Initial value of investment} \]
\[ \text{FV= Final value of investment} \]
\[ \Rightarrow 0.88 = \left[ \left( \frac{25000}{15000} \right)^\frac{1}{n} - 1 \right] \times 100 \Rightarrow 0.0888 = \left( \frac{5}{3} \right)^\frac{1}{n} - 1 \]
\[ \Rightarrow 1.089 = (1.667)^{\frac{1}{n}} \]
\[ \Rightarrow \frac{1}{n} \log(1.667) = \log(1.089) \Rightarrow n(0.037) = 0.2219 \]
\[ \Rightarrow n = 5.99 \approx 6 \text{ years} \]

**Section –C**

Let the company produces $x$ and $y$ gallons of alkaline solution and base oil respectively, also let $C$ be the production cost.
\[ \text{Min } C = 200x + 300y \]
subject to constraints:
\[ x + y \geq 3500 \ldots (1) \]
\[ x \geq 1250 \ldots (2) \]
\[ 2x + y \leq 6000 \ldots (3) \]
\[ x, y \geq 0 \]

<table>
<thead>
<tr>
<th>Corner Points</th>
<th>$C = 200x + 300y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(1250, 2250)</td>
<td>₹9,25,000</td>
</tr>
<tr>
<td>Q(1250, 3500)</td>
<td>₹13,00,000</td>
</tr>
<tr>
<td>---------------</td>
<td>------------</td>
</tr>
<tr>
<td>R(2500, 1000)</td>
<td>₹8,00,000</td>
</tr>
</tbody>
</table>

Minimum cost is ₹8,00,000 when 2500 gallons of alkaline solutions & 1000 gallons of base oil are manufactured.

12. The amount of sinking fund S at any time is given by

\[ S = R \left( \frac{(1+i)^n-1}{i} \right) \]

Where \( R = \text{Periodic payment}, \ i = \text{Interest per period}, \ n = \text{number of payments} \)

\( S = \text{Cost of machine} - \text{Salvage value} \)

\( = 50,000 - 5000 = ₹45,000 \)

\( i = \frac{8\%}{4} = 0.02 \)

\[ 45000 = R \left( \frac{(1+0.02)^{40}-1}{0.02} \right) \]

\[ 45000 = R \left( \frac{2.208-1}{0.02} \right) \]

\[ R = \frac{900}{1.208} \Rightarrow R = ₹745.03 \]

13. Amortized Amount i.e., P= Cost of house-Cash down payment

\( P = 15,00,000 - 4,00,000 = ₹11,00,000 \)

\( i = \frac{0.09}{12} = 0.0075 \)

\( n = 10 \times 12 = 120 \)

\[ EMI = R = \frac{P}{a_{n|i}} = \frac{11,00,000}{1-1.0075^{-120}} = \frac{8250}{1-0.4079} = ₹13933.5 \]

Total interest paid = \( nR - R = 13933.5 \times 120 - 11,00,000 \)

\( = ₹5,72,020 \)

OR

Face value of bond, F = ₹2000

Redemption value C = 1.05 \times 2000 = ₹2100

Nominal rate = 8%

\( R = C \times i_d = 2000 \times 0.08 = ₹160 \)

Number of periods before redemption i.e., n = 10

Annual yield rate, \( i = 10\% \) or 0.1

Purchase price \( V = R \left[ \frac{1-\left(1+i\right)^{-n}}{i} \right] + C \left(1 + i\right)^{-n} \)

\[ = 160 \left[ \frac{1-\left(1+0.1\right)^{-10}}{0.1} \right] + 2100 \left(1 + 0.1\right)^{-10} \]

\[ = 160 \times 6.14 + 2100 \times 0.3855 \]

\[ = 982.4 + 809.6 = 1792 \]
Thus present value of the bond is ₹1792.

| Case Study |
|------------|---|
| a) $\frac{dx}{dt} \propto x$, $\therefore \frac{dx}{dt} = -kx$ | 1 |
| $\Rightarrow \int \frac{dx}{x} = \int -k \, dt \Rightarrow \log x = -kt + c$ | 1 |
| $\Rightarrow x = e^{-kt + c} \Rightarrow x = \lambda e^{-kt}$ | 1 |
| Let $x = x_0$ at $t = 0$ | 1 |
| $\therefore x_0 = \lambda \Rightarrow x = x_0 e^{-kt}$ where $x_0$ = original quantity | 1 |
| $x = x_0 e^{-kt}$ ....(1) | 1 |
| Now, $\frac{x_0}{2} = x_0 e^{-5k}$ (∵ half life = 5 hours) | 1 |
| $\Rightarrow e^{-5k} = \frac{1}{2} \Rightarrow e^{k} = 2^{\frac{1}{5}}$ | 1 |
| The quantity of propofol needed in a 50 Kg adult at the end of 2 hours $= 50 \times 3 = 150 \text{ mg}$ $\Rightarrow 150 = x_0 e^{-2k}$ [using...(1)] | 1 |
| $\Rightarrow x_0 = 150 e^{2k} \Rightarrow x_0 = 150 (e^{k})^{2}$ | 1 |
| $\Rightarrow x_0 = 150(2^{\frac{1}{5}})^2 = 150 \times 1.3195 = 197.93 \text{ mg}$ | 1 |