Competency Focused Practice Questions

Mathematics (Volume 2) | Grade 12

Co-created by

CBSE Centre for Excellence in Assessment

and

Educational Initiatives

Preface

Assessments are an important tool that help gauge learning. They provide valuable feedback about the effectiveness of instructional methods; about what students have actually understood and also provide actionable insights. The National Education Policy, 2020 has outlined the importance of competency-based assessments in classrooms as a means to reform curriculum and pedagogical methodologies. The policy emphasizes on the development of higher order skills such as analysis, critical thinking and problem solving through classroom instructions and aligned assessments.

Central Board of Secondary Education (CBSE) has been collaborating with Educational Initiatives (Ei) in the area of assessment. Through resources like the Essential Concepts document and A- Question-A-Day (AQAD), high quality questions and concepts critical to learning have been shared with schools and teachers.

Continuing with the vision to ensure that every student is learning with understanding, Question Booklets have been created for subjects for Grade 10th and 12th. These booklets contain competency-based items, designed specifically to test conceptual understanding and application of concepts.

Process of creating competency-based items

All items in these booklets are aligned to the NCERT curriculum and have been created keeping in mind the learning outcomes that are important for students to understand and master. Items are a mix of Free Response Questions (FRQs) and Multiple-Choice Questions (MCQs). In case of MCQs, the options (correct answer and distractors) are specifically created to test for understanding and capturing specific errors/misconceptions that students may harbour. Each incorrect option can thereby inform teachers on specific gaps that may exist in student learning. In case of subjective questions, each question also has a detailed scoring rubric to guide evaluation of students' responses.

Each item has been reviewed by experts, to check for appropriateness of the item, validity of the item, conceptual correctness, language accuracy and other nuances.

How can these item booklets be used?

There are 137 questions in this booklet.

The purpose of these item booklets is to provide samples of high-quality competency-based items to teachers. The items can be used to–

- get an understanding of what good competency-based questions could look like
- give exposure to students to competency-based items
- assist in classroom teaching and learning
- get inspiration to create more such competency-based items

Students can also use this document to understand different kinds of questions and practice specific concepts and competencies. There will be further additions in the future to provide competency focused questions on all chapters.

The item booklets are aligned with the 2022-23 curriculum. However, a few questions from topic which got rationalized in 2023-24 syllabus are also there in the booklet which may be used as a reference for teachers and students.

Please write back to us to give your feedback.

Team CBSE

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Chapter - 1
Relations and Functions

Free Response Questions

Q: 1 *f* is a strictly increasing function while *g* is a strictly decreasing function. The range of *f* and *g* are the same as the codomain of *f* and *g* respectively. **[2]**

If (*fog*) is defined, will (*fog*) be an invertible function? Justify your answer.

Q: 2 *Prathibha Karanji* is an innovative program by the Government of Karnataka, India, where cultural and literacy competitions are held between schools at cluster, block, district and state levels. **[5]**

One of those competitions - Yogasana, is conducted under two categories - Middle school and High school. From a certain district, three students from middle school and two students from high school were selected for the state level.

Let M = { m_1, m_2, m_3 }, H = { h_1, h_2 }, represent the set of students from middle school and high school respectively who got selected for the state level from that district.

i) A relation R:M -> M is defined by R = {(x, y): x and y are students from the same category}. Show that R is an equivalence relation.

ii) A function *f* :M -> H is defined by *f* = {(m_1, h_1), (m_2, h_2), (m_3, h_2)} . Show that *f* is onto but not one-one.

Q: 3 Two functions *f* : R -> R and *g* : R -> *R* , where R is the set of real numbers, are defined as follows: **[5]**

$f(x) = (x + \sin x)$
$g(x) = (-x - \cos x)$

i) Find (*fog*)(*x*).
ii) Find (*gof*)(*x*).
iii) Using i) and ii), show that (*gof*)(0) - sin(1) = (*fog*)(0).

Show your steps.

Q: 4 If $f(x) = 3 + \left(\frac{e^{3x}+e^{-3x}}{e^{3x}-e^{-3x}}\right)$ and $f^{-1}(x) = \frac{1}{A}g(x)$, find : **[5]**

i) the value of A.
ii) $g(x)$.

Show your steps.

Q.No	What to look for	Marks
1	Writes that, *f* and *g* are one-to-one functions since they are strictly increasing and strictly decreasing respectively.	0.5
	Writes that, (*fog*) is also one-to-one since *f* and *g* are one-to-one.	0.5
	Writes that, (*fog*) is onto since it is given that *f* and *g* are onto functions.	0.5
	Concludes that, (*fog*) is a bijective function and therefore invertible.	0.5
2	i) Shows that R is reflective. The working may look as follows: x and x are from the same category => $(x, x) \in$ R for every $x \in$ M	0.5
	Shows that R is symmetric. The working may look as follows: $(x, y) \in$ R => x and y are from the same category. => y and x are from the same category. => $(y, x) \in$ R	1
	Shows that R is transitive. The working may look as follows: $(x, y) \in$ R and $(y, z) \in$ R => x and y are from the same category and, y and z are from the same category. => x, y and z are from the same category. => $(x, z) \in$ R	1
	Uses steps 1, 2 and 3 to conclude that R is an equivalence relation.	0.5
	ii) Writes that the Range of $f = \{ h_1, h_2 \}$ = codomain of f. Hence concludes that f is onto.	1

Q.No	What to look for	Marks
	Writes that $f(m_2) = f(m_3) = h_2$, but $m_2 \neq m_3$. Hence concludes that f is not one-one.	**1**
3	**i) Finds (*fog*)(*x*) as follows:** $f[g(x)] = f(-x - \cos x) = -x - \cos x + \sin(-x - \cos x)$	**1.5**
	ii) Finds (*gof*)(*x*) as follows: $g[f(x)] = g(x + \sin x) = -(x + \sin x) - \cos(x + \sin x)$	**1.5**
	iii) Substitutes $x = 0$ in the expression obtained in step 1 to find (*fog*)(0) as [-1 - sin(1)].	**0.5**
	Substitutes $x = 0$ in the expression obtained in step 2 to find (*gof*)(0) as (-1).	**0.5**
	Finds (*gof*)(0) - sin(1) as [-1 - sin(1)]. **Concludes that (*gof*)(0) - sin(1) = (*fog*)(0).**	**1**
4	**Equates f(x) to y and writes:** $f(x) = y = 3 + \left(\frac{e^{6x}+1}{e^{6x}-1}\right)$	**0.5**
	Simplifies the above equation to get: $y = \frac{4e^{6x}-2}{e^{6x}-1}$	**1**
	Simplifies the above equation to get: $e^{6x} = \frac{y-2}{y-4}$	**1**

Q.No	What to look for	Marks
	Simplifies the above equation to get: $x = f^{-1}(y) = \frac{1}{6}\log_e \frac{y-2}{y-4}$	**1.5**
	Rewrites the above equation in terms of *x* as: $f^{-1}(x) = \frac{1}{6}\log_e \frac{x-2}{x-4}$	**0.5**
	Compares the above equation with the given $f^{-1}(x)$ and writes: $A = 6$ and $g(x) = \log_e \left(\frac{x-2}{x-4}\right)$	**0.5**

Chapter - 2
Inverse Trigonometric Functions

Multiple Choice Questions

Q: 1 **Which of the following is equal to $-\tan^{-1}\left(\frac{2}{3\pi}\right)$?**

1. $\cot^{-1}\left(\frac{3\pi}{2}\right)$
2. $-\cot^{-1}\left(\frac{3\pi}{2}\right)$
3. $\frac{\pi}{2} - \cot^{-1}\left(\frac{3\pi}{2}\right)$
4. $\frac{\pi}{2} + \cot^{-1}\left(\frac{3\pi}{2}\right)$

Free Response Questions

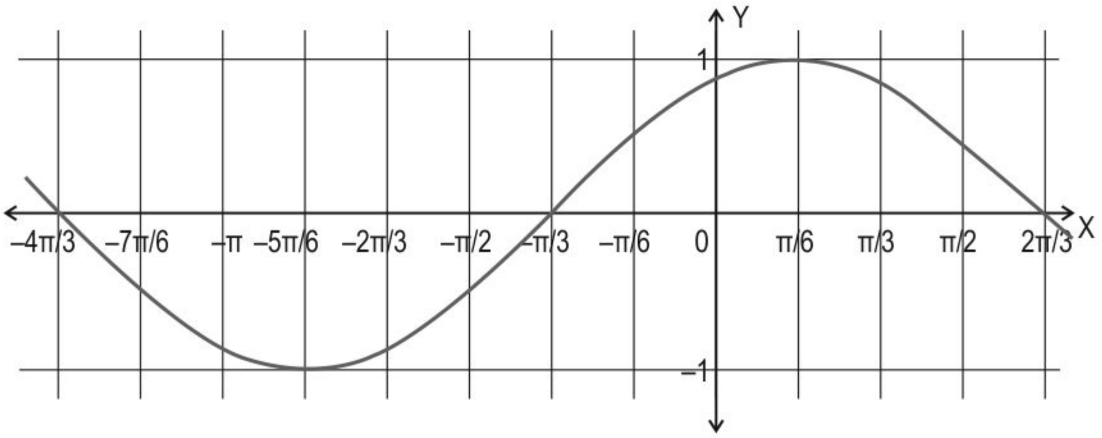
Q: 2 **Draw the graph of $\cos^{-1}(2x)$ in the domain $[-\frac{1}{2}, \frac{1}{2}]$.** **[1]**

Q: 3 **State whether the following statements are true or false. Explain your reasoning.** **[2]**

i) $\cos(\sec(x))^{-1} = \frac{1}{x}$

ii) $\sin^{-1}(-x) + \cos^{-1}(-x) = \sin^{-1}(x) - \cos^{-1}(x)$

Q: 4 **Shown below is a portion of the graph for $\sin(x + a)$, where $a \in R$ and $a > 0$.** **[2]**



What should the domain of the function be restricted to such that the function is invertible? Give valid reasons for your answer.

Q: 5 **If $4x + \frac{1}{x} = 4$, $x \in [0, \frac{\pi}{2}]$, find $\cos^{-1}x - \sin^{-1}x$. Show your work.** **[2]**

Q: 6 Show that $\cot^{-1}\left(\frac{1-x^2}{2x}\right) + \tan^{-1}\left(\frac{x^3-3x}{1-3x^2}\right) = -\tan^{-1}x$. **[3]**

Q: 7 **$f(x) = \cos(2\sin^{-1} x)$ and $g(x) = \sin^2(2\cos^{-1} x)$, $-1 \le x \le 1$.** **[3]**

If $h(x) = (f(x) + g(x))$, find $h(0.1)$. Show your work.

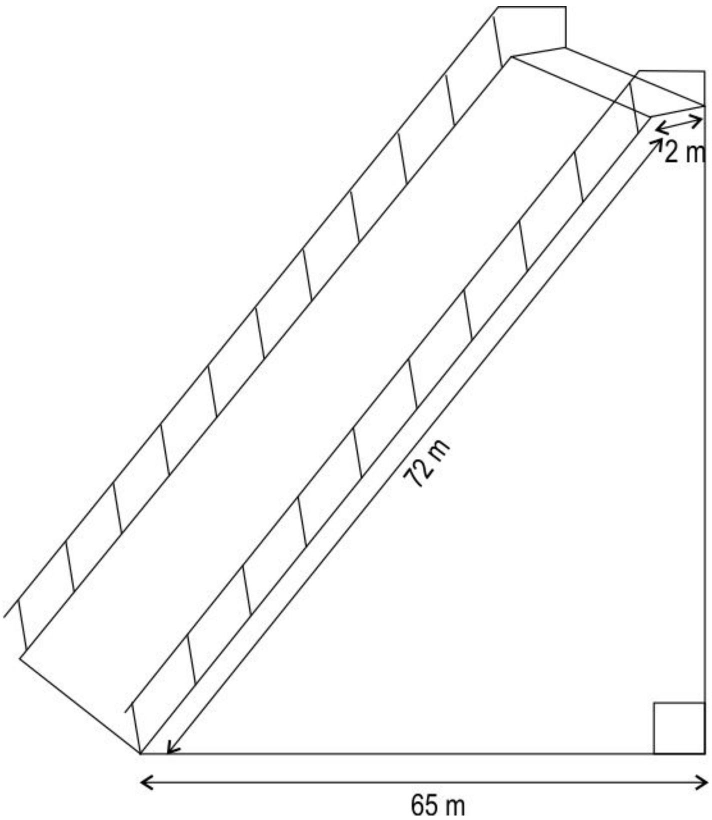
Q: 8 $\text{cosec}\,\theta = \frac{1}{\sin\theta}$ **[3]**

For $y \in \mathrm{R}$ where $\text{cosec}^{-1} y$ and $\sin^{-1} y$ are defined,
is $\text{cosec}^{-1} y = \frac{1}{\sin^{-1} y}$? If not, write the correct statement.

Q: 9 **Find the domain and range of the function $y = \sin^{-1}(|x| - 1)$. Show your work and give valid reasons.** **[3]**

Q: 10 **As per guidelines issued by the Home Ministry of India, in order for a ramp to be accessible to persons with disability, it must have a minimum slope ratio of 1:12. That is, for every unit of height, there must be at least 12 units of ramp.** **[3]**
(Source: https://www.mha.gov.in/sites/default/files/PublicNoticeforDraftStandards_22112021.pdf)

Abbas measured the dimensions of a ramp at his school and noted them in a rough diagram, as shown in the figure below.



(Note: The figure is not to scale.)

Is the given ramp meeting the Home Ministry's accessibility standards? Justify your answer.

(Note: Assume that the top of the ramp is parallel to the ground.)

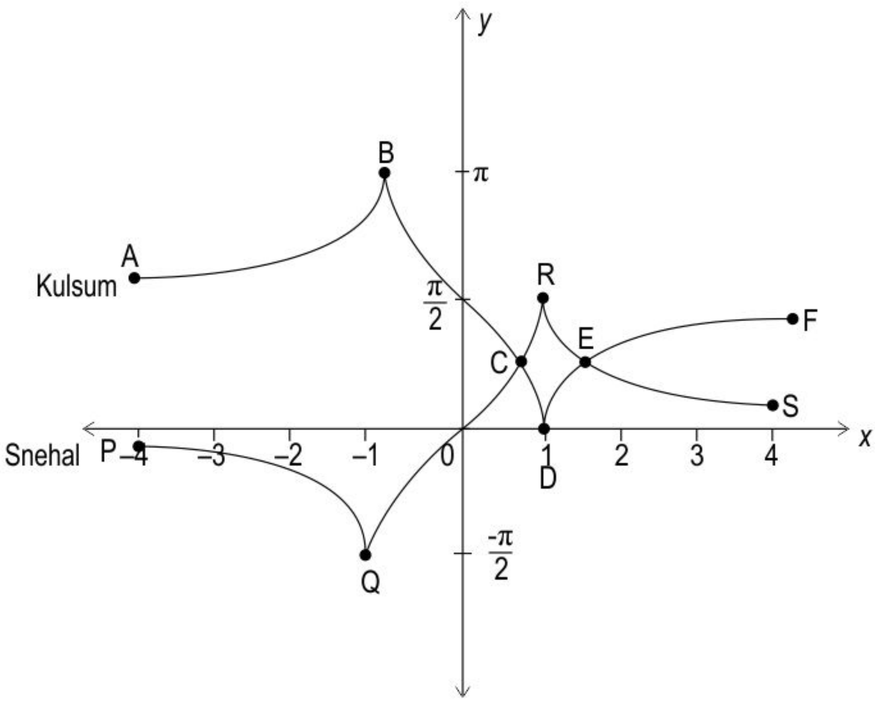
Q: 11 Evaluate $\tan\left(\frac{1}{2}\cos^{-1}\frac{5}{7}\right)$. Show your work. **[5]**

Case Study

Answer the questions based on the given information.

Madhu created a design on his floor using a combination of graphs of inverse trigonometric functions in the domain [-4, 4]. He also represented the coordinate axes for reference. He asked his friends, Kulsum and Snehal, to choose a path to walk on. Kulsum chose path ABCDEF, while

Snehal chose path PQCRES. They both started walking at the same time and with the same speed.



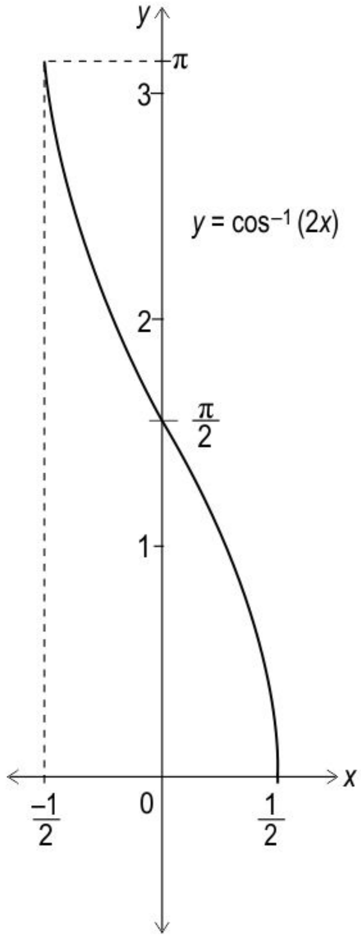
Q: 12 **Write the range of each of the two functions that Kulsum chose as her path to walk on.** **[2]**

Q: 13 **The graphs of which of the two functions are combined to form the path that Snehal chose?** **[1]**

Q: 14 **What is the *x* -coordinate of Kulsum's and Snehal's first meeting point? Show your steps.** **[2]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	2

Q.No	What to look for	Marks
2	Draws the graph of $\cos^{-1}(2x)$ in the given domain as follows: 	1
3	i) Writes that the given statement is false.	0.5
	Gives reason that $(\sec(x))^{-1} = \frac{1}{\sec x}$ and not $\sec^{-1}(x)$.	0.5
	ii) Writes that the given statement is false.	0.5

Q.No	What to look for	Marks
	Gives reason that $\sin^{-1}(-x) = -\sin^{-1}(x)$, and $\cos^{-1}(-x) = \cos^{-1}(x)$. So, $\sin^{-1}(-x) + \cos^{-1}(-x) = \cos^{-1}(x) - \sin^{-1}(x)$.	0.5
4	Writes that the domain should be restricted to $[\frac{-5\pi}{6}, \frac{\pi}{6}]$. (Award full marks for any domain that satisfies the conditions.)	1
	Reasons that if its domain is restricted to $[\frac{-5\pi}{6}, \frac{\pi}{6}]$, then the function becomes one-one and onto and hence, invertible.	1
5	Rewrites $4x + \frac{1}{x} = 4$ as $(2x - 1)^2 = 0$. Hence, finds the value of x as $\frac{1}{2}$.	1
	Evaluates $\cos^{-1} x - \sin^{-1} x$ as $\frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ or 30°.	1
6	Writes that: $\cot^{-1}\left(\frac{1-x^2}{2x}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$	0.5
	Takes $x = \tan\theta$. Then $\theta = \tan^{-1} x$. Solves the LHS by substituting the expression from step 1 as follows: $\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) + \tan^{-1}\left(\frac{\tan^3\theta - 3\tan\theta}{1-3\tan^2\theta}\right)$ $= \tan^{-1}(\tan 2\theta) + \tan^{-1}(-\tan 3\theta)$	1.5
	Finds $\tan^{-1}(-\tan(3\theta)) = \tan^{-1}(\tan(-3\theta))$.	0.5
	Simplifies the expression from step 2 as $\tan^{-1}(\tan(2\theta)) + \tan^{-1}(\tan(-3\theta))$ $= 2\theta - 3\theta = -\theta = -\tan^{-1} x$.	0.5

Q.No	What to look for	Marks
7	**Using the formula $\cos 2\theta = 1 - 2\sin^2 2\theta$, writes $f(x)$ as follows:** $\cos(2\sin^{-1} x) = 1 - 2\sin^2(\sin^{-1} x) = 1 - 2x^2$	1
	Using the formula $\sin 2\theta = 2\sin\theta.\cos\theta$, writes $g(x)$ as follows: $\sin^2(2\cos^{-1} x) = 4\sin^2(\cos^{-1} x).\cos^2(\cos^{-1} x)$ $= 4\cos^2(\cos^{-1} x)(1 - \cos^2(\cos^{-1} x))$ $= 4x^2(1 - x^2) = 4x^2 - 4x^4$	1
	Finds $h(x) = (1 - 2x^2) + (4x^2 - 4x^4) = 1 + 2x^2 - 4x^4$	0.5
	Calculates $h(0.1)$ as $1 + 2(0.1)^2 - 4(0.1)^4 = 1.0204$	0.5
8	Writes: $\text{cosec}^{-1}\, y = x$ $\Rightarrow \text{cosec}\, x = y$ $\Rightarrow \frac{1}{\sin x} = y$ $\Rightarrow \sin x = \frac{1}{y}$ $\Rightarrow \sin^{-1} \frac{1}{y} = x = \text{cosec}^{-1}\, y$	1.5
	Uses the given relation and the above step to write: $\sin^{-1} \frac{1}{y} = \frac{1}{\sin^{-1} y}$ which is not true for $y = 1$. Hence, $\text{cosec}^{-1}\, y = \frac{1}{\sin^{-1} y}$ is not true.	1.5
9	**Since the domain of inverse of sine function is [-1, 1], finds the domain of the given function as follows:** $-1 \le \lvert x \rvert - 1 \le 1$ $\Rightarrow 0 \le \lvert x \rvert \le 2$ $\Rightarrow \lvert x \rvert \le 2$ $\Rightarrow -2 \le x \le 2$	1.5

Q.No	What to look for	Marks
	Concludes the domain of $\sin^{-1}(\lvert x \rvert - 1)$ as [-2, 2].	**0.5**
	Finds the range of the function as an interval of real numbers where sin *y* is bijective. **Some examples are:** $[-\frac{\pi}{2}, \frac{\pi}{2}]$, $[-\frac{3\pi}{2}, -\frac{\pi}{2}]$, ...	**0.5**
	Gives a valid reason. For example, since sin *y* is bijective in the domain $[-\frac{\pi}{2}, \frac{\pi}{2}]$, the range of the given function is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.	**0.5**
10	**Assumes the angle of inclination of the given ramp as θ.** **Finds the base length of the given ramp as 63 m, and hence finds θ as $\cos^{-1}\frac{63}{72} = \cos^{-1}\frac{7}{8}$.**	**0.5**
	Notes that, for a ramp to meet the given accessibility standards, the angle of inclination must be less than or equal to $\sin^{-1}\frac{1}{12}$.	**0.5**
	Writes $\sin^{-1}\frac{1}{12}$ as follows: $\sin^{-1}\left(\frac{1}{12}\right) = \cos^{-1}\left(\sqrt{1-\left(\frac{1}{12}\right)^2}\right)$ $\Rightarrow \sin^{-1}\left(\frac{1}{12}\right) = \cos^{-1}\left(\frac{\sqrt{143}}{12}\right)$	**1**
	Notes that : $\cos^{-1}\left(\frac{7}{8}\right) > \cos^{-1}\left(\frac{\sqrt{143}}{12}\right)$ **Gives a valid reason. For example, the value of cos θ decreases as θ increases, when 0° ≤ θ ≤ 90°.** **Concludes that the ramp is not meeting the accessibility standards set by the Home Ministry.** **(Award no marks if student simply writes that the ramp does not meet the accessibility standards, if no valid reason is given.)**	**1**

Q.No	What to look for	Marks
11	Assumes that $\frac{1}{2}\cos^{-1}\frac{5}{7} = \theta$. Then, $\cos^{-1}\frac{5}{7} = 2\theta$ $\Rightarrow \cos 2\theta = \frac{5}{7}$	0.5
	Writes cos 2θ as follows: $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{5}{7}$	1
	Solves the equation from step 3 to get: $\tan^2\theta = \frac{1}{6}$ $\Rightarrow \tan\theta = \left(\sqrt{\frac{1}{6}}\right)$ or $\left(-\sqrt{\frac{1}{6}}\right)$	1
	Writes that $0 \leq \cos^{-1}\frac{5}{7} \leq \pi$. $\Rightarrow 0 \leq \frac{1}{2}\cos^{-1}\frac{5}{7} \leq \frac{\pi}{2}$ $\Rightarrow 0 \leq \tan\theta \leq \infty$	2
	Rejects the negative value for tan θ, and writes that: $\tan\theta = \frac{1}{\sqrt{6}}$	0.5
12	**Identifies two functions that Kulsum chose to walk on as $\cos^{-1}(x)$ and $\sec^{-1}(x)$.**	1
	Writes that the range of $\cos^{-1}(x)$ is $[0, \pi]$.	0.5
	Writes that the range of $\sec^{-1}(x)$ is $[0, \pi] - \frac{\pi}{2}$.	0.5
13	**Identifies the trigonometric functions that Snehal chose to walk on as $\sin^{-1}(x)$ and $\text{cosec}^{-1}(x)$.**	1

Q.No	What to look for	Marks
14	**Mentions that both friends met at point C which is the point of intersection of two functions, $\sin^{-1}(x)$ and $\cos^{-1}(x)$ and writes:** $\cos^{-1}(x) = \sin^{-1}(x)$	0.5
	Simplifies the above equation as: $\cos(\cos^{-1}(x)) = \cos(\sin^{-1}(x))$ $\Rightarrow x = \cos(\sin^{-1}(x))$ **Substitutes $\sin^{-1}(x)$ as u and simplifies the above equation as:** $\sin u = \cos u$ $\Rightarrow \sin u = \sin(\frac{\pi}{2} - u)$ $\Rightarrow u = \frac{\pi}{4}$ **(Award full marks if $\cos^{-1}(x)$ and $\sin^{-1}(x)$ are shown to be equal at $y = \frac{\pi}{4}$ directly.)**	1
	Finds the x-coordinate of Kulsum's and Snehal's first meeting point as: $x = \sin u = \sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$	0.5

Chapter - 3
Matrices

Multiple Choice Questions

Q: 1 $M = \begin{bmatrix} 2 & 3 \\ 6 & 4 \end{bmatrix}$

Given below are two statements in relation with the above matrix- one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).

Assertion *(A)* : The matrix M cannot be expressed as the sum of a symmetric and a skew-symmetric matrix.

Reason *(R)* : The matrix M is neither a symmetric matrix nor a skew-symmetric matrix.

1. Both (A) and (R) are true and (R) is the correct explanation for (A).
2. Both (A) and (R) are true but (R) is not the correct explanation for (A).
3. (A) is false but (R) is true.
4. Both (A) and (R) are false.

Free Response Questions

Q: 2 **The following are two non-zero matrices M and N:** **[1]**

$$M = \begin{bmatrix} 0 & a \\ 0 & b \end{bmatrix} \quad N = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

What is the necessary condition for their product to be a zero matrix? Show your work and give valid reason.

Q: 3 **Shown below is matrix P.** **[3]**

$$P = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$$

Find one such matrix Q such that (P + Q) is a skew-symmetric matrix. Show your work.

Q: 4 **Drona is solving a bank robbery. He has a note from one of his informants, Arjun, which contains the robber's name in the form of an encoded matrix N.** **[5]**

♦ The decoded name is stored in the form of a matrix M with numbers representing letters, such that 1 = A, 2 = B, 3 = C, and so on until 26 = Z.
♦ The entries are to be read column-wise.
♦ Arjun's notes to Drona are encoded using an encoder matrix, E.
♦ Matrix E is multiplied with Matrix M, to get the matrix in the note, N.

Matrices N and E are given below.

$$\begin{bmatrix} 62 & 52 \\ 25 & 21 \\ -51 & -51 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ -1 & -2 & -2 \end{bmatrix}$$

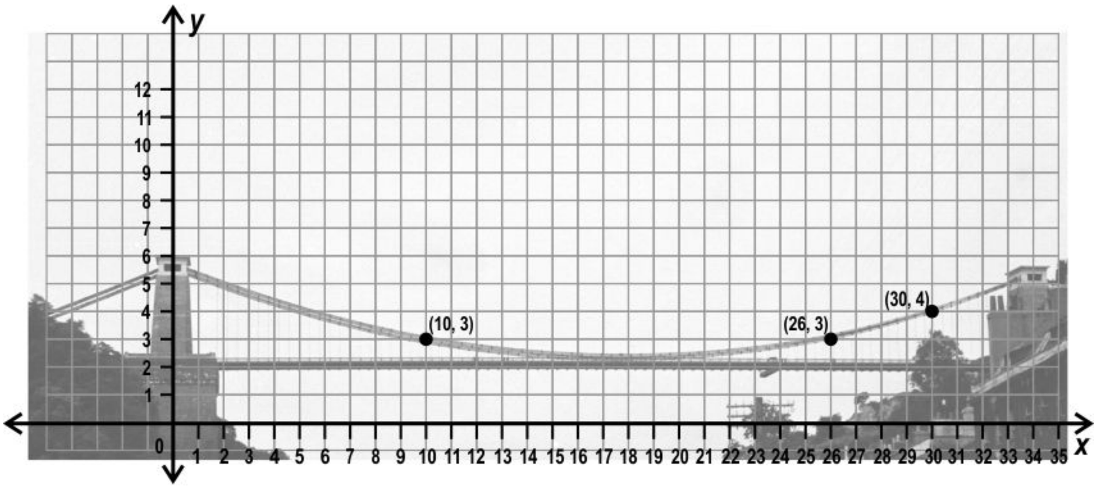
Matrix the note (N) | Encoder Matrix (E)

Find the key to decode the matrix N, and use it to find the robber's name. Show your work.

(Note: Encoding a matrix means to convert it to a coded form; Decoding a matrix means to convert it from coded form to normal form.)

Q: 5 **As part of an architecture project, Henry needs to create a miniature model of the Clifton Bridge, located in Bristol, UK. In order to do this, he must first find the equation of a parabola equivalent to the one made in Clifton Bridge.** **[5]**

He found a picture of the Clifton bridge, and placed it on a coordinate grid, as shown below. He noted that the parabola crossed the points (10, 3), (26, 3) and (30, 4) on the grid.



If the required parabola is of the form $y = ax^2 + bx + c$, then help Henry by finding *a*, *b* and *c*, and hence finding the equation of the parabola. Show your work.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3

Q.No	What to look for	Marks
2	**Writes the product of the matrices M and N as follows:** $$MN = \begin{bmatrix} ar & as \\ br & bs \end{bmatrix}$$	0.5
	Writes that the necessary condition for MN to be a zero matrix is that the values of *r* and *s* must always be 0 because *a* and *b* cannot be zero as M is a non-zero matrix.	0.5
3	**Represents the matrix (P + Q) as shown below:** $$P + Q = \begin{bmatrix} a+3 & b-1 \\ c+1 & d-2 \end{bmatrix}$$ where $Q = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	0.5
	Uses the skew-symmetric relation (P + Q) = -(P + Q)' to write the equation as: $$\begin{bmatrix} a+3 & b-1 \\ c+1 & d-2 \end{bmatrix} = \begin{bmatrix} -(a+3) & -(c+1) \\ 1-b & 2-d \end{bmatrix}$$	1
	Finds the value of *a* as (-3), *d* as 2 and the relation *b* = - *c* using the equality of matrices in the above step and solve the following equations: $a + 3 = -(a + 3)$ $d - 2 = 2 - d$ **and** $b - 1 = -(c + 1)$	1

Q.No	What to look for	Marks
	Finds the matrix Q as shown below where the numerical values of *b* and *c* are such that *c* = - *b* : $$Q = \begin{bmatrix} -3 & b \\ -b & 2 \end{bmatrix}$$	0.5
4	**Notes that the determinant of Matrix E ≠ 0.** **Writes that, since EM = N, we can multiply both sides by E^{-1} to get E^{-1} EM = E^{-1} N.** **Simplifies the above expression to get M = E^{-1} N.**	1

Q.No	What to look for	Marks
	Finds the inverse of matrix E using elementary row operations. **The solution may look as follows:** $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ -1 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} E$ $R_3 \to R_3 + R_1$ $\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} E$ $R_1 \to R_1 - 2R_2 - R_3$ $\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} E$ $R_2 \to R_2 - R_3$ $\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -1 \\ -1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} E$ $\Rightarrow E^{-1} = \begin{bmatrix} 0 & -2 & -1 \\ -1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ **(Award full marks if inverse is found using adj(E) ÷ det(E).)**	2

<table>
<tr><th>Q.No</th><th>What to look for</th><th>Marks</th></tr>
<tr><td></td><td>Multiplies E^{-1} with N as follows to get M:
$$\begin{bmatrix} 0 & -2 & -1 \\ -1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 62 & 52 \\ 25 & 21 \\ -51 & -51 \end{bmatrix}$$
$$= \begin{bmatrix} 0 + (-50) + 51 & 0 + (-42) + 51 \\ (-62) + 25 + 51 & (-52) + 21 + 51 \\ 62 + 0 + (-51) & 52 + 0 + (-51) \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 9 \\ 14 & 20 \\ 11 & 1 \end{bmatrix}$$</td><td>1.5</td></tr>
<tr><td></td><td>Decodes the letters representing the numbers and reads them column-wise to get the robber's name as ANKITA.</td><td>0.5</td></tr>
<tr><td>5</td><td>Writes the system of equations as:

100 a + 10 b + c = 3
676 a + 26 b + c = 3
900 a + 30 b + c = 4</td><td>0.5</td></tr>
<tr><td></td><td>Writes the above system of equations in the form AX = B as follows:
$$\begin{pmatrix} 100 & 10 & 1 \\ 676 & 26 & 1 \\ 900 & 30 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}$$</td><td>0.5</td></tr>
<tr><td></td><td>Finds |A| as 1(20280 - 23400) - 1(3000 - 9000) + 1(2600 - 6760) = -1280.
Writes that A^{-1} exists as |A| ≠ 0.</td><td>0.5</td></tr>
<tr><td></td><td>Finds A^{-1} as:
$$A^{-1} = \begin{pmatrix} \frac{1}{320} & -\frac{1}{64} & \frac{1}{80} \\ -\frac{7}{40} & \frac{5}{8} & -\frac{9}{20} \\ \frac{39}{16} & -\frac{75}{16} & \frac{13}{4} \end{pmatrix}$$
(Award 1 mark if only all the cofactors are found correctly.)</td><td>2</td></tr>
</table>

Q.No	What to look for	Marks
	Finds the values of *a*, *b* and *c* as $\frac{1}{80}$, - $\frac{9}{20}$ and $\frac{25}{4}$ respectively by solving for matrix X = A $^{-1}$ B as follows: $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \frac{1}{320} & -\frac{1}{64} & \frac{1}{80} \\ -\frac{7}{40} & \frac{5}{8} & -\frac{9}{20} \\ \frac{39}{16} & -\frac{75}{16} & \frac{13}{4} \end{pmatrix} \times \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{80} \\ -\frac{9}{20} \\ \frac{25}{4} \end{pmatrix}$	1
	Writes that the equation of a parabola equivalent to that of the Clifton bridge is $\frac{1}{80} x^2 - \frac{9}{20} x + \frac{25}{4}$.	0.5

Chapter - 4
Determinants

Multiple Choice Questions

Q: 1 **U and V are two non-singular matrices of order *n* and *k* is any scalar.**

Given below are two statements based on the above information - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).

Assertion (A) **: det(*k* U) = k^n × det(U)**

Reason (R) **: If W is a matrix obtained by multiplying any one row or column of V by the scalar *k*, then det(W) = *k* × det(V).**

1. Both (A) and (R) are true and (R) is the correct explanation for (A).
2. Both (A) and (R) are true but (R) is not the correct explanation for (A).
3. (A) is true but (R) is false.
4. (A) is false but (R) is true.

Free Response Questions

Q: 2 [1]

Show that for any positive integer n, $\begin{vmatrix} (n-1)! & n! \\ -n & n \end{vmatrix} = (n+1)!$.

(Note: n! denotes factorial n.)

Q: 3 **Given below is a system of linear equations in three variables.** [2]

***tx* + 3 *y* - 2z = 1**
***x* + 2 *y* + *z* = 2**
- *tx* + *y* + 2 *z* = -1

For what value of *t*, will the system fail to have a unique solution? Show your work.

Q: 4 **P(*k*, -1), Q(3, -5) and R(6, -1) are vertices of ΔPQR. The lengths of side QR and altitude PS are 5 units and 4 units respectively.** [2]

Use determinants to find the value(s) of *k*. Show your work.

Q: 5 [2]

If $A = \begin{pmatrix} 4 & -8 \\ 12 & 16 \end{pmatrix}$ and $B = \begin{pmatrix} -16 & -8 \\ 12 & -4 \end{pmatrix}$,

identify what type of matrix is (Adj A) × (Adj B).

Show your work.

Q: 6 **[3]**

Consider the matrix $X = \begin{bmatrix} p & -2 & -3 \\ -2 & 2 & 6 \\ 1 & 3 & q \end{bmatrix}$, where p and $q \in R$.

The co-factor of element 6 is (-11) and the minor of element 2 is 0.

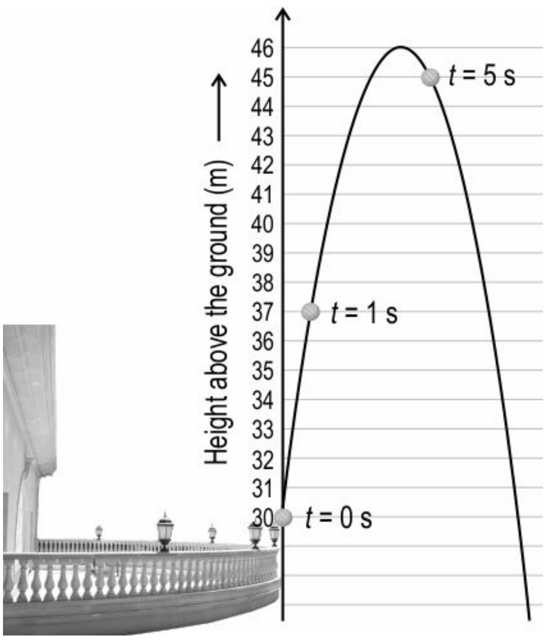
Find the values of *p* and *q.* Show your work.

Q: 7 **Without expanding the determinant, show that Δ is a perfect square for any integer values of *a*, *b* and *c*.** **[5]**

$$\Delta = \begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}$$

Q: 8 **A ball is thrown from a balcony. Its height above the ground after *t* seconds is given by $h(t) = pt^2 + qt + r$, where p, q, and $r \in$ R, and $h(t)$ is in meters.** **[5]**

Shown below is the trajectory of the ball with its height from the ground at *t* = 0 s, *t* = 1 s and *t* = 5 s.



Solve for *p*, *q* and *r* and find *h(t)* using the matrix method. Show your work and give valid reasons.

Q: 9 **A and B are invertible matrices of same order such that:** **[5]**

$$(AB)^{-1} = \frac{1}{48}\begin{bmatrix} 0 & 0 & 16 \\ 0 & 24 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

$$\text{and } A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Find B. Show your work.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	1

Q.No	What to look for	Marks
2	**Expands the determinant as:** $n\,(n-1)! + n\,(n\,!)$	0.5
	Simplifies the above expression as: $n\,! + n\,(n\,!) = n\,!(n+1) = (n+1)!$	0.5
3	**Writes the coefficient matrix and finds its determinant as:** $\begin{vmatrix} t & 3 & -2 \\ 1 & 2 & 1 \\ -t & 1 & 2 \end{vmatrix} = t\,(3) - 3(2+t) - 2(1+2t) = -4t - 8$	1
	Solves the equation -4 t - 8 = 0 for t and writes that for t = (-2), the system fails to have a unique solution.	1
4	**Uses QR and PS to find the area of the triangle as 10 sq units. The working may look as follows:** $\frac{1}{2} \times 5 \times 4 = 10$	0.5
	Equates the area found to that using determinants as follows: $\frac{1}{2}\begin{vmatrix} k & -1 & 1 \\ 3 & -5 & 1 \\ 6 & -1 & 1 \end{vmatrix} = 10$	0.5
	Expands the above determinant to write the equation as: $\frac{1}{2}\left\|k(-5+1) + 1\,(3-6) + 1(-3+30)\right\| = 10$ $\left\|-4k + 24\right\| = 20$	0.5

Q.No	What to look for	Marks
	Solves the above equation to find the values of *k* as 1 or 11.	**0.5**
5	**Finds (Adj A) as:** $\begin{pmatrix} 16 & 8 \\ -12 & 4 \end{pmatrix}$	**0.5**
	Finds (Adj B) as: $\begin{pmatrix} -4 & 8 \\ -12 & -16 \end{pmatrix}$	**0.5**
	Finds the product of (Adj A) and (Adj B) as: $\begin{pmatrix} -160 & 0 \\ 0 & -160 \end{pmatrix}$	**0.5**
	Identifies the matrix (Adj A) × (Adj B) as a scalar matrix. **(Award full marks if diagonal matrix is written instead of scalar matrix.)**	**0.5**
6	**Frames an equation in *p* using the co-factor of element 6 as** $A_{23} = (-1)^{2+3} \begin{vmatrix} p & -2 \\ 1 & 3 \end{vmatrix} = -3p - 2 = -11$	**1**
	Solves the above equation to find the value of *p* as 3.	**0.5**
	Frames an equation in *q* using the minor of element 3 as $M_{22} = \begin{vmatrix} p & -3 \\ 1 & q \end{vmatrix} = \begin{vmatrix} 3 & -3 \\ 1 & q \end{vmatrix} = 3p + 3 = 0$	**1**
	Solves the above equation to find the value of *q* as (-1).	**0.5**

Q.No	What to look for	Marks
7	**Performs the row operation $R_1 \to R_1 - R_2 - R_3$ on Δ to get:** $$\Delta = \begin{vmatrix} 0 & -2c^2 & -2b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$	1
	Takes (-2) common from the first row and performs the row operation $R_3 \to R_3 - R_2$ on Δ to get: $$\Delta = (-2)\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2-b^2 & -a^2 & a^2 \end{vmatrix}$$	1
	Performs the row operation $R_2 \to R_2 - R_1$ on Δ to get: $$\Delta = (-2)\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & a^2 & 0 \\ c^2-b^2 & -a^2 & a^2 \end{vmatrix}$$	1
	Performs the row operation $R_3 \to R_3 + R_2$ on Δ to get: $$\Delta = (-2)\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & a^2 & 0 \\ c^2 & 0 & a^2 \end{vmatrix}$$	1

Q.No	What to look for	Marks
	Expands the determinant to get: $\Delta = (-2)[-c^2(b^2a^2) + b^2(-c^2a^2)]$ $= 4a^2b^2c^2$ $= (2abc)^2$ **Hence concludes that Δ is a perfect square for any any integer value of *a*, *b* and *c*.**	1
8	**Finds $r = 30$ by substituting $t = 0$ in $h(t)$.**	0.5
	Frames the following equations in *p* and *q* by substituting $t = 1$ and $t = 5$ respectively: $p + q = 7$ $5p + q = 3$	0.5
	Writes the above system of equations in the matrix form using AX = B as: $\begin{bmatrix} 1 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$	0.5
	Finds |A| as -4 (≠ 0). Hence, writes that A^{-1} exists and the system has a unique solution.	0.5
	Finds *adj* A as: $\begin{bmatrix} 1 & -1 \\ -5 & 1 \end{bmatrix}$	0.5
	Finds A^{-1} using |A| and *adj* A as: $A^{-1} = \frac{1}{\lvert A \rvert} \times \text{adj } A$ $= \frac{1}{-4} \begin{bmatrix} 1 & -1 \\ -5 & 1 \end{bmatrix}$	1

Q.No	What to look for	Marks
	Writes that X = A^{-1} B and finds X as: $\begin{bmatrix} -1 \\ 8 \end{bmatrix}$ **Concludes that *p* = -1 and *q* = 8.**	1
	Writes $h(t) = -t^2 + 8t + 30$.	0.5
9	**Finds B^{-1} as $(AB)^{-1}$ A as follows:** $\frac{1}{48}\begin{bmatrix} 0 & 0 & 16 \\ 0 & 24 & 0 \\ 3 & 0 & 0 \end{bmatrix}\begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $= \begin{bmatrix} 0 & 0 & 1 \\ 0 & \frac{-1}{2} & 0 \\ \frac{-1}{8} & 0 & 0 \end{bmatrix}$	1.5
	Finds adj (B^{-1}) as follows: $adj(B^{-1}) = \begin{bmatrix} 0 & 0 & \frac{-1}{16} \\ 0 & \frac{1}{8} & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{8} & 0 \\ \frac{-1}{16} & 0 & 0 \end{bmatrix}$	1.5
	Finds det (B^{-1}) as 0 - 0 + 1($\frac{-1}{16}$) = $\frac{-1}{16}$.	1
	Finds the matrix B as follows: $B = \frac{1}{\frac{-1}{16}}\begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{8} & 0 \\ \frac{-1}{16} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$	1

Chapter - 5
Continuity and Differentiability

Multiple Choice Questions

Q: 1 **The function *f* : R -> R is defined by:**

$$f(x) = \begin{cases} |\sin x|, \text{ when } x \text{ is rational} \\ -|\sin x|, \text{ when } x \text{ is irrational} \end{cases}$$

Read the statements carefully and then choose the option that correctly describes them.

Statement 1 **: *f* (*x*) is continuous at *x* = 0.**

Statement 2 **: *f* (*x*) is discontinuous for all *x* ∈ R - {0}.**

1. Statement 1 is true but Statement 2 is false.
2. Statement 1 is false but Statement 2 is true.
3. Both Statement 1 and Statement 2 are true.
4. Both Statement 1 and Statement 2 are false.

Q: 2 **The Signum function *f* : R -> R, defined below, is discontinuous at *x* = 0. The identity function *g* : R -> R, defined below, is continuous everywhere.**

$$f(x) = \begin{cases} 1, \text{ for } x > 0 \\ 0, \text{ for } x = 0 \\ -1, \text{ for } x < 0 \end{cases}$$

$$g(x) = x, \text{ for all } x \in \text{R}$$

Which of these is true about the product of two functions (*f*.*g*)?

1. ($f.g$) is discontinuous at x = 0 but continuous elsewhere in R.
2. ($f.g$) is not differentiable anywhere in R.
3. ($f.g$) is differentiable everywhere in R.
4. ($f.g$) is continuous everywhere in R.

Q: 3 **Read the statements carefully and choose the option that correctly describes them.**

Statement 1: The function $\sqrt{x-5}$ is continuous in $(5, \infty)$.

Statement 1: The function $\sqrt{x-5}$ is differentiablein $(5, \infty)$.

1. Both statements are true and statement 2 explains why statement 1 is true.
2. Both statements are true but statement 2 does NOT explain why statement 1 is true.
3. Statement 1 is true but statement 2 is false.
4. Statement 1 is false but statement 2 is true.

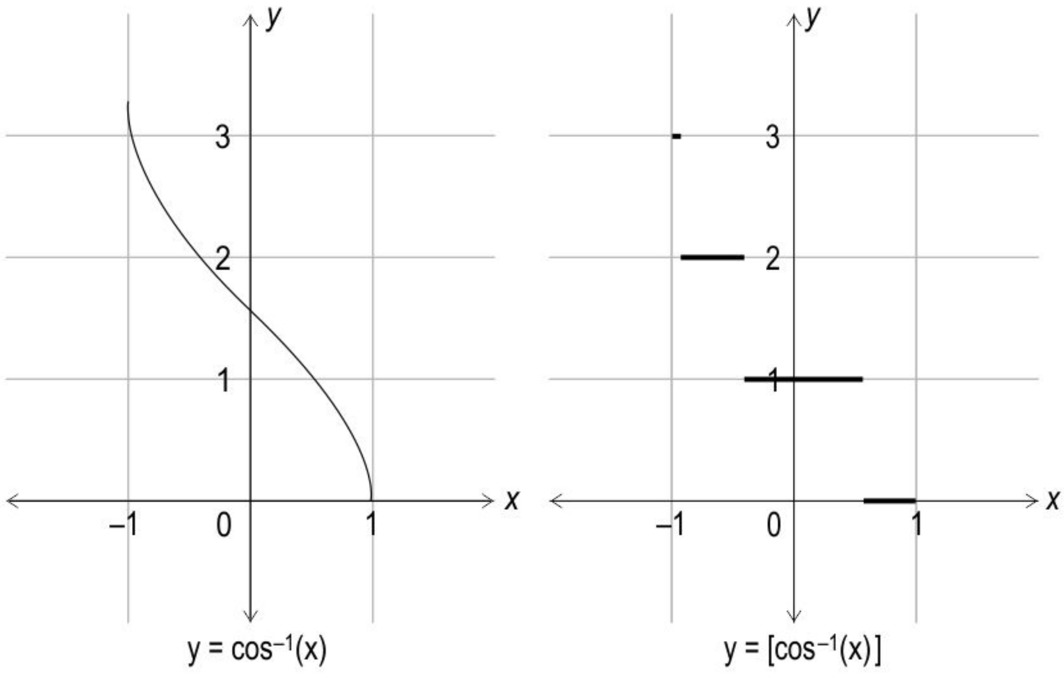
Free Response Questions

Q: 4 **Look at an inverse function below.** **[1]**

$y = \text{cosec}^{-1}(4x^4)$; $|4x^4| > 1$

Find $\frac{dy}{dx}$. Show your steps.

Q: 5 **Shown below are the graphs of two functions $y = \cos^{-1} x$ and $y = [\cos^{-1} x]$, where $[\cos^{-1} x]$ denotes the greatest integer function.** **[2]**



Find the points of discontinuity of the function $y = [\cos^{-1} x]$. Show your work.

Q: 6 If $y = ax^{n+2} + \frac{b}{x^{n+1}}$, where $n \in \mathbb{N}$ and $a, b \in \mathbb{R}$, **[3]**

prove that $x^2y'' = (n+1)(n+2)y$.

Q: 7 **Find the value of $(f \circ g)'$ at $x = 4$ if $f(u) = u^3 + 1$ and $u = g(x) = \sqrt{x}$. Show your work.** **[3]**

Q: 8 **[5]**

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^{10}}{10!}$$

If $f(x)$ is differentiated successively 10 times, what is $f^{(10)}(x)$? Show your work.

Q: 9 **[5]**

Find $\frac{d^2y}{dx^2}$ if, $y = tan^{-1}\left[\frac{log\left(\frac{e}{x^5}\right)}{log\left(ex^5\right)}\right] + tan^{-1}\left[\frac{4 + 5\, log\, x}{1 - 20\, log\, x}\right]$.

Show your work.

(*Note : Consider the base of logarithm to be e*.)

Q: 10 **Find $\frac{dm}{dn}$ at (m, n) = (-1, 1) where:** **[5]**

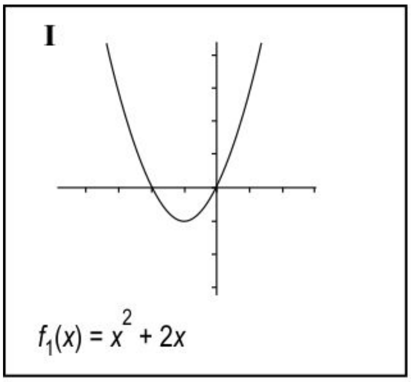
$$5m^2 + 2m - n^{\frac{-2}{3}} = 0$$

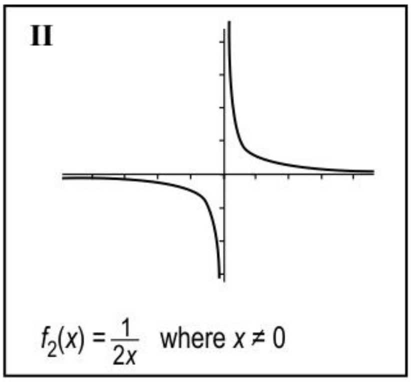
Show your work.

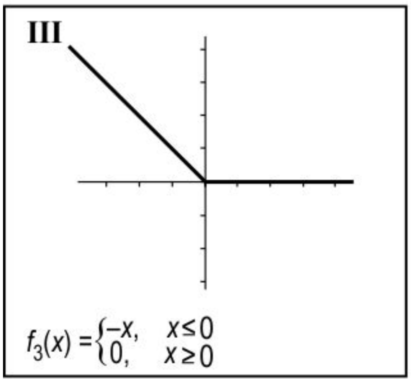
Case Study

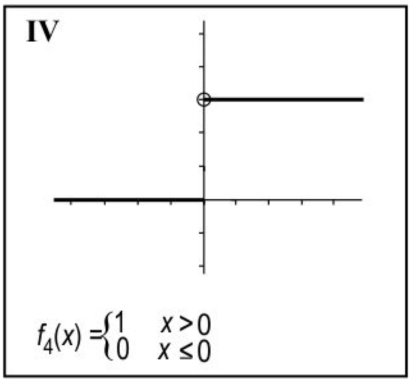
Answer the questions based on the information given below.

Ambika, a mathematics teacher is conducting a practice session on calculus where she is discussing continuity and differentiability of several functions in their domains. Shown below are four cards that contain a function each along with their domains and graphs.









While discussing in the group, four students claimed as follows:

♦ Leela: "As the function on card I is both continuous and differentiable, we can say that every continuous function is differentiable."
♦ Irfan: "As the graph is not in one piece, the function on card II is discontinuous."
♦ Deepak: "The function on card III is continuous."
♦ Kiran: "The function on card IV is discontinuous."

Q: 11 **Check whether Deepak and Kiran's claims are correct. Justify your answer.** **[2]**

Q: 12 **Is Irfan's claim correct? Does the reason given by him support his claim? Justify.** **[1]**

Q: 13 **Is Leela's claim true for all continuous functions? Justify with a valid reason or provide a counterexample.** **[2]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	4
2	4
3	1

Q.No	What to look for	Marks
4	**Differentiates the given function as:** $$\frac{dy}{dx} = \frac{-1}{4x^4\sqrt{(4x^4)^2 - 1}} \frac{d(4x^4)}{dx}$$	0.5
	Simplifies the above equation as: $$\frac{dy}{dx} = \frac{-16x^3}{4x^4\sqrt{(4x^4)^2 - 1}} = \frac{-4}{x\sqrt{16x^8 - 1}}$$	0.5
5	**With the help of the graphs, finds the values of x for which $\cos^{-1} x$ attains integer values as:** **$\cos^{-1} x = 3$** **=> $x = \cos 3$** **$\cos^{-1} x = 2$** **=> $x = \cos 2$** **$\cos^{-1} x = 1$** **=> $x = \cos 1$**	1.5
	Concludes that the points of discontinuity of the function $y = [\cos^{-1} x]$ are cos 3, cos 2 and cos 1.	0.5
6	**Finds the first derivative of y as:** $$y' = a(n+2)x^{n+1} - b(n+1)x^{-n-2}$$	1
	Finds the second derivative of y as: $$y'' = a(n+2)(n+1)x^{n} + b(n+1)(n+2)x^{-n-3}$$	1

Q.No	What to look for	Marks
	Multiplies x^2 on both sides of the above equation to get: $x^2y'' = (n+1)(n+2)\left[ax^{n+2} + \frac{b}{x^{n+1}}\right]$ $\Rightarrow x^2y'' = (n+1)(n+2)y$	1
7	**Finds (*f* o *g*)(*x*) as:** $(f \circ g)(x)$ $= f(g(x))$ $= f(\sqrt{x})$ $= (\sqrt{x})^3 + 1$ $= x^{\frac{3}{2}} + 1$	1
	Finds the derivative of (*f* o *g*)(*x*) as: $\frac{d}{dx}(f \circ g)(x)$ $= \frac{d}{dx}(x^{\frac{3}{2}} + 1)$ $= \frac{3}{2}x^{\frac{1}{2}}$	1
	Uses the above step to find the value of (*f* o *g*)' at *x* = 4 as 3.	1

Q.No	What to look for	Marks
8	**Differentiates *f* (*x*) once to get:** $$f^{(1)}(x) = 0 + 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^9}{9!}$$	1
	Rewrites $f^{(1)}(x)$ as: $$f^{(1)}(x) = f(x) - \frac{x^{10}}{10!}$$	0.5
	Finds the second derivative of *f* (*x*) as: $$f^{(2)}(x) = f(x) - \frac{x^{10}}{10!} - \frac{x^9}{9!}$$	1
	Finds the third derivative of *f* (*x*) as: $$f^{(3)}(x) = f(x) - \frac{x^{10}}{10!} - \frac{x^9}{9!} - \frac{x^8}{8!}$$	1
	Generalises the pattern as: $$f^{(10)}(x) = f(x) - \frac{x^{10}}{10!} - \frac{x^9}{9!} - \cdots - \frac{x^2}{2!} - x$$ **(Award full marks if the expression is correctly generalised at the second derivative stage.)**	1

Q.No	What to look for	Marks
	Finds the value of $f^{(10)}(x)$ as $f^{(10)} = 1$.	**0.5**
9	Simplifies $\log\left(\frac{e}{x^5}\right)$ as $1 - 5\log x$.	**0.5**
	Simplifies $\log(ex^5)$ as $1 + 5\log x$.	**0.5**
	Rewrites the given equation as: $y = \tan^{-1}\left[\frac{1-5\log x}{1+5\log x}\right] + \tan^{-1}\left[\frac{4+5\log x}{1-20\log x}\right]$	**0.5**
	Uses the trigonometric identities $\tan^{-1}A - \tan^{-1}B = \tan^{-1}\frac{A-B}{A+B}$ and $\tan^{-1}A + \tan^{-1}B = \tan^{-1}\frac{A+B}{1-AB}$ and rewrites the above equation as: $y = \tan^{-1}1 - \tan^{-1}(5\log x) + \tan^{-1}4 + \tan^{-1}(5\log x)$	**2**
	Simplifies above step to get: $y = \tan^{-1}1 + \tan^{-1}4$	**0.5**
	Differentiates the above equation to get $y' = 0$.	**0.5**
	Differentiates y' to get $y'' = 0$.	**0.5**
10	**Differentiates the equation given in the question with respect to *n* as follows:** $\frac{d}{dn}\left(5m^2 + 2m - n^{\frac{-2}{3}}\right) = \frac{d}{dn}(0)$ $\Rightarrow \frac{d}{dn}(5m^2) + \frac{d}{dn}(2m) - \frac{d}{dn}\left(n^{\frac{-2}{3}}\right) = 0$	**1**
	Simplifies the differentiation in the previous step as follows: $\frac{d}{dm}(5m^2)\frac{dm}{dn} + 2\frac{dm}{dn} + \frac{2}{3}n^{\frac{-5}{3}} = 0$	**1**

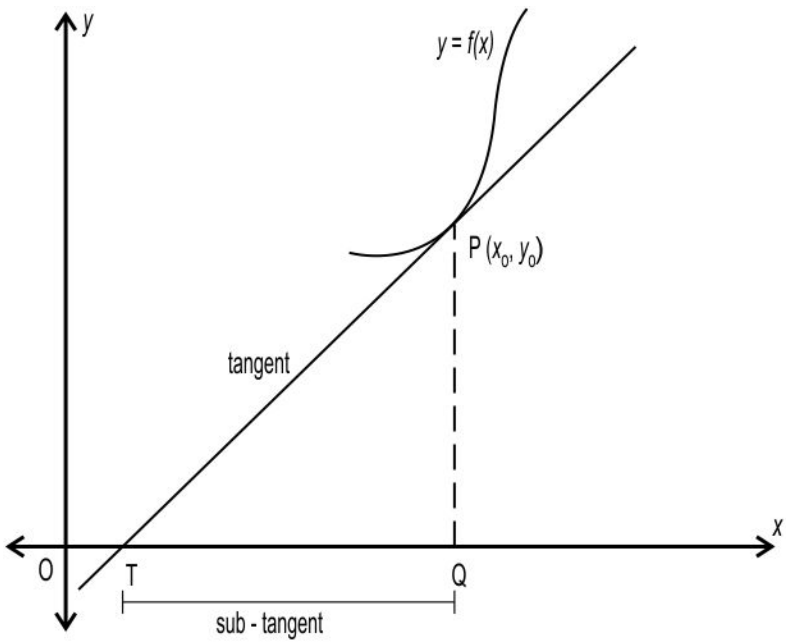
Q.No	What to look for	Marks
	Simplifies the previous step as: $\frac{dm}{dn}(10m+2)+\frac{2}{3}n^{\frac{-5}{3}}=0$ **(Award full marks if equivalent solving techniques without some intermediate steps are followed.)**	**1**
	Concludes that: $\frac{dm}{dn}=-\frac{n^{\frac{-5}{3}}}{15m+3}$	**1**
	Finds the value of $\frac{dm}{dn}$ at (-1,1) as $\frac{1}{12}$.	**1**
11	**Writes that Deepak is right and justifies it as follows:** **As f_3 (x) satisfies all the 3 conditions given below, it is continuous.** ♦ **$f_3(x)$ is defined for all real numbers.** ♦ **At $x = 0$, left hand limit = right hand limit = 0** ♦ **$f(0) = 0$**	**1.5**
	Writes that Kiran is right and justifies it as follows: **As $f_4(x)$ fails to satisfy that at $x = 0$, left hand limit (0) is not equal to the right hand limit (1), it is discontinuous.**	**0.5**
12	**Writes that the claim made by Irfan is wrong and the reason given by him does not support his claim.**	**0.5**
	Justifies that when $x = 0$, the function is not defined. So, $\frac{1}{2x}$ is continuous at all points except when $x = 0$. Hence, writes that the reason given by Irfan is not correct.	**0.5**
13	**Writes that Leela's claim is not true for all continuous functions.**	**0.5**

Q.No	What to look for	Marks
	Gives an example. Foe example: $f_3(x)$ is continuous, but not differentiable as it it has a corner point. **(Award full marks if any other valid examples are given.)**	**1.5**

Chapter - 6
Application of Derivatives and multiconcepts

Multiple Choice Questions

Q: 1 **The sub-tangent of a curve at a point is the projection on the *x* -axis of the portion of the tangent to the curve between the *x* -axis and the point of tangency. The sub-tangent of a curve *y* = *f* (*x*) at a point P(x_o, y_o) is illustrated below.**



Among the given slopes of tangents of a curve at a given point, which will result in the longest sub-tangent?

1 30°

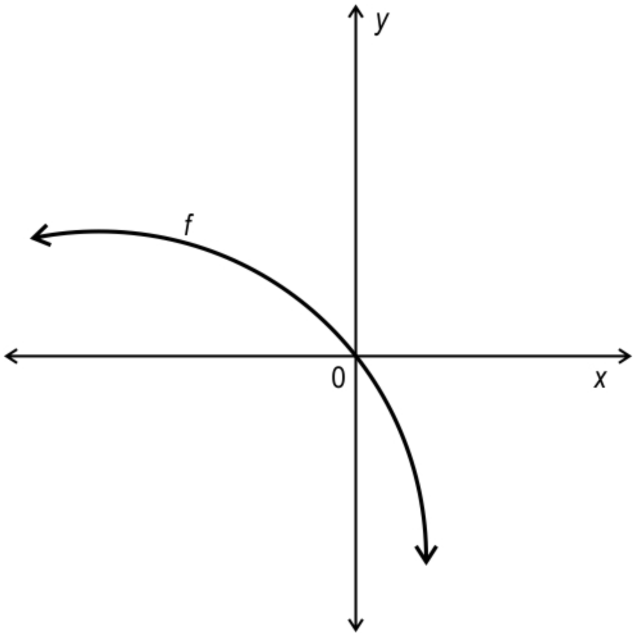
2 45°

3 60°

4 90°

Free Response Questions

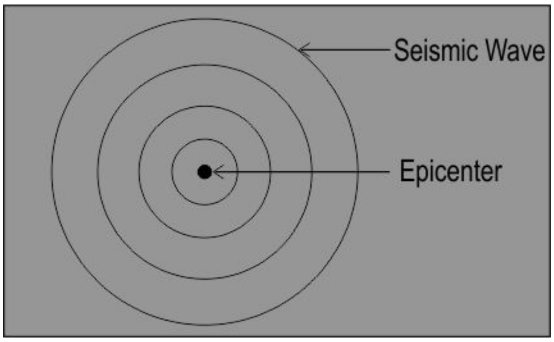
Q: 2 **The graph of a function, *f* : R -> R, is shown below.** **[1]**



Nadeem said that *f* '(0) > *f* (0).

Is Nadeem right? Give a valid reason.

Q: 3 **During an earthquake, seismic waves radiate from the epicenter of an earthquake in a circular pattern, as shown in the figure below.** **[1]**



If seismic waves travel at a speed of approximately 6 km/sec, then what is the rate of change of the area affected by the earthquake when the radius of the affected area is 25 km? Show your steps.

(Note: Take π as 3.14.)

Q: 4 **f(x) is an increasing function on the interval [0, 4], and f'(5) > 0.** **[1]**

Based on this information, is f(x) an increasing function on the interval [0, 5]? Justify your answer.

Q: 5 **A drone is an unmanned remotely operated aerial vehicle, often used for target practice or surveillance.** **[2]**

One such drone is flying according to the equation $s = t^n + 10$, where s is the distance of the drone from its remote's location at time t and n is a real number. The position of the remote is fixed.

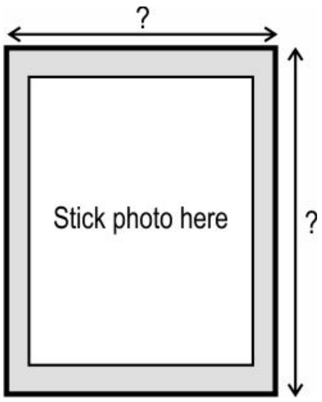
If the velocity of the drone is equal to its acceleration at 3 seconds, find n. Show your work.

Q: 6 **Find the equation of the tangent to the curve $x^2 y^2 = 4$ at the point (1, -2). Show your steps.** **[2]**

Q: 7 **When the diameter of a circle is 8 cm, by what factor does a small change in diameter affect its area? Show your work.** **[2]**

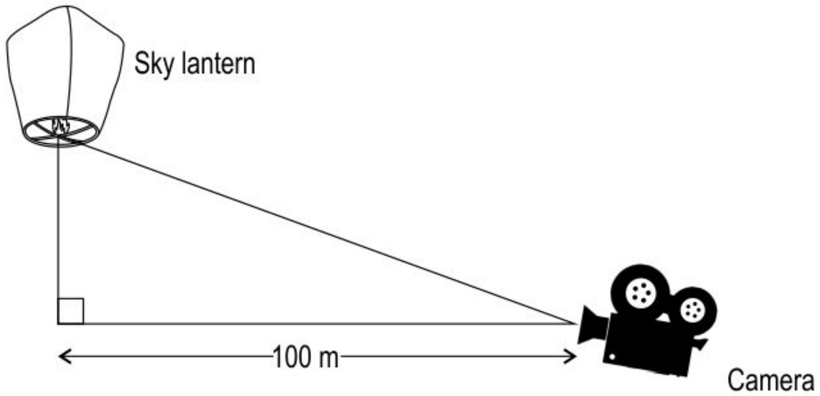
Q: 8 **Mr Aithal, a mathematics teacher, announces the following activity in his classroom and assures grand prizes for the winners.** **[5]**

Instructions:
- ♦ **Make a rectangular photo frame of total area 80 cm^2 using a chart paper.**
- ♦ **The frame should have a margin of 1.25 cm each at the top and the bottom.**
- ♦ **The frame should have a margin of 1 cm each on the left and the right sides.**
- ♦ **The area available at the centre to stick the photo should be maximum.**



What must be the dimensions of such a photo frame? Show your work.

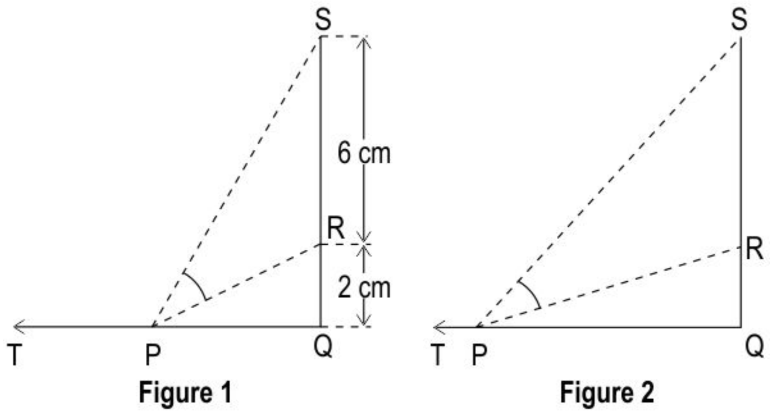
Q: 9 **A camera positioned on the ground recorded a sky lantern that was located 100 meters away. The lantern raised vertically from the ground into the sky at a constant rate of 25 meters per minute, and this entire process was captured on camera.** **[5]**



(Note: The figure is not to scale.)

When the lantern was at 75 m from the ground, what was the rate of change of angle of elevation, in radians/min? Show your steps.

Q: 10 **In the figures shown below, points Q, R and S are fixed. Point P can move forward and backward along ray QT.** **[5]**



(Note: The figures are not to scale.)

What should be the length of PQ such that ∠RPS is maximum? Show your work.

Case Study

Answer the questions based on the given information.

The total cost $C(n)$ of manufacturing n earphone sets per day in the House of Spark Electronics Limited is given by:

$C(n) = 400 + 4n + 0.0001n^2$ dollars.

Each earphone set is sold at:

$q = 10 - 0.0004n$ dollars where $n \geq 0$, $q \geq 0$

The daily profit in dollars is determined by the equation:

$P(n) = qn - C(n)$

Q: 11 **The marginal cost, $M(n)$ is the change in total production cost that comes from making or producing one additional unit. It is determined by the instantaneous rate of change of the total cost.** **[2]**

Find the marginal cost $M(n)$ of 10 earphone sets. Show your work.

Q: 12 **What quantity of daily production maximizes the profit? Show your work.** **[3]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	1

Q.No	What to look for	Marks
2	Writes that Nadeem is wrong.	0.5
	Gives a reason. For example, f (0) = 0, and since f is decreasing, f '(0) < 0. Hence, f '(0) < f (0).	0.5
3	Finds the rate of change of the affected area as $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ km^2/sec.	0.5
	Finds the rate of change of the area affected by the earthquake when the radius of the affected area (r) is 25 km as 2 × 3.14 × 25 × 6 = 942 km^2/sec.	0.5
4	Writes that f(x) need not be an increasing function on [0, 5].	0.5
	Gives a reason. For example, even though f'(5) > 0, there may be an x in (4, 5), such that f'(x) < 0.	0.5
5	Differentiates s with respect to time to find the velocity of the drone as: $s' = nt^{(n-1)}$	0.5
	Differentiates s' with respect to time to find the acceleration of the drone as: $s'' = n(n-1)t^{(n-2)}$	0.5
	Equates velocity of the drone to its acceleration at 3 seconds to find n as: $n \times 3^{(n-1)} = n(n-1) \times 3^{(n-2)}$ $\Rightarrow n \times 3^{(n-1)} = n(n-1) \times 3^{(n-1)} \times 3^{-1}$ $\Rightarrow n = 4$	1
6	Differentiates the given equation using the chain rule as follows: $2xy^2 + (x^2)\,2y\left(\frac{dy}{dx}\right) = 0$	0.5
	Simplifies the above equation as: $\frac{dy}{dx} = \frac{-y}{x}$	0.5

Q.No	What to look for	Marks
	Substitutes (1, -2) in the above equation to get the value of slope (m) as 2.	0.5
	Finds the equation of tangent as: $y - (-2) = 2(x - 1)$ $\Rightarrow y = 2x - 4$	0.5
7	Writes the area of a circle in terms of diameter, D as: $A = \frac{\pi}{4} D^2$	0.5
	Finds the rate of change of area with respect to diameter as follows: $\frac{dA}{dD} = \frac{\pi}{2} D$	1
	Finds $\frac{dA}{dD}$ when D = 8 cm as 4π cm^2/cm. Concludes that for a small change in diameter, the area changes by a factor of 4π. (Award full marks if the problem is solved correctly using approximation concept to obtain 4π x as the answer, where x is the small change in diameter.)	0.5
8	Assumes the width of photo frame to be x cm and its length to be $\frac{80}{x}$ cm.	0.5
	Subtracts the specified margins from the width and length to find the area (A) available to stick the photo as: $A = (x - 2)(\frac{80}{x} - \frac{5}{2})$ $\Rightarrow A = 85 - \frac{5}{2}x - \frac{160}{x}$	1
	Differentiates area with respect to x as: $\frac{dA}{dx} = -\frac{5}{2} + \frac{160}{x^2}$	1

Q.No	What to look for	Marks
	Equates the above derivative to zero and finds the critical point as: $-\frac{5}{2} + \frac{160}{x^2} = 0$ $\Rightarrow x^2 = 64$ $\Rightarrow x = 8$ (as x being a length cannot be negative)	**1**
	Finds $\frac{d^2A}{dx^2}$ at $x = 8$ as: $\frac{d^2A}{dx^2}_{(\text{at } x=8)} = -\frac{320}{x^3}_{(\text{at } x=8)} = -\frac{5}{8} < 0$ **Concludes that by second derivative test, the area is maximum at *x* = 8 cm.**	**1**
	Finds the length of the frame as $\frac{80}{8}$ = 10 cm. **Concludes that the required dimensions of the photo frame are 8 cm and 10 cm.**	**0.5**
9	**Takes *x* as the distance between the lantern and the ground, θ as camera's angle of elevation in radians and *t* as the time in minutes.** **Writes that $\frac{dx}{dt}$ = 25 m/min.**	**0.5**
	Uses tangent function and writes: $\tan\theta = \frac{x}{100}$	**0.5**
	Differentiates the above equation with respect to *t* to get: $\sec^2\theta \times \frac{d\theta}{dt} = \frac{1}{100} \times \frac{dx}{dt}$	**1**
	Uses steps 1 and 3 to write: $\frac{d\theta}{dt} = \frac{1}{4\sec^2\theta}$	**0.5**

Q.No	What to look for	Marks
	Uses secant function and writes: **sec θ =** $\frac{y}{100}$ **where *y* is the distance between the camera and the lantern.**	0.5
	Uses the Pythagoras theorem to find *y* as: $y^2 = x^2 + 100^2$ => $y^2 = 75^2 + 100^2$ => $y = 125$, as $y > 0$.	1
	Substitutes *y* = 125 to get sec θ as $\frac{5}{4}$ **.**	0.5
	Substitutes the value of sec θ in the equation obtained in step 3 to get: $\frac{d\theta}{dt} = \frac{4}{25}$ **or 0.16 radians/min.**	0.5
10	**Considers the length of PQ as *x* cm and finds ∠QPR and ∠QPS as:** $\angle QPR = \tan^{-1}\left(\frac{2}{x}\right)$ $\angle QPS = \tan^{-1}\left(\frac{8}{x}\right)$	0.5
	Considers ∠RPS as θ finds θ in terms of *x* as: $\theta = \tan^{-1}\left(\frac{8}{x}\right) - \tan^{-1}\left(\frac{2}{x}\right)$	0.5
	Finds the derivative of θ with respect to *x* as follows: $\frac{d\theta}{dx} = \frac{d}{dx}\left(\tan^{-1}\left(\frac{8}{x}\right) - \tan^{-1}\left(\frac{2}{x}\right)\right)$ $= \frac{d}{dx}\left(\tan^{-1}\left(\frac{8}{x}\right)\right) - \frac{d}{dx}\left(\tan^{-1}\left(\frac{2}{x}\right)\right)$	0.5

Q.No	What to look for	Marks
	Applies chain rule and differentiates as follows: $\frac{1}{\left(\frac{8}{x}\right)^2+1}\cdot\frac{d}{dx}\left(\frac{8}{x}\right)-\frac{1}{\left(\frac{2}{x}\right)^2+1}\cdot\frac{d}{dx}\left(\frac{2}{x}\right)$ $=\frac{2}{\left[\left(\frac{4}{x^2}+1\right)x^2\right]}-\frac{8}{\left[\left(\frac{64}{x^2}+1\right)x^2\right]}$	**1.5**
	Equates the derivative to 0 to find that the maximum value of θ will occur when *x* is either 4 or (-4). States that the minimum value of θ is 0, which cannot occur at *x* = 4 and hence it must be the maxima. The working may look as follows: $\frac{8}{\left[\left(\frac{64}{x^2}+1\right)x^2\right]}=\frac{2}{\left[\left(\frac{4}{x^2}+1\right)x^2\right]}$ Cancelling x^2 on both sides, $4\left(\frac{4}{x^2}+1\right)=\frac{64}{x^2}+1$ Rearranges terms to obtain, $x^2=16$ $\Rightarrow x=+4$ or -4	**1.5**
	Ignores *x* = -4 as the length of PQ cannot be negative and writes ∠RPS will be maximum when length of PQ = 4 cm.	**0.5**
11	**Writes the marginal cost function as:** $M(n)=\frac{dC}{dn}=\frac{d}{dn}(400+4n+0.0001n^2)$	**0.5**
	Finds the marginal cost function by completing the differentiation in the above step as: $M(n)=4+0.0002n$	**1**
	Finds the marginal cost of 10 earphone sets as 4.002 dollars by substituting 10 for *n* in the equation obtained in the above step.	**0.5**
12	**Writes the daily profit function as:** $P(n)=(10-0.0004n)n-(400+4n+0.0001n^2)$	**0.5**

Q.No	What to look for	Marks
	Simplifies the above equation to find the daily profit function as: $P(n) = -0.0005n^2 + 6n - 400$	**0.5**
	Differentiates the daily profit function obtained in the previous step as follows: $P'(n) = \frac{d}{dn}(-0.0005n^2 + 6n - 400)$ $= -0.001n + 6$	**0.5**
	Finds the critical point of $P(n)$ as $n = 6000$ by equating $P'(n)$ to 0 and solving for n as shown below: $P'(n) = -0.001n + 6 = 0$ $\Rightarrow n = 6000$	**0.5**
	Finds the second derivative as: $P''(n) = -0.001$ **Writes that this means that the function will be at its maximum at $n = 6000$.**	**0.5**
	Concludes that since the maximum value is obtained at $n = 6000$, hence the profit is maximized when the daily production is 6000 earphones.	**0.5**

Chapter - 7
Integrals

Multiple Choice Questions

Q: 1 **Look at the integral given below.**

$$\int_{\frac{a}{4}}^{\frac{b}{4}} f(4x)dx$$

If *f* (*x*) is continuous for all real values of *x*, then which of these is equal to the above integral?

1. $4\int_a^b f(x)dx$
2. $\frac{1}{4}\int_a^b f(x)dx$
3. $\frac{1}{4}\int_{4a}^{4b} f(x)dx$
4. $4\int_{4a}^{4b} f(x)dx$

Q: 2 **What is the value of the following integral?**

$$\int_{-2}^{2} (2-|x|)\,dx$$

1. 0
2. 4
3. 8
4. 12

Q: 3 **Varath says the following:**

$$\int_{-3}^{3}\left(\sqrt{x^2-4}\right)dx = F(3) - F(-3),$$

where $F(x)$ is the antiderivative of $\left(\sqrt{x^2-4}\right)$.

Which of the following can be said about Varath's statement?

1. It is true, as the function is continuous in [-3,3].
2. It is true, as per the fundamental theorem of calculus.
3. It is false, as the integral is not defined over the interval.
4. It is false, as the antiderivative of any function within a square root does not exist in R.

Free Response Questions

Q: 4 **Integrate the following function with respect to *x*.** **[1]**

$$e^{6-\ln x}$$

Show your steps.

Q: 5 **Given:** **[1]**

$$\int \frac{dx}{f(x)} = \frac{1}{3}\tan^{-1}\left(\frac{x-4}{3}\right) + C$$

where C is an arbitrary constant.

Find *f* (*x*). Show your work.

Q: 6 Check whether the given statement is true or false. **[1]**

For any function $f(x)$ that satisfies the condition $f(-3) = -f(3)$,
$\int_{-3}^{3} f(x) = 0$

Justify your answer.

Q: 7 **Ankit's partial solution for a question on integration is given below.** **[2]**

Question: Solve $\int \cos^3 x \sin^2 x \, dx$.

Ankit's solution:
Step 1: Let I = $\int \cos^3 x \sin^2 x \, dx$
Step 2: I = $\int \cos^3 x(1 - \cos^2 x) \, dx$
Step 3: I = $\int (\cos^3 x - \cos^5 x) \, dx$
Step 4: Let $\cos x = t$
Step 5: I = $\int (t^3 - t^5) \, dt$

Is Ankit correct? If yes, complete the integration. If no, in which step is the error present? Explain your reasoning.

Q: 8 **If *h* '(*x*) = *g* (*x*) and *g* is a continuous function for all real values of *x*, then prove that:** **[2]**

$$\int_{-1}^{1} g(6x)dx = \frac{1}{6}h(6) - \frac{1}{6}h(-6)$$

Q: 9 **Solve:** **[3]**

$$\int \frac{1}{m^2} \cos^2 \left(\frac{1}{m} - 1\right) dm$$

Show your steps.

Q: 10 **Evaluate the integral:** **[3]**

$$\mathrm{I} = \int_0^{\frac{\sqrt{3}}{2}} \left[\frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx\right]$$

Show your work.

Q: 11 **Solve the following integration and write your answer in its most simplified form.** **[5]**

$$\int \frac{x^2}{e^{2x}} dx$$

Show your steps.

Q: 12 **Integrate the given function. Show your steps.** **[5]**

$$\int \cot^{-1} \left(\frac{5}{x}\right) dx$$

Q: 13 Integrate $\frac{2e^x}{(e^x-1)(2e^x+3)}$. **[5]**

Show your work.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	2
2	2
3	3

Q.No	What to look for	Marks
4	**Rewrites the given expression as:** $\frac{e^6}{x}$	**0.5**
	Integrates the above expression with respect to *x* as: $e^6 \ln\lvert x\rvert + C$ **where, C is the constant of integration.**	**0.5**
5	**Finds *f* (*x*) as:** $9 + (x - 4)^2$ **by differentiating the RHS of the given equation using the differentiation of** $\tan^{-1} x$ **as follows:** $\frac{d}{dx}\left(\frac{1}{3}\tan^{-1}\left(\frac{x-4}{3}\right)\right) = \frac{1}{9+(x-4)^2}$	**1**
6	**Writes false.**	**0.5**
	Reasons that this would only be true if $f(-x) = -f(x)$ **for every value of *x.***	**0.5**
7	**Writes that Ankit is incorrect.**	**0.5**
	Writes that Ankit made an error in step 5.	**0.5**
	Gives a reason. For example, on substituting cos *x* as *t*, one also has to substitute *dx* in terms of *dt,* which Ankit has not done. Instead, he has simply replaced *dx* with *dt*.	**1**
8	**Substitutes** $u = 6x$ **and finds *du* as 6 *dx*.**	**0.5**
	Finds the limit as: **When** $x = -1$**, then** $u = -6$**.** **When** $x = 1$**, then** $u = 6$**.**	**0.5**

Q.No	What to look for	Marks
	Rewrites the integral and proves the given statement as: $\int_{-1}^{1} g(6x)dx$ $= \frac{1}{6}\int_{-6}^{6} g(u)du$ $= \left[\frac{1}{6}h(u)\right]_{-6}^{6}$ $= \frac{1}{6}h(6) - \frac{1}{6}h(-6)$	1
9	**Substitutes ($\frac{1}{m}$ - 1) as *u* to get:** $du = -\frac{1}{m^2}dm$	0.5
	Rewrites the given integral as: $-\int \cos^2 u\, du$	0.5
	Substitutes: $\cos^2 u = \frac{(1+\cos 2u)}{2}$ **in the above integral and rewrites it as follows:** $-\frac{1}{2}\int (1 + \cos 2u)\, du$	0.5
	Integrates the above integral to get the following expression where C is the arbitrary constant: $-\frac{1}{2}\left(u + \frac{1}{2}\sin 2u\right) + C$	1

Q.No	What to look for	Marks
	Substitutes *u* as ($\frac{1}{m}$ - 1) in the above expression to get the following expression as the solution: $-\frac{1}{2}\left[\frac{1}{m}-1+\frac{1}{2}\sin 2\left(\frac{1}{m}-1\right)\right]+C$	**0.5**
10	**Takes *u* = sin $^{-1}$ *x*** **Finds du as:** $du = \frac{dx}{\sqrt{1-x^2}}$	**0.5**
	Finds the change in limit when *x* = 0 and *x* = $\frac{\sqrt{3}}{2}$ to *u* = 0 and *u* = $\frac{\pi}{3}$ respectively.	**0.5**
	Rewrites the given integral using the above substitution and integrates the same as: $I = \int_0^{\frac{\pi}{3}} \frac{u}{\cos^2 u} du$ $\Rightarrow I = \int_0^{\frac{\pi}{3}} u \sec^2 u \, du$ $\Rightarrow I = u \tan u \,\|_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} \tan u \, du$ $\Rightarrow I = u \tan u \,\|_0^{\frac{\pi}{3}} + \log \cos u \,\|_0^{\frac{\pi}{3}}$	**1.5**
	Applies the limit to find the value of the given definite integral as: $\frac{\pi}{\sqrt{3}} - \log 2$	**0.5**
11	Let $I = \int \frac{x^2}{e^{2x}} dx$ **Evaluates integral using formula for integral by parts to get the following:** $I = x^2\left(\int e^{-2x}dx\right) - \int\left[2x\int e^{-2x}dx\right]dx$	**1**

Q.No	What to look for	Marks
	Solves the integration of e^{-2x} by substituting u as ($-2x$). **Gets dx as $\frac{-1}{2}du$.** **The integration may look as follows:** $-\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + c$ $\Rightarrow \int e^{-2x}dx = -\frac{1}{2}e^{-2x} + c$ **where c is a constant.**	1.5
	Substitutes the value of the above integral in the equation from step 2 to get the following: $I = -\frac{x^2}{2e^{2x}} + \int xe^{-2x}dx$	0.5
	Applies integration by parts to solve the integration of xe^{-2x} in a similar way as in step 3. **The integration may look as follows:** Let $h(x) = x$ and $g(x) = e^{-2x}$ $\Rightarrow \int xe^{-2x}dx = \frac{x}{-2e^{2x}} - \int \left[\int e^{-2x}dx\right] dx$ $\Rightarrow \int xe^{-2x}dx = -\frac{x}{2e^{2x}} + \frac{1}{2}\int e^{-2x}dx$ $\Rightarrow \int xe^{-2x}dx = -\frac{x}{2e^{2x}} - \frac{1}{4e^{2x}} + c_1$ where c_1 is a constant	1.5
	Writes answer as follows: $I = -\frac{2x^2+2x+1}{4e^{2x}} + C$ **where C is a constant.**	0.5

Q.No	What to look for	Marks
12	**Uses integration by parts to rewrite the given integral as:** $\cot^{-1}\left(\frac{5}{x}\right)\int dx - \int\left[\frac{d}{dx}\left(\cot^{-1}\left(\frac{5}{x}\right)\right)\int dx\right]dx$	1
	Simplifies the differentiation of cot $^{-1}$ ($\frac{5}{x}$) in the above expression as: $\frac{d}{dx}\cot^{-1}\left(\frac{5}{x}\right)$ $= -\frac{1}{1+\left(\frac{5}{x}\right)^2}\frac{d}{dx}\left(\frac{5}{x}\right)$ $= \frac{\frac{5}{x^2}}{1+\left(\frac{5}{x}\right)^2}$ $= \frac{5}{x^2+25}$	1.5
	Substitutes the above expression in step 1 and integrates the integral in step 1 to get the following expression: $x\cot^{-1}\left(\frac{5}{x}\right) - 5\int\frac{x}{x^2+25}dx$	1
	Completes integrating the above expression to get: $x\cot^{-1}\left(\frac{5}{x}\right) - \frac{5}{2}\log\left\|x^2+25\right\| + C$ **where C is an arbitrary constant.** **(Award full marks even if modulus is not used in log function as (x^2 + 25) is always positive.)**	1.5
13	Considers $e^x = t$ and finds $dx = \frac{dt}{t}$.	1

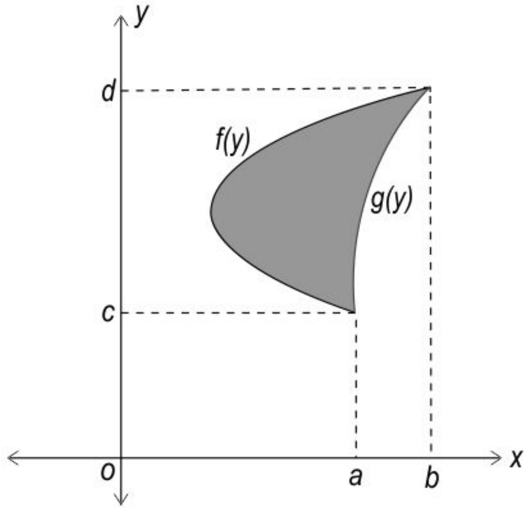
Q.No	What to look for	Marks
	Writes the function to be integrated as $\int \frac{2}{(t-1)(2t+3)} dt$.	1
	Uses partial fractions to rewrite the function as follows: $\frac{2}{5} \int \left(\frac{1}{t-1} - \frac{2}{2t+3} \right) dt$	1.5
	Integrates the function to obtain the following: $\frac{2}{5} \left[\ln \lvert t-1 \rvert - \ln \lvert 2t+3 \rvert \right] + C$ **where C is the constant of integration.**	1
	Replaces t with e^x to obtain $\frac{2}{5} \ln \left\lvert \frac{e^x - 1}{2e^x + 3} \right\rvert + C$.	0.5

Chapter - 8

Application of Integrals

Multiple Choice Questions

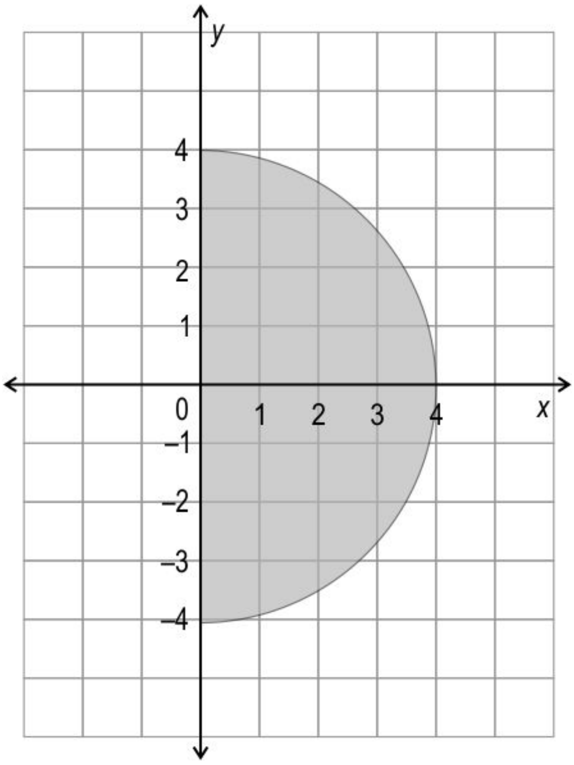
Q: 1 Shown below are partial graphs of two distinct functions, *f* (*y*) and *g* (*y*). The region between them is shaded.



Which expression gives the area of this shaded region?

1. $\int_a^b |f(y) - g(y)|\, dy$
2. $\int_a^b |f(x) - g(x)|\, dx$
3. $\int_c^d |f(y) - g(y)|\, dy$
4. $\int_c^d |f(x) + g(x)|\, dx$

Q: 2 **Shown below is the graph of the function $x^2 + y^2 = 16$, $0 \le x \le 4$.**



Which of the following expressions represents the area of the shaded region?

1 $2\int_0^4 \sqrt{16-y^2}dy$

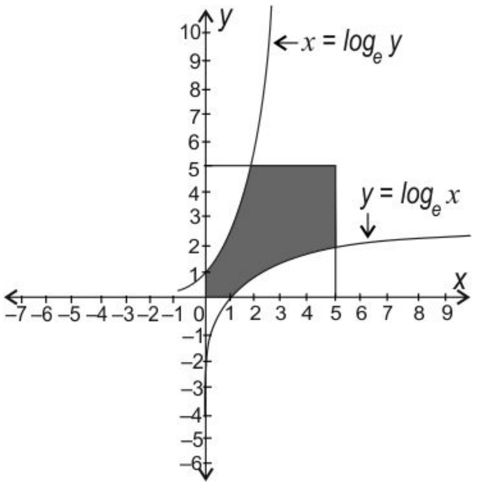
2 $\left|\int_{-4}^{0} (16-y^2)\,dy\right| + \int_0^4 (16-y^2)\,dy$

3 $\left|\int_{-4}^{0} \sqrt{16-x^2}\,dx\right| + \int_0^4 \sqrt{16-x^2}\,dx$

4 $\int_0^4 \sqrt{16-x^2}\,dx$

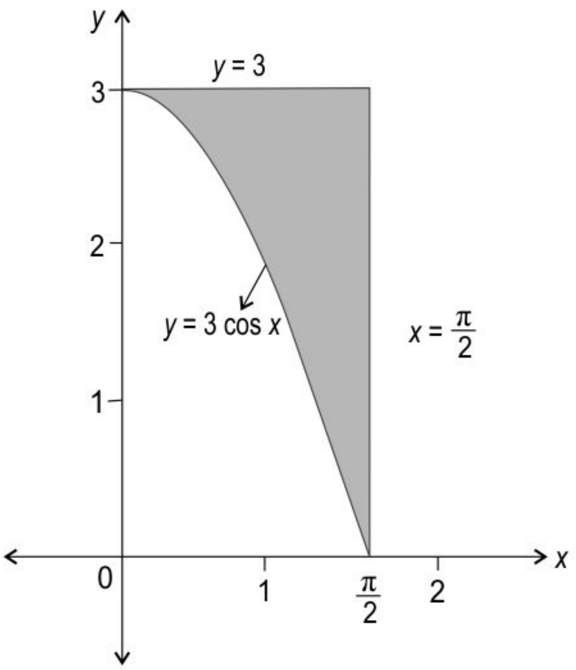
Free Response Questions

Q: 3 **Look at the graph below.** **[1]**



Write an expression for the area of the shaded region.

Q: 4 **Find the area of the shaded region in the graph shown below.** **[1]**



Show your work.

Q: 5 **The area bounded by the lines 2 y = x - 2, x = 1 and x = 4 can be found as follows:** **[1]**

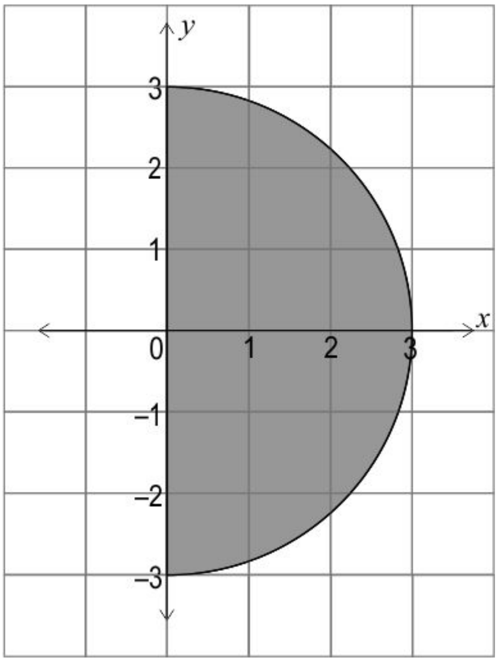
$$\int_1^4 \left(\frac{1}{2}x - 1\right)dx$$

State true or false. If true, justify your answer. If false, write the correct expression for the area.

Q: 6 **Roshan wants to calculate the area bounded by the curve $x^2 + |y| = 4$.** **[1]**

Write an expression that can be used to calculate the area correctly. Justify your answer.

Q: 7 **Shown below is the graph of the function $x^2 + y^2 = 9$, $0 \le x \le 3$.** **[1]**



Rajiv, Swara and Zaman represented the area of the shaded region in the following ways:

Rajiv: $\int_{-3}^{3} \sqrt{9 - x^2}dx$

Swara: $2\int_0^3 \sqrt{9 - y^2}dy$

Zain: $\left|\int_{-3}^{0} \sqrt{9 - x^2}dx\right| + \int_0^3 \sqrt{9 - x^2}dx$

Who represented it correctly and who did not? Justify your answer with a valid reason in each case.

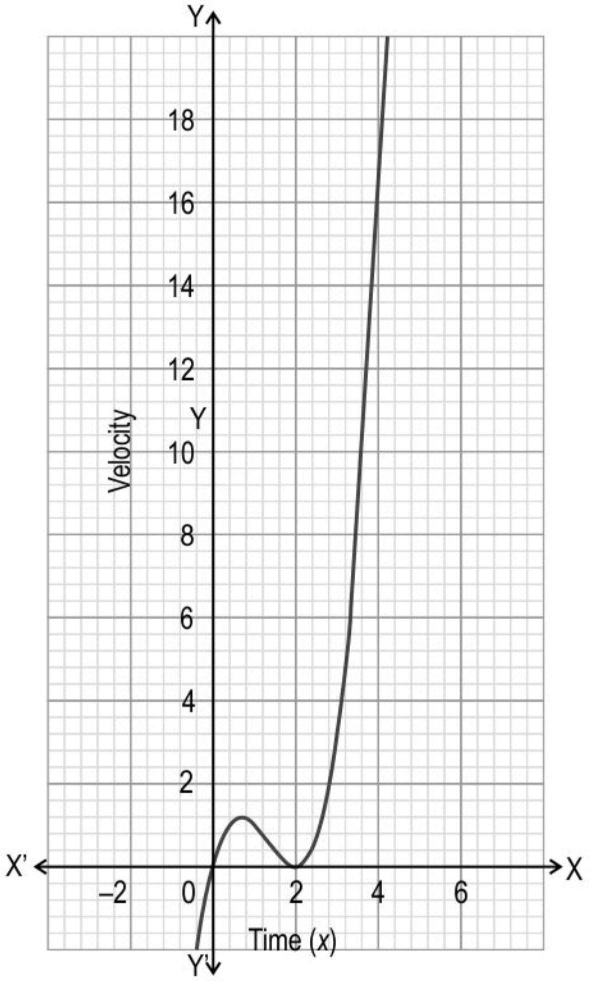
Q: 8 **The velocity function of an object is $v(t) = t^2 + 4t - 5$, where t is in hours and $v(t)$ is in kilometers per hour.** **[2]**

Find the displacement of the object during the first 3 hours. Show your steps.

Q: 9 **The area of the region bounded by $y = 4x^3 - kx^2 + 1$ and the x-axis, between the lines $x = 0$ and $x = 2$ is 10 sq units, where $k \in R$.** **[2]**

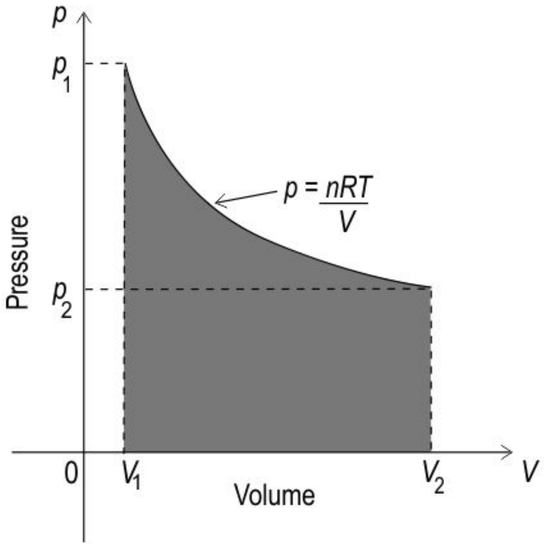
Find the value of k. Show your steps.

Q: 10 **The velocity of a particle, $v(x)$ is given by $(x - 2)^3 + 2(x - 2)^2$ m/s, where x is the time, as shown in the graph below.** **[2]**



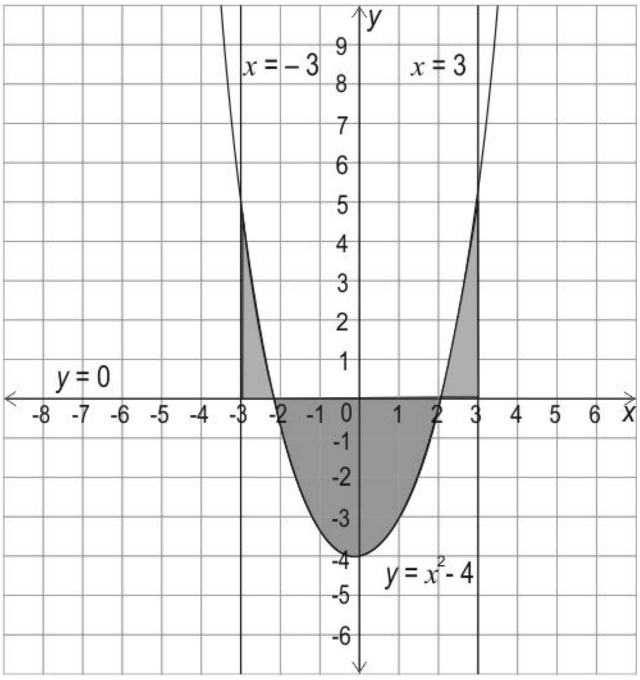
Find the displacement of the particle from 0 to 4 seconds. Show your steps.

Q: 11 **As n moles of a gas undergoes an expansion under constant temperature T, its volume V increases and it does some work on its surroundings. This work is represented by the area under the curve shown in the pressure-volume (pV) diagram below:** **[2]**



Find the expression for the amount of work done by the gas if $pV = nRT$, where R is the gas constant. Show your work.

Q: 12 **The shaded region shown in the graph below represents the area bounded by the graph of $f(x) = x^2 - 4$, the lines $x = -3$, $x = 3$ and the x-axis.** **[5]**



Find the area of the shaded region. Show your steps.

Q: 13 **A certain region is bounded by the x-axis and the graph of $y = \sin\frac{x}{2}$ between $x = 0$ and $x = 2\pi$. It is then divided into 2 regions by the line $x = k$.** **[5]**

If the area of the region between $x = 0$ and $x = k$ is thrice the area of the region between $x = k$ and $x = 2\pi$, find k. Draw a rough figure and show your steps.

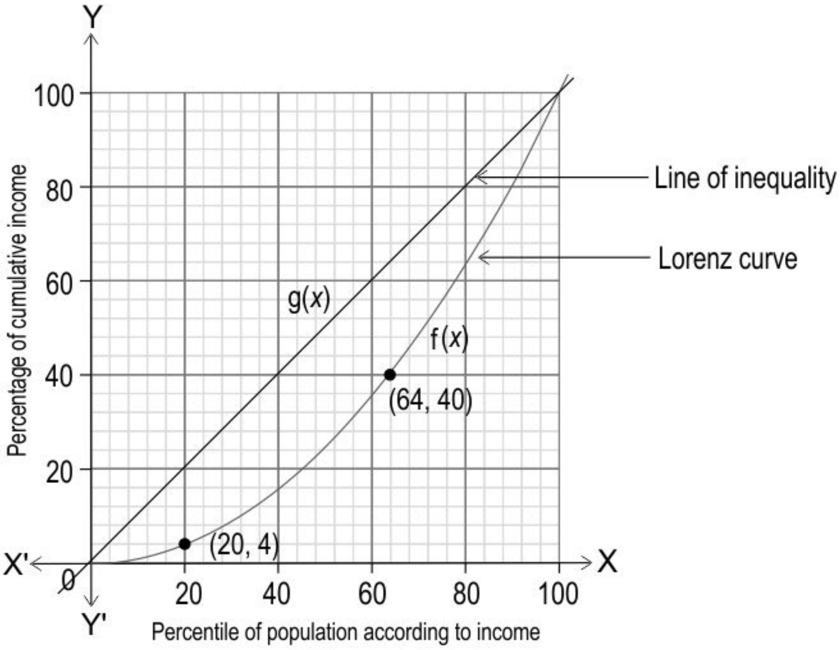
Case Study

Answer the following questions based on the given information.

A Lorenz curve is used to graphically represent income inequality in a society. It was developed by Max Lorenz in 1905.

In this curve, the percentile of the population according to their income is plotted on the x axis, and on the y axis, the percentage of cumulative income from that percentile of the population is plotted.For example, in the graph below, the point (20, 4) denotes that people with more income than that of 20% of the total population contribute 4% of the total income for the country. Similarly, the point (64, 40) denotes that people with more income than that of 64% of the country's population contribute 40% of the total income for the country.
On the graph, there is also a line of equality, given by the function $g(x) = x$. The further away the Lorenz curve of a society is from the line of equality, the more unequal its income distribution is.

In order to compare this data for multiple countries, the area under the Lorenz Curve is used to find the Gini coefficient (G), whose value ranges between 0 and 1. The closer G's value is to 1, the more unequal the income distribution of the society is.



The Lorenz curve shown above can be approximated by the following function:

$$f(x) = \frac{\sqrt{x}}{100} + \frac{(x-1)^2}{100}$$

Q: 14 **Find the area under the Lorenz Curve from 0 to 100. Show your work.** **[3]**

Q: 15 **The Gini coefficient is given by G = $\frac{A}{A+B}$, where A is the area between the Lorenz curve and the line of equality, and B is the area under the Lorenz curve.** **[2]**

Find the Gini coefficient for the given Lorenz curve, using integration. Show your work.

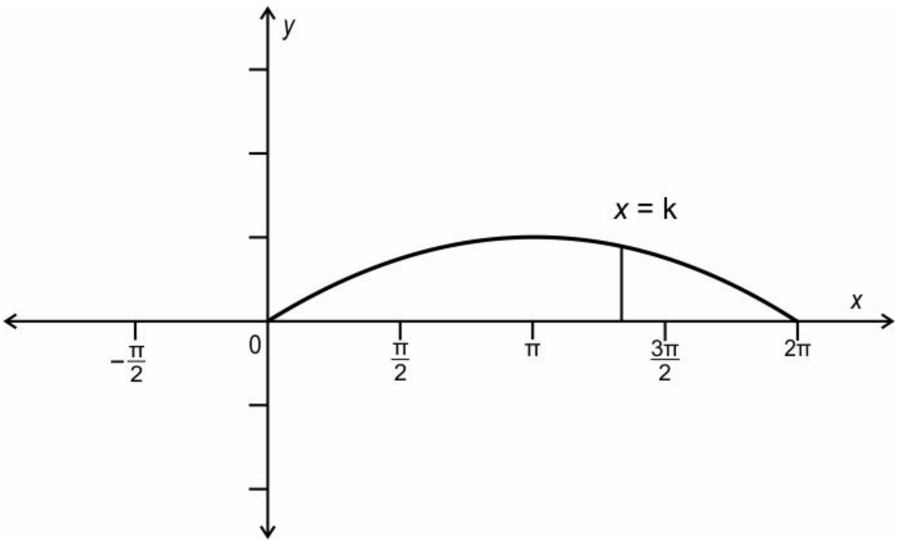
The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	1

Q.No	What to look for	Marks
3	**Writes an expression for the area of the shaded region as:** $\text{Shaded area} = 25 - 2\int_1^5 \log_e x\, dx$ **(Award full marks for any equivalent expression.)**	1
4	**Represents the area of the shaded region as:** $\frac{3\pi}{2} - \int_0^{\frac{\pi}{2}} 3\cos x dx$	0.5
	Simplifies the above expression to find the area of the shaded region as follows: $\frac{3\pi}{2} - 3\,[\sin x]_0^{\frac{\pi}{2}}$ $= 3\left[\frac{\pi}{2} - 1\right]$	0.5
5	**Writes false.**	0.5
	States that the line 2 *y* = *x* - 2 crosses the *x* -axis at *x* = 2 and hence the area under the curve will have to be found as follows: $\left\lvert\int_1^2 (\frac{1}{2}x - 1)\mathrm{d}x\right\rvert + \int_2^4 (\frac{1}{2}x - 1)\mathrm{d}x$	0.5
6	**Writes that the area is equivalent to the area bounded by $x^2 + y = 4$ and $x^2 - y = 4$.**	0.5
	Writes the following expression to calculate the bounded area: $\int_{-2}^{2}(4 - x^2)\ \mathrm{d}x + \left\lvert\int_{-2}^{2}(x^2 - 4)\ \mathrm{d}x\right\rvert$ **(Award full marks for any equivalent expression.)**	0.5

Q.No	What to look for	Marks
7	**Writes that only Swara represented the area of the shaded region correctly. Justifies the answer. For example, the parts of shaded region above and below the *x* -axis are equal. Hence the required area can be computed by integrating the function with respect to *y* -axis from 0 to 3 and then doubling it.**	**0.5**
	Writes that Rajiv and Zaman represented the area of the shaded region incorrectly. Justifies the answer. For example, if integrated along *x* -axis, the limit must range from 0 to 3. But Rajiv and Zaman have integrated from (-3) to 3.	**0.5**
8	**Writes the expression for the displacement of the object using the velocity function as follows:** $\int_0^3 (t^2 + 4t - 5)dt$	**0.5**
	Evaluates the above definite integral as: $\left[\frac{t^3}{3} + \frac{4t^2}{2} - 5t\right]_0^3$	**1**
	Applies the given limit to find the displacement as 9 + 18 - 15 = 12 km.	**0.5**
9	**Sets the integral equation as:** $\int_0^2 (4x^3 - kx^2 + 1)dx = 10$	**0.5**
	Simplifies the above equation as: $\left[\frac{4x^4}{4} - \frac{kx^3}{3} + x\right]_0^2 = 10$	**0.5**
	Applies the limit and simplifies the above equation as: $16 - \frac{8k}{3} + 2 = 10$	**0.5**

Q.No	What to look for	Marks
	Simplifies the above equation to get *k* as 3.	**0.5**
10	Integrates $\int_0^4((x-2)^3+2(x-2)^2)dx$ as $\left[\frac{(x-2)^4}{4}\right]_0^4+\left[\frac{2(x-2)^3}{3}\right]_0^4$	**1**
	Evaluates the above integral as $\frac{32}{3}$ and hence finds the displacement of the particle from 0 to 4 seconds as $\frac{32}{3}$ m.	**1**
11	**Represents the work done by the gas as:** $\int_{V_1}^{V_2} pdV$ $= nRT\int_{V_1}^{V_2}\frac{1}{V}dV$	**1**
	Integrates the above expression to find the work done by the gas as: $nRT\,[\log V]_{V_1}^{V_2}$ $= nRT\,[\log V_2 - \log V_1]$ $= nRT\log\left(\frac{V_2}{V_1}\right)$	**1**
12	**Sets up the integrals as follows:** Area = $\int_{-3}^{-2}(x^2-4)dx+\left\|\int_{-2}^{2}(x^2-4)dx\right\|+\int_{2}^{3}(x^2-4)dx$	**1**
	Evaluates the area from *x* = -3 to *x* = -2 as follows: $\int_{-3}^{-2}(x^2-4)dx=\left[\frac{x^3}{3}-4x\right]_{-3}^{-2}=\frac{7}{3}$	**1**

Q.No	What to look for	Marks
	Evaluates the area from x = -2 to x = 2 as follows: $\left\|\int_{-2}^{2}(x^2-4)dx\right\| = \left\|\left[\frac{x^3}{3}-4x\right]_{-2}^{2}\right\| = \left\|\frac{-32}{3}\right\| = \frac{32}{3}$	**1.5**
	Evaluates the area from x = 2 to x = 3 as follows: $\int_{2}^{3}(x^2-4)dx = \left[\frac{x^3}{3}-4x\right]_{2}^{3} = \frac{7}{3}$	**1**
	Adds the area found in the above 3 steps to find the area of the shaded region as: $\frac{7}{3}+\frac{32}{3}+\frac{7}{3}=\frac{46}{3}$ or 15.33 sq units	**0.5**
13	**Draws a rough graph of sin $\frac{\pi}{2}$ and x = k.**  **(Award only 0.5 marks if the range on the x -axis is not from 0 to 2π.)**	**1**

Q.No	What to look for	Marks
	Writes the equation for the given situation as: $\int_0^k sin\frac{x}{2}dx = 3\int_k^{2\pi} sin\frac{x}{2}dx$	**1**
	Simplifies the above equation as: $\left[-2\ cos\frac{x}{2}\right]_0^k = 3\left[-2\ cos\frac{x}{2}\right]_k^{2\pi}$	**1**
	Simplifies the above equation as: **$-2\cos\frac{k}{2} + 2\cos 0 = -6\cos\pi + 6\cos\frac{k}{2}$**	**1**
	Simplifies the above equation to get *k* as: $\frac{k}{2} = \cos^{-1}\frac{-1}{2} = \frac{2\pi}{3}$ **=>** $k = \frac{4\pi}{3}$	**1**
14	**Rewrites the integral of f(*x*) as follows:** $\int_0^{100}(\frac{\sqrt{x}}{100} + \frac{(x-1)^2}{100})dx$ $= \int_0^{100}\frac{\sqrt{x}}{100}dx + \int_0^{100}\frac{(x-1)^2}{100}dx$	**0.5**
	Solves $\int_0^{100}\frac{\sqrt{x}}{100}dx = \left[\frac{2x^{\frac{3}{2}}}{300}\right]_0^{100}$ **Evaluates this to find the answer as** $\frac{20}{3}$ **.** **(Award only 0.5 marks for correctly solving the integration, but incorrectly evaluating the expression.)**	**1**

Q.No	What to look for	Marks
	Solves $\int_0^{100} \frac{(x^2-2x+1)}{100}dx = \left[\frac{x^3}{300}\right]_0^{100} - \left[\frac{2x^2}{200}\right]_0^{100} + \left[\frac{x}{100}\right]_0^{100}$ **Evaluates this to find the answer as $\frac{10000}{3}$ - 100 + 1 = $\frac{10000}{3}$ - 99.** **(Award only 0.5 marks for correctly solving the integration, but incorrectly evaluating the expression.)**	**1**
	Adds the values from steps 2 and 3 to get: **$\frac{20}{3}$ + $\frac{10000}{3}$ - 99 = 3241 sq units.**	**0.5**
15	**To find A + B, integrates the area under g(x) = x as 5000 sq units.** **The integration may look as follows:** $\int_0^{100} x dx = \left[\frac{x^2}{2}\right]_0^{100} = 5000$	**1**
	Finds A as (5000 - 3241) = 1759 sq. units.	**0.5**
	Calculates G = 1759/5000 = approximately 0.35.	**0.5**

Chapter - 9
Differential Equations

Free Response Questions

Q: 1 **Sumit finds the general solution of the differential equation $yy' = e^{3x}$ as $y = \frac{1}{3}e^{3x} + C$, where C is the arbitrary constant.** **[1]**

Did Sumit find the correct general solution? Show your work and justify.

Q: 2 **The bottom valve of a conical tank is opened to remove sugarcane juice in a factory. The rate at which the juice pours out from the conical tank is directly proportional to the cube root of the rate of change of height of the juice present in the tank.** **[1]**

If *k* is the constant of proportionality, write a differential equation depicting the scenario.

Q: 3 **A differential equation is given below.** **[1]**

$$\left(\frac{d^3y}{dx^3}\right)^3 + \left(\frac{d^4y}{dx^4}\right)^2 - \left(\frac{dy}{dx}\right)^4 + 7y = 21$$

State whether the following statement is true or false. Justify your answer.

The degree of the given differential equation is equal to its order.

Q: 4 **[1]**

$$\left(\frac{d^4y}{dx^4}\right)^5 + \left(\frac{d^3y}{dx^3}\right)^6 - \left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 25$$

A differential equation is given above. State whether the following statement is true or false. Justify your answer.

The general solution of the given differential equation will have five arbitrary constants.

Q: 5 **If the tangent to a curve at every single point on it is given by $y - \frac{2y}{x+1}$ then find the equation of the curve. Show your steps.** **[2]**

Q: 6 **In a controlled condition within a laboratory, a spherical balloon is being deflated at a rate proportional to its surface area at that instant. The spherical shape of the balloon is maintained throughout the process.** **[2]**

Form a differential equation that represents the rate of change of its radius. Show your work.

Q: 7 **What family of curves does the solution of the differential equation $2x\frac{dy}{dx} = 7 + y$ represent? Show your work.** **[2]**

Q: 8 **The solution of the differential equation $xdy = (3x - 2y)\,dx$ is in the form:** **[3]**

$x^3 = C.\,f(x, y)$, where C is an arbitrary constant.

Find $f(x, y)$. Show your work.

Q: 9 **Birds are sensitive to microwaves that are emitted by mobile phones, which has resulted in a decline in the bird population, especially sparrows.** **[5]**

The population of sparrows in a certain region is decreasing according to the following equation due to the extensive use of mobile phones.

$\frac{dy}{dt} = ky$

where, y represents the population of sparrows at time t (in years) and k is a constant.

The population, which was e^{10} five years ago, has decreased by 25% in that time. Find k. Show your steps.

(Note: Take ln 3 ≈ 1.09 and ln 4 ≈ 1.38.)

Q: 10 **i) Find the differential equation representing the family of curves $(y - c)^2 = x^3$, where c is arbitrary constant.** **[5]**

ii) Find the order of the differential equation found in part i).

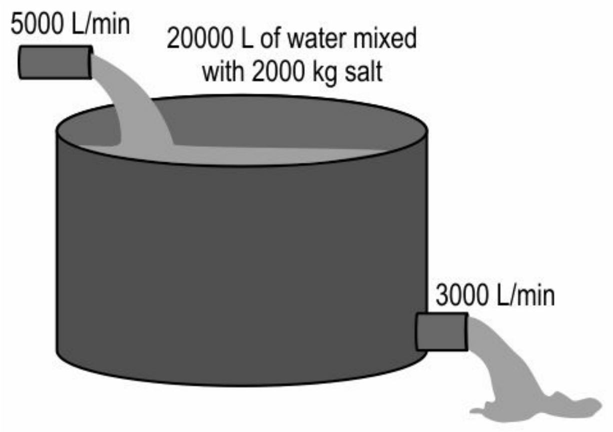
iii) Find the degree of the differential equation found in part i) if defined.

Show your work.

Case Study

Answer the questions based on the given information.

The mixing tank shown below generates saline water (a mixture of salt and water) for the cooling of a thermoelectric power plant.



The tank initially holds 20000 L of water in which 2000 kg of salt has been dissolved. Then, pure water is poured into the tank at a rate of 5000 L per minute. The mixture in the tank, which is stirred continuously, flows out at a rate of 3000 L per minute.

The quantity of salt in the tank at time t is denoted by Q_t, where t is in minutes and Q_t is in kilograms.

The rate of flow of salt into the tank is measured as:

$$\left(\frac{dQ_t}{dt}\right)_{in} = 0 \text{ kg/min}$$

The rate of flow of salt out of the tank is measured as:

$$\left(\frac{dQ_t}{dt}\right)_{out} = \text{Rate of flow of water out of the tank} \times \frac{\text{Quantity of salt in the tank at time } t}{\text{Amount of water in the tank at time } t}$$

The rate of change of quantity of salt in the tank with respect to time is given by

$$\left[\left(\frac{dQ_t}{dt}\right)_{in} - \left(\frac{dQ_t}{dt}\right)_{out}\right]$$

Q: 11 **Find the expression for the rate of change of quantity of salt in the tank with time t. Show your work.** **[1]**

Q: 12 **Find the general solution of the differential equation corresponding to the rate of change of quantity of salt in the tank with time t. Show your work.** **[2]**

Q: 13 **Use the initial conditions to determine the particular solution for the differential equation obtained. Show your work.** **[2]**

Q.No	What to look for	Marks
1	**Rewrites the given differential equation as** $ydy = e^{3x}\,dx$.	**0.5**
	Concludes that Sumit's general solution is incorrect by integrating both sides of the above equation to find the general solution as: $y^2 = \frac{2}{3} e^{3x}$ **+ C, where C is the arbitrary constant.**	**0.5**
2	**Writes a differential equation that depicts the given scenario as:** $\frac{dV}{dt} = k\sqrt[3]{\frac{dh}{dt}}$ **where,** V **is the volume,** h **is the height of the juice and** t **is the time.**	**1**
3	**Writes that the statement is false.**	**0.5**
	Gives a reason. For example, the order of the given differential equation is 4, but its degree is 2.	**0.5**
4	**Writes that the statement is false.**	**0.5**
	Gives a reason. For example, the number of arbitrary constants in the general solution of a differential equation is determined by its order, not its degree. Since the order of the given equation is 4, it will have only 4 arbitrary constants.	**0.5**
5	**Writes that** $\frac{dy}{dx} = y - \frac{2y}{x+1} = y\,(1 - \frac{2}{x+1})$. **Separates the variables as follows:** $\frac{dy}{y} = (1 - \frac{2}{x+1})\,dx$	**0.5**
	Integrates the above equation to get: $\log_e y = x - 2\log_e(x+1) + c$ $\Rightarrow \log_e y = x - \log_e(x+1)^2 + c$ $\Rightarrow \log_e y + \log_e(x+1)^2 = x + c$ **where c is a constant of integration.**	**1**

Q.No	What to look for	Marks
	Solves the above equation to find the equation of the curve as follows: $\log_e (y .(x +1)^2) = x + c$ $\Rightarrow y .(x +1)^2 = e^{x+c}$ $\therefore y = \frac{ke^x}{(x+1)^2}$, where k is a constant	**0.5**
6	**Takes V, S and *r* to be the volume, surface area and radius of the balloon at time *t*.** **Writes that since the volume of the balloon is decreasing with time, the rate of change of its radius is negative.**	**0.5**
	Uses the given information and expresses the relationship between volume and surface area as: $\frac{dV}{dt} = - k \times S$, **where *k* is a positive real number**	**0.5**
	Differentiates the above equation after substituting the expressions for volume and surface area of a sphere to get the required differential equation as: $\frac{4}{3} \pi(3 r^2) \frac{dr}{dt} = -k \times 4\pi r^2$ $\Rightarrow \frac{dr}{dt} = - k$	**1**
7	**Integrates the given differential equation as:** $\int \frac{dy}{7+y} = \int \frac{dx}{2x}$ $\Rightarrow \log(7 + y) = \frac{1}{2}\log x + \log c$ $\Rightarrow (7 + y) = \sqrt{x}\, c$ $\Rightarrow (7 + y)^2 = x\, c^2$ **where *c* is the constant of integration.**	**1.5**

Q.No	What to look for	Marks
	Compares the above equation with the general equation of parabola and concludes that the solution of the given differential equation represents the family of parabola.	**0.5**
8	**Rewrites the given differential equation as $\frac{dy}{dx} = 3 - 2\frac{y}{x}$.** **Takes $y = vx$ and differentiates it to get $\frac{dy}{dx} = v + x\frac{dv}{dx}$.**	**1**
	Rewrites the differential equation using the substitution in the above step as: $\frac{dv}{(1-v)} = 3\frac{dx}{x}$	**0.5**
	Integrates the above equation as: $-\log\|1 - v\| + \log\|C_1\| = 3\log\|x\|$ **where C_1 is the arbitrary constant.** **(Award full marks for equivalent appropriate answers.)**	**0.5**
	Simplifies the equation further and substitutes v as $\frac{y}{x}$ to obtain the general solution of the given differential equation as: $\left\|\frac{C_1}{1-v}\right\| = x^3$ $\Rightarrow \pm C_1 . \frac{x}{(x-y)} = x^3$ $\Rightarrow x^3 = \frac{x}{(x-y)}C$, where $C = \pm C_1$ **(Award full marks for equivalent appropriate answers.)**	**0.5**
	Concludes that: $f(x, y) = \frac{x}{(x-y)}$	**0.5**
9	**Uses the variable separable form to rewrite the given equation as:** $\frac{dy}{y} = k\,dt$	**0.5**

Q.No	What to look for	Marks
	Integrates the above equation on both sides to get: **ln $y = kt + c$, where c is a constant.**	**0.5**
	Writes the above equation in terms of e as: $y = e^{(kt+c)}$ => $y = e^{kt} \times e^{c}$	**1**
	Uses the given conditions to write: **At $t = 0$, $y = e^{c} = e^{10}$, where e^{10} is the initial population.**	**1**
	Uses the given condition to write: **At $t = 5$, $y = \frac{3}{4} e^{10} = e^{5k} \times e^{10}$** => $e^{5k} = \frac{3}{4}$	**1**
	Takes the natural logarithm on both sides to find the value of *k* as -0.058. The working may look as follows: $5k = \ln 3 - \ln 4$ => $5k = 1.09 - 1.38$ => $5k = -0.29$ => $k = \frac{-0.29}{5} = -0.058$	**1**
10	**i) Differentiates the given equation with respect to *x* as:** $2(y - c)y' = 3x^2$	**0.5**
	Simplifies the above differential equation to express *c* in terms of *x*, *y* and *y'* as: $c = y - \frac{3x^2}{2y'}$	**1**

Q.No	What to look for	Marks
	Uses the expression obtained in the above step to eliminate *c* from the original equation as: $[y-(y-\frac{3x^2}{2y'})]^2 = x^3$ $\Rightarrow 9x^4 = 4x^3(y')^2$ $\Rightarrow 4x^3(y')^2 - 9x^4 = 0$	**1.5**
	ii) Writes the order of the above differential equation as 1.	**1**
	iii) Writes the degree of the above differential equation as 2. **(If the differential equation obtained by the student is incorrect but have identified the order and degree of the incorrect equation correctly, then award 2 marks.)**	**1**
11	**Uses the information given to find the rate of change of quantity of salt as follows:** $\frac{dQ_t}{dt} = 0 - 3000 \times \frac{Q_t}{20000 + 5000t - 3000t} = \frac{-3Q_t}{20 + 2t}$	**1**
12	**Separates the variables of the differential equation as follows:** $\frac{dQ_t}{dt} = \frac{-3Q_t}{20 + 2t}$ $\Rightarrow \frac{dQ_t}{-3Q_t} = \frac{dt}{20 + 2t}$	**1**
	Integrates both sides to obtain the following: $\int \frac{dQ_t}{-3Q_t} = \int \frac{dt}{20 + 2t}$ $\Rightarrow -\frac{1}{3} \ln Q_t = \frac{1}{2} \ln (10 + t) + C$ **where C is an arbitrary constant.**	**1**

Q.No	What to look for	Marks
13	**Takes initial condition $Q_t(0)$ = 2000 to obtain C as follows:** $-\frac{1}{3}\ln 2000 = \frac{1}{2}\ln 10 + C$ $C = -\frac{1}{3}\ln 2 - \frac{3}{2}\ln 10$	**1**
	Frames the equation as: $-\frac{1}{3}\ln Q_t = \frac{1}{2}\ln(10 + t) - \frac{1}{3}\ln 2 - \frac{3}{2}\ln 10$	**1**

Chapter - 10
Vector Algebra

Multiple Choice Question

Q: 1 $\vec{u}$, $\vec{v}$ and $\vec{w}$ are three non-zero vectors that are neither parallel nor perpendicular to each other.
For which of the following will the product DEFINITELY be a vector?

(i) $(\vec{u} \cdot \vec{v}) \times \vec{w}$
(ii) $(\vec{u} \times \vec{v}) \cdot \vec{w}$
(iii) $(\vec{u} \times \vec{v}) \times \vec{w}$

1. only (ii)
2. only (iii)
3. only (i) and (iii)
4. all - (i), (ii) and (iii)

Free Response Questions

Q: 2 $\vec{p}$ and $\vec{q}$ are two collinear vectors. **[1]**

State whether the statement below is true or false.
Give a valid reason for your answer.

$(\lambda\vec{p} + \beta\vec{q}) \times \vec{q} = 0,$ where λ and β are scalars.

Q: 3 There are two vectors $\vec{u}$ and $\vec{v}$ such that $\vec{u} \cdot \vec{v} = 0$. **[1]**

Find the projection of the vector $\vec{u}$ on the vector $\vec{v}$.
Give a valid reason for your answer.

Q: 4 **State whether the following statement is true or false. Give a valid reason.** **[1]**

If the angle between $\vec{p}$ and $\vec{q}$ is obtuse, then $\vec{p} \cdot \vec{q} < 0$.

Q: 5 If $\vec{p}$ and $\vec{q}$ are unit vectors, show that the angle between $\vec{p}$ and $\vec{p} + \vec{q}$ is the same as the angle between $\vec{q}$ and $\vec{p} + \vec{q}$. **[1]**

Q: 6 The adjacent sides of a parallelogram, PQRS, are represented by $\vec{a}$ and $\vec{b}$. **[2]**

If $\vec{a} \cdot \vec{b} = 0$, what type of parallelogram is PQRS? Give a valid reason.

Q: 7 Show that any two non-zero vectors $\vec{u}$ and $\vec{v}$ are perpendicular if $\left|\vec{u} - \vec{v}\right| = \left|\vec{u} + \vec{v}\right|$. **[5]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3

Q.No	What to look for	Marks
2	Writes true.	0.5
	Gives a valid reason. For example, since $\vec{p}$ and $\vec{q}$ are collinear, $(\lambda\vec{p} + \beta\vec{q})$ is collinear to $\vec{p}$. This means that $(\lambda\vec{p} + \beta\vec{q})$ is parallel to $\vec{p}$. Hence, their cross product is zero.	0.5
3	**Writes that the projection vector will be a zero vector.**	0.5
	Gives the reason that the projection of vector $\vec{u}$ on vector $\vec{v}$ is given by $\frac{\vec{u}\cdot\vec{v}}{\lvert\vec{v}\rvert}$ and since the dot product is 0, the projection vector is a zero vector.	0.5
4	**Writes true.**	0.5
	Justifies as follows: $\vec{p}\cdot\vec{q} = p \times q \times \cos\theta$, where $\cos\theta$ is negative when $90° < \theta < 180°$.	0.5
5	Writes that $\vec{p} + \vec{q}$ represents the third side of the triangle whose other two sides are $\vec{p}$ and $\vec{q}$ with magnitude 1 unit each(as they are unit vectors). (Award full marks if pictorial/vector explanation is provided.)	0.5
	Concludes that both $\vec{p}$ and $\vec{q}$ make equal angles with $\vec{p} + \vec{q}$ as they form an isosceles triangle.	0.5
6	Writes that as $\vec{a}\cdot\vec{b} = 0$, $\lvert\vec{a}\rvert \times \lvert\vec{b}\rvert \times \cos\theta = 0$.	0.5

Q.No	What to look for	Marks
	Reasons that $\|\vec{a}\| \neq 0$ and $\|\vec{b}\| \neq 0$ as they need to form a parallelogram.	0.5
	Deduces that cos θ = 0 and hence θ = 90°.	0.5
	Concludes that PQRS is a rectangle as the sides are intersecting at 90°.	0.5
7	Considers vectors $\vec{u}$ and $\vec{v}$ as: $\vec{u} = p\hat{i} + q\hat{j} + r\hat{k}$ $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$	0.5
	Uses the condition $\|\vec{u} - \vec{v}\| = \|\vec{u} + \vec{v}\|$ to write the following: $(p-x)^2 + (q-y)^2 + (r-z)^2 = (p+x)^2 + (q+y)^2 + (r+z)^2$	1.5
	Simplifies the above equation to obtain $px + qy + rz = 0$.	1.5
	Uses the above step to find the dot product of the two vectors $\vec{u}$ and $\vec{v}$ as: $\vec{u} \cdot \vec{v} = px + qy + rz = 0$	1
	Writes that the vectors are perpendicular since their dot product is equal to zero.	0.5

Chapter - 11
Three Dimensional Geometry

Multiple Choice Questions

Q: 1 **Given below are the vector equations of two planes.**

$$\left[\vec{r} - \left(4\hat{i} + 3\hat{j} - 4\hat{k}\right)\right] \cdot \left(\hat{i} + 4\hat{j} - \hat{k}\right) = 0$$

$$\left[\vec{r} - \left(6\hat{i} + \hat{j} - 5\hat{k}\right)\right] \cdot \left(\hat{i} + 4\hat{j} - \hat{k}\right) = 0$$

Which of the following is true about these planes?

1. They coincide with each other.
2. They are parallel to each other.
3. They are perpendicular to each other.
4. They are not perpendicular but intersect each other along a unique line.

Q: 2 **Shown below are equations of four planes.**

P_1 : $-2x - 3y - 4z - 20 = 0$
P_2 : $2x + 3y + 4z - 9 = 0$
P_3 : $-4x - 6y + 8z = 0$
P_4 : $4x + 6y + 8z = 0$

Which of the provided planes does NOT share parallelism with the rest?

1. P_1
2. P_2
3. P_3
4. P_4

Free Response Questions

Q: 3 **There are three points P, Q and R. The direction ratios of line PQ are (-2), 1 and 3 and that of line QR are 6, (-3) and (-9).** **[1]**

Is there a unique plane that contains the points P, Q and R? Justify your answer.

Q: 4 **The equations for line *m* and line *n* are given below.** **[1]**

Line *m*: $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{2}$

Line *n*: $\frac{x-1}{2} = \frac{y-1}{1} = \frac{1-z}{k}$

If line *m* is perpendicular to line *n*, find the value of *k*. Show your steps.

(Note: $k \neq 0$.)

Q: 5 **Line *m* and line *n* are two lines which are not parallel, and do not intersect each other.** **[1]**

Asha, Ravi and Saquib tried to find the shortest distance between lines *m* and *n*.

♦ Asha found a line *k* parallel to line *m* on the same plane as line *n*. She found the shortest distance as the distance between lines *k* and *n*.

♦ Ravi found the length of a line segment that is perpendicular to both line *m* and line *n* as the shortest distance.

♦ Saquib said that it is impossible to find the distance between the two lines, since the lines are not parallel, and do not intersect.

Which of them is/are using the correct approach? Justify your answer.

Q: 6 $\vec{b}$ is a vector parallel to a line $\vec{c}$. **[1]**

$\vec{n}$ is the normal from the origin to the plane in which $\vec{c}$ lies.

Find $\vec{b} \cdot \vec{n}$. Give a valid reason.

Q: 7 **Shown below are the equations of two planes.** **[1]**

$P_1 : 3x - 3y - 3z = 1$
$P_2 : -3x + ny - z = 4$

If P_1 is perpendicular to P_2, find the value of *n*. Show your work.

Q: 8 **A line given by $\frac{x+1}{3} = \frac{y+1}{2} = \frac{2-z}{2}$ is parallel to the plane $6x + 4y + Gz = 12$.** **[2]**

Find the value of G. Show your steps.

Q: 9 **The equation of a plane can be given by $px - 2y + z = q$, where p and q ∈ R, such that:** **[2]**

♦ the plane is parallel to $-6x + 4y - 2z = 12$.

♦ the point (2, -3, 5) lies on the plane.

Find p and q. Show your work.

Q: 10 Shown below is the equation of a plane P. **[2]**

$\vec{r}.(0\hat{i} - 5\hat{j} + 0\hat{k}) = 8$

WITHOUT using the formula for distance of a point from a plane, find out how far the plane P is from the origin.

Q: 11 **Two lines are given below in vector form:** **[3]**

$$\vec{r_1} = (\hat{i} - \hat{j}) + \lambda(-\hat{i} + \hat{j} - \hat{k})$$

$$\vec{r_2} = (4\hat{i} - 3\hat{j} + \hat{k}) + \mu(-\hat{i} + 2\hat{j} - 3\hat{k})$$

Prove that the lines are non-skew lines. Give valid reasons.

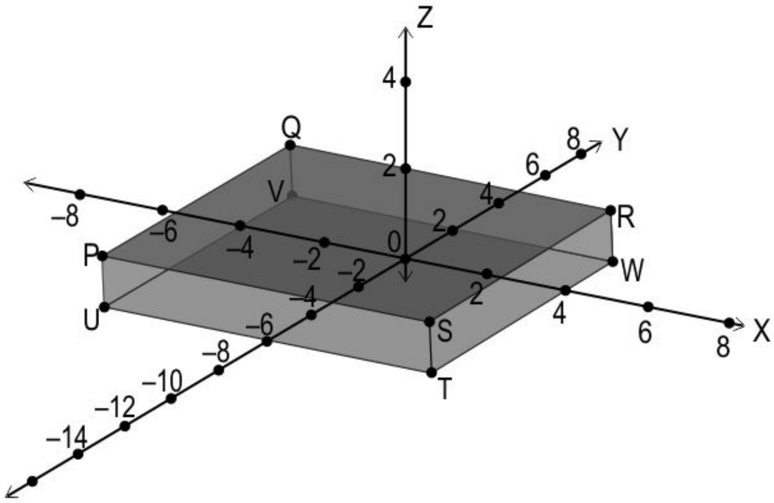
Q: 12 **Shown below are the equations of a line and a plane.** **[3]**

Line: $\vec{r} = \hat{i} + 2\hat{j} + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$
Plane: $8x - 2y + 6z = 1$

where λ is some real number.

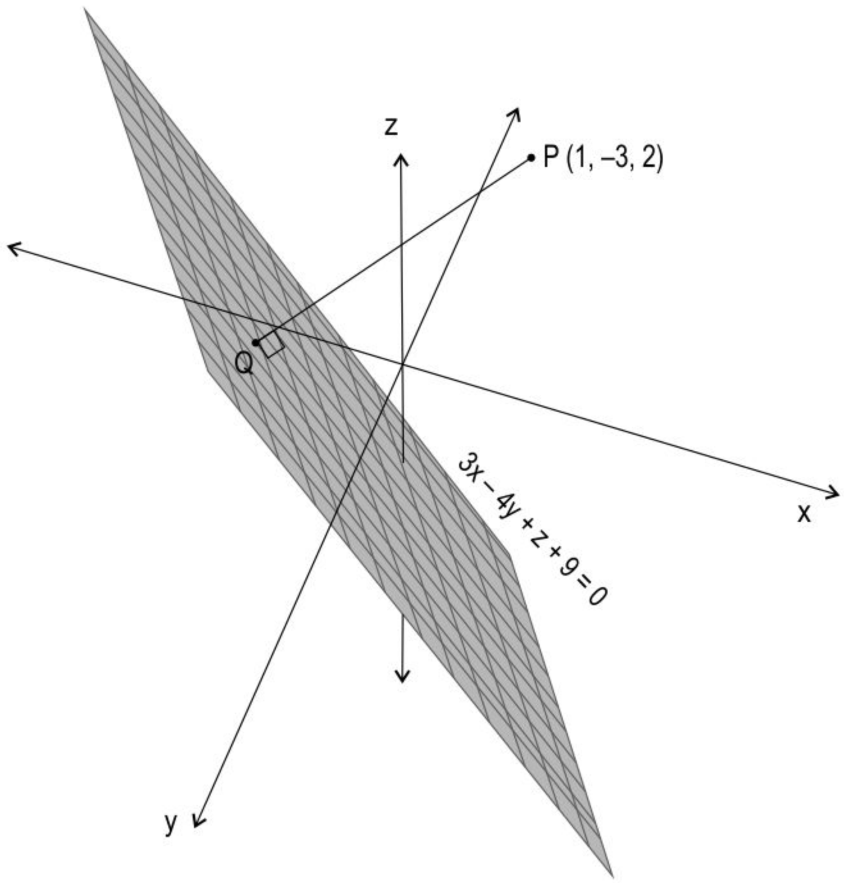
Check whether they intersect. If yes, find the point of intersection. If not, state a valid reason. Show your work.

Q: 13 **A cuboid is shown below, such that face TUVW lies on the *x* - *y* plane. The height of the cuboid is 2 units.** **[3]**



Find the equation of the line joining P and W. Does it pass through the origin? Show your work and give valid reason(s).

Q: 14 **Point P meets the plane shown below at point Q.** **[5]**



Find the coordinates of Q and the distance PQ. Show your work.

Q: 15 **The equation of line *m* is given by $\frac{x+8}{2} = \frac{5-y}{3} = \frac{z-4}{5}$. A line segment PQ is to be drawn perpendicular to line *m*, such that line *m* bisects PQ.** **[5]**

If P's coordinates are given by (-2, -7, 2), then find the coordinates of point Q. Show your steps.

Q: 16 **Shown below is an equation of a plane, where P is a constant.** **[5]**

$$3y + 4z + P = 0$$

The distance from the origin to the plane is the same as the distance from the point (n, 2, -4) to the plane. The distance from the point (2, m, 4) to the plane is six times the distance from the origin to the plane.

Find all possible values of:
i) P
ii) m
iii) n

Show your work. Give a valid reason.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	2
2	3

Q.No	What to look for	Marks
3	**Finds that the direction ratios of PQ and QR are proportional. The working may look as follows:** $\frac{6}{-2} = \frac{-3}{1} = \frac{-9}{3}$ **= (-3), where (-3) is the constant of proportionality.**	**0.5**
	States that there is no unique plane because P, Q and R are collinear and infinitely many planes that contain three collinear points exist.	**0.5**
4	**Since the two lines are perpendicular, writes the equation:** **3(2) + 2(1) + 2(- *k*) = 0**	**0.5**
	Solves the above equation to find the value of *k* as 4.	**0.5**
5	**Writes that only Ravi is using the correct approach. Gives a reason.** **For example, lines *m* and *n* are skew lines, and the distance between skew lines is given by the length of a line segment that is perpendicular to both lines. Hence, Saquib is incorrect.** **Asha is incorrect as the shortest distance between two non-parallel lines on the same plane is 0.**	**1**
6	**States that the normal to a plane is perpendicular to every line on the plane and the lines parallel to the lines on the plane.**	**0.5**
	Finds the value of the dot product as: $\vec{b} \cdot \vec{n} = 0$	**0.5**
7	**Writes that since P_1 is perpendicular to P_2, their normals are also perpendicular. Thus equates the dot product of the normals to zero and finds the value of *n* as (-2). The working may look as follows.** **3(-3) + (-3) *n* + (-3)(-1) = 0** **=> -9 - 3n + 3 = 0** **=> *n* = -2**	**1**

Q.No	What to look for	Marks
8	**Writes the equation of the line in vector form as follows:** $\vec{r} = \vec{a} + \lambda\vec{b}$, where $\vec{a} = (-\hat{i} - \hat{j} + 2\hat{k})$ and $\vec{b} = (3\hat{i} + 2\hat{j} - 2\hat{k})$	**0.5**
	Finds the equation of the plane in vector form as $\vec{r}.\hat{n} = d$, where $d = 12, \hat{n} = \frac{\vec{n}}{\lvert\vec{n}\rvert}$, and $\vec{n} = (6\hat{i} + 4\hat{j} + G\hat{k})$. Hence finds the equation of the plane as $\frac{\vec{r}.(6\hat{i}+4\hat{j}+G\hat{k})}{\lvert\sqrt{6^2+4^2+G^2}\rvert} = 12$	**0.5**
	As the line is parallel to the plane, notes that $\sin\phi = 0$, where ϕ is the angle between the given line and plane. $\therefore\ \vec{b}.\hat{n} = 0.$ Hence finds $\vec{b}.\hat{n} = (3\hat{i} + 2\hat{j} - 2\hat{k}).\frac{(6\hat{i}+4\hat{j}+G\hat{k})}{\lvert\sqrt{6^2+4^2+G^2}\rvert} = 0$ $\Rightarrow (3\hat{i} + 2\hat{j} - 2\hat{k}).(6\hat{i} + 4\hat{j} + G\hat{k}) = 0$ $\Rightarrow 18 + 8 - 2G = 0$	**0.5**
	Solves the above equation to find G = 13.	**0.5**
9	**Finds p = 3 by writing the following as the planes are parallel:** $\frac{p}{-6} = \frac{-2}{4} = \frac{1}{-2}$	**1**
	Finds q = 17 by substituting the coordinates of the point (2, -3, 5) in the equation of the plane as: **3(2) - 2(-3) + (5) = q**	**1**

Q.No	What to look for	Marks
10	**Finds the magnitude of the normal vector to the plane from the origin as:** $\left\|\vec{n}\right\| = \sqrt{0^2 + 5^2 + 0^2} = 5$	0.5
	Rewrites the given equation as: $\vec{r} \cdot \frac{(0\hat{i} - 5\hat{j} + 0\hat{k})}{\|\vec{n}\|} = \frac{8}{\|\vec{n}\|}$ $\Rightarrow \vec{r} \cdot \hat{n} = \frac{8}{5}$	1
	Compares the above with the equation of a plane in normal form and finds the distance between the plane and the origin as $\frac{8}{5}$ units.	0.5
11	**Considers the lines in vector form as:** $\vec{r_1} = \vec{a_1} + \lambda \vec{b_1}$ $\vec{r_2} = \vec{a_2} + \lambda \vec{b_2}$ which denotes that $\vec{r_1}$ passes through the point A, with position vector $\vec{a_1}$, and is parallel to $\vec{b_1}$, and $\vec{r_2}$ passes through the point B, with position vector $\vec{a_2}$, and is parallel to $\vec{b_2}$.	0.5
	Writes the vector of the lines joining A and B as: $\vec{AB} = (1 - 4)\hat{i} + (-1 - (-3))\hat{j} + (0 - 1)\hat{k} = -3\hat{i} + 2\hat{j} - 1\hat{k}$	0.5
	Writes that if the lines are coplanar, they will definitely be non-skew lines.	0.5

Q.No	What to look for	Marks
	Hence, proves coplanarity as follows: $\overrightarrow{AB}.(\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0$ the LHS of which translates to: $\begin{vmatrix} -3 & 2 & -1 \\ -1 & 1 & -1 \\ -1 & 2 & -3 \end{vmatrix}$	0.5
	Finds the value of the determinant as 0.	0.5
	Hence, concludes that the lines are coplanar and therefore non-skew lines.	0.5
12	Rearranges the equation of the line as: $\overrightarrow{r} = (1 + \lambda)\hat{i} + (2 - 3\lambda)\hat{j} + 2\lambda\,\hat{k}$ and finds a point P ((1 + λ), (2 - 3λ), 2λ) on the line.	0.5
	Substitutes the coordinates of P in the equation of the plane as: 8(1 + λ) - 2(2 - 3λ) + 6(2λ) = 1	0.5
	Finds the value of λ as (- $\frac{3}{26}$) by simplifying the above equation as follows: 8 +8λ - 4 + 6λ + 12λ = 1 => 4 + 26λ = 1 => λ = - $\frac{3}{26}$ Writes that there exists a value of λ for which the point P satisfies the equations of both the line and the plane. Hence concludes that the line and the plane intersect.	1

Q.No	What to look for	Marks
	Substitutes the value of λ in point P to find the point of intersection as: **P = ($\frac{23}{26}$, $\frac{61}{26}$, - $\frac{3}{13}$)**	**1**
13	**Finds the coordinates of P as (-4, -6, 2).** **Finds the coordinates of W as (4, 2, 0).**	**1**
	Finds the direction ratios of PW as follows: ***l* = 4 - (-4) = 8** ***m* = 2 - (-6) = 8** ***n* = 0 - 2 = (-2)**	**0.5**
	Assumes that (*x, y, z*) is an arbitrary point on the line joining P and W. **Finds the equation of the line as:** $\frac{x+4}{8} = \frac{y+6}{8} = \frac{2-z}{2}$ **or** $\frac{x-4}{8} = \frac{y-2}{8} = \frac{-z}{2}$	**1**
	Writes that the line does not pass through the origin, and gives a valid reason. **For example, in the equation from step 3,** $\frac{0+4}{8} \neq \frac{0+6}{8} \neq \frac{2-0}{2}$	**0.5**
14	**Identifies the direction ratios of the perpendicular to the plane as 3, -4 and 1.**	**0.5**
	Takes Q(x_1, y_1, z_1), identifies that the direction ratios of the perpendicular and the direction ratios of PQ are proportional and writes the following: $\frac{x_1-1}{3} = \frac{y_1+3}{-4} = \frac{z_1-2}{1} = k$ $\Rightarrow x_1 = 3k + 1;\ y_1 = -4k - 3;\ z_1 = k + 2$	**1**

Q.No	What to look for	Marks
	Substitutes the above values in the equation of the plane to obtain $k = -1$. The working may look as follows: **$3(3k + 1) - 4(-4k - 3) + 1(k + 2) + 9 = 0$** **$\therefore k = -1$**	1
	Hence, finds the coordinates of Q by substitution as Q(-2, 1, 1).	1
	Uses the distance formula to express PQ as follows: **$PQ = \sqrt{(1+2)^2 + (-3-1)^2 + (2-1)^2} = \sqrt{3^2 + (-4)^2 + 1^2}$ units**	1
	Solves the above to find the distance PQ as $\sqrt{26}$ units.	0.5
15	**Assumes that O(x, y, z) is the point of intersection of PQ and line m.** **Writes that $\frac{x+8}{2} = \frac{5-y}{3} = \frac{z-4}{5} = \lambda$, where λ is a constant.** **Using the equations from step 1, finds x, y, and z as follows:** **$x = 2\lambda - 8$** **$y = 5 - 3\lambda$** **$z = 5\lambda + 4$**	1
	Notes that O(x, y, z) and P both lie on line PQ. **Finds the direction ratios of PQ as:** **$(2\lambda - 8 - (-2), 5 - 3\lambda - (-7), 5\lambda + 4 - 2)$** **$= (2\lambda - 6, 12 - 3\lambda, 5\lambda + 2)$**	0.5
	Since PQ is perpendicular to line m, equates the dot product of their direction ratio to 0 to find λ as 1. **$(2\lambda - 6, 12 - 3\lambda, 5\lambda + 2).(2, -3, 5) = 0$** **$\Rightarrow 2(2\lambda - 6) - 3(12 - 3\lambda) + 5(5\lambda + 2) = 0$** **$\Rightarrow 4\lambda - 12 - 36 + 9\lambda + 25\lambda + 10 = 0$** **$\Rightarrow 38\lambda = 38$** **$\Rightarrow \lambda = 1$**	1.5

Q.No	What to look for	Marks
	Finds the O(x, y, z), the coordinates of the foot of the perpendicular from P to line *m* as (-6, 2, 9).	**0.5**
	Assumes the coordinates of Q as (*a, b, c*). Finds Q as (-10, 11, 16) using the midpoint theorem as: **-2 + *a* = 2(-6)** **=> *a* = -10** **-7 + *b* = 2(2)** **=> *b* = 11** **2 + *c* = 2(9)** **=> *c* = 16**	**1.5**
16	**Finds the distance from the origin to the plane as follows:** $D_1 = \frac{\lvert P \rvert}{5}$	**0.5**
	Finds the distance from the point (n, 2, -4) to the plane as follows: $D_2 = \frac{\lvert 0(n)+3(2)+4(-4)+P \rvert}{5} = \frac{\lvert -10+P \rvert}{5}$	**0.5**
	i) Uses the given condition that $D_1 = D_2$ to find the value of *P* as 5. The working may look as follows: **| *P* | = |-10 + *P* |** *P* = -10 + *P* / 0 = 10 / which is not possible. OR - *P* = 10 - *P* / 0 = 10 / which is not possible. OR *P* = 10 - *P* / => 2 *P* = 10 / => *P* = 5 OR - *P* = -10 + *P* / => -2 *P* = -10 / => *P* = 5 **(Award full marks if two of the unique cases are shown rather than 4 cases.)**	**1.5**
	ii) Finds the distance of the point (2, *m*, 4) from the plane as follows: $D_3 = \frac{\lvert 0(2)+3(m)+4(4)+P \rvert}{5}$	**0.5**

Q.No	What to look for	Marks
	Uses the given condition that $D_3 = 6D_1$ to find the value of *m* as 3 or -17. The working may look as follows: $\frac{\lvert 0(2)+3(m)+4(4)+P\rvert}{5} = \frac{6\lvert P\rvert}{5}$ **=> |3 *m* + 16 + 5| = 30** 3 *m* + 16 + 5 = 30 OR 3 *m* + 16 + 5 = -30 => 3 *m* = 9 ; => 3 *m* = -51 => *m* = 3 ; => *m* = -17	1
	iii) Writes that *n* can take any value as the coefficient of *x* in the given plane is 0.	1

Inner working table:

3 *m* + 16 + 5 = 30	OR	3 *m* + 16 + 5 = -30
=> 3 *m* = 9		=> 3 *m* = -51
=> *m* = 3		=> *m* = -17

Chapter - 12
Linear Programming

Multiple Choice Questions

Q: 1 **Shown below is a linear programming problem (LPP).**

Maximise Z = $x + y$

subject to the constraints:

$x + y \leq 1$
$-3x + y \geq 3$
$x \geq 0$
$y \geq 0$

Which of the following is true about the feasible region of the above LPP?

1 It is bounded.
2 It is unbounded.
3 There is no feasible region for the given LPP.
4 (cannot conclude anything from the given LPP)

Q: 2 **Two statements are given below - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).**

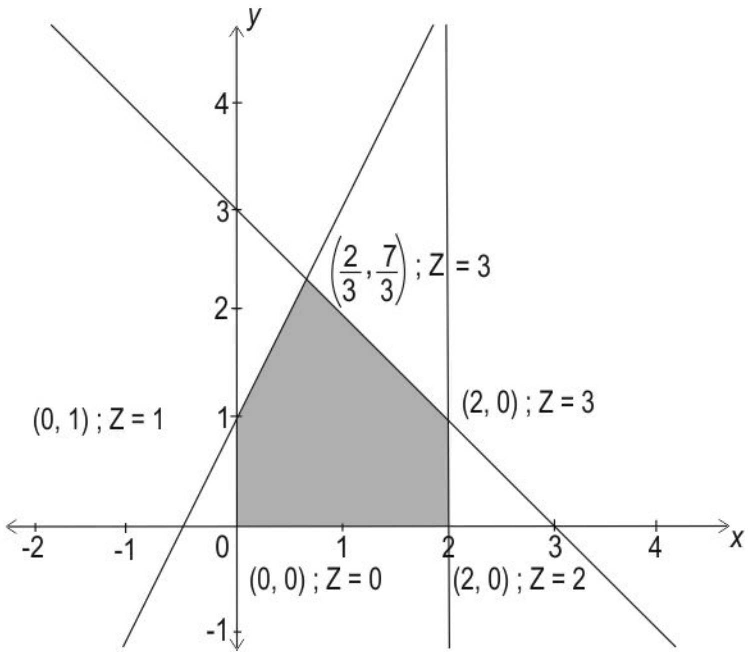
***Assertion* (*A*): All the points in the feasible region of a linear programming problem are optimal solutions to the problem.**

***Reason* (*R*): Every point in the feasible region satisfies all the constraints of a linear programming problem.**

1 Both (A) and (R) are true and (R) is the correct explanation for (A).
2 Both (A) and (R) are true but (R) is not the correct explanation for (A).
3 (A) is false but (R) is true.
4 Both (A) and (R) are false.

Free Response Questions

Q: 3 **Shown below is the feasible region of a linear programming problem(LPP) whose objective function is maximize Z = x + y.** **[1]**



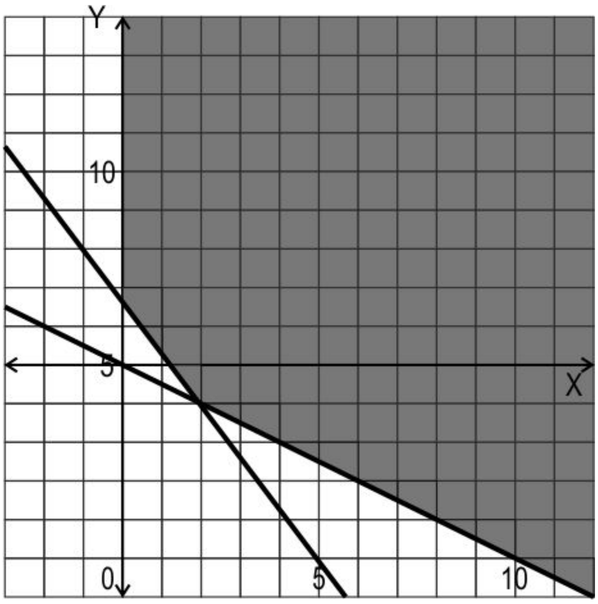
Sarla claimed that there exists no optimal solution for the LPP as there is no unique maximum value at the corner points of its feasible region.

Is her claim true? Give a valid reason.

Q: 4 **State whether the following statement is true or false. Explain your reasoning.** **[1]**

In a linear programming problem, it is possible for a non-corner point to have the optimal value of the objective function, *Z* = a *x* + b *y*.

Q: 5 **The shaded region in the graph shown below represents the feasible region for a linear programming problem with objective function Minimize: Z = 4 *x* - 3 *y*.** **[1]**

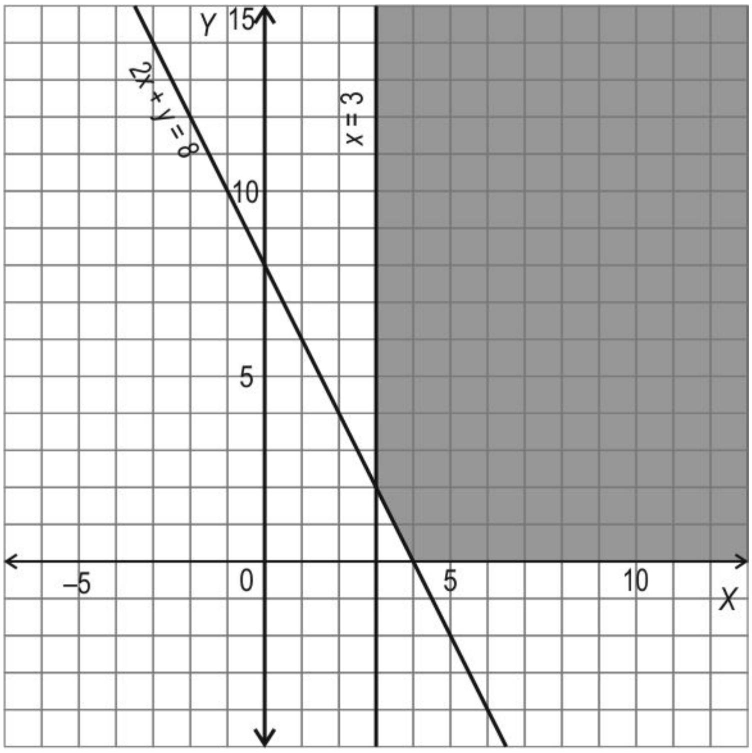


The values of the objective function at the corner points are given below:

Corner points	Z = 4x –3y
$(0, \frac{20}{3})$	–20
(2, 4)	–4
(10, 0)	40

Suhas says that the minimum value of Z is (-20). Is he correct or incorrect? Justify your answer.

Q: 6 **A feasible region with respect to certain constraints is shaded in the graph below.** **[1]**



Reece says that the objective function Z = *x* + *y* will give an optimal solution when maximized. Is he correct or incorrect? Justify your answer.

Q: 7 **The objective function of a linear programming problem is given by:** **[2]**

Maximise Z = *ax* + *by*

The following are known about the linear programming problem:

♦ The problem has an unbounded feasible region.
♦ (x_0, y_0) is a corner point of the feasible region such that $ax_0 + by_0$ has the maximum value among all the values of Z evaluated at the corner points of the feasible region.
♦ The optimal solution of this problem does not exist.

When and why does this occur?

Q: 8 Benjamin, a professional runner, follows a rigorous workout routine designed to improve his athletic performance. Running on the treadmill is a part of his daily routine. **[3]**

If he runs at 10 km/hr, he burns 90 cal/km. If he runs at a faster speed of 16 km/hr, the calories burnt increases to 150 cal/km. He wishes to run maximum distance in not more than an hour at only two speeds. He does not want to burn more than 1000 calories as it could be detrimental for his health.

Express the above optimisation problem as a linear programming problem.

(Note: Treadmill is a device used for exercise, consisting of a continuous moving belt on which one can walk or run.)

Q: 9 A government bond or sovereign bond is a form of bond issued by the government to raise money for public works such as parks, libraries, bridges, roads, and other infrastructure. There are multiple variants of bonds issued by Government of India (GOI) at fixed interest rates and for a fixed period. **[5]**

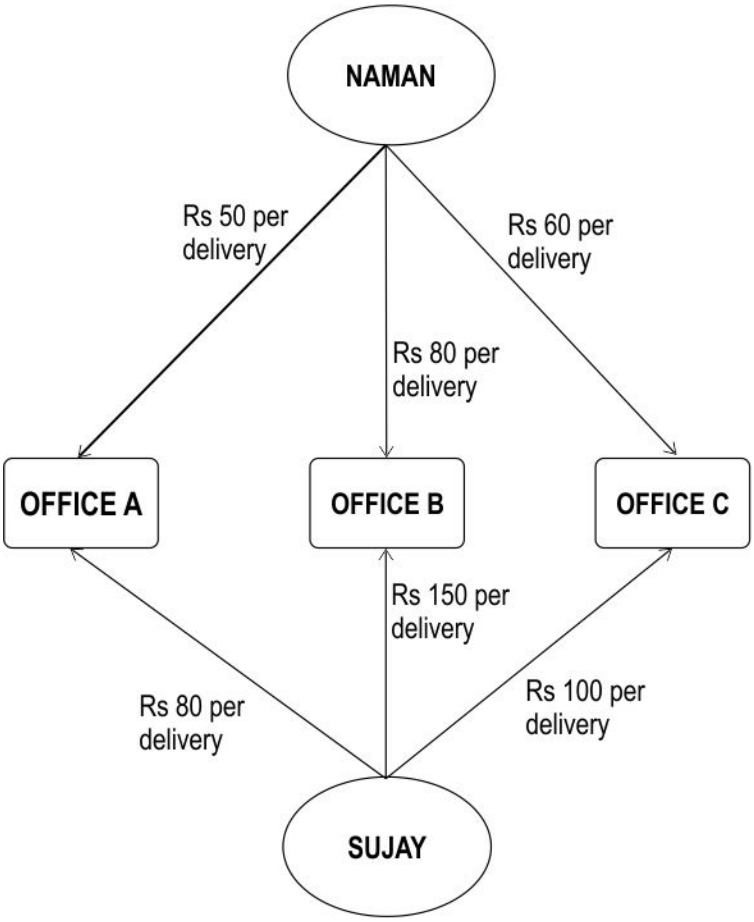
Jayesh Runjhunwala, an investor, wants to invest a maximum of Rs 20000 in government bonds for one year. He decides to invest in two types of bonds X and Y, bond X yielding 10% simple interest on the amount invested and bond Y yielding 15% simple interest on the amount invested. He wants to invest at least Rs 5000 in bond X and no more than Rs 8000 in bond Y.

Formulate the linear programming problem and determine the amount he must invest in two bonds to maximise his return. Show your work.

Q: 10 **Naman and Sujay work for Megha's tiffin service as delivery people. In their area, there are 3 offices that order lunch from Megha's tiffin service. Naman and Sujay's daily targets are to deliver 30 and 40 tiffins, respectively. They can each carry only one tiffin at a time to deliver to an office, to prevent the food from spilling out.** **[5]**

On average, the number of tiffins ordered by offices A, B and C daily are 20, 30 and 20 respectively.

The cost of travelling to/from each office to deliver a tiffin is shown in the tree diagram below:



Megha wants to minimise the cost of delivery for her tiffins.

i) Using the given tree diagram, form a system of inequalities to represent the above situation, in order to find out how many tiffins Naman and Sujay should each deliver to a respective office in a day.

ii) Find the function of the daily cost of delivery for Megha's tiffin service.

iii) Graphically solve the system of inequalities obtained in step i) to minimise the daily cost of delivery.

Show your work.

Q: 11 **Sudhanva has two kinds of areca nuts - premium and economy. He sells them to two stores, A and B. The profit margin per kg for premium and economy nuts are Rs 80 and Rs 50 respectively.** **[5]**

Both stores buy an equal quantity of premium nuts. However, Store B buys three times the economy nuts as compared to Store A. Store A can buy a maximum of 30 kg of nuts and store B can buy a maximum of 60 kg of nuts.

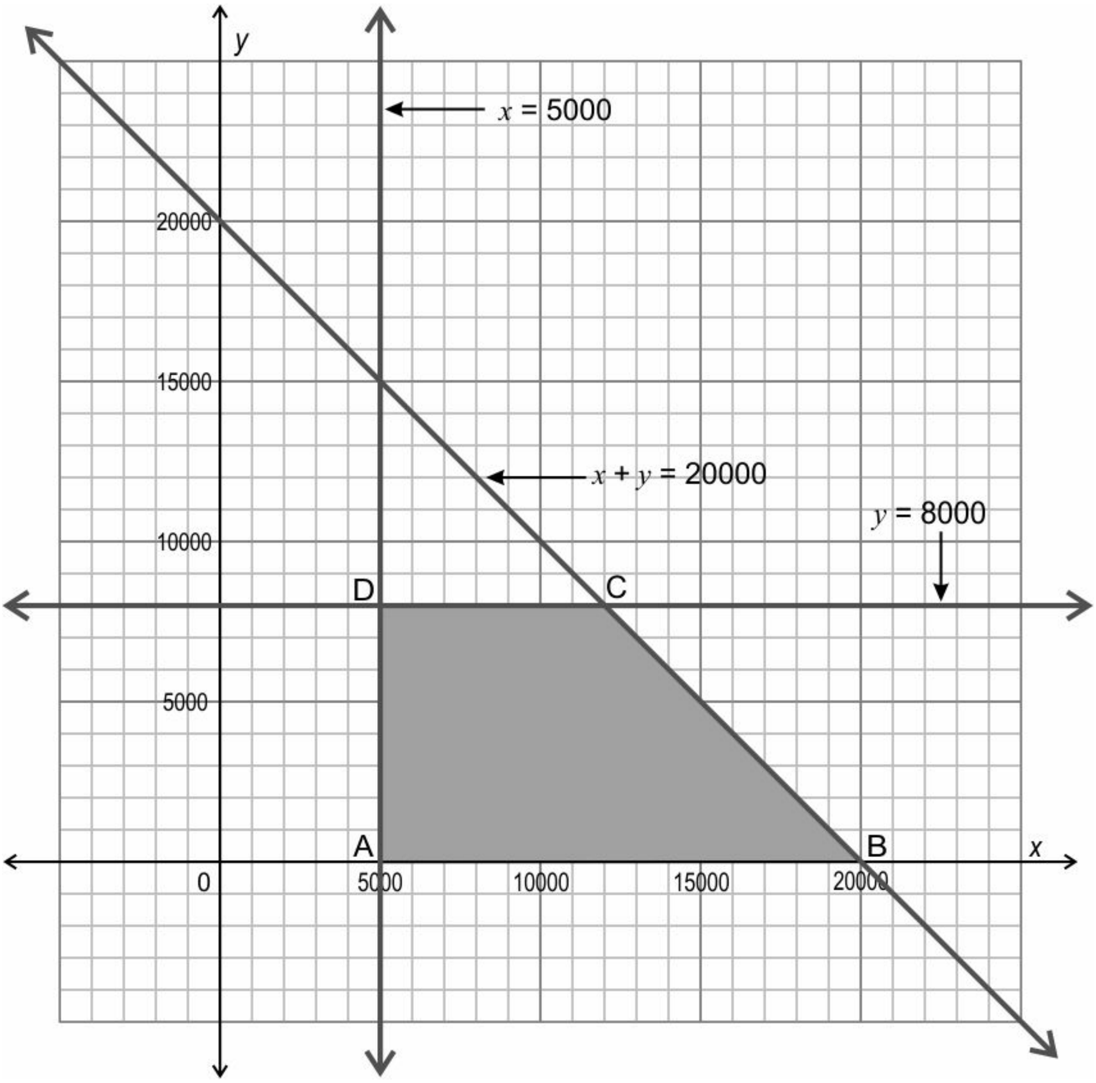
What should be the maximum weight of premium and economy nuts that Sudhanva should sell to obtain maximum profit? Show your steps.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	3

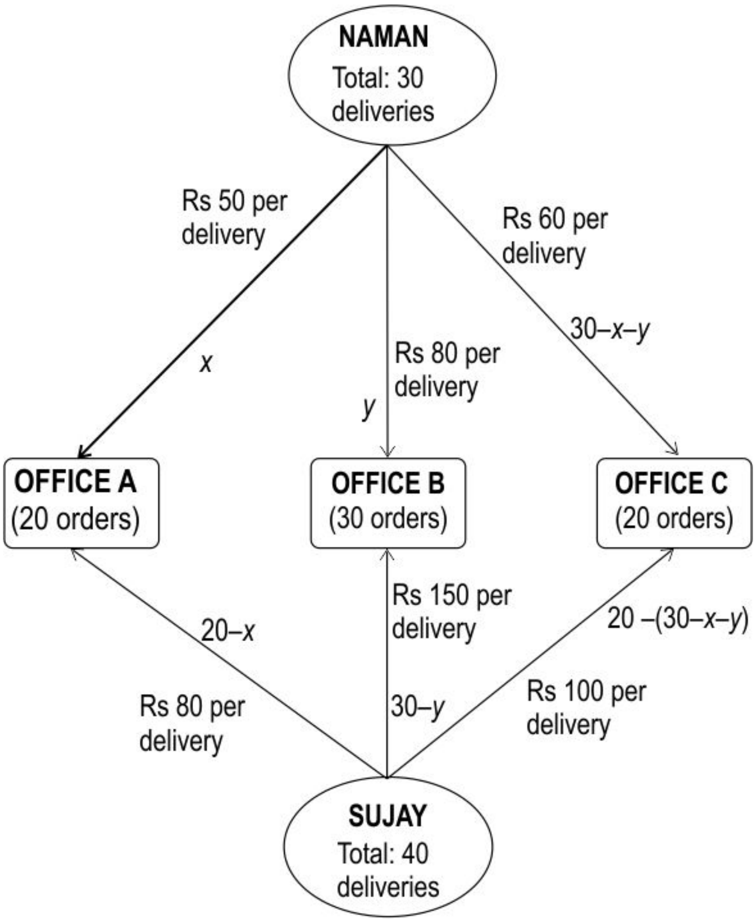
Q.No	What to look for	Marks
3	Writes that Sarla's claim is false.	0.5
	Gives a reason. For example, every point on the line joining ($\frac{2}{3}$, $\frac{7}{3}$) and (2, 1) gives the maximum value of Z which is 3. Hence, any point on the line joining ($\frac{2}{3}$, $\frac{7}{3}$) and (2, 1) is an optimal solution.	0.5
4	Writes that the statement is true.	0.5
	Gives a valid reason. For example, if two adjacent corner points yield the optimal value for the objective function, then every point on the line joining them also yields the optimal value.	0.5
5	Writes that Suhas is incorrect.	0.5
	Reasons that since the feasible area is unbounded, the minimum value of the objective function will exist only if the graph of $4x - 3y < (-20)$ has no point in common with the feasible region. Shows as an example that (1, 10) satisfies the inequality $4x - 3y < (-20)$ and is in common with the feasible region and hence, the minimum value of Z does not exist.	0.5
6	Writes that he is incorrect.	0.5
	Finds the value of the objective function at the corner points (2, 3) and (4, 0) as 5 and 4 respectively, and writes that $x + y > 5$ is overlapping with the feasible region and hence, no maximum value exists.	0.5
7	Writes that when $Z > ax_0 + by_0$ has at least one point common with the feasible region, the optimal solution of this problem does not exist.	0.5
	Gives reason that when $Z > ax_0 + by_0$ has at least one point common with the feasible region, it means that there is at least one non-corner point (x_1, y_1) in the feasible region such that $ax_1 + by_1 > ax_0 + by_0$.	1
	Further reasons that Z has no maximum value at (x_0, y_0) and thus the optimal solution of this problem does not exist as a maximum value, if exists, has to be obtained at a corner point .	0.5

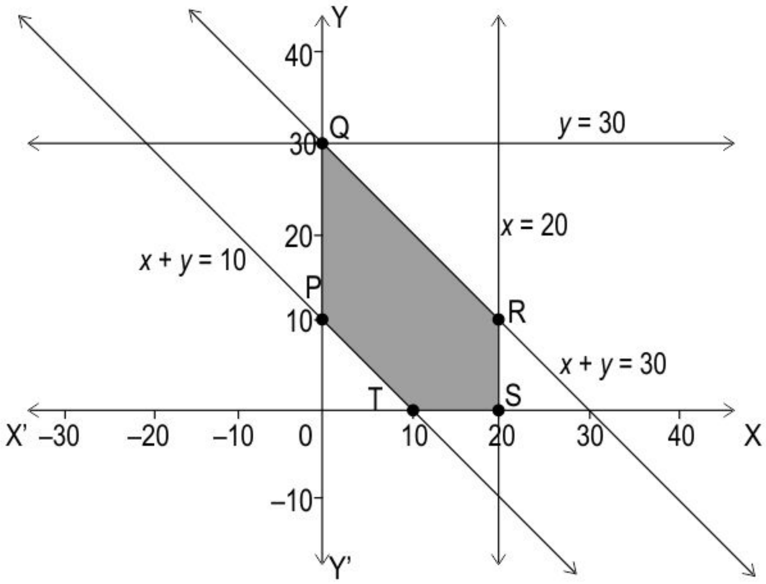
Q.No	What to look for	Marks
8	Takes *x* and *y* to be the distances (in km) covered by Benjamin at the speeds of 10 km/hr and 16 km/hr respectively. Finds the time consumed in covering these distances as $\frac{x}{10}$ hr and $\frac{y}{16}$ hr respectively.	1
	Writes the objective function of the given LPP as: Maximise Z = $x + y$, where Z is the total distance covered.	0.5
	Finds the calorie constraint corresponding to the given LPP as: $90x + 150y \leq 1000$	0.5
	Finds the time constraint corresponding to the given LPP as: $\frac{x}{10} + \frac{y}{16} \leq 1$	0.5
	Writes the non-negativity constraints corresponding to the given LPP as: $x \geq 0$ $y \geq 0$	0.5
9	Assumes the amount to be invested in bonds X and Y to be Rs *x* and Rs *y* respectively. Frames the objective function of the given problem as: Maximise Z = $\frac{x}{10} + \frac{3y}{20}$ (Award full marks if the objective function is framed based on the total amount as Maximise Z = $\frac{11x}{10} + \frac{23y}{20}$.)	1
	Writes the constraints of the given LPP as: $x + y \leq 20000$ $x \geq 5000$ $y \leq 8000$ $x, y \geq 0$	1

Q.No	What to look for	Marks
	Draws the graph of the system of inequalities and shades the feasible region. The graph may look as follows: 	**1.5**

Q.No	What to look for	Marks
	Evaluates the objective function at the corner points to find the optimal solution as follows: Corner points | $Z = \frac{x}{10} + \frac{3y}{20}$ A(5000, 0) | 500 B(20000, 0) | 2000 C(12000, 8000) | 2400 D(5000, 8000) | 1700 **(Award full marks if correct Z values are obtained corresponding to the objective function $\frac{11x}{10} + \frac{23y}{20}$.)**	**1**
	Writes that the maximum value of Z is obtained at the corner point C. Hence, concludes that (12000, 8000) is the optimal solution and Jayesh should invest Rs 12000 in bond X and Rs 8000 in bond Y.	**0.5**

Q.No	What to look for	Marks
10	**i) Considers *x* as the number of deliveries made by Naman to office A, and *y* as the number of deliveries made by Naman to office B.** **Forms the following inequalities:** $x \geq 0, y \geq 0$ $x + y \leq 30$ $x \leq 20$ $y \leq 30$ $x + y \geq 10$ **The completed tree diagram may look as follows:**	1

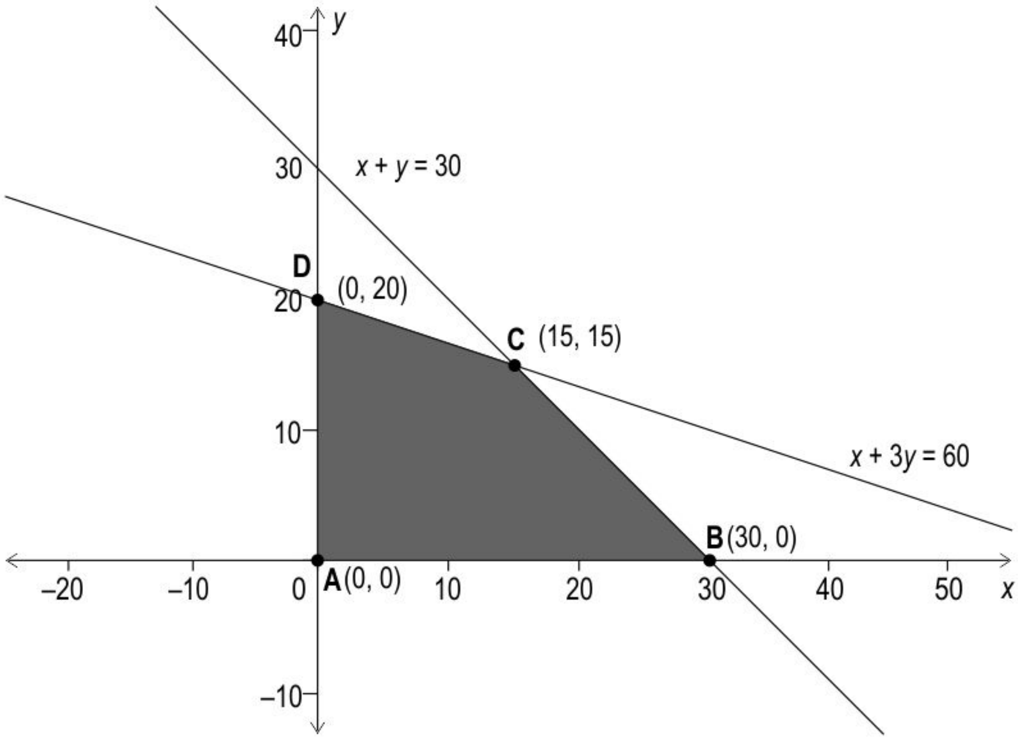


Q.No	What to look for	Marks
	ii) Finds the delivery cost function, Z as 10 x - 30 y + 6900. **The working may look as follows:** **Delivery cost = 50 x + 80 y + 60(30 - x - y) + 80(20 - x) + 150(30 - y) + 100(x + y - 10)** **= 50 x + 80 y + 1800 - 60 x - 60 y + 1600 - 80 x + 4500 - 150 y - 1000 + 100 x + 100 y** **= 10 x - 30 y + 6900**	1
	iii) Plots lines x + y = 30, x = 20, y = 30 and x + y = 10 on a graph and finds the area enclosed by polygon PQRST as the feasible region. **The graph may look as follows:** 	1

Q.No	What to look for	Marks
	Finds the value of the cost function at each of the corner points as follows: Point / x-coordinate / y-coordinate / Cost (Z): P / 0 / 10 / Rs 6600 Q / 0 / 30 / Rs 6000 R / 20 / 10 / Rs 6800 S / 20 / 0 / Rs 7100 T / 10 / 0 / Rs 7000	**1**
	Finds that the cost is minimum when x = 0 and y = 30. **Writes that Naman should deliver 30 tiffins to office B, and Sujay should deliver 20 tiffins each to offices A and C.**	**1**
11	**Assumes the weight of premium nuts for each store as x kg and the weight of economy nuts for store A as y kg. And, writes the constraints of the given problem as:** $x + y \leq 30$ $x + 3y \leq 60$ $x \geq 0$ $y \geq 0$	**1.5**
	Frames the objective function of the given problem as: **Maximize** $Z = (80 \times 2x) + (50 \times 4y) = 160x + 200y$	**0.5**

Corner point table:

Point	x -coordinate	y -coordinate	Cost (Z)
P	0	10	Rs 6600
Q	0	30	Rs 6000
R	20	10	Rs 6800
S	20	0	Rs 7100
T	10	0	Rs 7000

Q.No	What to look for	Marks
	Draws the graph of the system of inequalities and shades the feasible region. The graph may look as follows: 	**1.5**
	Evaluates the objective function at the corner points to find the optimal solution as follows: Corner points / $Z = 160x + 200y$: A(0, 0) → 0; B(30, 0) → 4800; C(15, 15) → 5400; D(0, 20) → 4000	**1**
	Writes that the maximum value of Z is obtained at the corner point C(15, 15) and concludes that the maximum profit of Rs 5400 is obtained when 15 kg of premium nuts are sold and 15 kg of economy nuts are sold.	**0.5**

Corner points	$Z = 160x + 200y$
A(0, 0)	0
B(30, 0)	4800
C(15, 15)	5400
D(0, 20)	4000

Chapter - 13
Probability

Free Response Questions

Q: 1 **Mumbai has a public transport system consisting of local trains which connect numerous stations across Mumbai. Among them are Andheri and Dadar.** **[1]**

Vani takes the local train scheduled at 8:55 AM from Andheri station to Dadar station every morning.

♦ **The probability that the train is late is $\frac{3}{4}$.**
♦ **The probability that Vani gets a seat in the train is $\frac{1}{15}$.**

What is the probability that the train is on-time, and Vani gets a seat in it? Show your work.

Q: 2 **A and B are events in the sample space S such that P(A) > 0 and P(B) > 0.** **[1]**

Check whether the following statement is true or false. If true, justify your conclusion; if false, state the right expression.

"If A and B are mutually exclusive, the probability of at least one of them occurring is: P(A).P(B)"

Q: 3 **A company conducts a mandatory health check up for all newly hired employees, to check for infections that could affect other office-going employees. A blood infection affects roughly 5% of the population. The probability of a false positive on the test for this infection is 4%, while the probability of a false negative on the test is 3%.** **[3]**

If a person tests positive for the infection, what is the probability that they are actually infected? Show your work.

(Note: A false positive on a test refers to a case when a person is not infected, but tests positive for the infection. A false negative on a test refers to a case when a person is infected, but tests negative for the infection.)

Q: 4 **A survey says that approximately 30% of all products ordered online across various e-commerce websites are returned. Two products that are ordered online are selected at random.** **[5]**

If X represents the number of products that are returned,

i) find the probability distribution of X.
ii) find the mean or expectation of X.

Show your work.

Q: 5 **Akshay is solving an exam paper with 50 multiple choice questions, each for 1 mark. In the paper, each multiple choice question has 4 options and exactly one correct answer.** **[5]**

Akshay solves only 30 questions, and randomly guesses the answers for the rest.

If exactly 40% of the questions Akshay solved are solved correctly, then what is the probability that Akshay scores at least 30 marks on the exam? Show your work.

Q.No	What to look for	Marks
1	**Finds the probability that the train is on-time as $1 - \frac{3}{4} = \frac{1}{4}$.**	**0.5**
	Finds the probability that the train is on-time, and Vani gets a seat in it as $\frac{1}{4} \times \frac{1}{15} = \frac{1}{60}$.	**0.5**
2	**Writes false.**	**0.5**
	Writes the correct expression. For example, since A and B are mutually exclusive, A ∩ B = Φ, which means that P(A ∩ B) = 0. Hence, the probability that at least one of them occurs is P(A ∪ B) = P(A) + P(B). **(Award full marks if just P(A ∪ B) = P(A) + P(B) is written without any explanation.)**	**0.5**
3	**Considers:** **X = Event that employee is affected by the infection.** **Y = Event that employee tests positive for the infection.** **Writes that:** **P(Y|X') = 0.04** **P(Y'|X) = 0.03** **P(X) = 0.05**	**0.5**
	Writes that: **P(Y|X) = 1 - P(Y'|X) = 0.97**	**0.5**
	Uses the law of total probability to find P(Y) as follows: **P(Y) = P(Y|X)P(X) + P(Y|X')P(X')** **=> P(Y) = (0.97)(0.05) + (0.04)(0.95)** **=> P(Y) = 0.0485 + 0.038 = 0.0865**	**1**

Q.No	What to look for	Marks
	Uses Bayes' Theorem to find P(X|Y) as follows: $P(X\|Y) = \frac{P(Y\|X)P(X)}{P(Y)}$ $\Rightarrow P(X\|Y) = \frac{0.97\times0.05}{0.0865} = \frac{0.0485}{0.0865}$ $\Rightarrow P(X\|Y) = \frac{97}{173}$	**1**
4	**i) Writes that the possible values of X are 0, 1 and 2.**	**0.5**
	Takes R to be the event that the product is returned and N to be the event that the product is not returned. **Uses the given information and finds the probability that the product is returned as P(R) = 0.3 and the probability that the product is not returned as P(N) = 1 - 0.3 = 0.7.**	**1**
	Finds the probability of each possible values of the random variable X as: **P(X = 0) = P(NN) = (0.7)(0.7) = 0.49** **P(X = 1) = P(RN, NR) = (0.3)(0.7) + (0.7)(0.3) = 0.21 + 0.21 = 0.42** **P(X = 2) = P(RR) = (0.3)(0.3) = 0.09**	**2**
	Finds the probability distribution of X as: X or x_i: 0, 1, 2; P or p_i: 0.49, 0.42, 0.09	**0.5**
	ii) Finds the mean or expectation of X as: **E(X) = $\sum x_i p_i$ = (0 × 0.49) + (1 × 0.42) + (2 × 0.09) = 0.6**	**1**

X or x_i	0	1	2
P or p_i	0.49	0.42	0.09

Q.No	What to look for	Marks
5	**Finds the number of questions Akshay solved correctly as 40% of 30 = 12.**	**1**
	Finds the number of questions Akshay randomly guessed the answer for as 20. **∴ The number of trials, $n = 20$** **Assumes p as the probability that the answer is correct, and q as the probability that the answer is incorrect.** **Finds $p = \frac{1}{4}$ and $q = \frac{3}{4}$.**	**1.5**
	Finds that the required probability, P(at least 30 marks) = P(at least 18 correct guesses) = P(exactly 18 correct guesses) + P(exactly 19 correct guesses) + P(exactly 20 correct guesses).	**0.5**
	Evaluates this using the Binomial Distribution as follows: P(exactly 18 correct guesses) = $^{20}C_{18}\left(\frac{1}{4}\right)^{18}\left(\frac{3}{4}\right)^{2}$ $= \frac{20!}{18!\times 2!} \times \frac{9}{4^{20}} = \frac{1710}{4^{20}}$ P(exactly 19 correct guesses) = $^{20}C_{19}\left(\frac{1}{4}\right)^{19}\left(\frac{3}{4}\right)^{1}$ $= \frac{20!}{19!\times 1!} \times \frac{3}{4^{20}} = \frac{60}{4^{20}}$ P(exactly 20 correct guesses) = $^{20}C_{20}\left(\frac{1}{4}\right)^{20}\left(\frac{3}{4}\right)^{0} = \frac{1}{4^{20}}$	**1.5**
	Finds the required probability as: $\frac{1710}{4^{20}} + \frac{60}{4^{20}} + \frac{1}{4^{20}} = \frac{1771}{4^{20}}$	**0.5**

14. Annexure

Correct Answer Explanation

Chapter Name	Q.No	Correct Answer	Correct Answer Explanation
Determinants	1	1	The "assertion" is a special case of "reasoning" where every row of V is multiplied by the scalar 'k'. Thus both assertion and the reasoning are true and latter is the correct explanation for former. Hence, option A is the correct answer.
Continuity and Differentiability	2	4	The product of the two functions is given by f.g = |x|. |x| is continuous everywhere in R. Hence, option D is the correct answer.
Application of Integrals	1	3	The shaded region is the difference of the areas covered by g(y) and f(y) with the y-axis. The limit of integration is from "c" to "d" since the integration is with respect to y-axis. Hence, option C is the correct answer.
Three Dimensional Geometry	2	3	In P1 and P2, (-2/2) = (-3/3) = (-4/4) = -1. In P1 and P3, (-2/-4) = (-3/-6) ≠ (-4/8). In P1 and P4, (-2/4) = (-3/6) = (-4/8). So, P1, P2 and P4 are parallel to each other and P3 does not share parallism with the rest. Hence, option B is the correct answer.

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