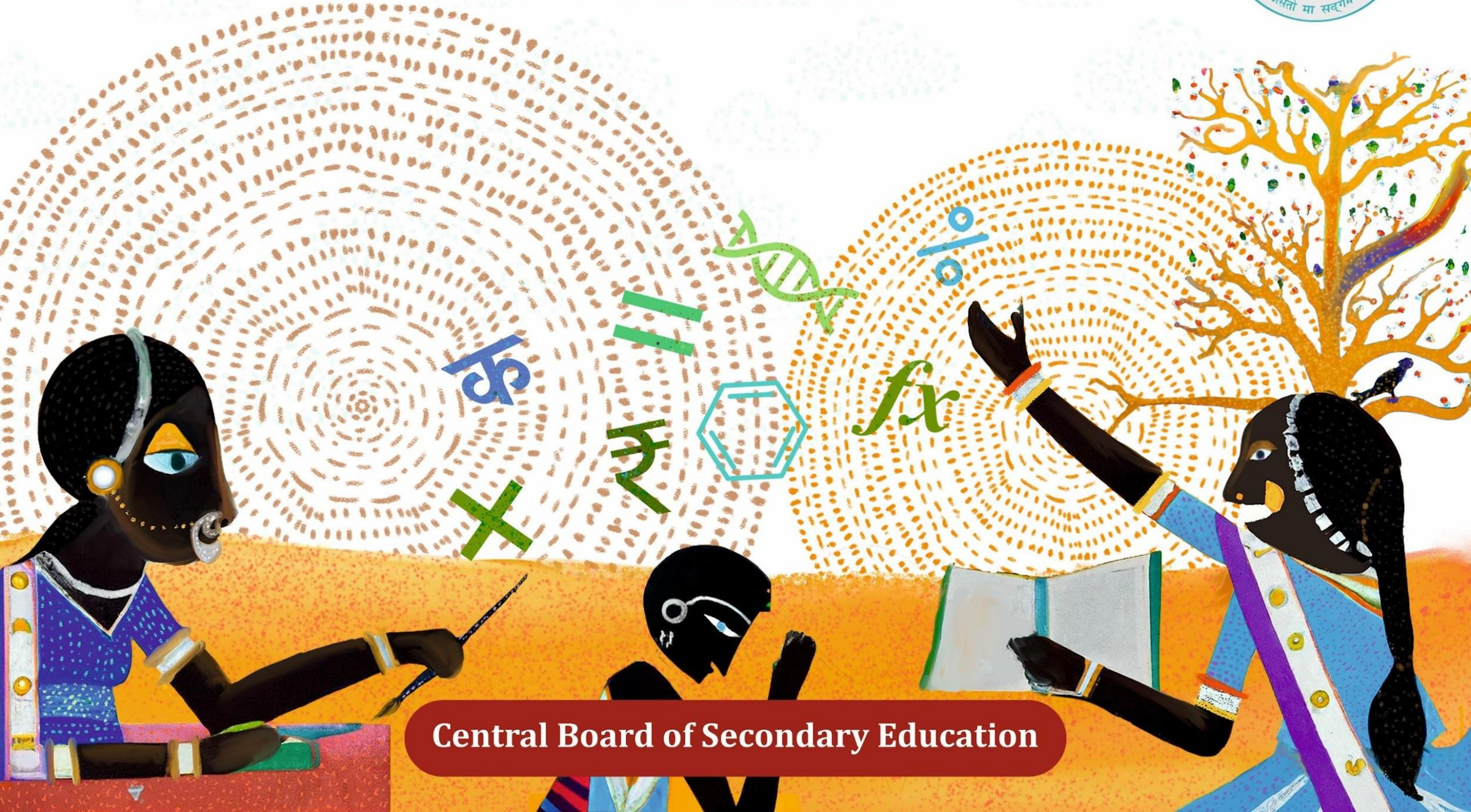
Learning Framework Classes 11-12
Mathematics



Central Board of Secondary Education

Ei

G20 | Azadi Ka Amrit Mahotsav

केन्द्रीय माध्यमिक शिक्षा बोर्ड
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Learning Framework Classes 11-12 Mathematics

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Co-created by
CBSE Centre for Excellence in Assessment
and
Educational Initiatives

FOREWORD

The vision of the National Education Policy (NEP) 2020 released by the Government of India, directs that children not only learn but more importantly learn how to learn. Education must move towards less content, and more towards learning about how to think critically and solve problems, how to be creative and multidisciplinary, and how to innovate, adapt, and absorb new material in novel and changing fields. Pedagogy must evolve to make education more experiential, holistic, integrated, inquiry-driven, discovery-oriented, learner-centred, discussion-based, flexible, and, of course, enjoyable. The policy has a clear mandate for competency-based education (CBE) to enhance the acquisition of critical 21st-century skills by the learners. The first determinant for implementing CBE is a curriculum which is aligned with defined learning outcomes and that clearly states the indicators to be achieved.

The Central Board of Secondary Education (CBSE) has collaborated with Educational Initiatives (Ei), to develop the Learning Framework for twelve subjects of Grades 11 and 12, i.e. English, Hindi, Mathematics, Physics, Chemistry, Biology, History, Geography, Economics, Accountancy, Business Studies and Computer Science. The Learning Frameworks comprise explicitly stated knowledge, skills and dispositions that an education system should try to achieve. These frameworks will help develop a common shared understanding among teachers, students and other stakeholders and would serve as a common benchmark for teaching, learning and assessment across the country.

These frameworks present indicators that are aligned with the CBSE curriculum and the NCERT learning outcomes. They further outline samples of pedagogical processes and assessment strategies to encourage curiosity, objectivity, and creativity with a view to nurturing scientific temper. This framework would be a key resource for teachers as they execute the curriculum. They have been developed to ensure that teachers align the learning to meet the set quality standards and also use it to track the learning levels of students. The effort has been to synchronize focus on quality education with uniformity in quality of standards across CBSE schools.

We hope, these frameworks not only become a reference point for competency-based education across the country but also facilitate planning and design of teaching-learning processes and assessment strategies by teachers and other stakeholders.

Please note that the learning frameworks have been drafted based on the 2022-23 curriculum. Certain chapters and topics that have been rationalized in the 2023-24 curriculum are retained in this document. The rationalized sections are referenced under Chapter#3 - Content Domains. Please note that the unit or content marked with * are partially rationalised whereas those with ** are the ones deleted in full.

Feedback regarding the framework is welcome. Any further feedback and suggestions will be incorporated in subsequent editions.

Team CBSE

PREFACE

The National Education Policy 2020 has outlined the importance of competency-focused education in classrooms, leading to curricular and pedagogical reforms in the school systems. The policy emphasizes on the development of higher-order skills such as analysis, critical thinking and problem-solving through classroom instructions and aligned assessments. These skills are important indicators which will further the dissemination of pedagogy and learning outcomes across schools and boards.

In order to propagate indicator-based learning through 'Learning Frameworks', the Central Board of Secondary Education (CBSE) has collaborated with Educational Initiatives (Ei). Learning frameworks are a comprehensive package which provides learning outcomes, indicators, assessment frameworks, samples of pedagogical processes, tools and techniques for formative assessment, blueprints, assessment items and rubrics. 12 such frameworks have been developed for English, Hindi, Mathematics, Physics, Chemistry, Biology, History, Geography, Economics, Accountancy, Business Studies and Computer Science in Grades 11 and 12.

The frameworks are adopted from the learning outcomes outlined in the NCERT which are mapped to key concepts of the content. These content domain-specific learning outcomes are broken down into indicators which define the specific skills a learner needs to attain. A clear understanding of these LOs will be immensely helpful for teachers and students to learn better. This document will help teachers to focus on skills of the subject in addition to concepts.

As per the National Focus group Position Paper on Teaching of Mathematics, "Principally, the higher secondary stage is the launching pad from which the student is guided towards career choices, whether they imply university education or otherwise. By this time, the student's interests and aptitudes have been largely determined, and mathematics education in these two years can help in sharpening their abilities. (Sec 6.4)" This document presents outcomes and indicators that focus on these Mathematics specific skills that students need to attain through different concepts addressed in the syllabus. In addition to this, sample pedagogical processes, formative assessment strategies and summative assessment items are also provided to enable teachers to make the most use of this document.

Contents

1. NATURE OF THE SUBJECT

"Mathematics is the queen of science, and arithmetic the queen of mathematics." - Carl Friedrich Gauss

"Mathematics is the most beautiful and most powerful creation of the human spirit." - Stephan Banach

The power of mathematics for describing and analysing the world around us is such that it has become a highly effective tool for solving problems. It is also recognized that students can appreciate the intrinsic fascination of mathematics and explore the world through its unique perceptions. Mathematics has been a part of everyone's life, be it estimates we make in our routine activities or precise calculations for various transactions and fairness in sharing or in describing objects around us. The relevance of mathematics is more than its utilitarian value. It helps us to think and reason about the world around us and make informed decisions, be it at the individual level to cope with life in various spheres of activity or at the societal level to contribute to technological and socio-economic development. Mathematics today is a diversified discipline, much more than arithmetic and geometry, which deals with data, measurement, and observations from science, with reference, deduction and proof, and with mathematical models, natural phenomena, human behaviour, and social systems. Mathematics makes our lives orderly and helps reduce chaos. Mathematics offers rationality to our thoughts. Some of the qualities nurtured by mathematics are the power of reasoning, creativity, abstract or spatial thinking, critical thinking, and problem-solving ability.

Mathematics is about a certain way of thinking and reasoning. It is a language of sciences helpful to express nature's phenomena. Mathematics reveals hidden patterns that help us to understand the world around us. Mathematics is a science of patterns and order.

The focus of school mathematics is developing the problem-solving and reasoning skills needed to have an organised and progressing society. This includes reflecting on and studying problems and topics which may be perceived as more of an intellectual exercise and not immediately useful at this stage. However, these have unforeseen, far-reaching benefits.

The selection of mathematics study material must be made in a manner such that mathematics will not be a burden to the learner but an engaging and joyful activity. The mathematics curriculum as well as teaching-learning processes should provide students with the opportunity to see themselves as "mathematicians", where they enjoy and are enthusiastic when exploring and learning about mathematics. It is intended that students become competent users of the language of mathematics and can begin to use it as a way of thinking, as opposed to seeing it as a series of facts and equations to be memorized. It is important that learners acquire mathematical understanding by constructing their own meaning through ever-increasing levels of abstraction, starting with exploring their own personal experiences, understandings and knowledge.

Secondary and in particular higher secondary school mathematics prepares students for possible study in three broad areas:

Mathematics for life: Knowing mathematics can be personally satisfying and empowering. It helps promote a rational way of thinking and problem-solving abilities. The underpinnings of everyday life are increasingly mathematical and technological. For instance, making purchasing decisions, choosing appropriate and cost-effective insurance or health plans, choosing appropriate investment schemes, managing finances, and voting knowledgeably, all call for quantitative sophistication.

Mathematics for the workplace: Just as the level of mathematics needed for intelligent citizenship has increased dramatically, so too has the level of mathematical thinking and problem-solving needed in the workplace, in professional areas ranging from health care to graphic design. Automation is replacing mechanical jobs and data analytics ensures the right information is made available to users at the right time. Coding as well as data analytics require mathematical thinking.

Mathematics for the scientific and technical community: Although all careers require a foundation of mathematical knowledge, some are mathematics intensive. Students may pursue an educational path that will prepare them for lifelong work as mathematicians, statisticians, engineers, and scientists.

School systems need to prepare students to think like a mathematician, a scientist, or a philosopher. The focus on conceptual understanding provides the underpinnings for a wide range of careers as well as for further study. Most advanced high school mathematics has rigorous, interesting applications in the professional world. For example, graphic designers routinely use geometry. Carpenters, surveyors, navigators, and architects apply the principles of trigonometry in their work. Algebra pervades computing and business modelling, from everyday spreadsheets to sophisticated scheduling systems and financial planning strategies. Statistics is a mainstay for economists, marketing experts, pharmaceutical companies, and political advisers.

By learning to think and communicate effectively in mathematics, students will be better prepared for changes in the workplace that increasingly demand teamwork, collaboration, and communication (21st century skills). Emphasis on fundamental concepts, thinking and reasoning, modelling, and communicating will lay foundation for more advanced Mathematics.

Assessment in mathematics has to encompass both the nature of mathematics and the difficulties which the learner faces because of it. When an assessment is cognizant of the limitations of the learner and the constraints and affordances of the nature of the subject, it enables the learner to harness the power of mathematics and the teacher to enable the learner to do this.

The indicators listed in the document should lead to development of mathematical thinking and thinking like a mathematician.

2. STAGE SPECIFIC CURRICULAR EXPECTATIONS

'Curricular expectations' define what a child should know and be able to do as well as the dispositions that should be acquired over a period of time. The learning outcomes derived from the curricular expectations and the syllabus may help all the stakeholders understand the goals to be achieved. The learning outcomes are generally treated as assessment standards or benchmarks for assessment.

The strands of mathematical proficiency specified in the National Research Council's report 'Adding it Up' are as follows:

- adaptive reasoning
- strategic competence
- conceptual understanding (comprehension of mathematical concepts, operations, and relations)
- procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately)
- productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy)

Learning Outcomes at Higher Secondary stage developed by National Council for Educational Research and Training (NCERT) mentions the following curricular expectations for mathematics. At this stage, learners are expected to develop ability and attitude for —

CE1. mathematization (ability to think logically, formulate and handle abstractions) rather than knowledge of procedures (formal and mechanical).

CE2. mathematical vocabulary (communicate mathematically using appropriate mathematical terms and symbols).

CE3. consolidation and generalisation of the concepts learnt so far.

CE4. understanding and proving mathematical statements.

CE5. addressing problems that come from other domains such as, science and social sciences.

CE6. integration of concepts and skills that the children have learnt into a problem-solving ability.

CE7. analysing and constructing the processes involved in mathematical reasoning.

CE8. establishing linkages between mathematics and daily life experiences and across the curriculum.

3. CONTENT DOMAINS

The content for Mathematics for Grades 11-12 in the CBSE curriculum has been organized for the content units.

Content units for the two grades, along with the chapters from the NCERT textbooks are mentioned in the tables below.

Please note that the units or content marked with * are partially rationalised whereas those with ** are the ones deleted in full, as per the academic year 2023-24 syllabus.

Table 3.1. Grade 11 Content units and textbook chapters

Content units	NCERT textbook chapters
Sets and Functions*	Sets*
	Relations and Functions
	Trigonometric Functions*
Algebra	Complex Numbers and Quadratic Equations*
	Linear Inequalities*
	Permutations and Combinations
	Binomial Theorem*
	Sequences and Series*
Coordinate Geometry	Straight Lines*
	Conic Sections*
	Introduction to Three-Dimensional Geometry*
Calculus	Limits and Derivatives
Statistics and Probability*	Statistics*
	Probability*

Table 3.2. Grade 12 Content units and textbook chapters

Content units	NCERT textbook chapters
Relations and Functions*	Relations and Functions*
	Inverse Trigonometric Functions*
Algebra	Matrices*
	Determinants*
Calculus	Continuity and Differentiability*
	Applications of Derivatives*
	Integrals*
	Application of Integrals*
	Differential Equations*
Vectors and Three-dimensional Geometry	Vector Algebra*
	Three-Dimensional Geometry*
Linear Programming	Linear Programming*
Probability	Probability*

4. SUBJECT SPECIFIC COGNITIVE DOMAINS

"As the Board is progressively allowing more space to 'learning outcome based' assessment in place of textbook driven assessment, question papers of Board examinations will have more questions based on real-life situations requiring students to apply, analyse, evaluate, and synthesize information as per the stipulated outcomes. The core-competencies to be assessed in all questions, however, will be from the prescribed syllabus and textbooks recommended therein. This will eliminate predictability and rote learning to a large extent."

[CBSE Curriculum for classes 11-12]

CATEGORIES OF COGNITIVE DOMAINS

Any concept or skill being assessed has layers of difficulty involved varying in the complexity of reasoning/thinking. It is expected of a learner to demonstrate a deeper understanding of core concepts and hence assessments assessing these concepts at various 'depths' as per the complexity of reasoning involved.

Revised Bloom's taxonomy (Anderson and Krathwohl, 2001) of cognitive process dimension has six categories, each associated with a set of specific cognitive processes. CBSE curriculum intends to have a balance of these categories of intellectual tasks in the teaching-learning and assessment of learning of a subject. These six categories as described in the revised Bloom's taxonomy, with their specific cognitive processes, are mentioned below.

COGNITIVE DOMAIN – REMEMBER

'Remember' involves retrieving relevant knowledge from the long-term memory. Recognising and recalling are the specific cognitive skills associated with this cognitive domain.

Asking students to provide a definition of a concept, e.g. What is the expression for the nth term of an arithmetic progression?

COGNITIVE DOMAIN – UNDERSTAND

'Understand' involves 'constructing meaning from instructional messages, including oral, written and graphic communication'. Interpreting, exemplifying, classifying, summarizing, inferring, comparing, explaining are the specific cognitive skills associated with this cognitive domain. Asking students to explain a phenomenon in terms of physical concepts/principles, e.g. How would you calculate the sum of the first 100 odd natural numbers using an A.P.?

COGNITIVE DOMAIN – APPLY

'Apply' involves carrying out or using a procedure in a given situation. Executing and implementing are the specific cognitive skills associated with this cognitive domain. Assessment tasks wherein students have to use the knowledge and/or procedures to solve a problem or to arrive at a decision in a given real-life situation cover this cognitive domain, e.g. A man starts repaying a loan as the first installment of Rs. 100. If he increases the instalment by Rs 5 every month, what amount he will pay in the 30th installment?

COGNITIVE DOMAIN – ANALYSE

'Analyse' involves breaking material into constituent parts and determining how parts relate to one another and to an overall structure and purpose. Differentiating, organising, and attributing are the specific cognitive skills associated with this cognitive domain. Asking students to compare and explain the relationship between two physical quantities from the same content domain, e.g. How many 3-digit numbers are multiples of 11?

COGNITIVE DOMAIN – EVALUATE

'Evaluate' involves making judgments based on criteria and standards. Checking and critiquing are the specific cognitive skills associated with this cognitive domain. Assessment tasks that require a deeper level of understanding wherein students are required to provide justification for their choice, e.g. Which of the two investment options would result in higher returns after 10 years and why? Option P: Rs 1000 deposited in a bank which pays an annual interest rate of 5% compounded annually for 10 years; Option Q: Rs 1000 invested in a post-saving scheme which pays an annual interest rate of 10% compounded annually for 5 years.

COGNITIVE DOMAIN – CREATE

'Create' involves putting elements together to form a coherent or functional whole; or reorganising elements into a new pattern or structure. Generating, planning, and producing are the specific cognitive skills associated with this cognitive domain. Tasks that require students to produce new artefacts based on what they have learnt, e.g. Conjecture: "If two A.P.s have the same common difference that is if a, a + d, a + 2 d and b, b + f, b + 2f, b + 3f… are two A.P., then the sequence (a + b), (a + d + b + f), (a + 2d + b + 2f)… is also an A.P. if d = f." Prove or disprove this.

(Note: In general, this way of classifying cognitive levels may not necessarily be the best and most useful for Mathematics. Various attempts have been made by researchers in the past to classify cognitive domains in Maths. There can be difficulty and hence some element of subjectivity in identifying the cognitive domain being assessed in a Mathematics question. A question may not necessarily assess only 1 cognitive level. At times one may not know what skills or thinking have been put to use to answer a question. There can be multiple ways to solve a problem (answer a question). A person may have rote learned and recalled from memory to answer a HOTS question if it is a familiar one. However, for practical purposes Bloom's taxonomy can be useful due to the lack of a better alternative.)

The following are some of the various levels of mathematical thinking demonstrated in understanding a concept/skill or solving a mathematical problem.

Recall formula, result, theorem, or a property (cognitive domain could be "recall" as per the revised Bloom's taxonomy if stated without any example and cognitive domain could be "understand" if stated by a learner along with his own example)

Apply a formula or a known procedure to compute using relevant information (cognitive domain: "understand" as per the revised Bloom's taxonomy)

Apply the concept in unknown situation/context to find an unknown (cognitive domain: "apply" as per the revised Bloom's taxonomy)

Apply the concept in an unknown situation along with other concepts to find an unknown (cognitive domain could be: "analyze" or "apply" as per the revised Bloom's taxonomy)

Analyzing different solution strategies and identifying an efficient one (cognitive domain could be "evaluate" or "analyze" as per the revised Bloom's taxonomy)

Form a conjecture/hypothesis or new relationship between quantities (cognitive domain could be "create" or "analyze" as per the revised Bloom's taxonomy)

Prove a conjecture/hypothesis (cognitive domain could be "create" as per the revised Bloom's taxonomy if a learner proves on his own even if it is already proved earlier by mathematicians, could be "recall or understand" if it is about reproducing the proof)

ASSESSMENT TASKS FOR DIFFERENT COGNITIVE DOMAINS

Some more examples of kinds of assessment tasks that can be associated with the different cognitive domains are given below. The following list should be taken as an indicative not an exhaustive one.

Table 4.1. Cognitive domains and processes with sample assessment tasks

Cognitive domain	Assessment tasks
Remember recognising Recalling defining	Identify, state, or define facts, relationships, formulae, or concepts. Identify or describe properties of mathematical ideas or concepts. Recognize and correctly use mathematical vocabulary, symbols, abbreviations, formatting, and scales. Identify the appropriate use for mathematical operations and procedures.
Understand interpreting exemplifying classifying summarizing inferring comparing explaining	Interpret information in the form of texts, graphs, or images in terms of physical concepts and their relationships. Provide examples related to specific mathematical concepts. Classify or compare numbers, terms or operations using mathematical concepts or principles. Provide a summary of development of a mathematical concept, model, or a principle. Derive a mathematical theorem, relationship or result representing relationship between variables, terms, and expressions. Infer results pertaining to mathematical concepts from given information – in quantitative or qualitative format.
Apply solving executing implementing	Use knowledge of mathematical concepts and their relationships to solve problems set in a variety of contexts. Use a known procedure to evaluate a variable or to find the relationship between different variables. Relate knowledge of an underlying concept to an observed or inferred property or use of new terms and concepts.

Analyse differentiating correlating concluding	Describe relationships between mathematical concepts from within the same or across different content domains. Differentiate between mathematical concepts, principles, or results from within the same content domain. Identify or formulate questions that can be answered by a given mathematical observation or result. Determining results using mathematical reasoning and process of logical deduction
Evaluate checking critiquing	Evaluate alternative explanations for a particular mathematical observation. Compare different solutions or approaches to a given problem. Evaluate conclusions drawn from the data using critical thinking.
Create designing planning producing	Devise solutions which involve consideration of numerous different or related concepts and principles. Plan and execute a logical proof with clear steps to accept or reject a result. Make a model to illustrate a mathematical concept, result, or theorem.

SPECIFIC TASKS FROM DIFFERENT COGNITIVE DOMAINS

Some specific examples of tasks from different cognitive domains are described below for two content chapters from classes 11 and 12 NCERT Mathematics textbooks. A chapter may not always cover all six cognitive domains. The following list of tasks should be taken as an indicative list not a comprehensive one.

Chapter 9. Sequences and Series – Class 11

Table 4.2. Cognitive domains with suggested sample tasks for class 11 for the chapter, sequences, and series

Cognitive domain	Sample tasks
Remember	Define: Arithmetic progression (or Geometric progression) Give the formula to find the sum of first n odd integers. Fill in the blank: Let a1, a2, a3, ... be the terms of an arithmetic progression. Then the sum expressed as a1 + a2 + a3 + ... is given by ______.
Understand	Give any two examples of A.P. Justify why each of them is an A.P. If the sum of the first n terms of an A.P. is 3n2 + 5n and its mth term is 164, find the value of m. (The relation am = Sm+1 – Sm needs to be applied here.) Consider a sequence, $\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{3}{4}, 2\frac{1}{4}$ *and* $2\frac{3}{4}$. Classify the given sequence as A.P. or G.P. $\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{3}{4}, 2\frac{1}{4}, 2\frac{3}{4}, \ldots$ *and* $0, \frac{1}{2}, 1,\ 1\frac{1}{2}, 2, 2\frac{1}{2}, 3$...are two A.P. The nth term of which of the two A.P. will be greater?
Apply	A farmer buys a used tractor for Rs 12000. He pays Rs 6000 cash and agrees to pay the balance in annual instalments of Rs 500 plus 12% interest on the unpaid amount. How much will the tractor cost him? If the sum of first n terms of an A.P. is 3n2 + 5n and its mth term is 164, find the value of m. $\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{3}{4}, 2\frac{1}{4}, 2\frac{3}{4}, \ldots$ *and* $0, \frac{1}{2}, 1,\ 1\frac{1}{2}, 2, 2\frac{1}{2}, 3$...are two A.P. Which of the two A.P. will have greater sum of its first n terms?

Analyse	A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively. Find such a quadratic equation. Show that $\frac{1\times 2^2 + 2\times 3^2 + \cdots + n\times(n+1)^2}{1^2\times 2 + 2^2\times 3 + \cdots + n^2\times(n+1)} = \frac{3n+5}{3n+1}$ The ratio of the A.M. and G.M. of two positive numbers a and b, is m : n. Show that $a : b = \left(m + \sqrt{m^2 + n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$.
Evaluate	Which of the two investment options would result in higher returns after 10 years and why? Option P: Rs 1000 deposited in a bank which pays an annual interest rate of 5% compounded annually for 10 years; Option Q: Rs 1000 invested in a post office saving scheme which pays an annual interest rate of 10% compounded annually for 5 years Elgar listed the returns on money he invested for each of the first two years in an investment scheme. Which of the two, arithmetic means of returns or the geometric means of the returns will help decide if the investment scheme was good or bad? Why?
Create	Is a sequence whose each term is derived by adding corresponding terms of two A.P. also an A.P.? e.g. if a, a + d, a + 2 d … and b, b + f, b + 2f, b + 3f… are two A.P., is a sequence (a + b), (a + d + b + f), (a + 2d + b + 2f)… an A.P. In what special case/condition such a sequence can be A.P.? Derive the general formula to find the sum of the first n even numbers. How could a geometric mean of n numbers a1, a2, … an be defined? (Note it should work for the case n = 2.) For two distinct whole numbers, will their geometric mean always be less than their arithmetic mean? What are the different ways to create new arithmetic progressions (A.P.) from any given A.P. or given two A.P.? Conjecture: A sequence whose each term is derived by adding corresponding terms of two A.P. is also an A.P., if both the A.P.s have the same common difference. e.g., if a, a + d, a + 2d … and b, b + f, b + 2f, b + 3f… are two A.P., the sequence (a + b), (a + d + b + f), (a + 2d + b + 2f) … is an A.P. if d = f. Prove or disprove this.

Table 4.3. Cognitive domains with suggested sample tasks for class 12, chapter "Continuity and Differentiability"

Cognitive domain	Sample tasks
Remember	State true or false: Every polynomial function is continuous over real numbers. State true or false: Every continuous function defined on an open interval I is also differentiable in the same interval I. Select all the functions below that are continuous from $(0, \frac{\pi}{2})$. (i) sin x (ii) cos x (iii) tan x (iv) cosec x
Understand	f(x) = sin x and g(x) = x2 are continuous functions on R. Is gof(x) = $sin^2\ x$ continuous in R? Why? Differentiate $x^2e^x\ w.r.t.x$ using the product rule. Find all the points of discontinuity of f defined by $f(x) = \|x + 1\|, x \in R$.
Apply	Which of the following intervals is the function f defined by $f(x) = \|x + 1\|, x \in R$ differentiable? Why? (i) $(-\frac{\pi}{2}, 0)$ (ii) $(-\frac{\pi}{2}, \frac{\pi}{2})$ (iii) $(-\infty, -\frac{\pi}{2})$ (iv) (-2, 1) Compare the continuity of the following functions, f(x) and g(x) over $x \in [-\pi, \pi]$. $f(x) = cos^2\ x\ and\ g(x) = cos\ cos\ x^2$ Find the derivative of the function given by $f(x) = (1-x)(x^2-1)\ and\ hence\ find\ f'(1)$.
Analyze	Find the derivative of the function given by $f(x) = (1-x)(x^2-1)\ and\ hence\ find\ f'(1)$. What can you say about the slope of f(x) at x = 1? Does there exist a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer. For the function, $f(x) = x^2 + 2x - 8, x \in [-4, 2]$, does there exist a tangent to the graph of f(x) that is horizontal (parallel to X-axis)? Justify your answer.

Evaluate	Three students were asked to differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$. Abdul differentiated using the product rule. Kabir expanding the product to obtain a single polynomial and then differentiated. Sam converted to equivalent logarithmic form and then differentiated (logarithmic differentiation). If they all differentiated correctly, will all of them get the same answer/result?
Create	Find a function f(x) such that it is first-order differentiable, but not second-order differentiable for all real numbers x. Construct at least 3 different real-valued functions that can be differentiated any number of times for any real number. Construct a real-valued function f(x) satisfying the following conditions: (a) It is defined on an interval in R that contains 2020 (b) It is continuous in the interval but not differentiable exactly at 2020.

5. LEARNING OUTCOMES

> "Competency based learning focuses on the student's demonstration of desired learning outcomes as central to the learning process. Learning outcomes are statements of abilities that are expected students will gain as a result of learning the activity. Learning outcomes are, thus, statements of what a learner is expected to know, understand and/or be able to demonstrate after completion of a process of learning. Therefore, the focus is on measuring learning through attainment of prescribed learning outcomes, rather than on measuring time."
>
> [Senior School Curriculum, CBSE]

Following learning outcomes for the senior secondary stage developed by the National Council for Educational Research and Training (NCERT) state important knowledge, skills and dispositions students need to attain at the end of an academic year in classes 11 and 12 in the context of learning Mathematics.

CLASS 11 LEARNING OUTCOMES FOR MATHEMATICS

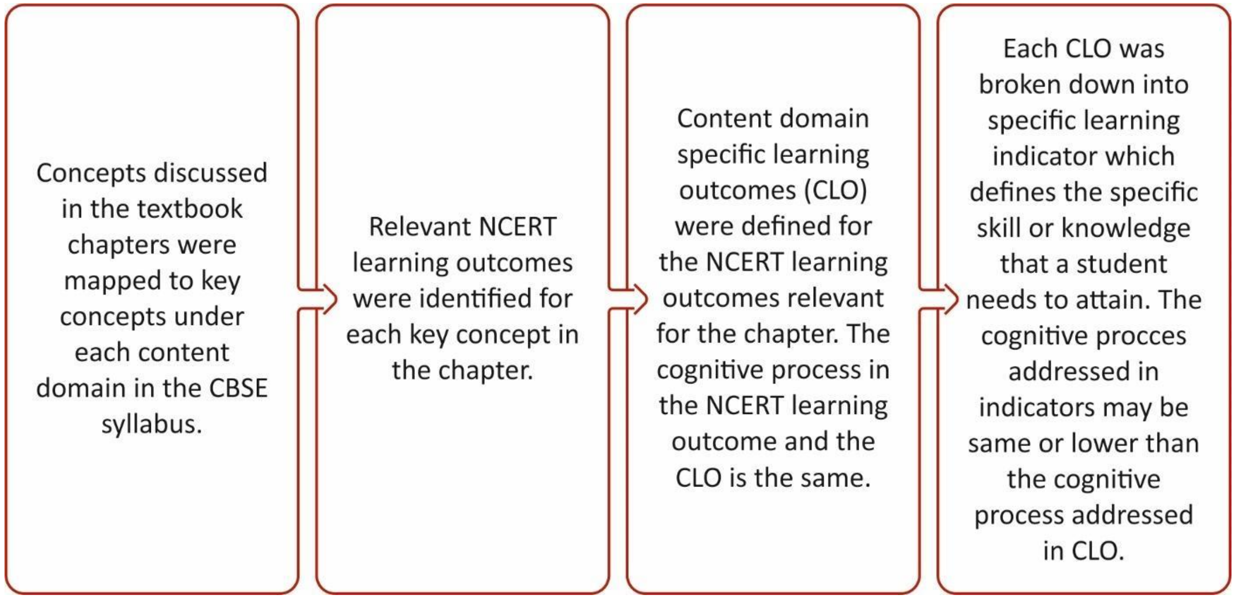
1. Develops the idea of set from the earlier learnt concepts in number system, geometry etc.
2. Identifies relations between different sets.
3. Relates earlier learnt concept of trigonometric ratios to functions and evolves the idea of trigonometric functions.
4. Extends the idea of real numbers to a larger system of complex numbers.
5. Demonstrates strategies for solving systems of linear inequalities.
6. Applies the ideas of permutations and combinations to daily life situations of arranging and grouping the objects.
7. Develops the idea of the Binomial Theorem for a positive integral index from the earlier learnt concepts of finding squares and cubes of binomials.
8. Extends the ideas related to arithmetic progressions learnt earlier to new types of sequences and their series.
9. Constructs different forms of a straight line using the earlier learnt concepts of coordinate geometry.
10. Analyses different curves like circles, ellipses, parabolas, and hyperbolas based on the ideas developed for straight lines using coordinates.
11. Develops strategies of locating a point in three dimensions based on the concepts of two-dimensional coordinate geometry.
12. Evolves the concepts of limit and derivative of a function by analyzing the behaviour of functions when the corresponding variable approaches a certain value.
13. Applies measures of dispersion to get a better interpretation of data of different daily life situations.
14. Builds up the axiomatic approach to probability through the terms, random experiment, sample space, events etc.

CLASS 12 LEARNING OUTCOMES FOR MATHEMATICS

1. Identifies different types of relations and functions.
2. Explores the values of different inverse trigonometric functions.
3. Evolves the idea of matrices as a way of representing and simplifying mathematical concepts.
4. Evaluates determinants of different square matrices using their properties.
5. Demonstrates ways to relate differentiability and continuity of a function with each other.
6. Develops the processes in Integral calculus based on the ideas of differential calculus learnt earlier.
7. Applies the concepts of Integral calculus to calculate the areas enclosed by curves.
8. Develops the concepts of differential equations using the ideas of differential.
9. Constructs the idea of vectors and their properties and relates them to earlier learnt concepts in different areas of mathematics such as geometry, coordinate geometry etc.
10. Evolves newer concepts in three-dimensional geometry from that learnt earlier, in the light of vector algebra, such as, direction cosines, equations of lines under different conditions etc.
11. Formulates and solves problems related to maximization/ minimization of quantities in daily life situations using systems of inequalities/inequations learnt earlier.
12. Calculates conditional probability of an event and uses it to evolve Bayes' theorem and multiplication rule of probability.

6. CONTENT DOMAIN SPECIFIC LEARNING OUTCOMES AND INDICATORS

The learning outcomes defined by NCERT are generic and broadly defined for the content defined in the curriculum. They articulate the discipline-specific skills that students need to attain through learning different concepts in the syllabus. A clear understanding of the scope of these learning outcomes for each concept dealt with in the NCERT textbook chapters will be very helpful for both teachers and students in planning teaching and learning better. The following process has been followed to list out the content domain-specific learning outcomes (CLOs) and indicators for all the content units and textbook chapters.



CLASS 11 CONTENT DOMAIN SPECIFIC LEARNING OUTCOMES AND INDICATORS

The table below lists all the content domain-specific learning outcomes and indicators for class 11 mathematics based on the NCERT class 11 textbook.

Table 6.1: Content domain-specific learning outcomes and indicators for class 11 mathematics

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Sets and Functions: 1. Sets	Using sets to represent information	Develops the idea of Set from the earlier learnt concepts in number system, geometry etc.	CLO1: Understands the structure & properties of sets	C1: Represents a set in a roster form and/or a set-builder form
				C2: Identifies if a given entity is an element of a set
			CLO2: Identifies different types of sets	C3: Identifies if a set is a null set (empty set), finite set or infinite set
				C4: Identifies if two sets are equal or finds equal sets among the collection of sets
			CLO3: Understands the composition of sets	C5: Identifies if a set is a subset of a given set and/or lists all the subsets of a set
				C6: Represents an interval of R in a set-builder form and vice-versa
				C7: Identifies or finds the universal set of given sets and/or finds the power set of a given set

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
				C8: Finds union of 2 or more sets
				C9: Demonstrates the properties of union of sets and applies them
Sets and Functions: 1. Sets			CLO4: Applies the algebraic operations on sets	C10: Finds the intersection of two or more sets
				C11: Demonstrates the properties of intersection of sets and applies them (includes distributivity of intersection over union of sets)
				C12: Identifies if two sets are disjoint sets
				C13: Finds the difference of two sets
				C14: Finds the complement of a subset of a universal set and/or applies properties/laws like De Morgan's law, double complementation etc.
				C15: Draws an appropriate Venn diagram and shows given operations on sets

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Sets and Functions: 2. Relations and Functions	Develop connections between sets	Identifies relations between different sets and applies the concept of a function.	CLO5: Understanding the concept of cartesian product and a relation	C16: Finds the Cartesian product of non-empty sets
				C17: Finds the number of elements in the Cartesian product of two finite sets
				C18: Finds the Cartesian product of the set of reals with itself (up to R × R × R)
				C19: Represents a relation from a set to another set in a set-builder, roaster form or an arrow diagram
				C20: Finds the domain and/or the range of a given relation
				C21: Finds the total number of relations possible from a set to another set
			CLO6: Differentiating relations and functions	C22: Identifies and explains if a given relation is a function
				C23: Finds the domain and/or the range of a given function

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Sets and Functions: 2. Relations and Functions			CLO7: Understanding and applying the concept of a function	C24: Finds the image of an element in a domain of a function or pre-image of an element in a codomain of a function
				C25: Finds domain and range of real functions like constant, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions
				C26: Interprets or draws the graph of a real function
				C27: Finds sum, difference, multiplication by scalar and/or another real function or quotients of given two real functions and applies their properties
Sets and Functions: 3. Trigonometric functions	Using trigonometric functions	Relates earlier learnt concept of trigonometric ratios to functions and evolves the idea of trigonometric functions.	CLO8: Understanding of functions built on trigonometric ratios	C28: Demonstrates the relationship between different units to measure an angle, and positive/negative angles
				C29: Converts radian measure of an angle into degree measure and vice versa; uses appropriate notations

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Sets and Functions: 3. Trigonometric functions				C30: Applies the relationship between angle subtended by an arc at the centre of a circle, the length of the arc and the radius of the circle to find an unknown quantity in the relationship
				C31: Finds domain and range of trigonometric functions and interprets their graphs
				C32: Finds value(s) of trigonometric function(s) given the value of one of the trigonometric functions by applying the relationship between them and their signs in different quadrants
				C33: Finds the value of a trigonometric function applying its periodicity and its signs in different quadrants
			CLO9: Applies trigonometric functions to prove identities and solve problems	C34: Verifies the trigonometric identities using properties of trigonometric functions

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
				C35: Applies trigonometric identities to verify a relationship involving trigonometric functions
				C36: Finds the value of a trigonometric function applying the trigonometric identities
Algebra: 5. Complex numbers and quadratic equations	Apply the understanding of complex numbers to solve quadratic equations	Extends the idea of real numbers to a larger system of complex numbers.	CLO10: Understands the algebra of complex numbers	C37: Represents a complex number in the standard form demonstrating the meaning of its real part and imaginary part; represents square roots of a negative real number as complex numbers
				C38: Applies the principle of equality of two complex numbers to find unknowns
				C39: Finds the sum and difference of two complex numbers and applies their properties
				C40: Finds the product of two complex numbers applying the definition of multiplication of two complex numbers

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Algebra: 5. Complex numbers and quadratic equations				C41: Applies distributive law of multiplication of two complex numbers to find products or verify identities
				C42: Applies the properties of multiplication of two complex numbers
				C43: Finds the quotient of two complex numbers applying the definition of division of two complex numbers
				C44: Finds products involving powers of i and/or expresses in the form of a + ib
				C45: Finds the square roots of a negative real number
			CLO11: Applies the algebra of complex numbers to prove identities and/or solve quadratic equations	C46: Finds the product and the quotient of complex numbers applying identities similar to that for real numbers and expresses in the form of a + ib.

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Algebra: 5. Complex numbers and quadratic equations				C47: Finds the modulus and argument/principal argument of a complex number
				C48: Finds the conjugate of a complex number expressing it in a + ib form
				C49: Applies the relation $z\bar{z}$ = |z|2 to find the multiplicative inverse of a non-zero complex number
				C50: Demonstrates the properties of modulus and conjugate of a complex number for any two complex numbers and applies them
				C51: Represents a complex number in the polar form
				C52: Finds the roots (imaginary roots) of a quadratic equation which are complex numbers
Algebra: 6. Linear equalities	Solve linear inequalities extending the strategies to	Demonstrates strategies for solving systems of linear inequalities.	CLO12: Solving linear inequalities in one variable	C53: Demonstrates the understanding that a solution of an inequality in one variable is the value of the variable which makes the inequality true and verifies if a given value is a solution;

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
	solve linear equations			applies to verify rules of inequalities of real numbers
Algebra: 6. Linear equalities				C54: Finds the solution set of an inequality in one variable with or without applying the rules for solving inequalities algebraically
				C55: Finds solutions of an inequality in one variable applying the rules for solving an inequality and represents the solutions graphically (on a number line)
				C56: Frames a linear inequality in one variable representing a given situation/real-life situation and finds solutions
Algebra: 7. Permutations and Combinations	Simple application of permutations & combinations and introduction to the	Applies the ideas of permutations and combinations to daily life situations of arranging and selecting the objects.	CLO13: Uses existing knowledge to develop an understanding of permutations and combinations	C57: Applies the fundamental principle of counting (multiplication principle) to find the total number of occurrences of the events in the given order
				C58: Evaluates a numerical expression involving factorial(s) or solves a linear equation in 1 variable

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
	fundamental principle of counting			involving factorials (not permutations)
Algebra: 7. Permutations and Combinations				C59: Finds the number of permutations of n different objects taken r at a time without repetition applying the formula for the number of permutations nPr = $\frac{n!}{(n-r)!}$
				C60: Finds the number of permutations when some numbers of objects are of same kind
				C61: Applies the formula nPr = $\frac{n!}{(n-r)!}$ to find the number of permutations
				C62: Applies the formula nPr = $\frac{n!}{(n-r)!}$ to solve an equation in n involving permutations
			CLO14: Applies permutation and combination formulae to solve simple problems	C63: Finds the number of permutations of n different objects taken r at a time, where repetition is allowed
				C64: Finds the number of permutations of n different objects

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
				where p objects are of the same kind and the rest are all different
Algebra: 7. Permutations and Combinations				C65: Finds the number of permutations of n different objects where pi, i = 2, 3, …, k < n objects are of the same kind and the rest are all different
				C66: Finds nCr applying the relation nPr = nCr r! $(0 < r \leq n)$
				C67: Applies the relation nPr = nCr r! to find the number of combinations of n different objects taken r objects at a time
				C68: Applies the relation nCr + nCr -1 = n+1Cr and nCn-r = nCr
				C69: Applies permutations and combinations to solve real world problems

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Algebra: 8. Binomial Theorem	Application of the binomial theorem	Develops the idea of the Binomial theorem for a positive integral index from the earlier learnt concepts of finding squares and cubes of binomials.	CLO15: Understanding the concept of binomial distribution and its application.	C70: Expands a given algebraic expression applying the binomial theorem
				C71: Applies the binomial theorem to evaluate a numerical expression
				C72: Applies the binomial theorem to prove or verify the given mathematical relationship
				C73: Applies the binomial theorem to find a particular term or compares the terms
Algebra: 9. Sequence and Series	Extension of existing understanding of progressions	Extends the ideas related to Arithmetic progressions learnt earlier to new types of sequences and their series.	CLO16: Applies the concepts of arithmetic progressions to solve problems	C74: Lists terms of a given sequence given the rule or the general formula for any term
				C75: Finds the series corresponding to a given sequence
				C76: Applies the formula of a general term in an A.P.
				C77: Applies the formula of the sum of first n terms in an A.P.

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Algebra: 9. Sequence and Series				C78: Applies formulae of the general term and sum of first n terms in an A.P. in real-life situations (word problems)
				C79: Inserts numbers between given two numbers to form an A.P.
			CLO17: Applies the concepts of geometric progressions to solve problems	C80: Identifies if a given sequence is A.P. or G.P.
				C81: Applies the formula of a general term of a G.P. to find terms in the G.P.
				C82: Applies the formula of the sum of the first n terms of a G.P.
				C83: Applies the formula of the general term and/or the sum of the first n terms of a G.P. in word problems based on real-life context
				C84: Inserts numbers between the two given numbers to form a G.P.
				C85: Applies the understanding of arithmetic and/or geometric mean of two numbers; applies the relationship between them

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
				C86: Applies the formulae to find the sum of first n natural numbers, their squares, or cubes
Coordinate Geometry: 10. Straight lines	Extend the concept & properties of straight lines using coordinate geometry	Constructs different forms of a straight line using the earlier learnt concepts of coordinate geometry.	CLO18: Applies distance, area, and section formulae to solve problems	C87: Finds the distance between two points in a coordinate plane applying the distance formula
				C88: Applies the formula of area of a triangle given coordinates of its vertices to find areas
				C89: Applies the section formula to find unknown coordinates when a point divides the line segment joining two points internally
				C90: Finds the slope of a line passing through the given two points
			CLO19: Understand how to find slope in the Cartesian coordinate system	C91: Finds the slope of a line making a given angle with the positive direction of x-axis
				C92: Determines if given pair of lines are parallel or perpendicular and applies the appropriate relation between their slopes

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Coordinate Geometry: 10. Straight lines				C93: Applies the relation between slopes of two lines and the angle between the two lines
				C94: Finds unknown coordinates given the three points are collinear and some of their coordinates
			CLO20: Applies the understanding of slope to form equation of a line	C95: Finds an equation of a line applying appropriate form of equation of a line satisfying given conditions
				C96: Converts an equation of a line from one form to another (point-slope form, slope-intercept form, two-point form, intercept form)
				C97: Finds the distance of a given point from the given line
				C98: Finds the distance between the given pair of parallel lines

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Coordinate Geometry: 11. Conic Sections	Understanding of standard equations of different conic sections	Analyses different curves like circles, ellipses, parabolas, and hyperbolas based on the ideas developed for straight lines using coordinates when a plane intersects a double napped right circular cone.	CLO21: Understands the formation of conic sections	C99: Knows what conic section is formed when a plane cuts a double-napped right circular cone
				C100: Finds an equation of a circle satisfying the given conditions
				C101: Finds the centre and/or radius of a circle when the equation of the circle is given
			CLO22: Understands different parts of conic sections & finds their standard equation	C102: Finds the coordinates of a focus, axis, the equation of the directrix and/or the length of the latus rectum of a parabola given the standard equation of the parabola
				C103: Finds the standard equation of a parabola given the coordinates of its focus and the equation of its directrix
				C104: Finds the standard equation of a parabola satisfying the given conditions on symmetry, point(s) that lie on it, etc.
				C105: Finds the coordinates of the foci, the vertices, the length of the major axis, the length of the minor

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Coordinate Geometry: 11. Conic Sections				axis, the eccentricity and/or the length of the latus rectum of an ellipse given its standard equation
				C106: Finds the standard equation of an ellipse given the coordinates of its vertices and foci
				C107: Finds the standard equation of an ellipse satisfying the conditions on vertices, foci, lengths of major axis and minor axis, points lying on the ellipse etc.
				C108: Finds the coordinates of the foci, the vertices, the eccentricity and/or the length of the latus rectum of a hyperbola given its standard equation
				C109: Finds the standard equation of a hyperbola satisfying the coordinates of vertices and foci
				C110: Finds the standard equation of a hyperbola satisfying the conditions like lengths of transverse axis, length of the conjugate axis, coordinates of foci, length of the latus rectum,

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
				eccentricity, points lying on the hyperbola etc.
Coordinate Geometry: 12. Introduction to 3-d geometry	Extension of understanding of 2-D coordinate system to 3-D coordinate system	Develops strategies of locating a point in three dimensions based on the concepts of two-dimensional coordinate geometry.	CLO23: Understand how to find coordinates of a point in 3-D coordinate system	C111: Finds the coordinates of a point in a 3-D coordinate system lying in one of the 3 coordinate planes
				C112: Finds the octant in which the given point lies in a 3-D space
				C113: Finds the distance between two given points in a 3-D coordinate system applying the distance formula
Calculus: 13. Limits and derivatives	Application of limits to build conceptual understanding of derivatives	Evolves the concepts of limit and derivative of a function by analyzing the behaviour of functions when the corresponding variable approaches a certain value.	CLO24: Understands the meaning of left-hand and right-hand limits	C114: Demonstrates that if the left-hand limit and the right-hand limit of a function at a point are equal, the limit of the function at the point exists and is the same as the left-hand or right-hand limit, otherwise the limit doesn't exist
				C115: Estimates the limit of a function at a point based on values of the function at points very near to the given point
				C116: Applies the rules of algebra of limits (sum, difference, product, or

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
				quotient of two functions) to find the limit when it exists
				C117: Finds the limit of a polynomial or a rational function at a given point when it exists
Calculus: 13. Limits and derivatives			CLO25: Determines the derivative of a given function using limits	C118: Demonstrates the derivative of a function at a point and state its definition; Finds the derivative of a given function at a point using the definition of derivative
				C119: Demonstrates the first principle of derivative and finds the derivative of a function by the first principle
			CLO26: Understands and applies the algebra of derivatives	C120: Applies the rules of algebra of derivatives to find the derivative. $(u \pm v)' = u' \pm v'$ $(uv)' = u'v + uv'$ (Leibnitz/product rule) $(\frac{u}{v})' = \frac{u'v-uv'}{v^2}$ (Quotient rule)
				C121: Applies the derivative of standard functions, xn, trigonometric functions to find the derivative of a

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
				function composed of such standard functions
Statistics & Probability: 15. Statistics	Application of statistics to analyse and interpret data	Applies Measures of dispersion to get a better interpretation of data of different daily life situations.	CLO27: Finds standard measures of dispersion for discrete frequency distributions	C122: Determines range of the given data as a measure of dispersion (variability)
				C123: Finds the mean deviation about mean/median for an ungrouped data
				C124: Finds the mean deviation about mean/median for discrete frequency distribution
			CLO28: Finds standard measures of dispersion for continuous frequency distributions	C125: Finds the mean deviation about mean/median for continuous frequency distribution
				C126: Finds the variance and standard deviation of ungrouped data
				C127: Finds the variance and standard deviation of a discrete frequency distribution efficiently
				C128: Finds the variance and standard deviation of a continuous frequency distribution efficiently

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Statistics & Probability: 16. Probability	Extension of discrete probability to continuous probability models	Builds up the axiomatic approach to Probability through the terms, random experiment, Sample space, events etc.	CLO29: Represents events as subsets of a sample space	C129: Examines if an experiment is a random experiment
				C130: Finds a sample space of a given random experiment and/or the number of sample points (elements) in the sample space
				C131: Determines if the given event associated with the sample space of a random experiment has occurred or not
				C132: Determines if the given event is an impossible or sure event
				C133: For events A and B of a random experiment, finds the set representing the events A or B, A and B or A but not B
			CLO30: Identifies complementary, mutually exclusive, and exhaustive events CLO31: Calculates the probability of an event in a random experiment	C134: Finds the complementary event of an event of a random experiment
				C135: Determines if given events of a random experiment are mutually exclusive events or identifies events that are mutually exclusive

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Statistics & Probability: 16. Probability				C136: Determines if given events of a random experiment are exhaustive
				C137: Determines if given events of a random experiment are mutually exclusive and exhaustive
				C138: Demonstrates the axiomatic definition of probability (determines if assignment of probability values to each outcome in a sample space is valid)
				C139: Computes the probability of an event (compound event) of a random experiment where each outcome is equally likely

CLASS 12 CONTENT DOMAIN SPECIFIC LEARNING OUTCOMES AND INDICATORS

The table below lists the content domain specific learning outcomes and indicators for class 12 based on chapters in NCERT class 12 textbook.

Table 6.2 Content domain specific learning outcomes and indicators

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Relations and Functions: 1. Relations and Functions	Different types of relations and functions	Identifies different types of relations and functions.	CLO1: Identifies or determines the type of a given relation or a function	C1: Demonstrates if a given relation is an empty relation or a universal relation (trivial relation)
				C2: Identifies if a given pair of elements of set(s) belongs to the given relation
				C3: Demonstrates if a given relation is reflexive, symmetric, or transitive with valid justifications; Gives example(s) of such relations satisfying given conditions (or non-examples)
				C4: Determines if a relation is an equivalence relation or not with valid justifications
				C5: Determines if subset of a set on which a relation is defined is an

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Relations and Functions: 1. Relations and Functions				equivalence class under the given relation or finds equivalence classes
				C6: Determines if a given function is a one-one (injective), an onto (surjective) function and/or a bijective function; Gives example(s) of a one-one (injective), an onto (surjective) function and/or bijective function with valid justifications; gives examples and non-examples of such functions
Relations and Functions: 2. Inverse Trigonometric Functions	Existence and definitions of inverse trigonometric functions	Explores the values of different inverse trigonometric functions	CLO2: Determines the range and domain of inverse trigonometric functions	C7: Finds the domain and the range of an inverse trigonometric function
				C8: Finds the principal value of a given inverse trigonometric function for the given point in its domain
				C9: Interprets graphs of inverse trigonometric functions

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Algebra: 3. Matrices	Understanding the properties of a matrix and its inverse	Evolves the idea of matrices as a way of representing and simplifying mathematical concepts.	CLO3: Understands different types of matrices	C10: Demonstrates representation of real-life data in matrix format
				C11: Identifies the characteristics of a matrix
				C12: Identifies the different types of matrices
				C13: Applies the equality of matrices to find unknown elements
			CLO4: Understands and applies the algebra of matrices	C14: Evaluates expressions that involve addition, subtraction, and scalar multiplication of matrices of order $m \times n$
				C15: Finds the product of matrices
				C16: Applies properties of operations of matrices in simplifying expressions
				C17: Applies the properties of transpose of a matrix, symmetric and skew symmetric matrices

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Algebra: 3. Matrices			CLO5: Understands and finds the inverse of a matrix	C18: Determines if a square matrix is invertible or not
				C19: Finds inverse of a square matrix if it exists (e.g., using adjoint of a matrix)
Algebra: 4. Determinants	Application of determinants to solve system of linear equations	Evaluates determinants of different square matrices using their properties.	CLO6: Understands the properties of determinants	C20: Finds determinant of a matrix of order 2×2 or 3×3
			CLO7: Applies the knowledge of determinants to solve system of linear equations	C21: Finds area of a triangle with given coordinates using determinants
				C22: Finds the minor and cofactor of an element of a determinant
				C23: Finds adjoint and inverse of a matrix
				C24: Applies the properties of matrices and determinants in solving the system of linear equations in two or three variables

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Calculus: 5. Continuity and Differentia-bility	Verifying if a function is continuous and finding derivatives	Extends the concept of limits to the continuity and the differentiability. Demonstrates ways to relate differentiability and continuity of a function with each other.	CLO8: Understands how to check continuity of a function	C25: Examines if a function is a continuous function at a given point; Identifies interval(s) in which a function is continuous and points of discontinuity if any
				C26: Demonstrates that polynomial functions and trigonometric functions are continuous functions
Calculus: 5. Continuity and Differentia-bility				C27: Applies the rule – the sum/difference/product/quotient of two continuous functions is also continuous
				C28: Demonstrates that if two functions are continuous, then their composite functions are also continuous
			CLO9: Understands the differentiability of a function and applies the properties of	C29: Demonstrates derivative of a function at a point and finds derivative of a given function at a point

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Calculus: 5. Continuity and Differentia-bility			derivates to differentiate a given function	C30: Applies rules related to algebra of derivatives to differentiate the given function $(u \pm v)' = u' \pm v'$ $(uv)' = u'v + uv'$ (Product rule) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ (Quotient rule)
				C31: Demonstrates understanding that a function differentiable at a point is also continuous at that point but the converse is not necessarily true (relationship between continuity and differentiability)
			CLO10: Applies rules and properties to find derivatives of complex function	C32: Applies the chain rule to differentiate composite functions
				C33: Differentiates a function defined implicitly
				C34: Applies the derivative of standard trigonometric inverse functions to differentiate

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Calculus: 5. Continuity and Differentia-bility				C35: Applies the derivative of exponential and logarithmic functions and their properties to differentiate; Carries out logarithmic differentiation on functions reducible to logarithmic form
				C36: Differentiates a function given in parametric form
				C37: Finds the 2nd order derivative of a given function; Verifies the given relationship involving first and 2nd order derivative
Calculus: 6. Applications of derivatives	Using differential calculus to solve problems relating to slopes or rates	Applies the concepts of Differential calculus to calculate the slopes of curves.	CLO11: Understands the application of derivative test to check if a function is increasing or decreasing	C38: Finds the rate of change of a quantity at a given instant/independent quantity applying differentiation
				C39: Applies differentiation and examines if a given function is increasing or decreasing in a given interval or finds an interval in which the function is increasing/decreasing determining critical points

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Calculus: 6. Applications of derivatives			CLO12: Applies derivatives to solve problems involving extrema	C40: Finds all the critical points of a function in a given domain
				C41: Finds the maxima, minima, or points of inflection of a given function; Finds the points of extrema in a given interval applying the first or 2nd derivative test (signs of the first or 2nd derivative)
				C42: Finds extrema or points of extrema for any real-life relation modelled or a mathematical problem applying the tests of derivatives
Calculus: 7. Integrals	Applying integration as a counter to differentiation	Develops the processes in Integral calculus based on the ideas of differential calculus learnt earlier.	CLO13: Understands integrals as anti-derivatives	C43: Finds a family of primitives (anti derivatives) or integral of a function given the derivative of the function or by the method of inspection
				C44: Finds the integrals (antiderivatives) of standard polynomial, trigonometric, inverse trigonometric, exponential functions

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
				and functions whose integrals are logarithmic functions
Calculus: 7. Integrals			CLO14: Applies calculus algebra to determine integrals of complex functions	C45: Finds the integrals of functions applying the properties of the sum/difference of integrals and integral of a constant multiple of a function
				C46: Finds the integrals by the substitution method knowing when it is applicable
				C47: Finds the integrals applying trigonometric identities
				C48: Finds the integrals applying standard formulae (integrals involving particular functions - $\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}},$ $\int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}$ $\int \frac{px + q}{ax^2 + bx + c} dx, \int \frac{px + q}{\sqrt{ax^2+bx + c}} dx,$

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Calculus: 7. Integrals				$\int \sqrt{a^2 \pm x^2}\, dx, \int \sqrt{x^2 - a^2}\, dx$, $\int \sqrt{ax^2+bx+c}\, dx$,)
				C49: Finds the integrals of rational functions reducing them to appropriate partial fractional forms
				C50: Find the integrals of product of two functions applying the method of integration by parts appropriately knowing when it is applicable
			CLO15: Determines the value of a definite integral within given limits	C51: Demonstrates the first and second fundamental theorems, interpreting integrals as area functions
				C52: Evaluates the definite integrals applying the 2nd fundamental theorem of integral calculus
				C53: Evaluates the definite integrals using the method of substitution

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Calculus: 7. Integrals				appropriately knowing when it is applicable
				C54: Evaluates the definite integrals applying its properties
Calculus: 8. Application of Integrals	Applies the understanding of integration to calculate area enclosed by curves	Applies the concepts of Integral calculus to calculate the areas enclosed by curves.	CLO16: Finds the area of a bounded region using integral calculus	C55: Finds the area of the region bounded by a simple curve (lines, circles, parabolas, ellipses in standard form) and axes
				C56: Finds the area of the region bounded by a simple curve (circle, parabola, ellipse in standard form) and a line(s)
Calculus: 9. Differential Equations	Representing a family of curves using differential equations	Develops the concepts of differential equations using differentiation and integration	CLO17: Develops the concept of a differential equation using derivatives	C57: Finds order and/or degree of a differential equation when defined and can identify when the degree is not defined
				C58: Verifies if a given function is a solution of a differential equation

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Calculus: 9. Differential Equations			CLO18: Differentiates between general and particular solution of a differential equation	C59: Understands if a given solution of a differential equation is a general solution or a particular solution and/or knows the number of arbitrary constants in a solution as per the order of the differential equation
				C60: Finds the general or the particular solution of given differential equation with variables separable
				C61: Identifies and shows if a given differential equation is homogenous
			CLO19: Determines general and/or particular solution of a differential equation	C62: Finds the general or the particular solution of a homogenous differential equation identifying the equation as homogenous
				C63: Identifies if a given differential equation is a first order linear differential equation

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Calculus: 9. Differential Equations				C64: Finds the integrating factor of the given first order linear differential equation. $(\frac{dy}{dx} + p(x)y = q(x)\ or\ \frac{dx}{dy} + p(y)x = q(y))$
				C65: Finds the general or the particular solution of a first-order linear differential equation
3-Dimensional Geometry: 10. Vectors	Understanding the vector algebra in 3-dimensional system	Constructs the idea of vectors and their properties and relates them to earlier learnt concepts in different areas of mathematics such as geometry, coordinate geometry etc.	CLO20: Understands direction cosines and angles between vectors	C66: Represents a vector using direction cosines and/or direction ratios or finds them; Demonstrates the relationship between direction cosines and angles made by the vector with the axes
				C67: Identifies type of a given pairs of vectors as collinear, equal, coinitial, negative of each other etc. and/or applies their properties
				C68: Demonstrates free vectors and position vectors

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
3-Dimensional Geometry: 10. Vectors			CLO21: Applies the laws of vector addition	C69: Demonstrates the triangle law and the parallelogram law of vector addition and the properties of vector addition (commutativity, associativity, existence of additive identity)
				C70: Computes the magnitude of a vector represented in the component form and/or finds components of a vector and/or unit vectors along the axes
				C71: Provides a vector with the same magnitude or the same direction as that of the given vector or finds a vector of given magnitude in the direction of the given vector
			CLO22: Understands the vector algebra	C72: Finds the sum or difference of two vectors represented in component form and applies the properties of addition of vectors and multiplication by a scalar.

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
				(i) $k\vec{a} + m\vec{a} = (k+m)\vec{a}$ (ii) $k(m\vec{a}) = (km)\vec{a}$ (iii) $k(\vec{a}+\vec{b}) = k\vec{a} + k\vec{b}$
				C73: Applies the condition of equality of two vectors represented in component form
3-Dimensional Geometry: 10. Vectors				C74: Finds a unit vector or a vector of given magnitude in the direction of the given vector represented in component form
				C75: Finds the vector joining two given points in the given direction
			CLO23: Applies different formulae to solve problems involving vectors in 3-d coordinate system	C76: Applies the section formulae and finds the position vector of a point dividing the line joining the given two points in the given ratio internally and/or externally or finds the midpoint
			CLO24: Finds scalar (dot) and cross (vector) products of	C77: Finds the scalar product of two vectors and/or the angle between them

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
3-Dimensional Geometry: 10. Vectors			vectors and applies their properties	C78: Applies the properties of the scalar product of two vectors to determine if the vectors are perpendicular to each other, distributivity of scalar product over addition of two vectors, the property $(\lambda\vec{a})\cdot\vec{b} = \lambda\left(\vec{a}\cdot\vec{b}\right) = \vec{a}\cdot\left(\lambda\vec{b}\right)$ etc.
				C79: Finds the scalar product of two vectors in component form
				C80: Finds the projection vector and/or projection of the given vector on the given directed line or another vector applying the scalar product
				C81: Demonstrates the vector product and finds the vector product of given two vectors or the angle between the two vectors
				C82: Finds the area of a triangle or a parallelogram applying vector product of two vectors

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
3-Dimensional Geometry: 10. Vectors				C83: Demonstrates the properties of the vector product like distributivity of vector product over addition, $\lambda\left(\vec{a}\vec{b}\right) = (\lambda\vec{a})\vec{b} = \vec{a}\ (\lambda\vec{b})$ etc. and applies them
				C84: Finds the cross product of two vectors represented in component form
3-Dimensional Geometry: 11. Three-Dimensional Geometry	Extend the knowledge of vectors to planes in 3-dimensional systems	Evolves newer concepts in the three-dimensional geometry from that learnt earlier, in the light of vector algebra, such as, direction cosines, equations of lines under different conditions etc.	CLO25: Understands the positional properties of vectors	C85: Gives the position vector of a point given its cartesian coordinates and vice-versa
				C86: Demonstrates the direction ratios and direction cosines of a line and the relationship between them, finds the direction ratios (direction cosines) of a line given the angles made with the axes
			CLO26: Applies the direction cosines to find the relationship between lines and planes	C87: Applies the direction ratios/direction cosines and the relationship between those for given two lines to determine if the two

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
3-Dimensional Geometry: 11. Three-Dimensional Geometry				lines are parallel. points on two lines are collinear
				C88: Finds the direction ratios/cosines of the line segment joining two points in a 3-D cartesian plane given their coordinates
			CLO27: Finds the vector equations and cartesian equations of given vectors, lines and/or planes	C89: Finds the vector equation of a line in a 3-D cartesian plane passing through the given point and parallel to the given vector or with the given direction ratios/cosines
				C90: Finds the vector equation of a line in 3-D cartesian plane passing through the given two points
				C91: Finds the cartesian equation of a line in 3-D cartesian plane passing through the given two points given their coordinates
				C92: Finds the vector equation for the line given its cartesian equation

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
3-Dimensional Geometry: 11. Three-Dimensional Geometry				and vice-versa applying the understanding of direction ratios
			CLO28: Applies vectors to solve problems pertaining to angles and distances in 3-dimensional coordinate system	C93: Finds the angle between the pair of lines applying the relationship between their direction ratios
				C94: Finds the shortest distance between the given two lines
				C95: Finds the shortest distance between the given pair of parallel lines
Linear Programming: 12. Linear Programming	Mathematical formulation of real-life optimization problems using systems of linear inequalities, and solving them	Formulates and solves problems related to maximization/ minimization of quantities in daily life situations using systems of inequalities/inequations learnt earlier.	CLO29: Understands the structure and components of linear programming problems	C96: Identifies objective function, constraints, decision variables and optimal solution/value in a linear programming problem
			CLO30: Understands the visual representation of feasible solution and finds the optimal value graphically	C97: Demonstrates the feasible region of the given linear programming problem
				C98: Knows that the optimum value if exists (maximum or minimum

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Linear Programming: 12. Linear Programming				value) occurs at a corner point of the feasible region
				C99: Solves the given mathematical linear programming problem graphically applying the Corner Point method
				C100: Understands that if two corner points of the feasible region are both optimal solutions of the same type, then any point on the line segment joining these two points is also an optimal solution of the same type and applies this understanding
Probability: 13. Probability	Develop the understanding of conditional probability and probability over continuous distributions	Calculates the conditional probability of an event and uses it to evolve Bayes' theorem and multiplication rule of probability. Determines mean and variance of a probability distribution using the concept of random variable.	CLO31: Understands the concept of conditional probability and relevant results	C101: Finds the conditional probability involving 2 events associated with the sample space of a random experiment applying the definition
				C102: Demonstrates the property of conditional probability, P(S|F) = P(F|F) = 1 where F is an event of the sample space S of an experiment

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Probability: 13. Probability			CLO32: Applies the results and properties of conditional probability	C103: Finds the conditional probability of an event applying the property P((A ∪ B)|F) = P(A|F) + P(B|F) – P((A) B)|F), where A, B and F are events of a sample space S
				C104: Finds the conditional probability of an event applying property P(E'|F) = 1 – P(E|F) where E and F are events of a sample space S of an experiment
				C105: Finds the probability of an event applying the multiplication rule of probability of two events
				C106: Finds the probability of an event applying the multiplication rule of probability of more than two events
				C107: Identifies or shows if given two events associated with the same random experiment are independent

Unit and chapter	Key concept	NCERT Learning Outcomes (LOs)	Content domain specific Learning Outcomes (CLOs)	Indicators
Probability: 13. Probability				C108: Finds the probability of an event applying the theorem of total probability
				C109: Finds the probability of an event applying Bayes' theorem
			CLO33: Understands the concept of random variables and continuous probability distributions	C110: Describes the random variable associated with the given sample space of a random experiment
				C111: Finds the probability distribution of the random variable associated with the given sample space of a random experiment
				C112: Finds the mean or expectation of the given random variable

7. SAMPLE PEDAGOGICAL PROCESSES AND ASSESSMENT STRATEGIES

"The pedagogical practices should be learner centric. It is expected of a teacher to ensure an atmosphere for students to feel free to ask questions. They would promote active learning among students with a focus on reflections, connecting with the world around them, creating and constructing knowledge. The role of a teacher should be that of a facilitator who would encourage collaborative learning and development of multiple skills through the generous use of resources via diverse approaches for transacting the curriculum."

[CBSE Curriculum for classes 11-R12]

CLASS 11

Content Domain: Calculus

Chapter 13: Limits and derivatives, Table 7.1

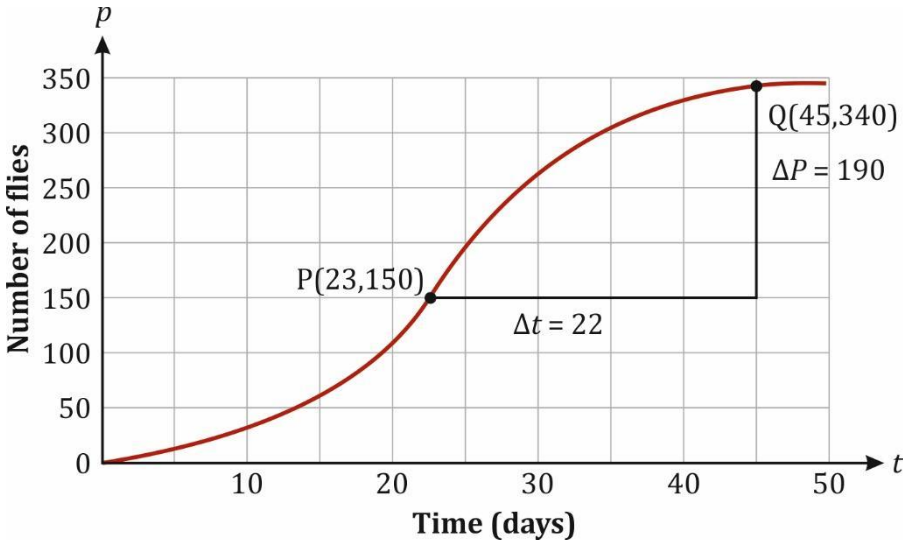
Learning outcomes and indicators	Pedagogical Processes	Assessment Strategies
Understands the meaning of left-hand and right-hand limits; finds the limit of a function at a point applying suitable properties. Indicators C114: Demonstrates that if the left-hand limit and the right-hand limit of a function at a point	Make students understand and appreciate the need for learning limits and derivatives of functions through activities. One of the activities (hooks) is suggested below in detail. Activity A: Estimating growth rate of population of fruit flies (Drosophila) The graph shows how a population of fruit flies (Drosophila) grew in a 50-day controlled experiment.	Ask students to examine if the limit of a function at a given point exists and hence find the limit when it exists function from the given graphs for a variety of functions (graphs can be generated from the Google search typing equations in the search bar and demonstrated) Give examples where left-hand limit and/or right-hand limit of a function at a point are different or one of them/both don't exist. Ask

are equal, the limit of the function at the point exists and is same as the left-hand or right-hand limit, otherwise the limit does not exist

C115: Estimates the limit of a function at a point based on values of the function at points very near to the given point

C116: Applies the rules of algebra of limits (sum, difference, product, or quotient of two functions) to find the limit when it exists

C117: Finds the limit of a polynomial or a rational function at a given point when it exists

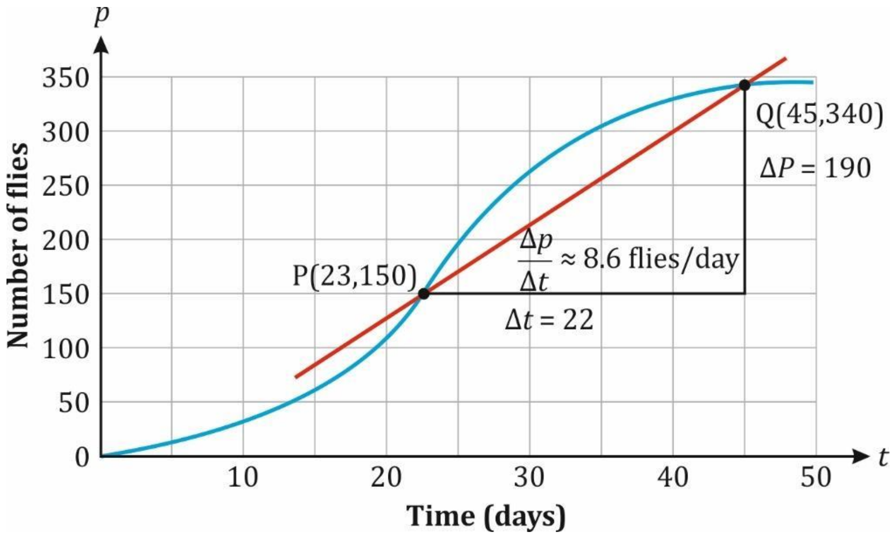


Question for discussion: In which time interval, the growth rate (number of flies/day) seems to be higher. Ask students to share the rationale for their answers.

Prompt them (if required) to observe the growth rates being lower in the intervals, 0-5 days, 5-10 days, 10-15 days, 40-45 days. Growth rates are higher between 20-30 days.

students to give an example where the limit of a function at a point doesn't exist. Ask them to justify their reasoning.

Ask students to find the limits of a function by applying the algebra of limits. Ask students to identify the rule and the individual functions while finding the limit. E.g., in finding the limit of h(x) = x2 + 2x at a point, help them see the given function is of the form f(x)·f(x) + g(x) where f(x) = x and g(x) = 2x and hence apply the algebra of limits



Ask students to estimate the growth rate (how fast the number of flies in the population was growing) on day 23 using the information in the graph and the table below.

Interval to estimate the growth	Point P on populati on curve	Point Q on populati on curve	Average Growth rate in the interval	Estimated growth rate (approximatel y) in number of flies/day
Day 23 – Day 45	(23, 150)	(45, 340)	$\frac{340-150}{45-23}$	8.6
Day 23 – Day 40	(23, 150)	(40, 330)	$\frac{330-150}{40-23}$	10.6

Day 23 – Day 35	(23, 150)	(35, 310)	$\frac{310-150}{35-23}$	13.3
Day 23 – Day 30	(23, 150)	(30, 265)	$\frac{265-150}{30-23}$	16.4

It may be good to connect with the average rates being slope of the secants i.e., the line connecting two points on a curve. (In general, encourage making such connections between concepts.)

Let students see that the estimated growth rate getting better if the interval to estimate growth rate is closer to 23 days.

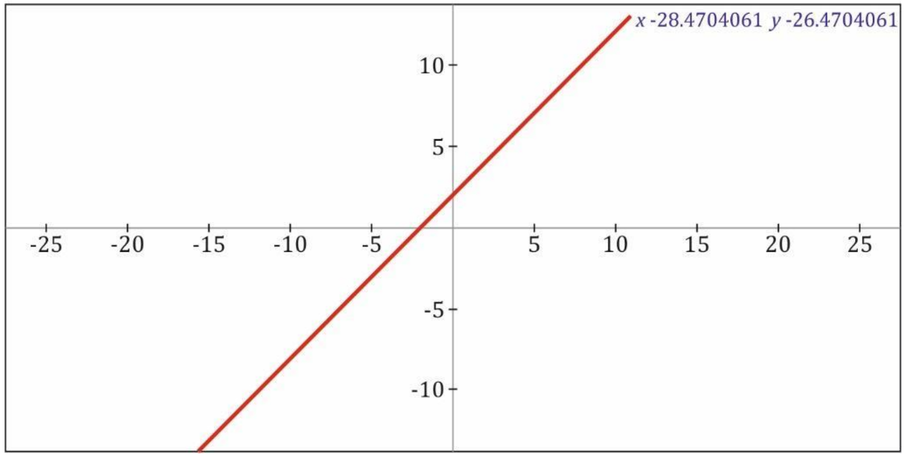
The above exercise can be repeated with any distance-time graph as well.

Lead them to the definition of tangent to a curve connecting with how tangents are defined for a circle.

Activity B to arrive at informal definition of limit of a function

Show students the graph of a function f(x) = $\frac{x^2-2^2}{x-2}$

Graph for $(x^2-4)/(x-2)$



It can be shown in google search bar (or a tool like GeoGebra) where values of x and f(x) are shown as mouse is hovered over a point on the curve.

Questions for discussion

Is the function defined at x = 2?

What value the function takes as x gets closer to 2? Asking them to make a table showing values of x and f(x) where x gets closer to 2 will help them to answer the question. (If required nudge students to populate table with values of x such as 1.9, 1.99, 1.999, 1.9999, 2.1, 2.01, 2.001, 2.0001.)

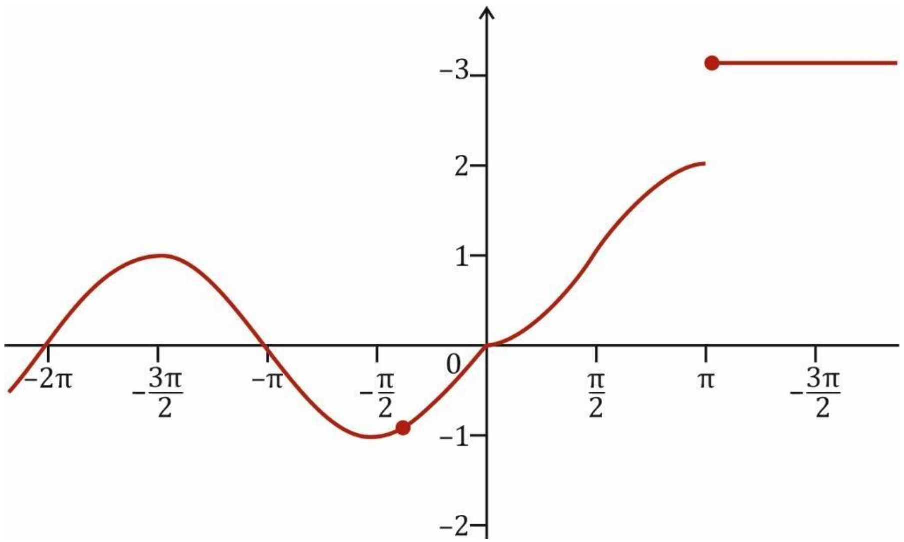
What value is f(x) getting closer and closer to when x gets closer to 2 from the left of x = 2.

What value is f(x) getting closer and closer to when x gets closer to 2 from the right of x = 2.

You can introduce notations of limit of a function and define left hand limit as well as right hand limit of a function.

Show a piece-wise function like this and ask to find left-hand limit as well as right-hand limit for x = 0 and x = π from its graph.

$$f(x) = \begin{cases} \sin(x) & : x < 0 \\ 1 - \cos(x) & : 0 \leq x \leq 3.14 \\ 3.14 & : \text{otherwise} \end{cases}$$



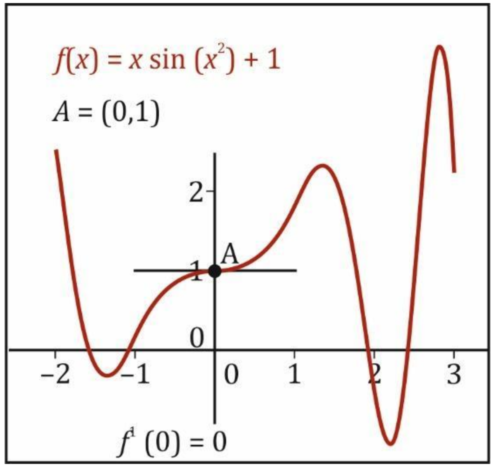
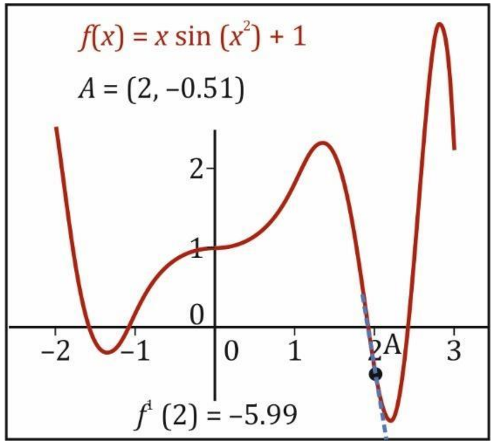
(Source: GeoGebra interactive)

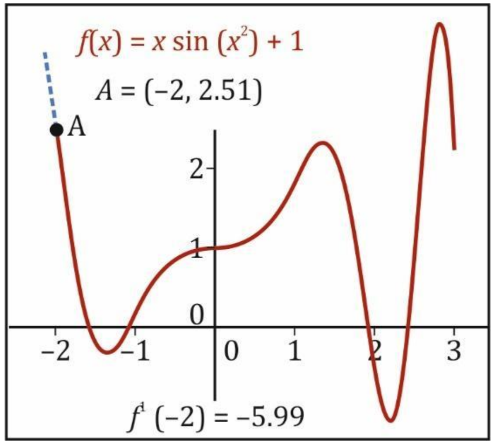
Question for discussion: What should be the limit of a function at a point if its left-hand and right-hand limit at that point is L?

Help students to see the need for the right-hand limit and the left- hand limit to be the same for the limit of a function to be defined uniquely.

Draw the attention to the observation that a function may not be defined at a point but the limit of the function at that point may be defined.

	Ask students to find limit of a function at a point from its graph or by tabulating functional values close to the point to strengthen the intuitive definition of limit of a function. Help students to arrive at the strategy of substituting variable say x with a to find $f(x)$ as f(a) wherever applicable (examining continuity of a function at x = a formally is not desired however can be nudged to see intuitively showing graphs). In general, encourage students to plot graphs of functions and guess the limit from their graphs before asking them to find the limit. (CAS software like GeoGebra or even google search page can be used to plot graphs.) Suggest taking contextual examples to make learning relatable and relevant. Students would have been exposed to graphs of – lines (linear), simple trigonometric functions, signum function etc. in earlier classes. Demonstrate how the algebra of limits work taking few examples of functions and their graphs. Make students apply the properties (algebra of limits) Make students find the limit of a polynomial function and a rational function applying the algebra of limits.	
Determines the derivative of a given function using limits. Indicators C118: Demonstrates the derivative of a function at a point and state its	Revisit the activity A on the growth of population of Drosophila flies. Ask them to relate finding growth rate on Day 23 with finding a limit as interval around day 23 becomes smaller and smaller and approaches 0. One may use appropriate distance-time graph describing a motion and discuss about (instantaneous) velocity of an object at a point. Lead them to the formal definition of derivate of a function from this notion of rate of change in extremely small interval. Help them connect the notion of derivative of a function with the slope of a tangent to a curve.	Show students this GIF.

definition; Finds the derivative of a given function at a point using the definition of derivative C119: Demonstrates the first principle of derivative and finds the derivative of a function by the first principle	Give opportunities to find the derivative of functions applying its definition and algebra of limits. Help them connect the result $\frac{sinx}{x}$ = 1 with the derivative of sin x at x = 0. Revisit the activity described earlier. Question for discussion: Is the rate of growth of population/instantaneous speed at a point t a function of t? Lead them to the first principle of derivative and introduce the notations dy/dx, f′(x). Give opportunities to find the derivatives of simple polynomial functions applying the first principle of derivative.	 Source: Wikipedia.org Ask to give the slope of the tangent to the graph at points like (0, 1), (2, -0.51), (-2, 2.51) 

		 Ask for the points where the tangent to the curve at each point is horizontal. Ask for the slopes of such tangents. Ask what the slope of the tangents to a function y = 2x + 5 is at any point on its graph. Ask to justify their answer using the first principle of derivative. A project or a group activity asking a student/group of students to graph few functions using a graphic utility software/google search, show the tangents at few points to the graph of a function and computing slope of the tangents at that point. Ask them to use the first principle of derivative

		and find the slopes of these tangents. A set of problems on derivative of functions applying the definition can be given. Some examples: e^{3x}; $\frac{x-1}{x+1}$; sin2x; sin(x2)
Understands and applies the algebra of derivatives. Indicators C120: Applies the rules of algebra of derivatives to find the derivative. $(u \pm v)' = u' \pm v'$ $(uv)' = u'v + uv'$ (Leibnitz/product rule) $(\frac{u}{v})' = \frac{u'v-uv\prime}{v^2}$ (Quotient rule) C121: Applies the derivative of standard functions, xn, trigonometric functions to find the derivative of a function composed of such standard functions	Activity to help students appreciate the need for algebra of derivatives: Ask students to find derivates of functions like x2 – 1; e^{2x}; $\frac{x-1}{x+1}$; sin2x; etc. using the first principles. Help them to see the derivative of a function being a specific case of limit of a function, algebra of limits holds true for derivates of functions as well. (Interested students can be encouraged to prove some of the results on algebra of derivatives applying the definition of derivative.) Lead students to find the derivative of a polynomial function applying algebra of derivatives. Lead students to find derivatives of standard trigonometric functions sin x, cos x, tan x etc. applying the first principle of derivatives. Ask students to find derivates of the functions they had found earlier applying the first principle now by applying algebra of derivatives. Ask which of the two methods (using the first principle or applying the algebra of derivatives) are effective and why. Give adequate practice in applying algebra of derivatives to reinforce the understanding and build fluency.	Ask to find derivatives of functions given above applying algebra of limits. Ask them to determine which of the two methods (using the first principle of derivatives) or using the algebra of derivatives is efficient in each case. Give them a set of functions to differentiate applying the most efficient method, like 2x2 – $\frac{1}{\sqrt{x}}$; $\frac{2020x^3+x^2-2022}{\sqrt{x}}$; $sec^2x - tan^2x$; e^{2x}; x2 + sin2x

Content Domain: Sets and functions

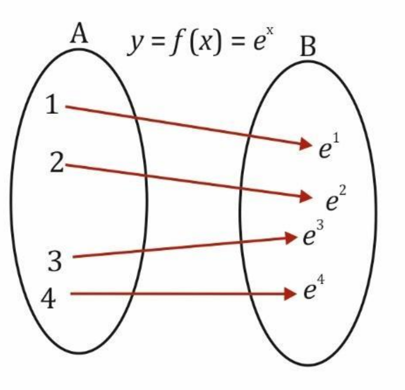
Chapter 13: Relations and functions, Table 7.2

Learning outcomes and Indicators	Pedagogical Processes	Assessment Strategies
Identifies or determines the type of a given relation and a function Indicators C1: Demonstrates if a given relation is an empty relation or a universal relation (trivial relation) C2: Identifies if a given pair of elements of set(s) belongs to the given relation C3: Demonstrates if a given relation is reflexive, symmetric, or transitive with valid justifications; Gives example(s) of such relations satisfying given conditions (or non-examples) C4: Determines if a relation is an equivalence relation or not with valid justifications	The key idea is enabling learning about equivalence relations and equivalence classes; how equivalence classes partition the domain. Questions for inquiry: (The questions are to build anticipation and interest to learn new things. All the learners are not necessary expected to answer.) What are examples of relation R in a set A which is a subset of A × A? Give examples where A = Z (integers), N (natural numbers) or R (real numbers). What could be different types of such relations? Are there examples of relations on Z or R which are 'symmetric' and 'transitive'? Are there examples of relations where every element in A is related to itself? Suggested activity: Define the relation R on set of integers Z as aRb if and only if |a-b| is divisible by 2. Questions for discussion Does (a, a) $\in R\ \forall a\ \in Z$? Does (a, b) $\in R$ $imply$ $(b,a) \in R, \forall\ (a,b)\ \in R$? Does (a, b) $\in R$ and $(b,c) \in R$ $imply$ $(a,c) \in R,\ \forall\ a,b,c\ \in Z$	Give different relations and ask to identify if given relations are reflexive, symmetric, and transitive or not. Ask them to give examples and non-examples of reflexive, symmetric, transitive relations. Ask them to identify if a given relation is an equivalence relation. Ask them to identify equivalence classes if the relation is an equivalence relation Ask students to give examples of different types of functions and ask to identify if each of them is one-one, onto, both one-one and onto and which ones is not. Ask them to justify. (They can be asked to take 2 finite sets and represent functions with arrow diagrams. They can be encouraged to graph known

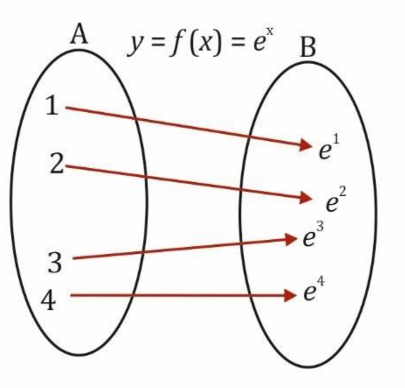
C5: Determines if subset of a set on which a relation is defined is an equivalence class under the given relation or finds equivalence classes C6: Determines if a given function is a one-one (injective), an onto (surjective) function and/or a bijective function; Gives example(s) of a one-one (injective), an onto (surjective) function and/or bijective function with valid justifications; gives examples and non-examples of such functions	What is the set of all elements in Z which are related to 0? Is this the set of all even numbers (referred as set E from now on)? What is the set of all elements in Z which are related to 1? Is this the set of all odd numbers (referred as set O from now on)? Are sets E and O mutually exclusive? Is $E \cup O = Z$? Are all elements in E related to each other? Are all elements in O related to each other? Is this same set as the set of integers whose elements leave remainder 1 if divided by 2? Is any element of E related to any element of O? Is there any other relation R on Z for which answers to Q1, 2 and 3 are affirmative? If yes, are there subsets of Z like sets of odd and even integers in the above case, which are mutually exclusive and exhaustive, all elements in each such subset are related to each other but not related to an element of another subset? Now you may introduce the formal definitions of reflexive, symmetric, transitive, equivalence relation, equivalence classes. Related them with the examples above. Help students to see that the set of even numbers is equivalence class containing 0, the set of odd numbers is equivalence class containing 1 for the relation R on Z in the above example. Ask students to give some more examples and non-examples of reflexive, symmetric and/or transitive relationships. Ask them to give examples of equivalence relations on Z and equivalence classes based on that relation.	functions like trigonometric functions, step functions, modulus functions, linear functions, quadratic functions etc.) Ask students to identify the sets between which mappings/functions become one-one, onto, or both one-one and onto in each of their examples of functions. Give more examples of functions like the ones in the activity for types of functions and ask students to identify if each function is one-one, onto, or both one-one and onto.

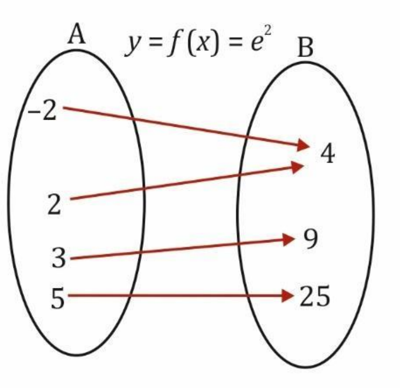
Revise the notion of functions as a special type of relation. Ask them to give examples of relations which are not functions with justifications. Help them to see the big picture as a function having 3 components: an input, rule/relationship, and an output.

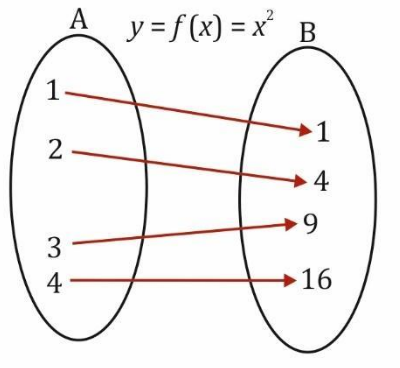
E.g., The set elements {1, 2, 3, 4} are input, the rule here is ex which maps every element in the set to the output i.e., set – {e, e2, e3, e4}.

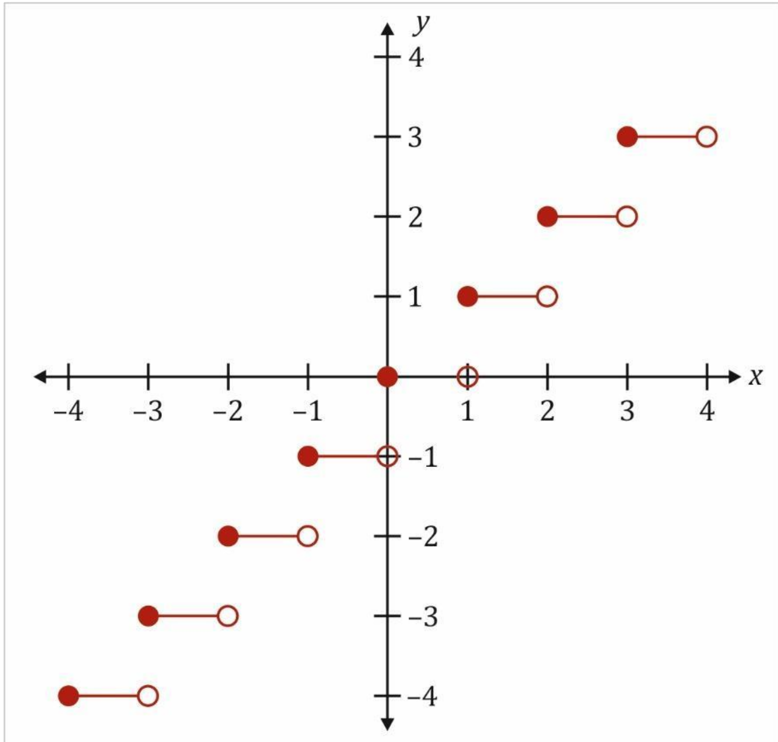


Activity for types of functions: Give examples and non-examples of functions one-one, onto, one-one as well as onto like these functions.

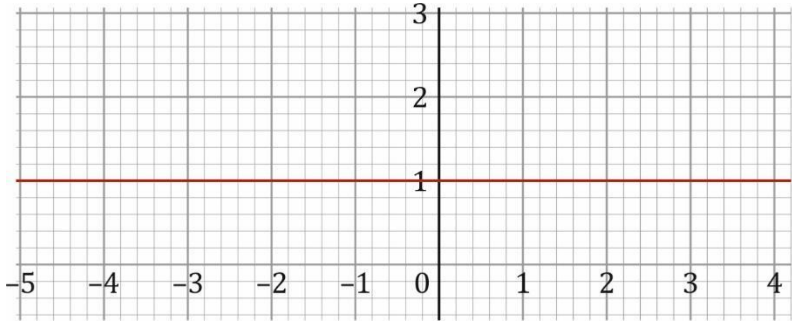




	 f : A B where A = (-4, 4) and B = {-4, -3, -2, -1, 0, 1, 2, 3} and f is defined as f(x) = [x] where [x] denotes the greatest integer less than or equal to x. the graph of f	

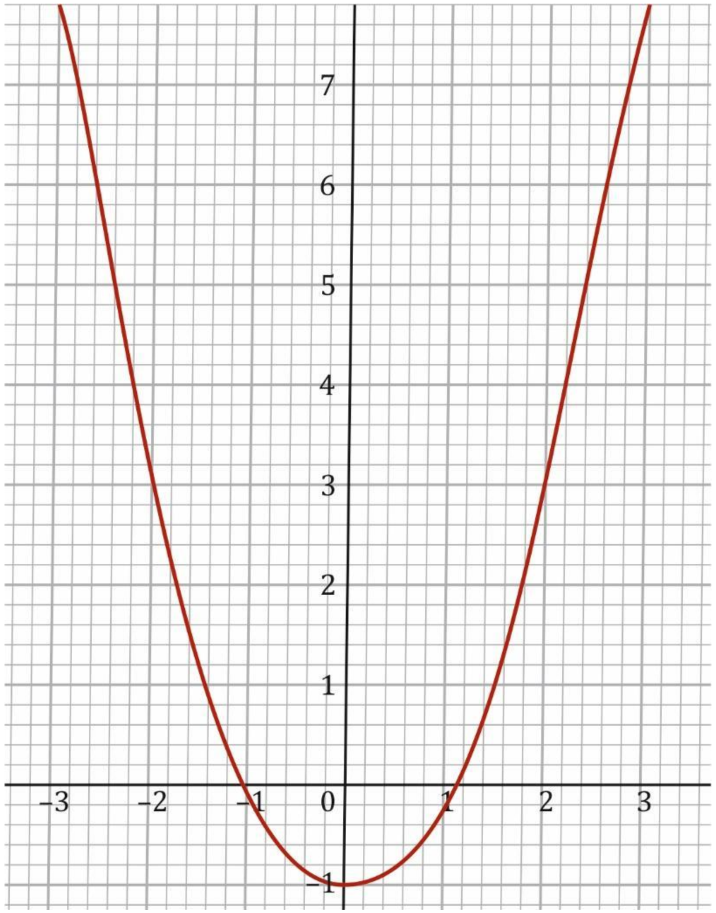


f: R▼ R, f(x) = 1, x R



- Ask them to observe any patterns in these functions.
- Ask them to make groups of functions mentioning the rule used to group these functions.

	• Prompt them to group functions based on comparing codomain and range of a function, uniqueness of preimages of images of functions etc. • Lead them to define one-one functions, onto functions. • Ask them to classify each of the above functions as one-one, onto, one-one as well as onto. • Ask students to give more examples and non-examples of functions which are one-one, onto and both one-one and onto. • Activity: Ask students to plot a graph of the function f(x) = x2 – 1 using google search, GeoGebra or such graphical utility software. • Ask in what interval is the function f(x) = x2 – 1 whose graph shown below is one-one and onto.	

	 Similarly, you can ask them to graph some trigonometric functions and ask to identify the intervals in which the functions are one-one and/or onto.	

8. TEST PAPER DESIGN

CLASS 11

The test papers for the final examination for class 11 should be balanced in terms of their coverage of content domains, cognitive domains, and types of questions. However, the blueprint governing the design of the test papers should not be very rigid and should provide sufficient latitude to the paper setter so that the focus while setting the paper remains on the quality of questions and the overall balance of the test paper.

Table 8.1. Distribution of marks across content domains

Content domains	Marks Distribution
Sets and Functions	23
Algebra	25
Coordinate Geometry	12

Table 8.2. Distribution of marks across cognitive domains

Cognitive domain (Typology of questions)	Marks distribution	% Weightage
Remembering: Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. Understanding: Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas.	44	55
Applying: Solve problems to new situations by applying acquired knowledge, facts, techniques, and rules in a different way.	20	25
Analysing: Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations. Evaluating: Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. Creating: Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	16	20

Calculus	08
Statistics and Probability	12
Total	**80**
Internal assessment	20

Total	**80**	**100**

Note: Classifying any question as per the cognitive domain may be subjective and at times not straightforward. So, the weightage of marks could be seen as indicative. It should be in some smaller range of the above-mentioned weightages practically. Keep checking circulars from CBSE on the same. Currently, the table is as per the CBSE circular.

CLASS 12

The test papers for the final examination for class 12 should be balanced in terms of its coverage of content domains, cognitive domains, and types of questions. However, the blueprint governing the design of the test papers should not be very rigid and should provide sufficient latitude to the paper setter so that the focus while setting the paper remains on the quality of questions and the overall balance of the test paper.

Table 8.3. Distribution of marks across content domains

Content domains	Marks Distribution
Relations and Functions	08
Algebra	10

Table 8.4. Distribution of marks across cognitive domains

Cognitive domain (Typology of questions)	Marks distribution	% Weightage
Remembering: Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. Understanding: Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas.	44	55
Applying: Solve problems to new situations by applying acquired knowledge, facts, techniques, and rules in a different way.	20	25

Calculus	35
Vectors and 3-D Geometry	14
Linear Programming	05
Probability	08
Total	80
Internal assessment	20

Analysing: Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations. Evaluating: Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. Creating: Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	16	20
Total	**80**	**100**

Note: Classifying any question as per the cognitive domain may be subjective and at times not straightforward. So, the weightage of marks could be seen as indicative. It should be in some smaller range of the above-mentioned weightages practically. Keep checking circulars from CBSE on the same. Currently, the table is as per the CBSE circular.

Table 8.5. Distribution of marks across types of questions for both Class 11 and 12

Question type	Marks distribution
MCQs with single option or multiple options as correct answer	15-20
Very short answer questions with 1 mark	8-10
Short answer questions with 2 or 3 marks	30-35
Long answer questions (including structured questions with sub-questions) with 5 or 6 marks	20-25

Total	80

Other details of the test paper

Maximum marks: 80

Duration of the test (writing time): 3 hours

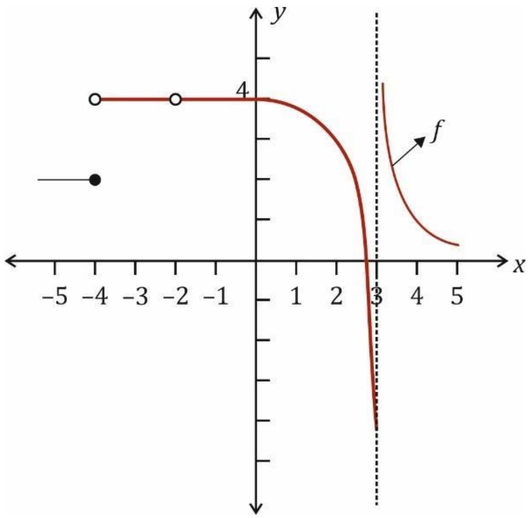
Time given for reading the test paper: 15 minutes

Total word count of the questions: 1600-2200 words

9. SAMPLE ASSESSMENT ITEMS WITH MARKING SCHEMES

MULTIPLE CHOICE QUESTION (MCQ)

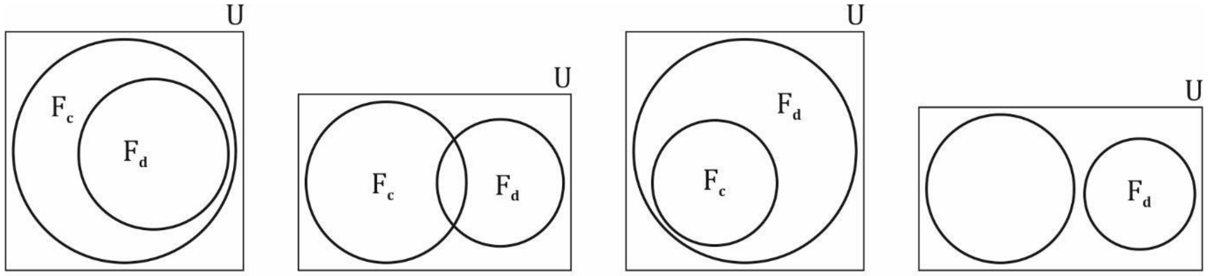
Content Domain (Chapter name)	**Limits and Derivatives**
Content Domain Learning outcome	Understands the meaning of left-hand limit and right-hand limit of a function. Determining the existence of limit of a given function at specific points in their domain
Indicator	Demonstrates that if the left-hand limit and the right-hand limit of a function at a point are equal, the limit of the function at the point exists and is same as the left-hand or right-hand limit, otherwise the limit does not exist
Cognitive level	Understand
Thinking Process	Identify different points of the functions where multiple values of limit could exist. For each such point, Find the left-hand limit of the function. Find the right-hand limit of the function. If left-hand limit is not equal to right-hand limit, identify this point as the point where limit does not exist.
Difficulty level	Medium
Marks	1
Time	2 minutes
Item Stem	Shown here is a graph of a function f defined in [-5, 5] – {-2, 3}.

<table>
<tr><td></td><td colspan="2">
(Here f(-4) = 2.)
For which of these values of a in the interval (-5, 5), $f(x)$ does NOT exist?
only a = -2
only a = -4 and a = 3
only a = -2 and a = 3
only a = -4, a = -2 and a = 3</td></tr>
<tr><td>Correct answer</td><td>B</td><td>At a = -4, the function's left-hand limit is 2 while right-hand limit is 4.
At a = 3, the function's left-hand limit tends to -∞ while right-hand limit tends to +∞.</td></tr>
<tr><td>Distractor 1</td><td>A</td><td>Student incorrectly identifies -2 as the point of no limit as the function is not defined at that point (Incorrect understanding of limit of a function).</td></tr>
<tr><td>Distractor 2</td><td>C</td><td>Student incorrectly identifies the point of no definition but correctly identifies the point of asymptotic values (Partial understanding of limits).</td></tr>
<tr><td>Distractor 3</td><td>D</td><td>Student correctly identifies the point of no limits but also included the point of no definition (Partial understanding of limits).</td></tr>
</table>

Content Domain (Chapter name)	**Matrices**
Content Domain Learning outcome	Understands and applies the algebra of matrices
Indicator	Applies properties of operations of matrices in simplifying expressions
Cognitive level	Application
Thinking Process	1. Expand the expression. 2. Club all terms with AB and all terms with BA separately. 3. Simplify the expression. 4. Conclude whose answer is correct.
Difficulty level	Easy
Marks	1
Time	1 minute
Item Stem	Kushal, Harsh, Imran and Joseph were asked to simplify 5AB + 2(BA + AB) – 3BA, where A and B are matrices of orders 3 × 3 and 3 × 3, respectiv ely. Also, $A \neq B$, $A \neq I$, $B \neq I$ and $B \neq A^{-1}$. Shown below are their responses. <table><tr><td>**Person**</td><td>Kushal</td><td>Harsh</td><td>Imran</td><td>Joseph</td></tr><tr><td>**Person's response**</td><td>6AB</td><td>7AB - BA</td><td>8AB</td><td>6AB - BA</td></tr></table> Only one of them gave the correct answer. Who gave the correct answer?

	A. Kushal B. Harsh C. Imran D. Joseph	
Correct answer	B	Correctly understands that for two matrices A and B of different orders, the product AB is not equal to product BA.
Distractor 1	A	Student assumes the commutative property for matrices, i.e., AB = BA
Distractor 2	C	Student incorrectly assumes that AB = -BA
Distractor 3	D	Student incorrectly applies distributivity to simplify as 5AB + 2BA + AB - 3BA = 6AB - BA

Content Domain (Chapter name)	**Continuity and differentiability**
Content Domain Learning outcome	Understands the differentiability of a function and applies the properties of derivates to differentiate a given function. (Understands the relationship between continuity and differentiability of a function)
Indicator	Demonstrates understanding that a function differentiable at a point is also continuous at that point but the converse is not necessarily true (relationship between continuity and differentiability) Demonstrates understanding that a function differentiable at a point is also continuous at that point but the converse is not necessarily true.
Cognitive level	Remember/Recall
Thinking Process	Recall the relationship between continuous functions and differentiable functions and understand that some continuous functions may not be differentiable, but all differentiable functions are continuous.

Difficulty level	Medium	
Marks	1	
Time	1 minute	
Item Stem	If F_c is the set of all continuous functions and F_d is the set of all differentiable functions, then which among the following Venn diagrams correctly represents the relationship between the two sets?  **A** **B** **C** **D**	
Correct answer	A	Student correctly understands that every differentiable function is continuous, but not vice-versa.
Distractor 1	B	Students who select this assume that some differentiable functions may be discontinuous.
Distractor 2	C	Students who select this incorrectly understand that any continuous function is also a differentiable function.
Distractor 3	D	Students who select this consider continuous and differentiable functions as mutually distinct from each other.

Content Domain (Chapter name)	Application of derivatives
Content Domain Learning outcome	Applies derivatives to solve problems involving extrema
Indicator	Finds all the critical points of a function in a given domain
Cognitive level	Application
Thinking Process	1. Recall that critical points are those where the function is either – a. not differentiable b. or has the first derivative as zero. 2. Find the first derivative of the function. 3. Find all the points where the first derivative is undefined. 4. Find the point where first derivative is zero. 5. List all the relevant points.
Difficulty level	Medium
Marks	1
Time	2-3 minutes
Item Stem	Which of the following are the critical points c of the function $y = tan^{-1}(sec\ x)$ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$? (Note: $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$ and $\frac{d}{dx}(\sec x) = \sec x \tan x$.) A. only $c = 0$ B. only $c = \frac{\pi}{2}$ and $c = -\frac{\pi}{2}$

	C. only $c = 0$, $c = \frac{\pi}{2}$ and $c = -\frac{\pi}{2}$ D. (The function has no critical points in the given interval.)	
Correct answer	C	Student correctly selects all the points where the derivative of the function is either undefined or is zero, i.e., identifies $c\, f'(c) = 0$.
Distractor 1	A	Student only takes the point where $f'(c) = 0$ (i.e., $c = 0$)
Distractor 2	B	Student only takes the points where $f'(c)$ is undefined. (i.e., $c = \pm\frac{\pi}{2}$ where $\cos x = 0$ and hence the first derivative is undefined)
Distractor 3	D	The student selecting this option does not know how to find critical points using derivatives or does not understand the meaning of critical points.

CONSTRUCTED RESPONSE QUESTIONS

Content domain (Chapter name)	**Relations and Functions**
Content Domain Learning outcome	Identifies or determines the type of a given relation or a function
Indicator	Determines if a given function is a one-one (injective), an onto (surjective) function and/or a bijective function; Gives example(s) of a one-one (injective), an onto (surjective) function and/or bijective function with valid justifications; gives examples and non-examples of such functions
Cognitive level	Understanding

Difficulty level	Medium
Marks	2
Time	3-5 minutes
Item stem	P = $\{x: x = 2n - 1, n$ N and $n < 1{,}00{,}000\}$ Q = $\{x: x = 2n, n$ N and $n < 1{,}00{,}000\}$ i. Define a one-one function from set P to set Q. ii. Show that the function you defined is one-one.

Marking Scheme

Step	Marks
Defines a one-function from P to Q, e.g., defines a function f: P → Q as $f(x) = x + 1$ (one of the possible examples of one-one functions)	1
Shows that the function defined is one-one. E.g., for $f(x) = x + 1$, shows, for any x, y P, $f(x) = f(y)$ ® $x + 1 = y + 1$ ® $x = y$	1

Content domain (Chapter name)	**Matrices**
Content Domain Learning outcome	Understands and applies the algebra of matrices
Indicator	Finds the product of matrices (Understands the conditions to find the product of matrices)
Cognitive level	Remember/Recall
Difficulty level	Easy
Marks	1
Time	2 minutes
Item stem	A and B are two matrices such that both products, AB and BA, exist. Write a condition on the order of the matrices A and B for the above statement to be true.

Marking Scheme

Step	Marks
Write that the above statement will be true if the orders of matrices A and B are of the form $m \times n$ and $n \times m$ respectively, where m and n are positive integers. (Award 0.5 marks if the condition 'both the matrices A and B are square matrices of the same order' is written.) OR (Award 0.5 marks if particular orders are written instead. For example, 2×3 and 3×2.)	1

Content domain (Chapter name)	Probability
Content Domain Learning outcome	Applies the results and properties of conditional probability
Indicator	Finds the conditional probability of an event applying property P(E'|F) = 1 – P(E|F) where E and F are events of a sample space S of an experiment
Cognitive level	Application
Difficulty level	Medium
Marks	2
Time	3-5 minutes
Item stem	X and Y are the two events such that P(X|Y) = 0.2 and P(Y) = 0.5. Find the value of P(X' ∩ Y) showing your steps.

Marking Scheme

Step	Marks
Correctly identifies the formula or a mathematical relation to be used. a) Writes that P(X' ∩ Y) = P(X'|Y) × P(Y). (OR writes that $P(X' \cap Y) = P(Y) - P(X \cap Y)$)	0.5
b) Uses the property of conditional probability and simplifies the above equation as P(X' ∩ Y) = [1 - P(X|Y)] × P(Y). (OR $P(X' \cap Y) = P(Y) - P(X \cap Y) = P(Y) - P(Y) \times P(Y)$ as $P(X \cap Y) = P(Y) \times P(Y)$)	1

Correctly computes probability substituting appropriate values in a formula or a mathematical relation Substitutes the given values in the above equation to find P(X' ∩ Y) as 0.8 × 0.5 = 0.4. (OR $P(X' \cap Y) = P(Y) - P(Y) \times P(Y)$ = 0.5 – 0.2 ⍰ 0.5 = 0.4)	0.5

Content domain (Chapter name)	**Probability**
Content Domain Learning outcome	Applies the results and properties of conditional probability
Indicator	Finds the conditional probability of an event applying the property P((A ∪ B)|F) = P(A|F) + P(B|F) – P((A ∩ B)|F), where A, B and F are events of a sample space S
Cognitive level	Application
Difficulty level	Medium
Marks	3
Time	3-5 minutes
Item stem	In a clinic, out of 40 patients who are waiting to see the doctor: ● 24 have cold. ● 18 have fever. ● 9 have migraine. ● 4 have both cold and fever. ● 6 have both fever and migraine. ● 3 have both migraines and cold. ● 2 have all the three. (Note: Assume that each of the 40 patients present is suffering from one or more of the 3 ailments - cold, fever or migraine only.)

	If a patient is selected at random, what is the probability that he/she has a cold or fever or both, given that he/she has a migraine? Show your work.

Marking Scheme

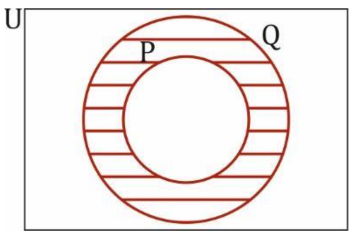
Step	Marks
Correctly model the real-life problem and represents mathematically. Assumes the set of patients having cold, fever and migraine to be C, F and M, respectively. Write that the required probability is given by $P(C \cup F \mid M) = P(C \mid M) + P(F \mid M) - P(C \cap F \mid M)$	0.5
Solves the mathematical problem by modelling the real-life problem correctly. a) Evaluates $P(C \mid M)$ as $\frac{P(C \cap M)}{P(M)} = \frac{3}{9}$.	0.5
b) Evaluates $P(F \mid M)$ as $\frac{P(F \cap M)}{P(M)} = \frac{6}{9}$.	0.5
c) Evaluates $P(C \cap F \mid M)$ as $\frac{P(C \cap F \cap M)}{P(M)} = \frac{2}{9}$.	0.5
d) Finds the probability that a patient selected at random has cold or fever, provided he/she has migraine as: $\frac{3}{9} + \frac{6}{9} \quad \frac{2}{9} = \frac{7}{9}$ Reports the required probability as $\frac{7}{9}$. (Award 1.5 marks if the problem is solved correctly without explicitly writing the formula in step 1.)	1

GRADE 11

Chapter 1 Sets

Chapter name	**Sets**
Essential Idea	Representation of a set in different forms, knowing and finding different types of sets and performing operations on sets
Item stem	If P ⊂ Q, then draw a Venn diagram representing Q – P.

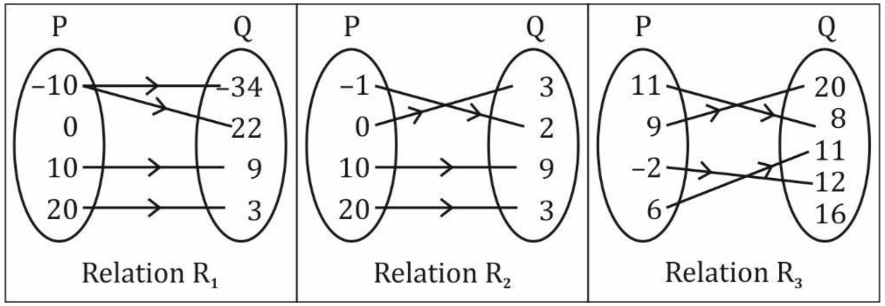
Marking Rubric

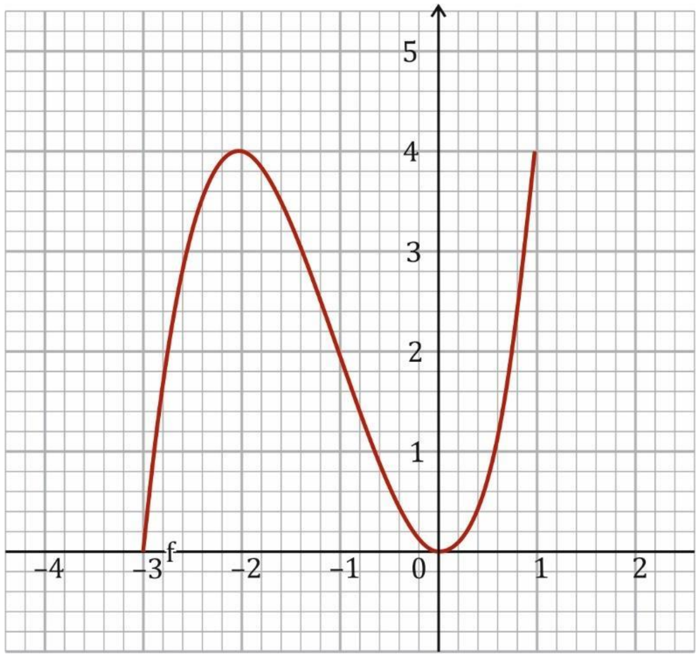
Description	**Marks**
Writes that P must be completely contained within the set Q.	0.5
Q – P shall contain all elements of Q that are not contained in P already.	0.5
Draws the correct Venn diagram as 	1

Chapter Name	Sets	
Essential Idea	There is a relation between complement of union of sets and intersection of complements of sets. (*The relation referred here is named as De Morgan's law.*) and applying that	
Item Stem	S and T are proper subsets of the universal set U. Which of these sets is **sufficient** to find S' ∩ T'?	
Correct answer	U and S ∪ T	Reason: S' ∩ T' = (S ∪ T)' = U – (S ∪ T). Knowing only U and (S ∪ T) is sufficient to find the given set.
Distractor 1	S' and T	Explanation: No intermediate expansion of the set expression has both S' and T.
Distractor 2	S' ∪ T'	Explanation: No intermediate expansion of the set expression has S' ∪ T.
Distractor 3	U and S ∩ T	Explanation: The final expansion of the given set expression has S ∪ T, not S ∩ T.

Chapter 2 Relations and Functions

Chapter Name	Relations and Functions
Essential Idea	Functions are a particular type of relations and demonstration of understanding of components of a function like domain, codomain, image, pre-image, range etc.
Item Stem	Which of the following relation/mapping diagram(s) represents a function? Share reasons why a given relation is a function or not.

	 A. only R_2 B. only R_1 and R_2 C. only R_2 and R_3 D. all – R_1, R_2 and R_3	
Correct answer	only R_2 and R_3	Reason: R_2 and R_3 are functions since every element of their domain is mapped uniquely to an element of their respective co-domains.
Distractor 1	only R_2	Explanation: R_1 does not represent a function since the element -10 in domain is mapped to two different elements in the co-domain.
Distractor 2	only R_1 and R_2	Explanation: R_2 is a function but R_1 is not.
Distractor 3	all – R_1, R_2, and R_3	Explanation: R_2 and R_3 are functions but R_1 is not.

Chapter name	**Relations and Functions**
Essential Idea	Graph of a function describes a functional relationship well and aids reasoning about the function *(Drawing and interpreting graph of a function is an essential skill.)*
Item stem	The figure below shows the graph of a function f defined in the domain $-3 \le x \le 1$.  1. Give any 2 points (as coordinate pairs) on the graph of the function. 2. What value does the function map 1 to? (In other words, what is $f(1)$)? 3. Find the range of the function.

Marking Rubric	
Description	**Marks**
Part 1: Writes any two ordered pairs (x, y) from the graph. For e.g. (-3, 0) or (0, 0) or (1, 4) etc.	1

Part 2: Writes that $f(1) = 4$ since graph's value at $x = 1$ is 4.	1
Part 3: Writes the range of the function $f(x)$ as [0, 4]	1

Chapter 3 Trigonometric Functions

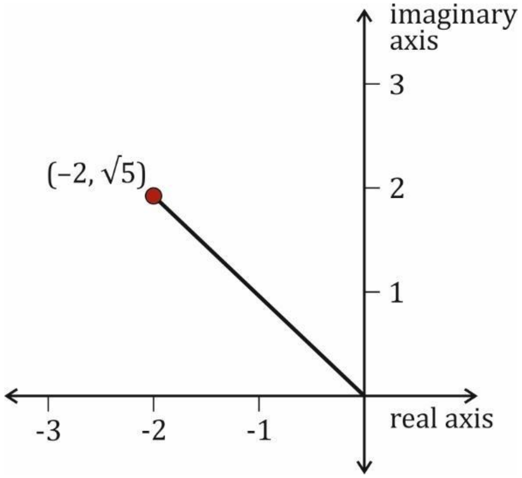
Chapter Name	**Trigonometric Functions**	
Essential Idea	Trigonometric functions are real functions which relate an angle of a right triangle to ratios of two side lengths, with a defined range and domain and thus different from trigonometric ratios though related. Their values repeat in specific intervals of domain and periodicity is defined based on that. (*Knowing domain, range, graph, and periodicity of each of the 6 trigonometric functions is an essential skill.)*	
Item Stem	Which of these is the same as sin 560°? A. -sin 20° B. -cos 20° C. sin 20° D. cos 20°	
Correct answer	-sin 20°	Reason: sin 560° = sin (360° + 180° + 20°) = sin (180° + 20°) = -sin 20° Since the sine function is negative in third quadrant.
Distractor 1	-cos 20°	Explanation: sin(180° + 20°) ≠ - cos 20°
Distractor 2	sin 20°	Explanation: sin(180° + 20°) ≠ sin 20°
Distractor 3	cos 20°	Explanation: sin(180° + 20°) ≠ cos 20°

Chapter name	Trigonometric Functions
Essential Idea	Trigonometric identities wherever functions are defined • $\cos(x + y) = \cos x \cos y - \sin x \sin y$ • $\sin(x + y) = \sin x \cos y + \cos x \sin y$ • $\tan(x + y) = \frac{tantan\, x + tantan\, y}{1 - tanxtantan\, y}$ • $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{\tan^2 x}{\tan^2 x}$ • $\sin 2x = 2\sin x \cos x = \frac{2tantan\, x}{\tan^2 x}$
Item stem	If $\sin x = \frac{1}{3}$, where $0 < x < \frac{\pi}{2}$, find the value of $\cos 2x$.

Marking Rubric

Description	Marks
Identifies appropriate mathematical result or formula to apply Writes $\cos 2x = 1 - 2\sin^2 x$	1
Applies the appropriate mathematical result or formula and correctly computes the required value. Substitutes $\sin x = \frac{1}{3}$ in the equation from step 1 to find $\cos 2x$. $\cos 2x = 1 - \frac{2}{9} = \frac{7}{9}$	1

Chapter 5 Complex numbers and quadratic equations

Chapter Name	**Complex numbers and quadratic equations**	
Essential Idea	The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a, b, c \in R$, $a \neq 0$, $b^2 - 4ac < 0$, are given by $x = \frac{-b \pm (\sqrt{4ac - b^2})i}{2a}$	
Item Stem	One of the roots of a quadratic equation $x^2 + mx + 9 = 0$ $(m > 0)$ is shown in the complex plane below.  What is the other root of the quadratic equation?	
Correct answer	$-2-\sqrt{5}i$	Reason: The complex roots are conjugates of each other.
Distractor 1	$2-\sqrt{5}i$	Explanation: Only the imaginary part has the sign reversed in the conjugate, not both real and imaginary parts.
Distractor 2	$\sqrt{5}+2i$	Explanation: Only the imaginary part has the sign reversed in the conjugate, not the real parts.
Distractor 3	$\sqrt{5}-2i$	Explanation: The conjugate of a complex number does not interchange the real and complex parts.

Chapter name	Complex numbers and quadratic equations
Essential Idea	Product of two complex numbers $z_1 = a + ib$ and $z_2 = c + id$, $z_1z_2 = (ac - bd) + i(ad + bc)$. It satisfies the properties similar to that of the multiplication of real numbers. The identities for squares and cubes which holds true for real numbers also holds for the complex number e.g. $(z_1 + z_2)^2 = z_1^2 + z_2^2 + 2z_1z_2$
Item stem	Verify, $(4 - 3i)(4 + 3i) = 25$. Using this relation $(4 - 3i)(4 + 3i) = 25$, find the complex number z such that $(4 - 3i)\cdot z = 1$.

Marking Rubric	
Description	**Marks**
Finds the product applying the definition or the identity $(a + ib)(a - ib) = a^2 + b^2$ correctly as 25.	1
Manipulates the equation correctly as $(4 - 3i)(4 + 3i) = 25$ ® $(4 - 3i)(4 + 3i)/25 = 1$ ® $(4 - 3i)(\frac{4}{25} + \frac{4}{25} i) = 1$	1
Finds the required complex number. So, $z = (\frac{4}{25} + \frac{4}{25} i)$ as $(4 - 3i)\cdot z = 1$.	1

Chapter 6 Linear Inequalities

Chapter Name	Linear Inequalities	
Essential Idea	A solution of an inequality in one variable is the value(s) of the variable which makes the inequality true.	
Item Stem	If $p < (-2)$, which of these can be $(- p)$?	
Correct answer	5	Reason: Since $p < -2$, we can conclude that $- p > 2$.
Distractor 1	- 5	Explanation: It can be a valid value for p, instead of $-p$.

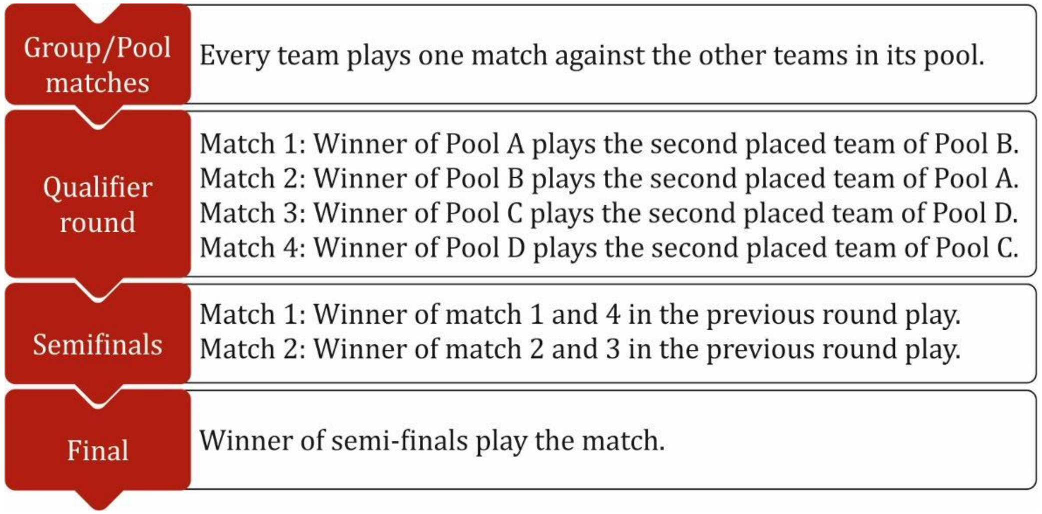
Distractor 2	- 1	Explanation: Student multiplied -1 on only one side of the inequality to get $-p > -2$.
Distractor 3	2	Explanation: Student did not understand that $-p > 2$ does not include $p = 2$.

Chapter Name	**Linear Inequalities**	
Essential Idea	A solution of an inequality in one variable is the value(s) of the variable which makes the inequality true.	
Item Stem	$-2y < 1000$, for some number y. **Assertion (A)**: y could be (-600). **Reasoning (R)**: $-2y < 1000$ ® $-y < 500$ ® $y < (-500)$	
Correct answer	Both A and R are false.	Reason: $-2y < 1000$ ® $-y < 500$ ® $y > (-500)$. $(-600) < (-500)$, so y cannot be (-600).
Distractor 1	A is true and R is the correct explanation of A.	Explanation: Student think the sign can be interchanged without reversing the inequality or multiplying both sides of inequality by -1 does not change the inequality.
Distractor 2	A is false but R is true.	Explanation: Student think the sign can be interchanged without reversing the inequality or multiplying both sides of inequality by -1 does not change the inequality. Additionally, she thinks the solution must be a set and hence A is false. Alternatively, she may be thinking $(-600) > (-500)$ and hence A is false.
Distractor 3	A is true and R is not the correct explanation of A.	Explanation: Student could reason out correctly as $-2y < 1000$ ® $-y < 500$ ® $y > (-500)$. She thinks $(-600) > (-500)$. (Students may be struggling to order integers correctly.)

Chapter name	Linear Inequalities
Essential Idea	The solution to a system of linear inequality is the region where the graphs of all linear inequalities in the system overlap. Some of the real-life problems can be solved by modelling them using linear inequalities and solving the problem mathematically.
Item stem	Rahul is asked to buy a common gift and 11 identical bouquets for the members of the school hockey team. ● He has the budget of ₹ 1000 to be spent completely. ● Bouquets can be purchased for any price from ₹ 20 onwards. ● He would like to spend at least ₹ 600 on the gift. Taking ₹ g to be the cost of the common gift, express the price range for the common gift in the form of an inequality.

Marking Rubric	
Description	**Marks**
Models the given real-life problem mathematically. (a) Writes that the minimum value of g is 600, so one part of the inequality would be $600 \leq g$	0.5
(b) Writes that the maximum amount that Rahul can spend on the gift is $1000 - 11 \times 20 =$ ₹ 780. That is, $g \leq 780$	1
(c) Combines the two inequalities as $600 \leq g \leq 780$	0.5
Finds the unknown Mentions the price-range for the common gift is ₹ 600 to ₹ 780.	0.5

Chapter Name	**Permutations and Combinations**	
Essential Idea	A permutation is an arrangement in a definite order of a number of objects taken some or all at a time whereas a combination refers to several ways of choosing items from a large set of objects, such that their order does not matters. A permutation is an ‘ordered combination.’	
Item Stem	4 cards from a pack of 52 playing cards can be selected in $^{52}C_4$ ways. A pile is made with these 4 cards placing cards one over another. Piles with the same 4 cards arranged in different order are considered as different piles. For example, piles of the same 4 cards in figure 1 and figure 2 are different as the order of the cards is different in them. Figure 1  Figure 2  In how many different ways can a pile can be made using any 4 cards from a pack of 52?	
Correct answer	$4!\,^{52}C_4$	Reason: 4 cards can be chosen in $^{52}C_4$ ways. These four cards can then be arranged in 4! Different ways to get different piles.
Distractor 1	$^{52}C_4$	Explanation: The student did not consider the ordering of the cards for different piles.
Distractor 2	$2!\,^{52}C_4$	Explanation: The student incorrectly calculated the number of ways 4 selected cards could be arranged.
Distractor 3	$3!\,^{52}C_4$	Explanation: The student incorrectly calculated the number of ways 4 selected cards could be arranged.

Chapter name	Permutations and Combinations
Essential Idea	A permutation is an arrangement in a definite order of a number of objects taken some or all at a time whereas a combination refers to several ways of choosing items from a large set of objects, such that their order does not matters. A permutation is an 'ordered combination.'
Item stem	16 teams participate in a world cup of cricket. The teams are placed into four pools (Pool A, B, C and D) of four teams each. Matches in the tournament are played in the following manner.  i.How many matches are played altogether? ii.How many matches does the winner in the final match play in the tournament totally?

Marking Rubric

Description	Marks
Models the given real-life problem mathematically and finds the required unknown quantity.	1

Part i: (a) Calculates that ${}^4C_2$ number of matches will be played in each group for the first stage, and hence, the total number of group/pool matches would be $4.{}^4C_2 = 24$	
Computes and reports the total number of matches played in the tournament as 31. (b) Adds all matches from the qualifier, semi-finals and final to get 24 + 4 + 2 + 1 = 31 matches as the answer.	1
Models the given real-life problem mathematically and finds the required unknown quantities. Part ii: (a) Writes that the final winner would have played following number of matches at each stage: Group/pool stage: 3 matches (one against each other team in their group) Qualifies stage: 1 match Semi-final stage: 1 match Final stage: 1 match	1
(b) Computes the required unknown quantity correctly So, total number of matches played = 3+1+1+1 = 6 matches	0.5
(c) Reports the total matches played by the final winner as 6 matches.	0.5

Chapter 8 Binomial Theorem

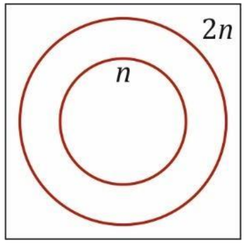
Chapter Name	**Binomial Theorem**	
Essential Idea	For any real numbers x, y, and a natural number n, $(x + y)^n$ can be expressed as sums of powers of x and y and is a more general case of an identities for $(x + y)^2$ and $(x + y)^3$. The identity is commonly known as Binomial theorem and it states the principle to expand $(x + y)^n$ as sums of powers of x and y (*Expanding algebraic expressions using Binomial theorem and its application is an essential skill.)*	
Item Stem	What is the term independent of x (constant term) in the expansion of $\left(\pi x + \frac{1}{\pi x}\right)^{100}$?	
Correct answer	${}^{100}C_{50}$	Reason: The n^{th} term in the binomial expansion is ${}^{100}C_n(\pi x)^n.(1/\pi x)^{100-n}$. The term independent of x should have same exponent for both parts. Hence, n = 50.

Distractor 1	$^{100}C_{48}$	Explanation: The understanding of the coefficients in binomial expansion is incorrect.
Distractor 2	$^{100}C_{49}$	Explanation: The understanding of the coefficients in binomial expansion is incorrect.
Distractor 3	$^{100}C_{100}$	Explanation: Incorrectly assumes that the last term will be independent of *x.*

Chapter name	**Binomial Theorem**
Essential Idea	For any real numbers x, y, and a natural number n, $(x + y)^n$ can be expressed as sums of powers of x and y and is a more general case of an identity for $(x + y)^2$ and $(x + y)^3$. The identity is commonly known as the Binomial theorem, and it states the principle to expand $(x + y)^n$ as sums of powers of x and y (*Expanding algebraic expressions using the Binomial theorem and its application is an essential skill.)*
Item stem	Find the coefficient of t^2 in the expanded form of $(2 + t)^4(1 + \frac{1}{t})$

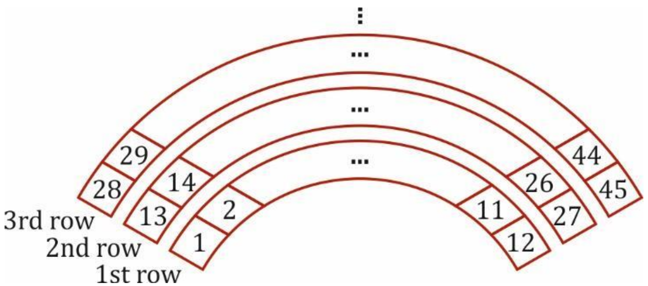
Marking Rubric	
Description	**Marks**
Manipulates the given mathematical relation appropriately. Rewrites the expression as $(2 + t)^4 + \frac{1}{t}(2 + t)^4$	0.5
Finds the appropriate mathematical result or relation to find the unknown Writes that the coefficient should be $2^2\ {}^4C_2 + 2\ {}^4C_1$	1
Finds the required unknown quantity. Simplifies to get the coefficient as 24 + 8 = 32.	0.5

Chapter 9 Sequences and Series

Chapter Name	**Sequences and Series**	
Essential Idea	Arithmetic patterns can be of several types based on the nature of the general rule to generate a pattern. The general term of two such types of patterns, arithmetic progressions and geometric progressions can be expressed as a relationship between the first term and the common difference or common ratio. (*An essential skill is to identify if a given sequence is an arithmetic progression or geometric progression and accordingly find any of the terms in the sequence or the sum of the first n terms in word problems based on real-life context or any mathematical problem.*)	
Item Stem	A large amphitheatre has a seating arrangement in concentric circles with the stage at the centre. The number of seats in each outer circle is double the number of seats in the corresponding inner circle as shown in the figure.  The first circle (the innermost circle) has 20 seats. How many seats will be there in total if there are 10 circles of seats?	
Correct answer	20,460	Reason: The sum of n terms of a GP is given by $S_n = a \times \frac{r^n - 1}{r - 1}$. For n = 10, a = 20 and r = 2, the sum will be $20 \times \frac{2^{10} - 1}{2 - 1} = 20 \times 1023 = 20{,}460$
Distractor 1	1,100	Explanation: Incorrectly identifies the sequence as an AP and finds the sum of n terms of AP.
Distractor 2	10,220	Explanation: Incorrectly writes the sum of n terms of GP as $S_n = a \times \frac{r^{n-1} - 1}{r - 1}$
Distractor 3	10,240	Explanation: Writes the nth term of GP instead of the sum of n terms.

<table>
<tr><th>Chapter name</th><th>Sequences and Series</th></tr>
<tr><td>Essential Idea</td><td>Arithmetic patterns can be of several types based on the nature of general rules to generate a pattern. The general term of two such types of patterns, arithmetic progressions and geometric progressions can be expressed as a relationship between the first term and the common difference or common ratio.

(An essential skill is to identify if a given sequence is an arithmetic progression or geometric progression and accordingly find any of the terms in the sequence or the sum of the first n terms in word problems based on real-life context or any mathematical problem.)</td></tr>
<tr><td>Item stem</td><td>(Case or context) The seats in an auditorium are numbered in numerical order from the first row to the last row, and from left to right as shown in the figure. The first row has 12 seats. Each succeeding row has 3 more seats than the previous one.



1) List the first 6 terms of a sequence of the number of seats in row 1, row 2, row 3 etc. Do the number of seats in row 1, row 2, row 3, … form an arithmetic progression?

2) Do the seat numbers of the last seat in each row form an arithmetic progression?

3) In the 3rd row, the first seat number is 28 and the last seat number is 45. What will the last seat number in the 10th row of the auditorium be?

4) What will the last seat number in the nth row of the auditorium be?</td></tr>
</table>

Marking Rubric	
Description	**Marks**
Part 1 (a) **Models the number of seats in each row mathematically as a sequence**. Writes the 6 terms as – 12, 15, 18, 21, 24, 27	0.5

(b) **Proves/Justifies the sequence is A.P with appropriate reasoning.** Demonstrates that the sequence is an A.P. since the common difference between the number of seats in each subsequent row is 3 (constant).	0.5
Part 2: Writes that the last seat number in rows do not form an AP with appropriate reason. The last seat numbers in rows in order are 12, 27, 45… and the differences between consecutive terms are 15, 18, 21… which is not constant. Hence the sequence of the last seat numbers is not in A.P.	0.5
Part 3: **Identifies the appropriate mathematical result or formula to be applied and finds the required unknown** The last seat number in the 10^{th} row will be the sum of the number of seats in the first 10 rows = $\frac{10}{2}(2 \times 12 + 9 \times 3) = 5 \times 51 = 255$	0.5
Part 4: (a) **Models the given real-life situation mathematically and finds the required unknown.** Writes that the last seat number in the *n*th row will be the sum of a number of seats in the first *n* rows. The number of seats in each row, 12, 15, 18, 21… forms an A.P. with a = 12 and d = 3. Expresses the required sum is the sum of the first n terms of an AP applying $S_n = \frac{n}{2}(2a + (n - 1)d)$ where a = 12 and d = 3. Applies the formula and expresses the sum of the first n terms as $\frac{n}{2}(2 \times 12 + (n - 1) \times 3)$ (b) **Finds the required unknown.** The last seat number in the nth row = $\frac{n}{2}(21 + 3n)$	1 0.5 0.5 0.5

Chapter 10 Straight Lines

Chapter Name	**Straight lines**
Essential Idea	Two lines are parallel if and only if their slopes are equal. Two lines are perpendicular if and only if the product of their slopes is -1.
Item Stem	What is the slope of the line perpendicular to $5x + 4y = 10$?

Correct answer	$\frac{4}{5}$	Reason: The slope of the given line is $m = \frac{-5}{4}$. The line perpendicular to this line would have a slope of -$1/m$. Hence, the perpendicular line's slope will be $\frac{4}{5}$.
Distractor 1	$\frac{-5}{4}$	Explanation: Student selects the slope of the given line.
Distractor 2	$\frac{-4}{5}$	Explanation: Student incorrectly applies the relation $m_1m_2 = 1$ instead of $m_1m_2 = -1$.
Distractor 3	$\frac{5}{4}$	Explanation: Student incorrectly applies the relation $m_1/m_2 = -1$ instead of $m_1m_2 = -1$.

Chapter name	**Straight lines**
Essential Idea	Various forms of an equation of a line: ● Equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ ● Equation of the line passing through (x_1, y_1) and slope m: $y - y_1 = m(x - x_1)$ ● Equation of the line with slope m and y-intercept c: $y = mx + c$ ● Equation of the line with x-intercept a and y-intercept b: $\frac{x}{a} + \frac{y}{b} = 1$ ● Equation of the line having the normal distance from origin p and angle between normal and the positive x-axis ω is given by $x\cos\omega - y\sin\omega = p$ ● General form of the equation of a line: Ax + Bx + C = 0, where A and B are not zero simultaneously
Item stem	Express the equation $5x + 4y = 10$ in intercept form. Hence find the slope, y-intercept, and x-intercept of the line.

Marking Rubric

Description	**Marks**

Divides both sides of the equation $5x + 4y = 10$ by 10 to get, $\frac{x}{2} + \frac{y}{2.5} = 1$ or such equivalent form	1
Comparing the intercept form of the given equation with the general form $\frac{x}{a} + \frac{y}{b} = 1$, arrives as x-intercept $a = 2$ and y-intercept $b = 2.5$.	1
Converts the equation in intercept form to slope-intercept form as $y = -\frac{5}{4}x + 2.5$ ($y = mx + c$ form) and arrives at the slope as $m = -\frac{5}{4}$	1

Chapter 11 Conic sections

Chapter Name	**Conic Sections**	
Essential Idea	The distance of any point on the curve of a parabola, ellipse, or a hyperbola, from its focus and the distance of the point from its directrix are in the same ratio (known as eccentricity). ● For an ellipse the ratio is less than 1. ● For a parabola the ratio is 1. ● For a hyperbola the ratio is greater than 1.	
Item Stem	Any point on the curve $y^2 + 2x = 0$ is equidistant from the point P and the line l. What are the coordinates of P and the equation of the line l?	
Correct answer	P($-\frac{1}{2}$, 0) and l: $x = \frac{1}{2}$	Reason: For the parabola $y^2 = -2x$, value of $a = -1/2$ and the centre is at origin (0, 0). Hence the focus is (-1/2, 0) and directrix line is $x = ½$.
Distractor 1	P(0, -1) and l: $y = 1$	Explanation: Incorrectly recalls the standard equation of parabola as $x^2 = 2ay$.
Distractor 2	P($\frac{1}{2}$, 0) and l: $x = -\frac{1}{2}$	Explanation: Incorrectly rewrites the parabola equation as $y^2 = 2x$.
Distractor 3	P(-1, 0) and l: $x = 1$	Explanation: Incorrectly recalls the standard equation of parabola as $y^2 = 2ax$.

Chapter name	Conic Sections
Essential Idea	(see table below)

Conic section	Equation	Eccentricity (e)	Focus	Semi-latus rectum
Ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a,b,>0)$	$\sqrt{1-\frac{b^2}{a^2}}$	$(\sqrt{a^2-b^2},0)$ & $(-\sqrt{a^2-b^2},0)$	$\frac{b^2}{a}$
Parabola	$y^2=4ax$	1	$(a,0)$	$2a$ (if $a>0$)
Hyperbola	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\ (a,b,>0)$	$\sqrt{1+\frac{b^2}{a^2}}$	$(\sqrt{a^2+b^2},0)$ & $(-\sqrt{a^2+b^2},0)$	$\frac{b^2}{a}$

Item stem	The equation of a conic section is given by $\frac{x^2}{144}+\frac{y^2}{25}=1$. 1. What type of conic is this? 2. Find the eccentricity and the latus rectum of the conic.

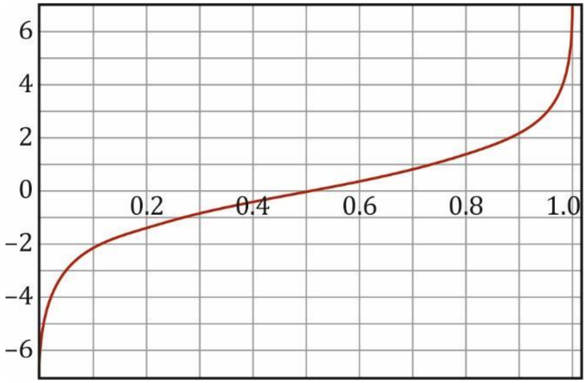
Marking Rubric

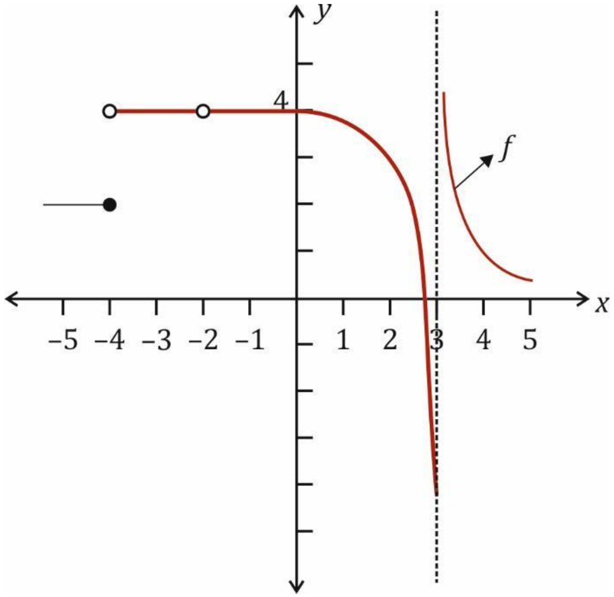
Description	Marks
Part 1: Identifies the conic section as an ellipse.	1
Part 2: Writes that for an ellipse in the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, the eccentricity is given by $\sqrt{1-\frac{b^2}{a^2}}\ and\ finds\ it\ as\ \sqrt{\frac{119}{144}}$	1

Writes that for an ellipse in the form $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$, the latus rectum is given by $\frac{2b^2}{a}$ *and finds it as* $\frac{25}{6}$	1

Chapter 12 Introduction to three-dimensional geometry

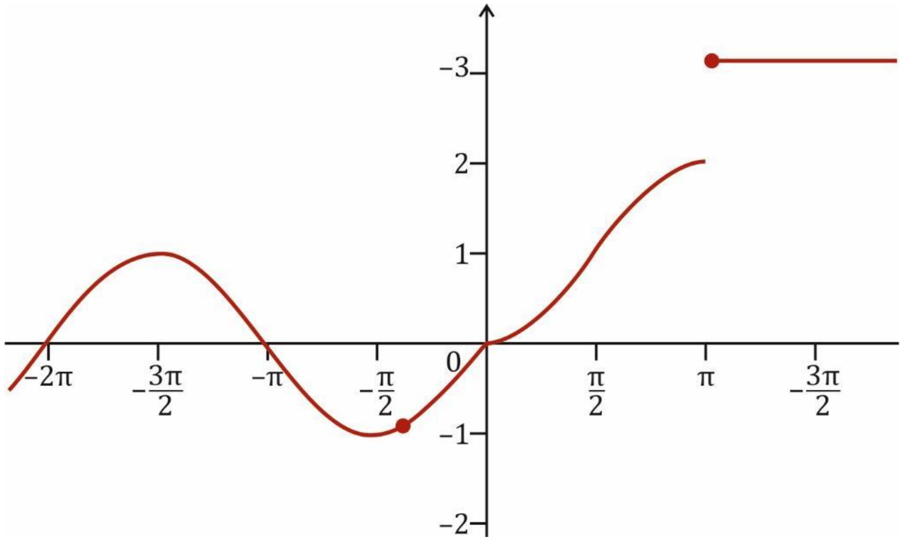
Chapter Name	**Introduction to 3-d geometry**	
Essential Idea	The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the 3-d coordinate system is given by $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.	
Item Stem	P(1, *b*, 3) is a point in the 4th octant in a 3-d space. P is at a distance of 5 units from Q(1, 0, 0). What is *b*?	
Correct answer	-4	Reason: The distance between the two points results in the equation $(1-1)^2 + b^2 + 3^2 = 5^2$, solving which given $b = \pm 4$. Since P is in 4th octant, that would make the y-coordinate negative. Hence, b = -4.
Distractor 1	-2	Explanation: Incorrectly writes distance formula as (1-1) + b + 3 = 5 and writes -2 since P is in 4th octant.
Distractor 2	2 44	Explanation: Incorrectly writes distance formula as (1-1) + b + 3 = 5.
Distractor 3	4	Explanation: Student did not pay attention to the 4th octant part of the question.

Chapter Name	**Limits and Derivatives**	
Essential Idea	The derivative of a function is a special case of a limit of a function. It quantifies the change in function $f(x)$ at point a in its domain as, $\frac{f(a+h)-f(a)}{h}$. Geometrically it is the slope of the tangent to graph of the function $f(x)$ at a.	
Item Stem	Study the graph of the function f and answer the question based on the graph.  At which of these points (a) in the domain of the function is the limit $\frac{f(a+h)-f(a)}{h}$ maximum?	
Correct answer	0.99	Reason: The given limit is the expression for the first order derivative, which is also the slope of the tangent to the curve (or slope of the curve). The slope is maximum at a point close to 1.
Distractor 1	0.4	Explanation: Incorrectly selects the point where the graph crosses the x-axis.
Distractor 2	0.69	Explanation: Does not understand the concept of limits and derivatives.
Distractor 3	0.8	Explanation: Does not understand the concept of limits and derivatives.

Chapter Name	**Limits and Derivatives**	
Essential Idea	Limit of a function at a point may exist even if the function is not defined at that point.	
Item Stem	Shown here is a graph of a function *f* not defined at x = -2 and x = 3.  **Assertion (A)**: $f(x)$ does NOT exist. **Reason (R)**: *f*(-2) is not defined.	
Correct answer	A is false but R is true. R is not the explanation for A.	Reason: $f(x)$ = 4 and thus exists though function is not defined at x = (-2). Both the left-hand limit and the right-hand limit exists and are the same. Hence the limit also exists at x = -2.
Distractor 1	Both A and R are true. R is the correct explanation of A.	Explanation: Students think the limit of a function at a given point cannot exist if the function is not defined at that point (a potential misconception).

Distractor 2	A is true but R is false.	Explanation: Students think the limit of a function does not exist. They may have arrived at different right-hand and left-hand limit at x = -2. Since according to them left-hand limit and/or right-hand limit exists, they think the function is defined.
Distractor 3	Both A and R are false.	Explanation: Students may have found the left-hand limit and the right-hand limit as the same and hence deduced that the limit of the function exists at x = -2. They may be thinking since the limit exists, the function is also defined at x = -2.

Chapter Name	**Limits and Derivatives**	
Essential Idea	Recognizing indeterminate form of a limit of a function at a point and applying appropriate strategies to find limit of the function accordingly when they exist. Knowing that the limit of a function of indeterminate form may exist.	
Item Stem	Find the limit: $\frac{sin\,(2021x)}{2021x}$	
Correct answer	$\frac{sin\,(2021)}{2021}$	Reason: $\lim_{x\to 1} sin\,sin\,2021x$ = sin 2021 (limit exists and is non-zero). $\lim_{x\to 1} 2021x = 2021$ (non-zero). Applying the algebra of limits (quotient of two functions), $\frac{f(x)}{g(x)}$ = $f(x)/\,g(x)$
Distractor 1	1	Explanation: Inappropriately applies the result considering the given limit as an indeterminate form. $\frac{sinsin\,x}{x}$ = 1
Distractor 2	sin 1	Explanation: Inappropriately cancels out 2021x considering as a common factor: $\frac{sin\,(2021x)}{2021x}$ = $\frac{sin\,1}{1}$ = sin 1
Distractor 3	$\frac{sin\,(20211)}{20211}$	Explanation: Merely substitutes x with 1 blindly.

Chapter name	**Limits and Derivatives**
Essential Idea	For a function $f(x)$, as x in its domain gets closer and closer to a, the value l to which $f(x)$ gets closer and closer to is the limit of the function $f(x)$ as x tends to a. Hence, the limit of the function as x tends to a is same as the left-hand limit and the right-hand limit.
Item stem	For a piece-wise function f defined as follows: $f\,(x) \; = \; \{sin\; sin\, x \; : \; x \; < \; 0 \; 1 \; -cos\; cos\; (x) \; : 0 \leq \times \leq 3.14 \; 3.14 \; : \; otherwise$ Its graph is as below.  Based on the graph, i. $f(x)$ tends to as x tends to 0 from the left. (*Fill in the blank and share reason for your answer*) ii. $f(x)$ tends to as x tends to 0 from the right. (*Fill in the blank and share reason for your answer*) iii. $f\,(x)$ = _____. (*Fill in the blank and share reason for your answer*) iv. *Does* $f\,(x)$ exist? Justify your answer.

Marking Rubric	
Description	**Marks**
Part i: writes 0 as $sin\ x\ = 0$	0.5
Part ii: writes 0 as $(1\ -\ cos\ x)\ = 0$	0.5
Part iii: writes 0 as left-hand limit and right-hand limits are equal.	0.5
Part iv: $f(x)$ does not exist because the left-hand limit = 2 and right-hand limit = 3.14 are different.	0.5

Chapter 15 Statistics

Chapter name	**Statistics**
Essential Idea	The computation of standard deviation, the most used measure of dispersion for a given set of data using an appropriate method
Item stem	242 students took a test of Mathematics consisting of four questions. The maximum possible marks were 24. The following sums were calculated to obtain some statistics. Sum of the scores = $S_x = \sum x_i = 3805$ and Sum of the squares of the scores = $T_x = \sum x_i^2 = 62741$, where x_i is marks scored by the i^{th} student $(1 \leq i \leq 242)$ Compute the mean and the standard deviation of the marks obtained.

Marking Rubric	
Description	**Marks**
Correctly identifies the formula of mean to be applied and computes the mean. Computes mean as $\frac{\sum xi}{n} = \frac{3805}{242} = 15.72$	1

Correctly identifies the formula of the standard deviation and computes the s.d. Writes the formula for standard deviation as $\sqrt{\frac{\Sigma\ (x_i-\underline{x})^2}{n}} = \sqrt{\frac{\Sigma\ \ x_i{}^2+n\underline{x}^2-2\underline{x}\Sigma\ \ x_i}{n}} = \sqrt{\frac{62741+59826-119653}{242}} \approx 3.47$	1

Chapter Name	**Statistics**
Essential Idea	Computing mean/median of grouped as well as ungrouped data
Item Stem	Mr. Joshi is calculating the mean of the scores/marks of 88 students in his school who took a test for the Science scholarship.

The frequency distribution of marks scored is found in the table below.

Marks (x)	$0 \leq x < 10$	$10 \leq x < 20$	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$
Frequency (f)	6	16	24	25	17

To calculate mean, he arrives at the following table in a spreadsheet.

Marks	Mid Interval Value (x)	F	fx	x^2	fx^2
$0 \leq x < 10$	5	6	30	25	150
$10 \leq x < 20$	15	16	240	225	3600
$20 \leq x < 30$	25	24	600	625	15000
$30 \leq x < 40$	35	25	875	1225	30325
$40 \leq x < 50$	45	17	765	2025	34425
Total		**88**	**2510**		**83800**

	What is the mean of the marks of these students?	
Correct answer	$\frac{2510}{88}$	Correctly computes mean as $\bar{x} = \sum \frac{fx}{n}$
Distractor 1	$\frac{125}{5}$	Takes mean of 5 mid interval values
Distractor 2	$\frac{2510}{50}$	Incorrectly takes n as 50 (the highest possible score)
Distractor 3	$\frac{83800}{88}$	Not clear about the formula for mean of grouped data

Chapter 16 Probability

Chapter Name	**Probability**
Essential Idea	Probability of an event can be interpreted as a real-valued function on power set of a sample space of a random experiment, satisfying certain axioms. (Axiomatic definition of probability of an event and its application to find probability of an event) Let S be the sample space of a random experiment. The probability P is a real-valued function with domain as the power set of S and range as the interval [0, 1] satisfying the following axioms. i) For any event, $P(E) \geq 0$. ii) $P(S) = 1$ iii) If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$.

Item Stem	A pack of cards contains 10 cards of club and 10 cards of hearts. Two cards are selected at random **with** replacement. Let C denote the outcome that the card drawn is of a club and H denote the outcome that the card drawn is of hearts. Which of the following could be the valid probability values of the outcomes in the sample space?	
Correct answer	A	Reason: The probability of each event is ¼ as the probability of drawing C is ½ and probability of drawing H is also ½. C and H are independent events.
Distractor 1	B	Explanation: Does not consider the total number of cards and the probabilities do not add up to 1.
Distractor 2	C	Explanation: Incorrectly adds the probabilities, and the probabilities do not add up to 1.
Distractor 3	D	Explanation: Incorrect understanding of probability, assumes that a total of 100 outcomes are possible.

Options	**P(CH)**	**P(CC)**	**P(HC)**	**P(HH)**
A.	¼	¼	¼	¼
B.	½	½	½	½
C.	$\frac{2}{20}$	$\frac{2}{20}$	$\frac{2}{20}$	$\frac{2}{20}$
D.	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$

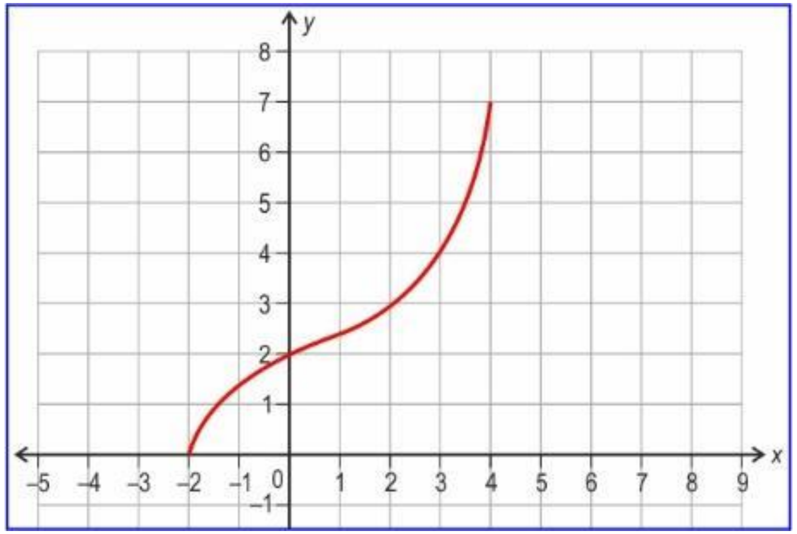
Chapter name	**Probability**
Essential Idea	(Axiomatic definition of probability of an event and its application to find probability of events) Let S be the sample space of a random experiment. The probability P is a real-valued function with domain as the power set of S and range as interval [0, 1] satisfying the following axioms. i) For any event, P(E) ≥ 0. ii) P(S) = 1

	iii) If E and F are mutually exclusive events, then P(E ∪ F) = P(E) + P(F).
Item stem	A pack of cards contains 10 cards of club and 10 cards of hearts. Two cards are selected at random **with** replacement. What is the sample space associated with the random experiment?) Let C denote the outcome of card being drawn as club and H denote the outcome of card being drawn as hearts. Which of the following could be the valid probability values of the outcomes in the sample space? <table><tr><th></th><th>P(CH)</th><th>P(CC)</th><th>P(HC)</th><th>P(HH)</th></tr><tr><td>**A.**</td><td>¼</td><td>¼</td><td>¼</td><td>¼</td></tr><tr><td>**B.**</td><td>½</td><td>½</td><td>½</td><td>½</td></tr><tr><td>**C.**</td><td>2/20</td><td>2/20</td><td>2/20</td><td>2/20</td></tr><tr><td>**D.**</td><td>1/100</td><td>1/100</td><td>1/100</td><td>1/100</td></tr></table>i) Find the probability of the event that exactly one of the two cards drawn is of hearts.

Marking Rubric

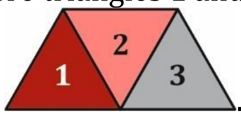
Description	**Marks**
Part i: Writes the sample space as = {CC, CH, HC, HH} where C denotes a club card and H denotes a hearts card.	1
Part ii: Writes that the favourable outcomes for the given event are CH and HC. Therefore, the probability is P(CH) + P(HC) = 0.5	1

Chapter 1 Relations and Functions

Chapter name	Relations and Functions
Essential Idea	Examination of a function being one-one and/or onto; identification of the subset of the domain in which the function is one-one and the subset of codomain in which it is onto.
Item stem	Shown here is a graph of a function f: [-2, 4] → [0, 7]  It can be seen from the graph that, $f(-2) = 0$, and $f(0) = 2$. Is the function one-one? Is the function onto?

Marking Rubric

Description	Marks
Identifies that the function is one-one with valid reasons.	0.5
Identifies that the function is onto with valid reasons.	0.5

Chapter Name	**Relations and Functions**	
Essential Idea	Identification of a relation as reflexive, symmetric and transitive; and identification if a relation is an equivalence relation	
Item Stem	Here are some relations defined on a set of all triangles. The relation R_1 on the set A of all scalene triangles as $R_1 = \{(T_1, T_2): T_1$ is similar to $T_2\}$ The relation R_2 on the set B of all triangles as $R_2 = \{(T_1, T_2)$: Area of T_1 is same as the area of $T_2\}$ The relation R_3 on the set C of all equilateral triangles as $R_3 = \{(T_1, T_2): T_1$ shares a common side with $T_2\}$ Which of these relations is an equivalence relation? A. only R_1 B. only R_2 C. both R_1 and R_2 D. both R_1 and R_3	
Correct answer	both R_1 and R_2	Triangles T_1 and T_2 are similar and if triangles T_2 and T_3 are similar, then the triangles T_1 and T_3 are also similar. So, R_1 is an equivalence relation. In particular, the relation of congruence of triangles (like similarity) is also an equivalence relation. Congruent triangles are of equal areas. So, R_2 is also an equivalence relation.
Distractor 1	only R_1	Cannot not find R_2 as equivalence relation.
Distractor 2	only R_2	Cannot not find R_1 as equivalence relation.
Distractor 3	both R_1 and R_3	Fails to see that R_3 is not a transitive relation. Here triangles 1 and 2 are related, triangles 2 and 3 are related but triangles 1 and 3 are not related.  .

Chapter Name	**Inverse Trigonometric Functions**	
Essential I dea	The inverse of a trigonometric function exists, and the following table shows their principal value branches along with their domains and ranges. $\sin^{-1}$: [-1, 1] $\rightarrow$ $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ $\cos^{-1}$: [-1, 1] $\rightarrow$ $[0,\pi]$ cosec^{-1} : R - [-1, 1] $\rightarrow$ $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$ $\sec^{-1}$: R - [-1, 1] $\rightarrow$ $[0,\pi]\ \left\{\frac{\pi}{2}\right\}$ $\tan^{-1}$: R $\rightarrow$ $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ $\cot^{-1}$: R $\rightarrow$ $[0,\pi]$ Their graphs can be obtained from the graph of the corresponding trigonometric function interchanging the axes.	
Item Stem	If $\sin^{-1}x = \theta$ (the principal value branch of $\sin^{-1}$), where $0 \le x \le 1$, what is the range in which θ lies?	
Correct answer	$[0,\ \frac{\pi}{2}]$	Reason: The principal branch of $\sin^{-1}$ ranges from 0 to $\frac{\pi}{2}$ when x lies within [0,1].
Distractor 1	$[\frac{-\pi}{2},\ 0]$	Explanation: Student does not understand the principal branch range, selects 4th quadrant.
Distractor 2	$[-\pi,\ \frac{-\pi}{2}]$	Explanation: Student does not understand the principal branch range, selects 3rd quadrant.
Distractor 3	$[\frac{\pi}{2},\ \pi]$	Explanation: Student does not understand the principal branch range, selects 2nd quadrant.

Chapter name	Inverse Trigonometric Functions
Essential Idea	Identities or properties of inverse of trigonometric functions can be derived from the corresponding identities involving trigonometric functions. e.g., $\sin 2\theta = \frac{2tan\theta}{1+tan^2\theta}$, so this gives corresponding identity as $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}$.
Item stem	Find the value of $\sin\frac{3\pi}{2} - \sin(\sec^{-1}t + \mathrm{cosec}^{-1}t)$ where $\|t\| \geq 1$.

Marking Rubric	
Description	**Marks**
Writes that $\sec^{-1}t + \mathrm{cosec}^{-1}t = \frac{\pi}{2}$, for $\|t\| \geq 1$.	1
Simplifies the expression as $\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}$	0.5
Solves the expression to get -2 as answer.	0.5

Chapter 3 Matrices

Chapter Name	Matrices
Essential Idea	i. Two matrices of the same order can be added. The sum of two matrices $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ is $A + B = [a_{ij} + b_{ij}]_{m\times n}$. Hence the addition of two matrices is commutative as well as associative. ii. Any two matrices cannot be multiplied. The product of two matrices A and B is defined if the number of columns of A is same as the number of rows of B. $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{n\times p}$, then $AB = [c_{ik}]_{m\times p}$ where $c_{ik} = \sum_{j=1}^{n} a_{ij}b_{jk}$
Item Stem	Let $P = \begin{bmatrix} -1.1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $Q = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ and R = [1 10 2].

	Which of the following is defined?	
Correct answer	Only PQ	Reason: The order of matrix P is 3x3 and that of Q is 3x1. Two matrix of orders *m* x *n* and *p* x *q* can be multiplied only if *n* = *p.*
Distractor 1	Only PR	Explanation: The order of matrix P is 3x3 and that of R is 1x3. ($3 \neq 1$)
Distractor 2	Only QP	Explanation: The order of matrix Q is 3x1 and that of P is 3x3. ($1 \neq 3$)
Distractor 3	(*All the products PQ, PR and QP are defined.*)	Explanation: Assumes that any two matrices can be multiplied.

Chapter name	**Matrices**
Essential Idea	Inverse of a matrix exists only for square matrix and is unique. The product of a matrix and its inverse is the identity matrix.
Item stem	Let P = [0 1 1 0]. Then P^2 = [1 0 0 1]. Does P^{-1} exist? If yes, find P^{-1}.
Marking Rubric	

Description	**Marks**
Writes that P^{-1} exists.	0.5
Simplifies the expression to get, $P^2 = I$ Þ $P = P^{-1}$	0.5

Chapter 4 Determinants

Chapter Name	**Determinants**	
Essential Idea	Determinant of a square matrix is a unique real number associated with the square matrix and defined uniquely. E.g., for a 2 × 2 square matrix A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant of A, |A| = ad - bc	
Item Stem	Determinant of a non-singular matrix P of order 2 is 9. What is the determinant of adj P?	
Correct answer	9	Reason: Let P = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then, Adj P = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. |Adj P| = $ad - bc$ = |P| = 9.
Distractor 1	-9	Explanation: Assumes that |Adj P| = |P|
Distractor 2	$\frac{1}{9}$	Explanation: Assumes that |Adj P| = $\frac{1}{\|P\|}$
Distractor 3	$\frac{1}{9}$	Explanation: Assumes that |Adj P| = $\frac{1}{\|P\|}$

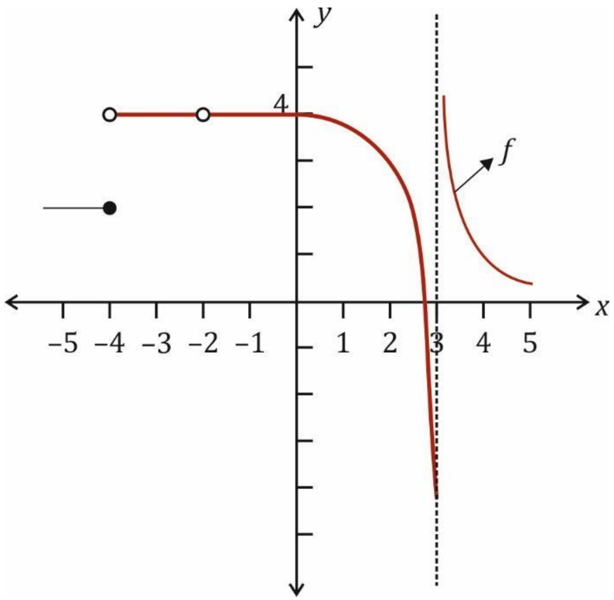
Chapter name	**Determinants**
Essential Idea	If A is non-singular matrix, A is invertible and $A^{-1} = \frac{1}{\|A\|}$ adj A.
Item stem	Determinant of a non-singular matrix P of order 2 is 12. Find the determinant of P^{-1}.

Marking Rubric

Description	**Marks**
Identifies the appropriate mathematical result or formula to be applied to find the required unknown. Writes that $\|P^{-1}\| = \frac{1}{\|P\|^2}$ |adj P| (Alternatively solves as $PP^{-1} = I$ ® | PP^{-1}| = 1)	0.5

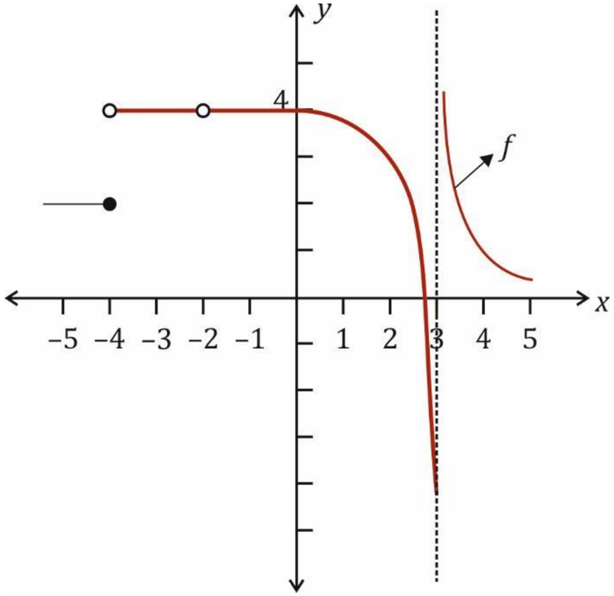
Writes that |adj P| = |P| for order 2 matrix. (Alternatively, |P| |P^{-1}| = 1)	0.5
Finds the required unknown. Solves to get |P^{-1}| = $\frac{1}{\|P\|} = \frac{1}{12}$ (Alternatively, |P^{-1}| = $\frac{1}{\|P\|} = \frac{1}{12}$)	1

Chapter 5 Continuity and Differentiability

Chapter Name	**Continuity and Differentiability**
Essential Idea	Continuity of a function $f(x)$ at a point a is a special form of a limit where $f(x)$ tends to the limit $f(a)$ as x tends to a. Hence all the properties of limit of a function holds for a continuous function. Examine the continuity of a function in a given interval by applying them.
Item Stem	The graph of a piece-wise function f defined in (-5, 5) – {3} is shown here.  (*It is asymptotic to the vertical line x = 3.*)

	Based on the graph, in which of the following intervals is the function continuous?	
Correct answer	(-5, 5) – {-4, 3}	Reason: x = -4 and x = 3 are points of discontinuity. Hence the function is continuous in the interval [-5, 5] – {-4, 3}.
Distractor 1	(-5, 5)	Explanation: Does not identify both points of discontinuity.
Distractor 2	(0, 5)	Explanation: Does not identify x = 3 as the point of discontinuity.
Distractor 3	(-5, 5) – {3}	Explanation: Does not identify x = -4 as the point of discontinuity.

Chapter name	**Continuity and Differentiability**
Essential Idea	A function differentiable at a point in the domain is also continuous at that point. The converse is not true.
Item stem	The graph of a piece-wise function f defined in [-5, 5] – {3} is shown here.

	 (*The curve is asymptotic to the vertical line x = 3 i.e., it does not intersect the vertical line x = 3 and extends to infinity as points get closer to x = 3.*) a. The function is differentiable in the interval (0, 3). Is the function continuous in (0, 3)? b. Is the function continuous at x = (-4)? Examine if the function is differentiable at x = (-4).

Marking Rubric

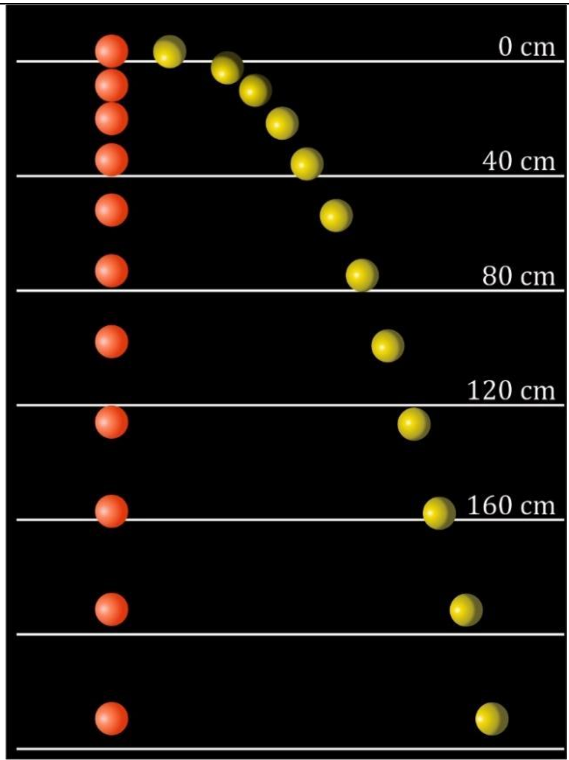
Description	**Marks**
Part a: Writes that the function is continuous in (0, 3) since it is differentiable in that interval.	1
Part b: Writes that function is discontinuous at x = -4 since the left-hand limit and right-hand limits are not the same.	0.5
Writes that function is not differentiable at x = -4 since it is discontinuous at that point.	0.5

Chapter name	Continuity and Differentiability
Essential Idea	Differentiation of a function applying the chain rule
Item stem	Apply the chain rule for the function $x^{1/4}$ and show that the power rule $\frac{dx^n}{dx} = nx^{n-1}$ works for n = ¼. Hint: $x^{1/4} = \sqrt{\sqrt{x}}$.

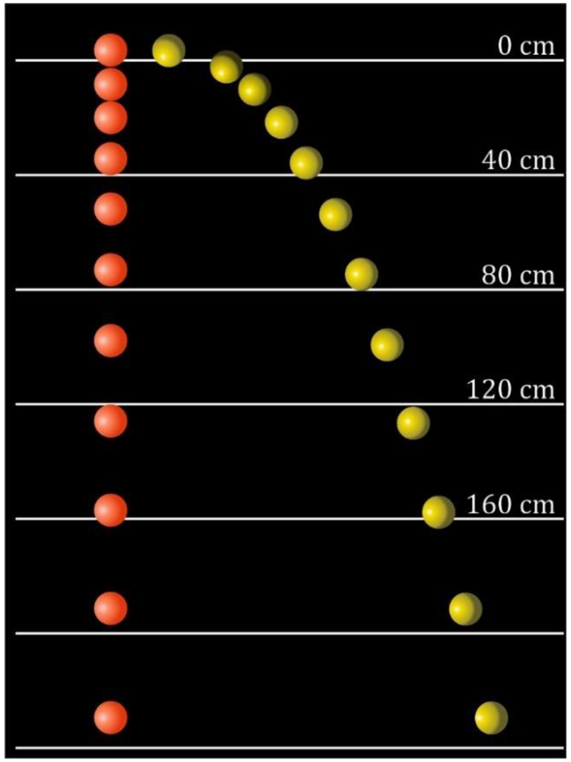
Marking Rubric	
Description	**Marks**
Applies chain rule on $x^{1/4} = \sqrt{\sqrt{x}}$ to get $\frac{d\sqrt{\sqrt{x}}}{dx} = \frac{d\sqrt{\sqrt{x}}}{d\sqrt{x}} \times \frac{d\sqrt{x}}{dx}$	1
Solves the expression to get $\frac{d\sqrt{\sqrt{x}}}{dx} = \frac{1}{2\sqrt{\sqrt{x}}} \times \frac{1}{2\sqrt{x}} = \frac{1}{4x^{\frac{3}{4}}}$	1
Rewrites the expression as following which is same as nx^{n-1} for n = ¼. $\frac{1}{4}x^{-\frac{3}{4}}$	1

Chapter 6 Applications of Derivatives

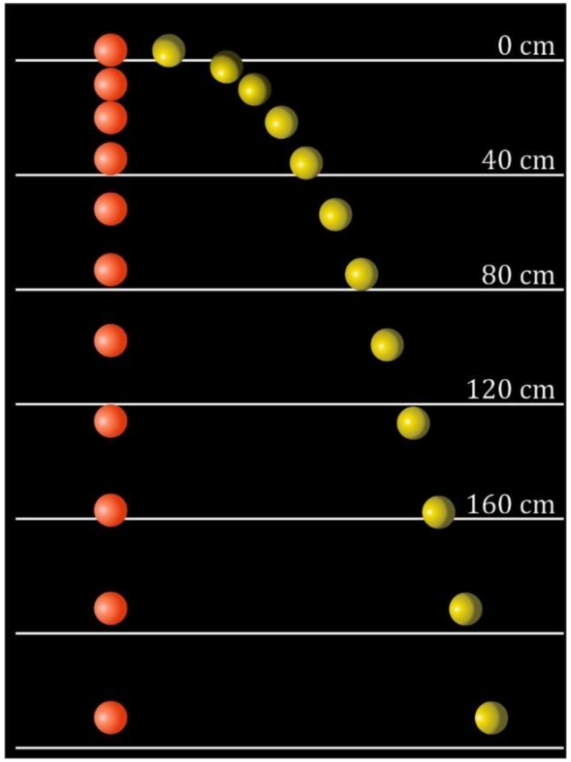
Chapter Name	Applications of Derivatives
Essential Idea	The derivative of a function can be used to: i. determine the rate of change of quantities. ii. find critical points and points of inflexion. iii. find if a function is increasing or decreasing at a point
Item Stem	Shown here is a multi-flash photograph of two balls falling from the rest from the same height released at the same time.

		Both the balls fall downward vertically at the same rate as also seen in the photograph. Let $s = 490t^2$ (the free fall equation) be the vertical distance (in cm) travelled by any of the two balls at time t seconds from the start. How fast were the balls falling (vertically downwards) when they reached 160-cm mark?
Correct answer	560 cm/s	Reason: Finds the time taken to cover 160 m as $t = 4/7$ s. Speed is calculated as $\frac{ds}{dt} = 490 \times 2t = 980\,t = 560$ cm/s
Distractor 1	160 cm/s	Explanation: Uses the familiar number from the question stem.
Distractor 2	280 cm/s	Explanation: Incorrectly differentiates s and misses multiplying by 2.
Distractor 3	320 cm/s	Explanation: Incorrectly calculates the time taken as 16/49 s.

A few case-based questions on the above case can also be framed. Sharing 2 such examples below.

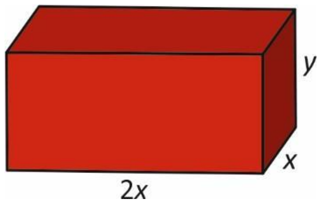
Chapter Name	**Applications of Derivatives**	
Essential Idea	The derivative of a function can be used to: i. determine the rate of change of quantities. ii. find critical points and points of inflexion. iii. find if a function is increasing or decreasing at a point	
Item Stem		Shown here is a multi-flash photograph of two balls falling from the rest from the same height released at the same time. Both the balls fall downward vertically at the same rate as also seen in the photograph. Let $s = 490t^2$ (the free fall equation) be the vertical distance (in cm) travelled by any of the two balls at time t seconds from the start. The horizontal acceleration of both the balls is 0 m/s^2. The horizontal distance (gap) between the two balls keeps increasing. Which of these is true about the rate at which the horizontal distance (gap) between the two balls changes?
Correct answer	remains the same	Reason: The 2nd ball (yellow ball) moves with the constant velocity in the horizontal direction as acceleration in the horizontal direction is 0.

Distractor 1	increases continuously	Explanation: The gap increases so the rate at which the distance increases is also increasing.
Distractor 2	decreases continuously	Explanation: (maybe randomly answering)
Distractor 3	(*Cannot be said from the given information*)	Explanation: Since the actual value of the horizontal velocity for the 2nd ball (yellow ball) is not explicitly given, students might think they cannot answer without knowing the actual horizontal velocity.

<table>
<tr><th>Chapter Name</th><th colspan="2">Applications of Derivatives</th></tr>
<tr><td>Essential Idea</td><td colspan="2">The derivative of a function can be used to:
i. determine the rate of change of quantities.
ii. find critical points and points of inflexion.
iii. find if a function is increasing or decreasing at a point</td></tr>
<tr><td>Item Stem</td><td colspan="2">Shown here is a multi-flash photograph of two balls falling from the rest from the same height released at the same time.


Both the balls fall downward vertically at the same rate as also seen in the photograph.
Let $s = 490t^2$ (the free fall equation) be the vertical distance (in cm) travelled by any of the two balls at time t seconds from the start. The horizontal acceleration of both the balls is 0 m/s^2.
Which of these is true about the vertical distance travelled by the yellow ball?
The ball covers _____ distance vertically within each second as it continues to fall.</td></tr>
<tr><td>Correct answer</td><td>More</td><td>Reason: The ball with vertical speed given by $\frac{ds}{dt} = 490 \times 2t = 980t$ cm/s. The speed increases with time. So, the ball covers more vertical distance in each second as it continues to fall vertically.</td></tr>
</table>

Distractor 1	Less	Explanation: Since the ball will come to a halt, students might reason that the distance travelled each second (speed) is decreasing with an increase in time.
Distractor 2	Equal	Explanation: Since the fall is shown for equal distances, students might reason it is falling vertically at the same rate.
Distractor 3	(Cannot be said from the given information)	Explanation: Since the vertical speed is not explicitly given or because they cannot see differentiating s(*t*) w.r.t. time *t* gives vertical speed, students might think the question cannot be answered.

<table>
<tr><th>Chapter name</th><th>Application of derivatives</th></tr>
<tr><td>Essential Idea</td><td>First derivative and 2nd derivative of a function can help determine an extreme value(s) of an increasing or decreasing function. (The essential skill is an application of the first derivate test for increasing or decreasing functions and the first as well as the 2nd derivative test for extreme values.)</td></tr>
<tr><td>Item stem</td><td>A small cuboid box has a rectangular base of length 2x cm and width x cm. Its height is y cm.


(Figure not to scale)
The sum of the length, width and height is 18 cm. Find the value of x for which the volume of the box is maximum.</td></tr>
</table>

Marking Rubric

Description	**Marks**
Finds the relationship between dependent and independent variables correctly. (a) Writes the sum of length, breadth, and height as $2x + x + y = 18 \Rightarrow y = 18 - 3x$	0.5

(b) Writes the expression for volume as V = $2x^2y = 2x^2(18 - 3x) = 36x^2 - 6x^3$	1
Differentiates the relation correctly. Differentiates V w.r.t. x and equates it to 0 to get $72x - 18x^2 = 0$.	1
Finds x such $f'(x) = 0$ (x at which the first order derivative is 0) Solves the equation to get $x = 0$ and $x = 4$ (rules out 0 since the length x must be a positive number)	0.5
Finds the point of maximum with appropriate justifications. Finds the second derivative at $x = 4$ to conclude that it is the point of maximum.	1
Reports the volume of the box is maximum at $x = 4$	0.5

Chapter 7 Integrals

Chapter Name	**Integrals**
Essential Idea	Fundamental theorems of integral calculus and their application 1. Let $f(x)$ be a continuous function on $[a, b]$ and the area function be $A(x) = \int_a^x f(x)dx$. Then $A'(x) = f(x)$ for $\forall x \in [a, b]$. 2. If f is a continuous function on $[a, b]$ and F is an antiderivative of f, $\int_a^b f(x)dx = F(b) - F(a)$.
Item Stem	Shami, Amresh and David are asked to find $\int 2sinx\ cosx\ dx$ and using that, find the definite integral, $\int_0^{\frac{\pi}{4}} 2\ sinx\ cosxdx$. **Shami's method** (without his final answer): $\int 2sinx\ cosx\ dx = \int 2u\ du$ (taking $u = \sin x$) $= u^2 + c = \sin^2 x + c$ So, $\int_0^{\frac{\pi}{4}} 2\ sinx\ cosxdx = \sin^2\frac{\pi}{4} - \sin^2 0$.

	Amresh's method (without his final answer): $\int \quad 2sinx\ cosx\ dx = \int \quad -2v\ dv$ (taking v = cos$x$) $= -v^2 + c = -\cos^2 x + c$ So, $\int_0^{\frac{\pi}{4}} \quad 2\ sinx\ cosxdx = -\cos^2\frac{\pi}{4} - (-\cos^2 0) = \cos^2 0 - \cos^2\frac{\pi}{4}$. **David's method** (without his final answer): $\int \quad 2sinx\ cosx\ dx = \int \quad sin2x\ dx = \frac{-cos2x}{2} + c$ So, $\int_0^{\frac{\pi}{4}} \quad 2\ sinx\ cosxdx = \frac{-cos\frac{\pi}{2}}{2} - \frac{-cos0}{2} = \frac{1}{2}(\cos 0 - \cos\frac{\pi}{2})$. Whose method (and answers) are correct? Why? A. only Shami's B. only Amresh's C. only David's D. all – Shami's, Amresh's and David's	
Correct answer	D	Reason: All three methods are correct since they all apply valid substitution and result in ½ as the final answer.
Distractor 1	A	Explanation: Assumes that only one method of substitution is valid.
Distractor 2	B	Explanation: Assumes that only one method of substitution is valid.
Distractor 3	C	Explanation: Assumes that only one method of substitution is valid.

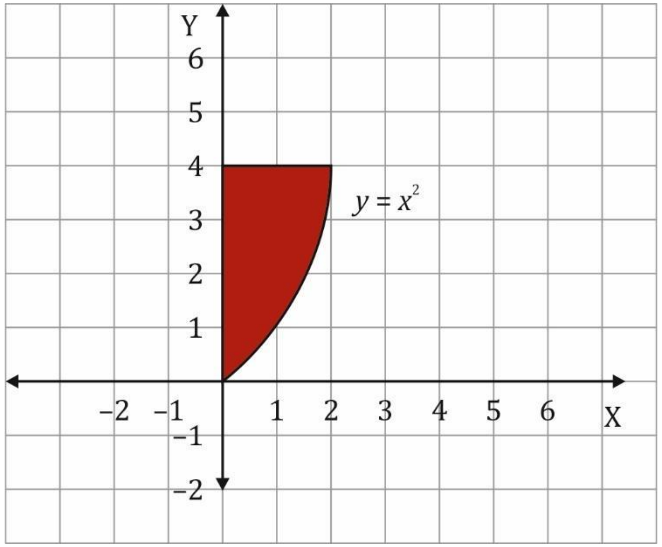
Chapter name	**Integrals**
Essential Idea	A function F(x) is an antiderivative of a function $f(x)$ if F'(x) = f(x) for all x in the domain of f.

Item stem	f and g are functions such that $f(x) = \frac{d}{dx}(2020 - \sqrt{x})$ and $g(x) = \frac{d}{dx}(x + 2020)$. Find $\int [f(x) + g(x)]dx$.

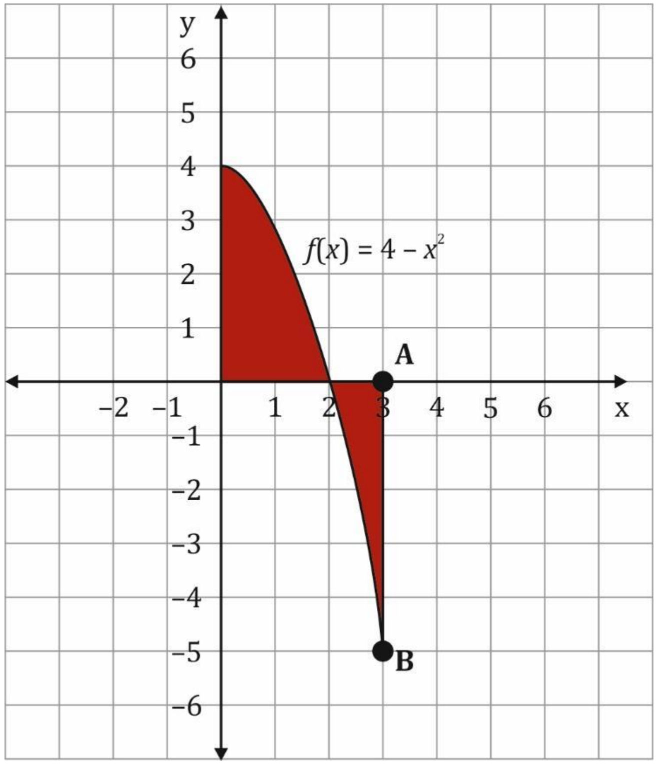
Marking Rubric	
Description	**Marks**
Rewrites the expression as $\int f(x)dx + \int g(x)dx$	0.5
Simplifies the expression to get $(2020 - \sqrt{x}) + (x + 2020) = 4040 - \sqrt{x} + x$ or such correct forms	0.5

Chapter 8 Application of Integrals

Chapter Name	**Application of Integrals**
Essential Idea	If $f(x) \geq 0$ is a continuous function on $[a, b]$, the area of the region between the graph of f and the x-axis is A = $\int_a^b f(x)dx$.
Item Stem	What is the area of the shaded region between the curve $y = x^2$, $0 \leq x \leq 2$ and Y-axis?

	 **A.** $\int_0^2 \quad x^2dx$ **B.** $\int_0^2 \quad \sqrt{y}dy$ **C.** $\int_0^4 \quad x^2dx$ **D.** $\int_0^4 \quad \sqrt{y}dy$

Correct answer	D	Reason: $y = x^2$ is rewritten as $x = \sqrt{y}$ and integrated from $y = 0$ to $y = 4$.
Distractor 1	A	Explanation: Does not realize that the shaded area in between the curve and y-axis, not x-axis.
Distractor 2	B	Explanation: Incorrectly applies the limits.
Distractor 3	C	Explanation: Does not realize that the shaded area in between the curve and y-axis, not x-axis, but applies correct limits.

Chapter name	Application of Integrals
Essential Idea	If $f(x) \geq 0$ is a continuous function on $[a, b]$, the area of the region between the graph of f and the x-axis is A = $\int_a^b f(x)dx$.
Item stem	Find the area of the shaded region between the curve, $y = 4 - x^2$, $0 \leq x \leq 3$ and the X-axis. 

Marking Rubric

Description	Marks

Identifies appropriate formula or mathematical relation/result to find the required unknown. Writes the required area is given by the absolute value of the integral I, where $$I = \int_0^2 (4 - x^2)dx + \int_2^3 -(4 - x^2)dx$$	1
Finds the required area correctly. Solves the integral to get $\frac{23}{3}$ and shares the required area as $\frac{23}{3}$ sq units	1
Note: If a student is expected to plot a graph and then find the area of the bounded region, then to use the standard equations as students can be expected to know plotting of standard forms of curves.	

Chapter 9 Differential equations

Chapter Name	**Differential equations**
Essential Idea	The family of functions satisfying the given differential equation is the general solution and a particular function satisfying the given differential equation is the particular solution.
Item Stem	As per the Newton's law of cooling, the rate at which an object cools is given by the following equation. $\frac{d\theta}{dt} = -k(\theta - \theta_s)$ where $\theta = \theta(t)$ is the temperature of the cooling object at time t, θ_s is the temperature of the environment (assumed to be constant) and k is a thermal constant related to the cooling object. If $\theta = \theta_0$ (initial temperature of the cooling object) at time $t = 0$, which of these is the particular solution of the given differential equation? A. $\theta(t) = \theta_s - (\theta_0 - \theta_s)$ B. $\theta(t) = \theta_s + (\theta_0 - \theta_s)e^{-kt}$ C. $\theta(t) = \theta_s - (\theta_0 - \theta_s)kt$ D. $\theta(t) = \theta_s - (\theta_0 - \theta_s)\sin kt$

Correct answer	B	Reason: The differential equation is written as $\frac{d\theta}{(\theta - \theta s)} = -kdt,$ integrating it to get $\theta(t) = \theta_s + (\theta_0 - \theta_s)e^{-kt}$ with initial condition.
Distractor 1	A	Explanation: Does not apply integral to find the solution.
Distractor 2	C	Explanation: Incorrectly applies the integral to solve differential equation.
Distractor 3	D	Explanation: Incorrectly applies the integral to solve differential equation.

Chapter name	**Differential equations**
Essential Idea	Based on the nature of a first order and first-degree differential equation, an appropriate method can be applied to find its general or particular solution. *(An essential skill is to find the general or the particular solution of a given first order, first degree differential equation applying appropriate method.)*
Item stem	As per the Newton's law of cooling, the rate at which an object cools is given by the following equation. $\frac{d\theta}{dt} = -k(\theta - \theta_s)$ OR $\frac{d\theta}{dt} + k\theta = k\theta_s$ where, $\theta = \theta(t)$ is temperature of the cooling object at time t, θ_s is the temperature of the environment (assumed to be constant), k is a thermal constant related to the cooling object. If $\theta = \theta_0$ (initial temperature of the cooling object) at time $t = 0$, find the particular solution of the given differential equation.
Marking Rubric	

Description	Marks
Solves the given differential equation and finds the general solution. (a) Writes the differential equation as $\frac{d\theta}{(\theta - \theta s)} = -kdt$	0.5
(b) Applies integration to find the general solution as $\theta(t) = \theta_s + Ce^{-kt}$	1.5
Substitutes correct values of variables in the general solution to find the particular solution. Applies initial condition $\theta_0 = \theta_s + C \Rightarrow C = \theta_0 - \theta_s$	0.5
Finds and shares the particular solution. Writes the particular solution as $\theta(t) = \theta_s + (\theta_0 - \theta_s)e^{-kt}$	0.5

Chapter 10 Vector Algebra

Chapter Name	**Vector Algebra**	
Essential Idea	A vector has both magnitude as well as direction. Any vector can be expressed as a sum of the component vectors along the x-axis, y-axis, and z-axis.	
Item Stem	Express the vector joining the two points A(0, 1, 2) and B(1, 4, 4) as the sum of the components along *x*, *y* and *z* axes using unit vectors $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$. *i.e.*, vector $\overrightarrow{AB} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$. A. $\hat{\imath} + 4\,\hat{\jmath} + 4\hat{k}$ B. $\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ C. $-\hat{\imath} - 3\hat{\jmath} - 2\hat{k}$ D. $\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$	
Correct answer	B	Reason: Subtracts the position vector of A from B to get vector $\overrightarrow{AB}$.

Distractor 1	A	Explanation: Selects B's position vector as the answer.
Distractor 2	C	Explanation: Subtracts B from A instead of A from B.
Distractor 3	D	Explanation: Adds the position vectors instead of subtracting them.

Chapter name	**Vector Algebra**
Essential Idea	There are two ways to find product of any 2 vectors, a scalar product (dot product) and vector product (cross product). The essential skill is understanding how the scalar product and vector product of two vectors are defined and the application of their properties.
Item stem	Consider the vectors $\vec{a} = 3\hat{\imath} + 2p\hat{\jmath}$ and $\vec{b} = (p+1)\hat{\imath} + 8\hat{\jmath}$. Find the possible values of p for which $\vec{a}$ and $\vec{b}$ are parallel.

Marking Rubric

Description	**Marks**
Finds the vector product of the two vectors as $(24 - 2p^2 - 2p)\,\hat{k}$	1.5
Writes that for a and b to be parallel, vector product should be equal to zero vector.	0.5
Solves the equation to find $p = -4$ and $p = 3$	1

Chapter name	Three-dimensional Geometry
Essential Idea	**Equations of lines in 3-d geometry**: (Forms of equations in 2-d cartesian systems are not generalizable to 3-d cartesian but vector equation form can be extended to 3-d cartesian system from 2-d). Knowing these forms of equations (vector forms, cartesian forms) and how to convert from one form to another is an essential skill.
Item stem	Fighter jets are flying in a formation for an aero show as shown in the figure. Taking their control tower as the reference point and the reference point being origin, the coordinates of two fighters in their flight path are A(10.5 km, 10 km, 1 km) and B (10 km, 10.5 km, 0.9 km). They are moving along the straight-line joining A and B at that point as seen in the figure.  What is the vector equation and the cartesian equation of the line passing through A and B?

Marking Rubric	
Description	**Marks**
Let *a* and *b* be the position vectors of A and B. $\vec{a} = 10.5\hat{\imath} + 10\hat{\jmath} + \hat{k}$ $\vec{b} = 10\hat{\imath} + 10.5\hat{\jmath} + 0.9\hat{k}$ $\vec{b} - \vec{a} = -0.5\hat{\imath} + 0.5\hat{\jmath} - 0.1\hat{k}$	0.5
Finds the vector equation: Let *r* be the position vector of a point on the line. Required equation: $\vec{r} = 10.5\hat{\imath} + 10\hat{\jmath} + \hat{k} + \lambda(-0.5\hat{\imath} + 0.5\hat{\jmath} - 0.1\hat{k})$	1
Finds the Cartesian equation: (*Use of vector equation* $\vec{b} - \vec{a} = -0.5\hat{\imath} + 0.5\hat{\jmath} - 0.1\hat{k}$ *to find the denominators in the Cartesian equation is recommended.*) Cartesian equation: $\frac{x-10.5}{10-10.5} = \frac{y-10}{10.5-10} = \frac{z-1}{0.9-1}$ $= \frac{x-10.5}{-0.5} = \frac{y-10}{0.5} = \frac{z-1}{-0.1}$	1

More questions using the given case (situation) can be framed.

Chapter name	**Three-dimensional Geometry**
Essential Idea	Direction cosines of a line are the cosines of the angles made by the line with the positive directions of the coordinate axes. **Direction cosines of a line joining two points** P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) are $\dfrac{x_2 - x_1}{PQ}, \dfrac{y_2 - y_1}{PQ}, \dfrac{z_2 - z_1}{PQ}$ where PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
Item stem	Fighter jets are flying in a formation for an aero show as shown in the figure. Taking their control tower as the reference point and the reference point being the origin, the coordinates of two fighters in their flight path are A (10.5 km, 10 km, 1 km) and B (10 km, 10.5 km, 0.9 km). They are moving along the straight line joining A and B at that point as seen in the figure.  1. What are the direction ratios and direction cosines of the line $AB^{\leftrightarrow}$? 2. What is the angle made by the line $AB^{\leftrightarrow}$ with the positive direction of z-axis? A. $\frac{-0.5}{\sqrt{0.51}}$ B. $\frac{0.5}{\sqrt{0.51}}$ C. $\frac{-0.1}{\sqrt{0.51}}$ D. (-0.1)

Marking Rubric		
Description		**Marks**
Let a and b be the position vectors of A and B. $\vec{a} = 10.5\hat{\imath} + 10\hat{\jmath} + \hat{k}$ $\vec{b} = 10\hat{\imath} + 10.5\hat{\jmath} + 0.9\hat{k}$ $\vec{b} - \vec{a} = -0.5\hat{\imath} + 0.5\hat{\jmath} - 0.1\hat{k}$ Direction ratios of the line AB are -0.5, 0.5, -0.1.		1
Direction cosines of the line AB are: $\frac{-0.5}{\sqrt{0.5^2 + 0.5^2 + 0.1^2}}, \frac{0.5}{\sqrt{0.5^2 + 0.5^2 + 0.1^2}}, \frac{-0.1}{\sqrt{0.5^2 + 0.5^2 + 0.1^2}}$ $\frac{-0.5}{\sqrt{0.51}}, \frac{0.5}{\sqrt{0.51}}, \frac{-0.1}{\sqrt{0.51}}$		1
Let the angle made by the positive direction of z-axis be f. $\frac{-0.1}{\sqrt{0.51}}$ $\emptyset = \frac{-0.1}{\sqrt{0.51}}$		1
Correct answer	C	Let the angle made by the positive direction of z-axis be f. $\frac{-0.1}{\sqrt{0.51}}$ $\emptyset = \frac{-0.1}{\sqrt{0.51}}$
Distractor 1	A	Explanation: Selects the direction cosine to find the angle made by the positive direction of x-axis instead of z-axis

Distractor 2	B	Explanation: Selects the direction cosine to find the angle made by the positive direction of y-axis instead of z-axis
Distractor 3	D	Explanation: Selects the direction ratio instead of direction cosine to find the angle made by the positive direction of z-axis

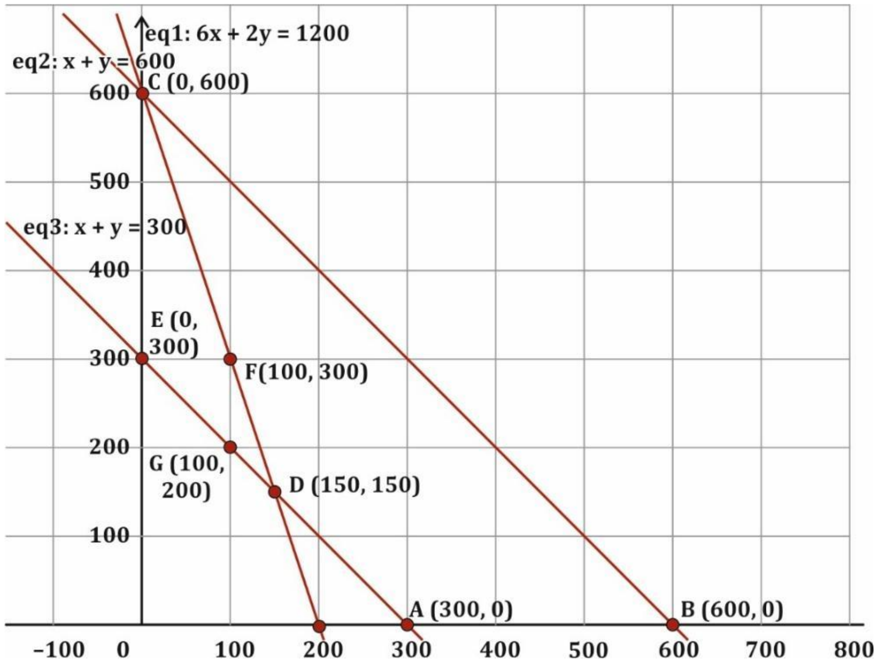
Chapter 12 Linear programming

<table>
<tr><th>Chapter Name</th><th>Linear Programming</th></tr>
<tr><td>Essential Idea</td><td>A linear programming problem in two variables is a special type of optimization problem where an objective function being optimized is a linear function in two variables and constrains are linear inequalities in two variables. (The essential skill is to identify the objective function and constraints and thereby represent a given linear programming problem mathematically.)</td></tr>
<tr><td>Item Stem</td><td>A mining company owns two different mines that produce ore which is graded in two classes: high and medium grade. The two mines have different operating characteristics as detailed below.
<table>
<tr><th rowspan="2">Mine</th><th rowspan="2">Cost per day (in lakhs)</th><th colspan="2">Production (tons/day)</th></tr>
<tr><th>High grade</th><th>Medium grade</th></tr>
<tr><td>X</td><td>105</td><td>5</td><td>3</td></tr>
<tr><td>Y</td><td>90</td><td>1</td><td>2</td></tr>
</table>
For a particular week, the company contracted to provide 10 tons of high-grade and 8 tons of medium-grade ore to a factory. The company wants to work out how many days per week it should operate each mine to fulfill the contract keeping the mining cost as low as possible.
Taking x to be the number of days per week that mine X is operated, and y to be the number of days per week mine Y is operated, what will be the objective function and constraints for this LPP (Assume $0 \le x, y \le 7$.)?
A. Objective function: $105x + 90y$; Constraints: $5x + y \ge 10$ & $3x + 2y \ge 8$
B. Objective function: $105x + 90y$, Constraints $5x + 3y \ge 10$ & $x + 2y \ge 8$ </td></tr>
</table>

	C. Objective function: $5x + 3y$, Constraints $5x + y \geq 105$ & $x + 2y \geq 90$ D. Objective function: $x + 2y$, Constraints $5x + 3y \leq 105$ & $x + 2y \leq 90$	
Correct answer	A	Reason: The objective function is to minimize the total cost, with minimum number of high-grade ores as 10 and minimum number of medium grade ore as 8. (As per the contract, the company must provide 10 tons of high-grade and 8 tons of medium grade. So, it must produce at least 10 tons of high-grade and 8 tons of medium grade.)
Distractor 1	B	Explanation: The constraints are incorrect with numbers from the table processed incorrectly.
Distractor 2	C	Explanation: The objective function is incorrect and constraints are also incorrect. Numbers taken from the table only.
Distractor 3	D	Explanation: The objective function is incorrect, and constraints are also incorrect. Numbers taken from the table only.

Chapter name	**Linear Programming**
Essential Idea	If the feasible region of a linear programming problem where Z = ax + by is an objective function and constraints are linear inequalities in x and y is bounded, then Z has both a maximum and minimum value in R and each of these occurs at a corner point of R.
Item stem	The linear programming problem is as follows: Minimize: $50x + 20y$ Subject to the constraints, $6x + 2y \geq 1200$ $300 \leq x + y \leq 600$, $x > 0$ and $y > 0$

Point	A (300, 0)	B (0, 600)	C (0, 600)	D (150, 150)	G (100, 200)
Value of objective function at the point	15000	30000	12000	10500	9000



(*Graph obtained using GeoGebra*)

(a) Find the values of x and y for which the value of the objective function $50x + 20y$ is the least. Justify your answer.

(b) Can the objective function have its minimum or maximum value at any of points E(0, 300), F(100, 300) or G(100, 200) and why?

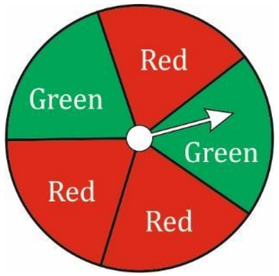
Marking Rubric

Description	**Marks**

(a) Mentions the objective function attains it optimum value at one of the 4 corner points A, B, C or D of the feasible region, quadrilateral ABCD in the graph. Identifies the point D (i.e., $x = y = 150$) as the optimal feasible solution of the given LPP specifying the value of the objective function is the least at D among the 4 corner points of the feasible region.	0.5 0.5
(b) Mentions the optimal feasible solution is one of the corner points of the solution region. Points E and G cannot be the optimal feasible solution of the given LPP as these two points are part of the infeasible region. Point F though a point in the feasible region is not one of the 4 corner points of the feasible region.	0.5

Chapter 13 Probability

<table>
<tr><th>Chapter Name</th><th>Probability</th></tr>
<tr><td>Essential Idea</td><td>Defining random variables appropriately to have their probability distribution similar to known probability distributions (e.g., Bernoulli distribution) helps to find probabilities of events of interest.
Finding the probability of an event using the probability distribution of a random variable</td></tr>
<tr><td>Item Stem</td><td>An unbiased circular spinner has a movable pointer and five equal sectors, two coloured green and three coloured red.



A game consists of spinning the wheel 5 times. Each time the spinner stops on green the player wins and receives one 10-rupee note, otherwise the player receives nothing. For example, if the outcome of one game is “green, red, green, red, red” the player wins two 10-rupee notes.
What is the probability of a player winning at least ₹40 in the game?
Note: If X = Amount (in Rs) earned by a player on spinning the wheel 5 times, then the probability distribution of the random variable X is as follows:
<table>
<tr><th>X</th><td>0</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td></tr>
<tr><th>P(X)</th><td>$(3/5)^5$</td><td>$5 \times (3/5)^4 \times (2/5)$</td><td>$10 \times (3/5)^3 \times (2/5)^2$</td><td>$10 \times (3/5)^2 \times (2/5)^3$</td><td>$5 \times (3/5) \times (2/5)^4$</td><td>$(2/5)^5$</td></tr>
</table>
</td></tr>
</table>

	A. $\frac{272}{3125}$ B. $\frac{162}{625}$ C. $\frac{16}{3125}$ D. $\frac{2}{5}$	
Correct answer	$\frac{272}{3125}$	Reason: P(Y = *4* successes) = ${}^5C_4\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^1 = \frac{240}{3125}$ and P(Y = *5* successes) = ${}^5C_5\left(\frac{2}{5}\right)^5 = \frac{32}{3125}$. P(Y ≥ 4 successes) = $\frac{240}{3125} + \frac{32}{3125} = \frac{272}{3125}$
Distractor 1	$\frac{16}{3125}$	Explanation: Student does not apply the combinations of the winning conditions.
Distractor 2	$\frac{162}{625}$	Explanation: Incorrectly applies the formula.
Distractor 3	$\frac{2}{5}$	Explanation: Does not understand the concept at all.

The above question can also be answered by another method. Each spin of the spinner is a Bernoulli trial. Taking Y = no. of 10-rupee notes won by a player in the game, the required probability can be obtained from the probability distribution of Y which is the binomial distribution.

Note: The other questions based on the case that can be used to assess the concept are as follows:

1. What is the probability that the spinner stops on the green in a spin?

   A. $\frac{1}{5}$

   B. $\frac{1}{2}$

   C. $\frac{2}{5}$

   D. $\frac{3}{5}$

2. Is each spin of the spinner a Bernoulli trial?
3. If X = no. of 10-rupee notes won by a player in the game, is the probability distribution of X the binomial distribution? If yes, what are its parameters?

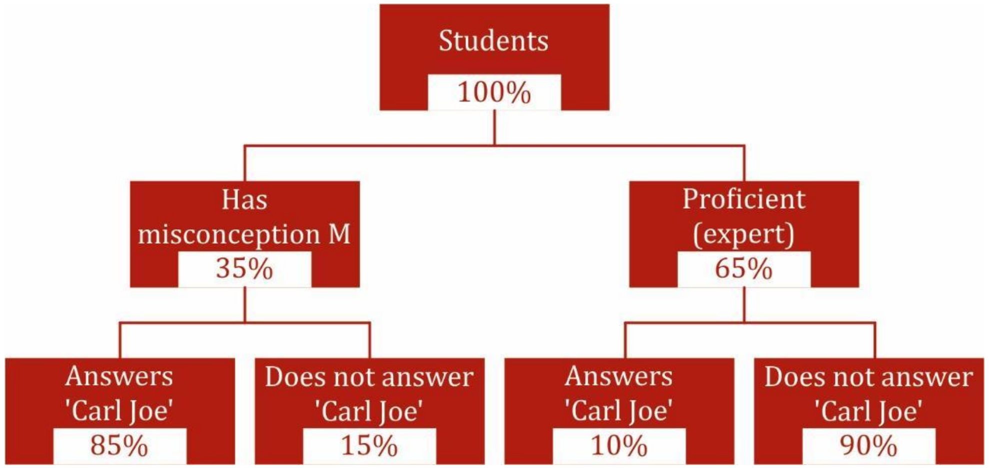
Chapter name	Probability
Essential Idea	Bayes theorem describes the probability of an event, based on prior knowledge of conditions that might be related to the event as follows: P(A|B) = $\frac{P(B\|A)P(A)}{P(B)}$ where A and B are events, P(B) ≠ 0. *(The essential skill is to interpret the result, identify it can be applied to find a given conditional probability of an event and apply it to find the conditional probability if applicable.)*
Item stem	Some students order decimals incorrectly due to flawed thinking. E.g., these students will say 6.25 > 6.3 as 625 > 63. Such students are said to have a misconception. Students are asked the following question in a test to diagnose if they have the said misconception (common error) M.

Question in a test: The table shows the time taken in seconds by each runner to finish a 100 m race. A runner who finishes the race in the shortest time wins the race. Who won the race?

Runner	Time taken to finish the race (in seconds)
Carl Joe	9.69
Baba Adams	9.069
Jay Armstrong	9.78
Balvan Pahelvan	9.078
Abbas Ali	11.407
Chin-Chin Chow	10.95
Huan Ki	11.013

A student answering as Carl Joe in the question maybe having the misconception/wrong notion M, "*9.69 < 9.069 as 969 < 9069*" (*comparing like whole numbers ignoring the decimal point*).

Based on the responses of students in the same test in the past, it is known that:



- 35% of students have the misconception M and the rest don’t (referred as proficient in the diagram).
- 85% of the students having the misconception M answers Carl Joe in the test question and
- 90% of students who are proficient don’t answer as Carl Joe. (Some students categorized as proficient may have answered as Carl Joe as a random guess or by mistake.)

a) What is the probability of a student answering as ‘Carl Joe’ in the given question of the test?

b) What is the probability of a student who answered as ‘Carl Joe’ in the question having the misconception M?

Marking Rubric

Description	**Marks**
Part a: Calculates the probability as P(“Carl Joe”) = 0.35×0.85 + 0.65×0.10 = 0.3625	1
Part b: Applies Bayes theorem to write the required probability as P(M | ”Carl Joe”) = $\frac{P(M\cap \text{"Carl Joe"})}{P(\text{"Carl Joe"})} = \frac{P(\text{"Carl Joe"} \mid M)\times P(M)}{P(\text{"Carl Joe"})}$	1
Substitutes the values in the equation to get P(M | ”Carl Joe”) = $\frac{0.85\times 0.35}{0.3625} = 0.82$	1

Note: Few more questions can be framed based on the above case.

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