Bridges for Exploration

(Link materials for Senior School Students)

- Mathematics
- Physics
- Chemistry
- Biology

CENTRAL BOARD OF SECONDARY EDUCATION

Shiksha Kendra, 2, Community Centre,
Preet Vihar, Delhi-110 301 India

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(Link materials for Senior School Students)

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भारत का संविधान

उद्देशिका

हम, भारत के लोग, भारत को एक सम्पूर्ण [1]प्रभुत्व-संपन्न समाजवादी पंथनिरपेक्ष लोकतंत्रात्मक गणराज्य बनाने के लिए, तथा उसके समस्त नागरिकों को:

सामाजिक, आर्थिक और राजनैतिक न्याय,
विचार, अभिव्यक्ति, विश्वास, धर्म
और उपासना की स्वतंत्रता,
प्रतिष्ठा और अवसर की समता
प्राप्त कराने के लिए
तथा उन सब में व्यक्ति की गरिमा
[2]और राष्ट्र की एकता और अखंडता
सुनिश्चित करने वाली बंधुता बढ़ाने के लिए

दृढ़संकल्प होकर अपनी इस संविधान सभा में आज तारीख 26 नवम्बर, 1949 ई० को एतद्द्वारा इस संविधान को अंगीकृत, अधिनियमित और आत्मार्पित करते हैं।

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1. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977) से "प्रभुत्व-संपन्न लोकतंत्रात्मक गणराज्य" के स्थान पर प्रतिस्थापित।
2. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977) से "राष्ट्र की एकता" के स्थान पर प्रतिस्थापित।

भाग 4 क

मूल कर्तव्य

51 क. मूल कर्त्तव्य – भारत के प्रत्येक नागरिक का यह कर्तव्य होगा कि वह –

(क) संविधान का पालन करे और उसके आदर्शों, संस्थाओं, राष्ट्रध्वज और राष्ट्रगान का आदर करे;

(ख) स्वतंत्रता के लिए हमारे राष्ट्रीय आंदोलन को प्रेरित करने वाले उच्च आदर्शों को हृदय में संजोए रखे और उनका पालन करे;

(ग) भारत की प्रभुता, एकता और अखंडता की रक्षा करे और उसे अक्षुण्ण रखे;

(घ) देश की रक्षा करे और आह्वान किए जाने पर राष्ट्र की सेवा करे;

(ङ) भारत के सभी लोगों में समरसता और समान भ्रातृत्व की भावना का निर्माण करे जो धर्म, भाषा और प्रदेश या वर्ग पर आधारित सभी भेदभाव से परे हों, ऐसी प्रथाओं का त्याग करे जो स्त्रियों के सम्मान के विरुद्ध हैं;

(च) हमारी सामासिक संस्कृति की गौरवशाली परंपरा का महत्त्व समझे और उसका परिरक्षण करे;

(छ) प्राकृतिक पर्यावरण की जिसके अंतर्गत वन, झील, नदी, और वन्य जीव हैं, रक्षा करे और उसका संवर्धन करे तथा प्राणी मात्र के प्रति दयाभाव रखे;

(ज) वैज्ञानिक दृष्टिकोण, मानववाद और ज्ञानार्जन तथा सुधार की भावना का विकास करे;

(झ) सार्वजनिक संपत्ति को सुरक्षित रखे और हिंसा से दूर रहे;

(ञ) व्यक्तिगत और सामूहिक गतिविधियों के सभी क्षेत्रों में उत्कर्ष की ओर बढ़ने का सतत प्रयास करे जिससे राष्ट्र निरंतर बढ़ते हुए प्रयत्न और उपलब्धि की नई उंचाइयों को छू ले;

[1](ट) यदि माता-पिता या संरक्षक है, छह वर्ष से चौदह वर्ष तक की आयु वाले अपने, यथास्थिति, बालक या प्रतिपाल्य के लिये शिक्षा के अवसर प्रदान करे।

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1. संविधान (छयासीवां संशोधन) अधिनियम, 2002 की धारा 4 द्वारा प्रतिस्थापित।

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the[2] unity and integrity of the Nation;

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949, do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

---

1. Subs, by the Constitution (Forty-Second Amendment) Act. 1976, sec. 2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs, by the Constitution (Forty-Second Amendment) Act. 1976, sec. 2, for "unity of the Nation" (w.e.f. 3.1.1977)

THE CONSTITUTION OF INDIA

Chapter IV A

FUNDAMENTAL DUTIES

ARTICLE 51A

Fundamental Duties - It shall be the duty of every citizen of India-

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wild life and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

[1](k) who is a parent or guardian to provide opportunities for education to his/her child or, as the case may be, ward between age of 6 and 14 years.

---

1. Subs. by the Constitution (Eighty - Sixth Amendment) Act, 2002

ACKNOWLEDGEMENT

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PREFACE

Central Board of Secondary Education prescribes curriculum for secondary and senior school level which advocates preclusion of specialization of subjects at classes IX – X. The curriculum of Science at classes IX – X, therefore, includes basic knowledge of all the subjects in the domain of Natural Sciences and similar is the case in Mathematics. However senior school curriculum demands specialization particularly in subjects Physics, Chemistry, Biology and Mathematics, because of which students suddenly feel a steep hike in the quantum and difficulty level of content.

Many of the students, especially those studying in the schools of remote areas find themselves unprepared to cope with a curriculum which demands high levels of rigor. Apropos to this mismatch, many of the students drop their dream of pursuing Sciences in senior school level even though they possess scientific aptitude and temper. Due to untenable difference in the levels, it was realized that there is a need for link courses to orient students to the requirement of subject.

The CBSE with a rich experience to its credit in handling the educational needs of millions of learners over the last several decades has to respond to this emerging need and hence took the initiative to develop such link courses initially in four subjects for sciences i.e. Mathematics, Physics, Chemistry and Biology.

This Link Material is aimed to act as a buffer for the new entrants in senior school curriculum, with an objective to provide adequate preparation for the transition to hardcore specialization of subjects. This gives them a breather, to groom themselves before senior school. By learning these concepts, the students will be equipped with the knowledge and confidence needed to take on bigger challenges in the course of learning.

This is a self learning module with appropriate technological inputs to further enhance and enrich learning and will be available as online and offline resource. Duration of the course in each subject is of 10 hours. The material incorporates related activities with different topics to engage learners in active learning. The material in every subject is also self evaluative in nature. With the related worksheets for self assessment after every topic, learners will be able to assess their progress in grasping various concepts and may decide to move ahead or revisit.

This material can also be used by teachers to connect students with their previous knowledge and for bringing them to the expectations of the present curriculum, and it may be referred during class transactions as and when required.

I appreciate the sincere efforts of Ms. Sugandh Sharma, Additional Director (I&R), CBSE and her team for bringing out this material.

I hope that the readers will find this material useful and would be able to use it in a meaningful way. Any suggestions for further improvement are always welcome.

Y. S. K. Seshu Kumar

Chairman, CBSE

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MATHEMATICS

Chapter 1 : Factorization and Quadratic Formula

Chapter 2 : Partial Fractions

Chapter 3 : Sets, Intervals, Absolute Value Function and Inequalities

Chapter 4 : What is Function?

Chapter 5 : Trigonometry

CHAPTER 1

FACTORIZATION AND QUADRATIC FORMULA

Dear Reader

There are large numbers of topics in class XI where you need to apply various methods of factorizing polynomials like splitting the middle term, Using Factor Theorem, zeroes of a polynomial and Quadratic Formula. Some of such topics in class XI are Relations and functions, Complex Numbers, Permutations and Combinations, Binomial Theorem, Sequence and Series, Straight Lines, Conic Sections, Limits and Derivatives etc. So, you are requested to revisit and revise all the important terms, concepts and methods.

Some of the terms and solved examples are given below for your ready reference followed by worksheets. You are also provided with links to some interactive and interesting websites and YouTube videos where you can have more practice and understanding.

Let's recap some basic terms and concepts.

Constant and Variable

A **constant** is a value which remains fixed e.g. 3, $-\frac{1}{2}, \pi$ etc., whereas a **variable** is represented by a symbol which can have different numerical values e.g. if r is the radius of a circle, then its area is given by $A = \pi r^2$. Here r is a variable which will assume the value of the radius of that circle whose area is to be found.

Algebraic Expression

An expression containing or using only algebraic symbols and some or all algebraic operations of addition (+), subtraction (-), multiplication ($\times$), division ($\div$), extraction of roots, raising to integral and fractional powers is called an **Algebraic Expression**.

For example $2x^3 - 5x, \sqrt{5}xy + x^3, y - \frac{1}{y^2}$ etc.

Polynomial

An algebraic expression in which the exponent of variables are whole numbers is known as a **Polynomial**.

For example, $5x^2-3$ is a polynomial in one variable x, $x^3+\sqrt{5}xy-y^3$ is a polynomial in two variables x and y.

Now consider algebraic expressions such as $x+\frac{1}{x}, \sqrt{x}-4, \sqrt[3]{x}-x^2$. We can write $x+\frac{1}{x}=x+x^{-1}$. Here, the exponent of the second term, i.e., x^{-1} is –1, which is not a whole number. So, this algebraic expression is not a polynomial in x. Again, we can write $\sqrt{x}-4=x^{\frac{1}{2}}-4$. Here the exponent of x is $\frac{1}{2}$, which is not a whole number. So, $\sqrt{x}-4$ is not a polynomial in x? What about $\sqrt[3]{x}-x^2$? No, it is also not a polynomial in x. (Why?).

Let us now define a general polynomial in one variable as follows:

Let x be a variable, n be a whole number and $a_0, a_1, a_2, ..., a_n$ be constants (real numbers). Then $a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+...+a_1x+a_0$ is known as a polynomial in the variable x.

Degree of a Polynomial in one variable:

The exponent of the highest degree term in a polynomial with non-zero coefficient is known as its degree. For example, the degree of $f(x)=3x^4-2x^3+x^2+2x-5$ is 4. The degree of constant polynomials, except for the polynomial 0, such as 5, -2, … is 0. The degree of the polynomial 0 is not defined.

Linear Polynomial:

A polynomial of degree one is known as a linear polynomial. For example, $f(x)=3x+5$ is a linear polynomial.

Quadratic Polynomial:

A polynomial of degree two is known as a quadratic polynomial. For example, $f(x)=4x^2-7x+8$ is a quadratic polynomial.

Zeroes of a Polynomial: A real number α is a zero of the polynomial

$f(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+...+a_1x+a_0$ if

$f(\alpha)=0$ i.e. $a_n\alpha^n+a_{n-1}\alpha^{n-1}+a_{n-2}\alpha^{n-2}+...+a_1\alpha+a_0=0$

Factorization

The process of expressing a given algebraic expression as a product of algebraic expressions, each of degree less than that of given algebraic expression, is called factorization.

When we factorize an algebraic expression, we write it as a product of factors. These factors may be numbers, algebraic symbols or algebraic expressions.

Expressions like $4xy, 2y(x+2), 3(x-1)(2y+3)$ are already in factored form. On the other hand consider expressions like $4x+12, x^3-4x, x^2-5x+6$ .It is not obvious what their factors are. We have already developed few systematic methods to factorize these expressions in class IX and class X.

Identity

An **identity** is an equality that holds true regardless of the values chosen for its variables. They are used in simplifying or rearranging algebra expressions. For example, the identity $(a+b)^2 = a^2+2ab+b^2$ is true for all the values chosen for a and b .

Some other identities are:

$(a-b)^2 = a^2-2ab+b^2$

$a^2 - b^2 = (a+b)(a-b)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$(a-b)^3 = a^3-3a^2b+3ab^2-b^3$

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Solved Examples\Various Methods for Factorization

(a) Method of common factors

Example 1: Factorize $5xy+15y$

Solution:

$5xy+15y=5y(x+3)$

Example 2: Factorize $27x^3y-81x^2yz$

Solution:

$27x^3y-81x^2yz=27x^2y(x-3z)$

(b) Factorization by regrouping terms

Example 3: factorize 6xy – 4y + 6 – 9x

Solution:

6xy – 4y + 6 – 9x

= 6xy – 4y – 9x + 6

= 2y (3x – 2) – 3 (3x – 2)

= (3x – 2) (2y – 3)

Example 4: Factorize $x^3-2x^2y+3xy^2-6y^3$

Solution:

$$\begin{aligned}&x^3-2x^2y+3xy^2-6y^3\\&=\left(x^3-2x^2y\right)+\left(3xy^2-6y^3\right)\\&=x^2(x-2y)+3y^2(x-2y)\\&=(x-2y)\left(x^2+3y^2\right)\end{aligned}$$

(c) Factorization using identities:

Example 5: factorize $4x^2-12x+9$

Solution:

$$4x^2 - 12x + 9$$
$$= (2x)^2 - 2(2x)(3) + (3)^2$$
$$= (2x-3)^2 = (2x-3)(2x-3)$$

Example 6: Factorize $x^8 - y^8$

Solution:

$$x^8 - y^8$$
$$= (x^4)^2 - (y^4)^2$$
$$= (x^4 + y^4)(x^4 - y^4)$$
$$= (x^4 + y^4)\left[(x^2)^2 - (y^2)^2\right]$$
$$= (x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$$
$$= (x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$$

Note: $(x^2 + y^2)$ and $(x^4 + y^4)$ can't be factorized further in the set of real numbers. Yet these factors can be further factorized in the set of complex numbers, about which you will study later.

(d) Factorization using splitting the middle term

Let $x^2 + bx + c$ be a quadratic polynomial and $(x+p)$ and $(x+q)$ are other two polynomials such that

$$x^2 + bx + c = (x+p)(x+q)$$
$$= x^2 + (p+q)x + pq$$

Comparing the coefficients of like terms, we get $p+q=b$ and $pq=c$

So, in order to factorize quadratic polynomial $x^2 + bx + c$, we have to find two terms p and q such that $p+q=b$ and $pq=c$.

Let us understand this process by considering an example.

Example 7: Factorize: $x^2 - 7x + 12$

Solution:

We note 12 = (-3) x (-4) and (-3) + (-4) = -7. Therefore,

$$x^2 - 7x + 12 = x^2 - 3x - 4x + 12 = x(x-3) - 4(x-3) = (x-3)(x-4)$$

Example 8: Factorize $12x^2 - 25x + 12$

Solution:

Observe that here coefficient of $x^2 \neq 1$.

Here we find two numbers such that their sum is -25 and product is $12 \times 12 = 144$.

Clearly, $-16 - 9 = -25$ and $(-16) \times (-9) = 144$.

$$\therefore 12x^2 - 25x + 12$$
$$= 12x^2 - 16x - 9x + 12$$
$$= 4x(3x-4) - 3(3x-4)$$
$$= (3x-4)(4x-3)$$

(e) Factorization using completing the square

Example 9: Factorize $x^2 - 7x - 12$

Answer: Consider $x^2 - 7x - 12$

adding $\left(\frac{1}{2}\text{coeff. of } x\right)^2$ i.e. $\left[\frac{1}{2} \times (-7)\right]^2 = \frac{49}{4}$ to both sides, we get

$$x^2 - 7x + \frac{49}{4} - \frac{49}{4} - 12$$
$$= \left(x - \frac{7}{2}\right)^2 - \frac{97}{4}$$
$$= \left(x - \frac{7}{2}\right)^2 - \left(\frac{\sqrt{97}}{2}\right)^2$$
$$= \left(x - \frac{7}{2} + \frac{\sqrt{97}}{2}\right)\left(x - \frac{7}{2} - \frac{\sqrt{97}}{2}\right)$$

(f) Factorization using Factor Theorem

Factor Theorem: If $p(x)$ is a polynomial of degree $n \geq 1$ and a is any real number, then $(i)\ x-a$ is a factor of $p(x)$, if $p(a)=0$, and $(ii)\ p(a)=0$, if $x-a$ is a factor of $p(x)$

Factor theorem can be used to factorize some polynomials. For this purpose, we try to find an integral zero of the polynomial. As soon as we get a zero of a polynomial, we get one of the factors of the polynomial. We repeat this process until we get a polynomial which can be further factorized easily.

Method

(i) Obtain the given polynomial $p(x)$.

(ii) Obtain the constant term in $p(x)$ and find its all possible factors.

(iii) Take one factor, say a_1 and replace x by a_1. If $p(a_1)=0$, then $(x-a_1)$ is one of the factor of $p(x)$. Otherwise, take another factor say a_2 and continue this process till you get as many factors as degree of the polynomial. Let $(x-a_1),(x-a_2),(x-a_3),\ldots$ are factors of $p(x)$.

(iv) Now, put $p(x)=k(x-a_1)(x-a_2)(x-a_3)\ldots$, where k is a constant term whose value is to be determined.

(v) Substitute any value of x other than $a_1, a_2, a_3, \ldots$ in the equation obtained in the step (iv) and get the value of k.

(v) Substitute the value of k in $p(x)=k(x-a_1)(x-a_2)(x-a_3)\ldots$

Let us consider the following solved example.

Example 10: Factorize $p(x)=x^2+2x-15$ using factor theorem.

Solution: Here coefficient of x^2 is 1. Integral factors of -15 are $\pm 1, \pm 3, \pm 5, \pm 15$.

We observe that $p(3)=3^2+2\times 3-15=0 \therefore (x-3)$ is a factor of $p(x)$.

Also, $p(-5)=(-5)^2+2\times(-5)-15=0 \therefore (x+5)$ is also a factor of $p(x)$.

$\therefore x^2+2x-15=(x-3)(x+5)$

Example 11: Factorize $f(x)=x^3-6x^2+11x-6$ using factor theorem.

Solution:

We shall now look for all the factors of constant term, which is -6 here. Factors of -6 are $\pm1, \pm2, \pm3, \pm6$. Putting $x=1$ in $f(x)$, we have $f(1)=0$, so $x-1$ is a factor of $f(x)$.

Dividing $f(x)$ by $(x-1)$, we get $f(x)=(x-1)(x^2-5x+6)$. The expression (x^2-5x+6) can now be factorized using splitting the middle term as $(x-2)(x-3)$. So, $f(x)=(x-1)(x-2)(x-3)$

Example 12: Factorize $p(y)=2y^3+y^2-2y-1$ using factor theorem.

Solution:

Here we observe that $p(1)=2(1)^3+1^2-2(1)-1=0$. $\therefore (y-1)$ is a factor of $p(y)$. Dividing $p(y)$ by $(y-1)$, we get

$$
\begin{aligned}
p(y)&=(y-1)(2y^2+3y+1)\\
&=(y-1)(2y^2+2y+y+1)\\
&=(y-1)[2y(y+1)+1(y+1)]\\
&=(y-1)(y+1)(2y+1)
\end{aligned}
$$

Equation

An **Equation** is an equality containing one or more variables which may be true for some values of the variable. Solving the equation consists of determining the values of the variables which when substituted for the variables make the equality true.

Linear Equation:

An algebraic *equation* in which the highest degree term in the variable or variables is of the first degree. For example $ax+b=0; a\neq 0$, $ax+by=c$; at least one of a or b is non-zero.

Note that $2x+5y+3xy=0$ is not a linear equation as it contains the product of variables x and y.

Quadratic Equation:

A quadratic equation is an equation of the second degree, meaning that the highest exponent is 2. The standard form of a quadratic equation is $ax^2+bx+c=0, a\neq 0$ with a, b and c being constants, or numerical coefficients, and x is an unknown variable.

If we can factorize ax^2+bx+c, $a\neq 0$ into a product of two linear factors, then the roots of the quadratic equation $ax^2+bx+c=0, a\neq 0$ can be found by equating each factor to zero.

Solution of a Quadratic Equation by the Method of Completion of Squares

Consider the following Quadratic Equation $x^2-3x-18=0$

Now, $x^2-3x-18=0 \Rightarrow x^2-3x=18$

adding $\left(\frac{1}{2}\text{coeff. of } x\right)^2$ i.e. $\left[\frac{1}{2}\times(-3)\right]^2=\frac{9}{4}$ to both sides, we get

$$\Rightarrow x^2-3x+\frac{9}{4}=18+\frac{9}{4}$$

$$\Rightarrow \left(x-\frac{3}{2}\right)^2=\frac{81}{4}=\left(\frac{9}{2}\right)^2$$

$$\Rightarrow x-\frac{3}{2}=\pm\frac{9}{2}$$

$$\Rightarrow x=\frac{3}{2}\pm\frac{9}{2}$$

$$\Rightarrow x=\frac{3}{2}+\frac{9}{2} \text{ or } \frac{3}{2}-\frac{9}{2}$$

$$\Rightarrow x=6 \quad \text{or} -3$$

Now consider the quadratic equation in general form $ax^2+bx+c=0, a\neq 0$

Divide both sides by a, we get $x^2+\frac{b}{a}x=-\frac{c}{a}$

Adding $\left(\frac{1}{2}\text{coeff. of } x\right)^2$ *i.e.* $\left(\frac{b}{2a}\right)^2$ to both sides, we ge

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2$$

$$\Rightarrow\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$\Rightarrow x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Hence the roots of the given quadratic equation are $x=\frac{-b+\sqrt{b^2-4ac}}{2a},\frac{-b-\sqrt{b^2-4ac}}{2a}$.

Here, $d=b^2-4ac$ is known as **discriminant**.

Solving Quadratic Equation

(a) Solution of a Quadratic Equation by Factorization:

Example 13: Find the roots of the equation $x^2-7x+12=0$ by factorization.

Answer: $x^2-7x+12=0\Rightarrow(x-3)(x-4)=0\Rightarrow x=3, x=4$

(b) Solution of a Quadratic Equation by the method of completing the squares

Example 14: Find the roots of $2x^2-7x+3=0$ (if exists) by the method of completing the squares.

Solution:

$$2x^2-7x+3=0$$

$$\Rightarrow 2x^2-7x=-3 \qquad [\text{transferring the constant term}]$$

$$\Rightarrow x^2-\frac{7}{2}x=-\frac{3}{2} \qquad [\text{dividing both sides by } 2]$$

adding to both sides the square of the $\frac{1}{2}$ coefficient of x *i.e.* $\left[\frac{1}{2}\times\left(-\frac{7}{2}\right)\right]^2$

$$x^2-\frac{7}{2}x+\left(-\frac{7}{4}\right)^2=\left(-\frac{7}{4}\right)^2-\frac{3}{2}$$

$$\Rightarrow\left(x-\frac{7}{4}\right)^2=\left(\frac{5}{4}\right)^2\Rightarrow x-\frac{7}{4}=\pm\frac{5}{4}$$

$$\Rightarrow x=\frac{7}{4}\pm\frac{5}{4}\Rightarrow x=3\text{ or }\frac{1}{2}$$

Example 15: Find the roots of $2x^2+x-4=0$ (if exists) by the method of completing the squares.

Solution:

$$2x^2+x-4=0$$

$$\Rightarrow 2x^2+x=4$$

$$\Rightarrow x^2+\frac{x}{2}=2$$

adding $\left(\frac{1}{2}\times\text{coeff. of }x\right)^2$ i.e. $\left(\frac{1}{2}\times\frac{1}{2}\right)^2$, to both sides

$$\Rightarrow x^2+\frac{x}{2}+\frac{1}{16}=2+\frac{1}{16}$$

$$\Rightarrow\left(x+\frac{1}{4}\right)^2=\left(\frac{\sqrt{33}}{4}\right)^2$$

$$\Rightarrow x+\frac{1}{4}=\pm\frac{\sqrt{33}}{4}$$

$$\Rightarrow x=\frac{-1\pm\sqrt{33}}{4}$$

$$\Rightarrow x=\frac{-1+\sqrt{33}}{4}\text{ or }\frac{-1-\sqrt{33}}{4}$$

Solution of a Quadratic Equation by using Quadratic Formula.

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 16: Solve $x^2 - 3x - 4 = 0$ using quadratic formula.

Solution:

Here $a = 1, b = -3, c = -4$, so $x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(1)(-4)}}{2(1)} = \dfrac{3 \pm 5}{2} = 4 \text{ or } -1$

Difference between Zero and a Root

In general, we say that a *zero* of a polynomial $p(x)$ is the number a such that $p(a) = 0$. You must have observed that the zero of the polynomial $x - 2$ is obtained by equating it to 0, i.e., $x - 2 = 0$, which gives $x = 2$. We say $p(x) = 0$ is a polynomial equation and 2 is the *root of the polynomial* equation $p(x) = 0$. So we say 2 is the zero of the polynomial $x - 2$, or a *root* of the polynomial equation $x - 2 = 0$.

In general, a real number α is called a root of the quadratic equation $ax^2 + bx + c = 0, a \neq 0$ if $a\alpha^2 + b\alpha + c = 0$. We also say that $x = \alpha$ **is a solution of the quadratic equation**, or that α **satisfies the quadratic equation**. Note that **the zeroes of the quadratic polynomial** $ax^2 + bx + c$ **and the roots of the quadratic equation** $ax^2 + bx + c = 0$ **are the same**.

SOME USEFUL WEB LINKS

Factorization

Finding common factors and making use of standard identities:
https://www.mathsisfun.com/algebra/factoring.html

Quadratic Equation

Quadratic Equation – standard form and methods to solve it:
https://www.mathsisfun.com/algebra/quadratic-equation.html

Input a, b, c in $ax^2 + bx + c = 0$ to get its solution:
https://www.mathsisfun.com/quadratic-equation-solver.html

Factoring Quadratic Equation by observation and hence using formula:
https://www.mathsisfun.com/algebra/factoring-quadratics.html

Method of completing the square:
https://www.mathsisfun.com/algebra/completing-square.html

Solved examples using Quadratic Formula:
http://www.purplemath.com/modules/quadform.htm

Factoring Quadratic – Solved examples
http://www.regentsprep.org/regents/math/algebra/ae5/lfaceq.htm

Youtube Videos

- https://www.youtube.com/watch?v=ZQ-NRsWhOGI
- https://www.youtube.com/watch?v=AMEau9OE6Bs
- https://www.youtube.com/watch?v=3ayhvAI3IeY
- https://www.youtube.com/watch?v=iII7E1m6f9M
- https://www.youtube.com/watch?v=1tIXSI-dxZQ
- https://www.youtube.com/watch?v=fBwRstnyOq4
- Using factor theorem
- https://www.youtube.com/watch?v=t9eKZG-QCZk
- https://www.youtube.com/watch?v=SEfQWHwZcvU

DO IT YOURSELF

I. 1. Few quadratic polynomials are factorized into their linear factors as given below. Complete the table given below:

S. No.	Factor 1	Factor 2	Quadratic Polynomial
1	$x+5$	$x+7$	

2	$x-5$	$x-7$	
3	$x+5$	$x-7$	
4	$x-5$	$x+7$	

2. Few quadratic polynomials are given below. Find their factors (if possible)

S. No.	Quadratic Polynomial	Factor 1	Factor 2	Observation
1	$x^2+7x+12$			
2	$x^2-7x+12$			
3	$x^2+7x-12$			
4	$x^2-7x-12$			

II. Factorizing using method of common factors, grouping of terms and using identities

Factorize the following:

(i) $4x^2y+12xy^2$

(ii) a^2-b^2-a-b

(iii) $x^2+xy-7x-7y$

(iv) $15xy-6x+5y-2$

(v) $121x^2-81$

(vi) $x^4-2x^2y^2+y^4$

(vii) x^3+27y^3

(viii) $\frac{x^3}{8}-y^3$

(ix) x^4+4x^2+3

(x) $9(2x-y)^2-4(2x-y)-13$

III. Factorizing using splitting the middle term and factor theorem

Factorize the following:

(i) $x^2-7x-18$

(ii) x^2+x-30

(iii) $x^2+12x+27$ (iv) $x^2-2x-99$

(v) $x^2-12x+20$ (vi) $9x^2-3x-2$

(vii) $3x^2-5x+2$ (viii) $6x^2+5x-6$

(ix) $15x^2-x-28$ (x) $4x^2+4\sqrt{3}x+3$

(xi) x^3-3x^2-9x-5 (xii) $x^3-6x^2+3x+10$

(xiii) $x^3+4x^2-11x-30$ (xiv) $x^3+13x^2+32x+20$

(xv) $2x^3-5x^2-x+6$

IV. Solve the following equations:

(i) $x^2-3x-28=0$ (ii) $3x^2-x-4=0$

(iii) $x^2+2x-5=0$ (iv) $x^2+12x+35=0$

(v) $4x^2+5x-6=0$ (vi) $x^2+5x+5=0$

(vii) $4x^2+7x-15=0$ (viii) $6x^2-x-12=0$

(ix) $3x^2-16x-12=0$ (x) $10x^2-27x+18=0$

(xi) $15x^2+4x-3=0$ (xii) $3x^2+13x-10=0$

(xiii) $4x^2+11x-20=0$ (xiv) $9x^2-6x-7=0$

(xv) $5x^2+3x+1=0$

ANSWERS

I. 1. (i) $x^2+12x+35$ (ii) $x^2-12x+35$

(iii) $x^2-2x-35$ (iv) $x^2+2x-35$

2. (i) $x+3, x+4$ (ii) $x-3, x-4$

(iii) $\left(x+\frac{7}{2}+\frac{\sqrt{97}}{2}\right), \left(x+\frac{7}{2}-\frac{\sqrt{97}}{2}\right)$ (iv) $\left(x-\frac{7}{2}+\frac{\sqrt{97}}{2}\right), \left(x-\frac{7}{2}-\frac{\sqrt{97}}{2}\right)$

II. (i) $4xy(x+3y)$ (ii) $(a+b)(a-b-1)$

(iii) $(x-7)(x+y)$ (iv) $(3x+1)(5y-2)$

(v) $(11x+9)(11x-19)$ (vi) $(x-y)^2(x+y)^2$

(vii) $(x+3y)(x^2+9y^2-3xy)$ (viii) $\left(\frac{x}{2}-y\right)\left(\frac{x^2}{4}+y^2+\frac{xy}{2}\right)$

(ix) $(x^2+1)(x^2+3)$ (x) $(2x-y+1)(18x-9y-13)$

III. (i) $(x+2)(x-9)$ (ii) $(x+6)(x-5)$

(iii) $(x+3)(x+9)$ (iv) $(x+9)(x-11)$

(v) $(x-2)(x-10)$ (vi) $(3x-2)(3x+1)$

(vii) $(x-1)\left(x-\frac{2}{3}\right)$ (viii) $\left(x+\frac{3}{2}\right)\left(x-\frac{2}{3}\right)$

(ix) $\left(x+\frac{4}{3}\right)\left(x-\frac{7}{5}\right)$ (x) $\left(x+\frac{\sqrt{3}}{2}\right)\left(x+\frac{\sqrt{3}}{2}\right)$

(xi) $(x+1)(x+1)(x-5)$ (xii) $(x+1)(x-2)(x-5)$

(xiii) $(x+2)(x-3)(x+5)$ (xiv) $(x+1)(x+2)(x+10)$

(xv) $(x+1)(x-2)\left(x-\frac{3}{2}\right)$

IV. (i) $7, -4$

(ii) $-1, \frac{4}{3}$

(iii) $-1+\sqrt{6}, -1-\sqrt{6}$

(iv) $-5, -7$

(v) $-2, \frac{3}{4}$

(vi) $\frac{-5 \pm \sqrt{5}}{2}$

(vii) $-3, \frac{5}{4}$

(viii) $\frac{3}{2}, -\frac{4}{3}$

(ix) $6, -\frac{2}{3}$

(x) $\frac{6}{5}, \frac{3}{2}$

(xi) $\frac{1}{3}, -\frac{3}{5}$

(xii) $-5, \frac{2}{3}$

(xiii) $-4, \frac{5}{4}$

(xiv) $\frac{1 \pm 2\sqrt{2}}{3}$

(xv) No real roots

PARTIAL FRACTIONS

Dear Reader

An algebraic fraction or a rational fraction can be, often, expressed as the algebraic sum of relatively simpler fractions called partial fractions. The application of partial fractions lies in evaluating the antiderivative of the rational functions, which is dealt with in std. XII.

Example 1: $\frac{2x-1}{x^2-5x+6} \equiv \frac{2x-1}{(x-2)(x-3)}$, which can be resolved into partial fractions as :

$$\frac{2x-1}{(x-2)(x-3)} = \frac{5}{x-3} - \frac{3}{x-2}$$

We will now explain how to carry out this process.

I) A Brief Recap

Definition: An algebraic fraction or a rational fraction is a fraction in which the numerator and the denominator are both polynomial expressions.

Example 1: $\frac{2x}{-x^2+5}, \frac{x^3-1}{x^4+2x^2+1}$

The above fractions are called **proper fractions** as the degree of the polynomial in the numerator is lower than the degree of the polynomial in the denominator.

Now consider the following fractions:

Example 2: $\frac{x^4+x^2-1}{-x^2-5x+6}, \frac{2x+7}{x+5}$

In these cases, the degree of the polynomial in numerator is more than or equal to the degree of the polynomial in the denominator.

Such fractions are called **improper fractions.**

II) Expressing a Rational Fraction as the sum of its Partial Fractions

CASE I

We shall consider the case where the denominator is expressed as product of its linear factors.

Example: Express $\frac{4x-1}{2x^2-5x-3}$ as the sum of its partial fractions.

Ans. To express this algebraic fraction as sum of its partial fractions, we would first express the polynomial in the denominator as product of its factors i.e. $2x^2 - 5x - 3 \equiv (2x+1)(x-3)$.

Therefore, $\frac{4x-1}{2x^2-5x-3} \equiv \frac{4x-1}{(2x+1)(x-3)}$

We may now write $\frac{4x-1}{(2x+1)(x-3)}$ as sum of its components (proper fractions) as

$\frac{4x-1}{(2x+1)(x-3)} \equiv \frac{A}{2x+1} + \frac{B}{x-3}$, where A and B are non-zero constants to be determined.

Now, $\frac{4x-1}{(2x+1)(x-3)} \equiv \frac{A}{2x+1} + \frac{B}{x-3}$ (i)

- $\frac{4x-1}{(2x+1)(x-3)} \equiv \frac{A(x-3)+B(2x+1)}{(2x+1)(x-3)}$

As the denominators are same, we need to find the values of 'A' and 'B' for which

- $4x-1 = A(x-3) + B(2x+1)$
- $4x-1 = x(A+2B) + (-3A+B)$

Comparing the coefficients on both sides we get, $4 = A + 2B$; $-1 = -3A + B$

Solving the equations simultaneously we get, $A = \frac{6}{7}$ and $B = \frac{11}{7}$

Substituting in (i), we get

$\frac{4x-1}{(2x+1)(x-3)} \equiv \frac{6}{7(2x+1)} + \frac{11}{7(x-3)}$,

CASE II

We will now consider the case where the denominator in the fraction has a repeated linear factor:

Example: Express $\frac{x+1}{(x-1)(x^2+x-2)}$ as sum of its partial fractions.

Solution: We would first express the denominator in terms of product of its factors, if possible.

The possible factors of $(x^2+x-2)$are$(x-1)$ and $(x+2)$

Therefore, $(x-1)(x^2+x-2) \equiv (x-1)^2(x+2)$

Therefore, the possible factors of the denominator are: $(x-1), (x-1)^2, (x+2)$.

Now we may write our algebraic fraction as the sum of its partial fractions as:

$\frac{x+1}{(x-1)(x^2+x-2)} \equiv \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$; where A, B, C are non-zero constants to be determined (i)

NOTE: If in case the denominator in the above expression was taken to be $(x-1)^2(x^2+x-2)$, then its factors would have been$(x-1), (x-1)^2$, $(x-1)^3$ and $(x+2)$. and a fourth fraction $\frac{D}{(x-1)^3}$would be added to the above, where D would be another constant to be determined.

From (i) we get $\frac{x+1}{(x-1)(x^2+x-2)} \equiv \frac{A(x-1)(x+2)+B(x+2)+c(x-1)^2}{(x-1)^2\,(x+2)}$

As the denominators are same we need to find the values of A and B for which

- $(x+1) = A(x-1)(x+2) + B(x+2) + C(x-1)^2$

(Note: We can also achieve the above equation by multiplying both sides of the equation by $(x-1)^2(x+2)$)

- $(x+1) = x^2(A+C) + x(A+B-2C) + (-2A+2B+C)$

On comparing the coefficients we get,

$\mathrm{A} + \mathrm{C} = 0,$ $\quad A+B-2C = 1,$ $\quad -2A+2B+C = 1$

Solving the above equations we get,

$$A = \frac{1}{9}, \qquad B = \frac{2}{3}, \qquad C = -\frac{1}{9}$$

Substituting in (i) we get,

$$\frac{x+1}{(x-1)(x^2+x-2)} \equiv \frac{1}{9(x-1)} + \frac{2}{3(x-1)^2} - \frac{1}{9(x+2)}$$

CASE III

We will now consider the case in which the denominator is a quadratic term which cannot be factorised further.

Example: Express $\frac{x}{(x^2+1)(x-1)}$ as sum of its partial fractions.

Solution:

The two denominators of partial fractions would be$(x^2+1) and\ (x-1)$.Here, the quadratic polynomial can't be resolved into further linear factors. We will now express the given fraction as the following

$$\frac{x}{(x^2+1)(x-1)} \equiv \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

Note: The numerator of the fraction having the quadratic polynomial is a linear polynomial in x.

$$=> \frac{x}{(x^2+1)(x-1)} \equiv \frac{A(x^2+1)+(Bx+C)(x-1)}{(x-1)(x^2+1)}$$

- $x = A(x^2+1) + (Bx+C)(x-1)$
- $x = x^2(A+B) + x(-B+C) + (A-C)$

Comparing both sides, we get

- $0 = A + B$ (i)

- $1 = -B + C$ (ii)
- $0 = A - C$
- Solving (i) and (ii) simultaneously we get,

$$A = \frac{1}{2}; B = -\frac{1}{2}; C = \frac{1}{2}$$

Therefore,

$$\frac{x}{(x^2+1)(x-1)} \equiv \frac{1/2}{x-1} + \frac{-1/2x+1/2}{x^2+1}$$

$$\frac{x}{(x^2+1)(x-1)} \equiv \frac{1}{2(x-1)} - \frac{x}{2(x^2+1)} + \frac{1}{2(x^2+1)}$$

CASE IV

We will now consider the case of improper fractions.

Example: Express $\frac{4x^3+10x+}{x(2x+1)}$ as sum of its partial fractions.

In the above case we note that the degree of the polynomial in the numerator is more than the degree of the polynomial in the denominator.

Degree of the numerator is 3; Degree of the denominator is 2

In such cases where the degree of term in numerator is more than or equal to the degree of the denominator, we need to perform the long division first.

Therefore, $\frac{4x^3+10x+}{x(2x+1)} \equiv 2x - 1 + \frac{11x+4}{x(2x+1)}$ (i)

Now, consider $\frac{11x+4}{x(2x+1)} \equiv \frac{A}{x} + \frac{B}{2x+1}$ (ii)

Therefore, $11x+4 = A(2x+1) + Bx$

- $11x+4 = x(2A+B) + A$

Comparing the coefficients on both sides we get: $A = 4$; $2A + B = 11$

On solving we get, $A = 4$; $B = 3$

Substituting in (ii) we get:

$$\frac{11x+4}{x(2x+1)} \equiv \frac{4}{x} + \frac{3}{2x+1}$$

Therefore (i) can be expressed as

$$\boxed{\frac{4x^3+10x+4}{x(2x+1)} \equiv 2x-1+\frac{4}{x}+\frac{3}{2x+1}}$$

NOTE: In general, in case of improper partial fractions, for example, the case above, we may split it into partial fractions as:

Ans. $$\frac{4x^3+10x+4}{x(2x+1)} \equiv Ax + \mathbf{B} + \frac{C}{x} + \frac{D}{2x+1}$$

Then we may solve the above by comparing the coefficients on both the sides.

That is to say, that in case of improper fractions we can start writing the expression in the decreasing powers of x to represent the quotient after division. In the case mentioned above the degree of numerator is 3 and the degree of denominator is 2 so we start with the decreasing power of x starting with $Ax + B$as the difference in the degrees of the polynomial in the numerator and the denominator is '1' and then solve the above.

Consider more such examples:

$$\frac{x^2+x+1}{x^2-x} \equiv A + \frac{B}{x} + \frac{C}{x-1};$$

In this case, the degree of the numerator and denominator is 2, which is same so we start with a constant to represent the quotient.

Considering yet another case,

$\frac{x^4+1}{x^2-1} \equiv Ax^2 + Bx + C + \frac{D}{x-1} + \frac{E}{x+1};$

In this case, the degree of the numerator is 4 and the degree of the denominator is 2 so we start writing the expression in the decreasing powers of $'x'$ starting with the degree 2.

DO IT YOURSELF

Express the following as sum of its partial fractions:

1) $\frac{2}{(1-x)(x^2+1)}$ Ans. $\frac{1}{(1-x)} + \frac{x+1}{(x^2+1)}$

2) $\frac{x^2+1}{(x-1)^2(x+3)}$ Ans. $\frac{3}{8(x-1)} + \frac{1}{2(x-1)^2} + \frac{5}{8(x+3)}$

3) $\frac{3x+5}{x^3-x^2-x+1}$ Ans. $\frac{1}{2(x+1)} - \frac{1}{2(x-1)} + \frac{4}{(x-1)^2}$

4) $\frac{1}{x^3+x^2+x+1}$ Ans. $\frac{1}{2(x+1)} - \frac{x}{2(1+x^2)} + \frac{1}{2(1+x^2)}$

5) $\frac{x^3+x+1}{(x^2-1)}$ Ans. $x + \frac{1}{2(x+1)} + \frac{3}{2(x-1)}$

REFERENCES

- https://en.wikipedia.org/wiki/Partial_fraction_decomposition
- https://en.wikibooks.org/wiki/Calculus/Integration.../Partial_Fraction_Decomposition

CHAPTER 3

SETS, INTERVALS, ABSOLUTE VALUE FUNCTION AND INEQUALITIES

Dear Reader

In day to day life we speak of collection of objects such as the collection of the months in a year, the collection of the novels written by the writer Mahadevi Verma, the collection of the rivers of India and so on.

In mathematics too, we come across collections of numbers, functions etc., for example, we have the collections of natural numbers, prime numbers, integers etc. The above collections of objects / numbers are well defined as we can identify/ decide whether a given particular object belongs to the collection or not.

Such a well-defined collection of objects is called a **set**.

Consider the cricket team of India comprising of 11 players.

It is a well-defined set as we can list the members of the team. But if asked to form a set of the best batsmen that would not be possible as the degree of potential may be measured in different spheres and vary from player to player.

NOTE: The following may not be considered as a set:

I. The collection of five most talented writers of the world, as the criterion for determining the talented authors may vary from person to person.

II. The collection of five intelligent students in a given class is also not a well-defined collection as intelligence of a student may be measured in different spheres which may vary from student to student.

There are two symbolic methods of representing a set:

I. Roster or Tabular form

The objects/ elements/ members of a set are enclosed within the braces {}, separating them by commas.

Example: The set of Vowels, V = {a, e, i, o, u}. Here all the vowels are listed within the braces.

NOTE: Sets are always denoted by capital letters.

II. Set- Builder form

In this form we represent the elements of the set by using the symbol x (or y, z), followed by '**:**' or '|' and then mention the characteristic property possessed by the elements of the set. The whole description is then enclosed within the braces.

Example The set of Vowels, V = {x: x is a vowel} or V = {x | x is a vowel}. Here only the characteristic property possessed by the elements is stated within the braces.

Let's try out some questions related to the representation of the sets. Also represent the information graphically.

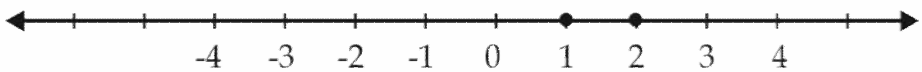
Question 1: Write the solution set of the equation $x^2 - 3x + 2 = 0$.

Solution: $x^2 - 3x + 2 = 0$

- $(x - 1)(x - 2) = 0$
- $x = 1, 2$

Therefore, in the set notation the solution set is written as: A = {1, 2}.

On the real number line it may be represented as:



Question 2: Represent the set of all the integers from -1 to 3, both in the set notation form and graphically on the number line.

Solution: Let us first form the set comprising of the above elements, which is,

A = {-1, 0, 1, 2, 3}.

Plotting on the number line, we get,

Let's consider the following example of plotting the real numbers on the number line from -1 to 3.

The set of real numbers from -1 to 3 is denoted by the set A = $\{y: -1 \leq y \leq 3\}$.

In this set it is impossible to list the real numbers as, between any two real numbers there lie infinite number of real numbers. Therefore, in order to denote the above set we introduce the concept of intervals.

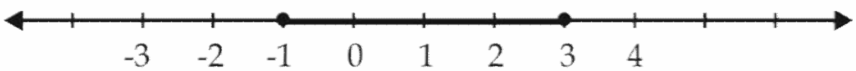
An interval is a set that consists of all real numbers between a given pair of numbers. It can also be thought of as a segment of the real number line. An end point of an interval is either of the two points that mark the end of the line segment. An interval can include both end points (closed interval), neither of the endpoints (open interval), one of the end points (semi closed or semi open interval).

Types of Intervals

Open interval: An open interval does not include the end points and the exclusion of the endpoints is indicated by round brackets ( ) in the interval notation, that is to say, (a, b) where $a<b$. When represented on the real number line, the exclusion of the end point is illustrated by an open dot.

Example: The interval of numbers between -1 and 3 is written as the open interval (-1, 3) = {x: -1 <x < 3}.

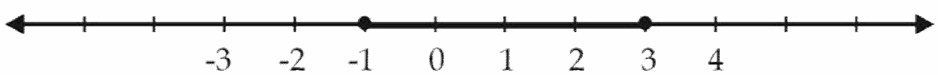
As a segment of the real number line, the representation is given by:



Closed interval: A closed interval includes the end points and the inclusion of the endpoints is indicated by square brackets [ ] in the interval notation, that is to say, [a, b] where a<b. When represented on the real number line, the inclusion of the end point is illustrated by a closed dot.

Example: The interval of numbers from -1 and 3 (both endpoints included) is written as the closed interval [-1, 3] = {x: -1 ≤x ≤ 3}.

As a segment of the real number line, the representation is given by:



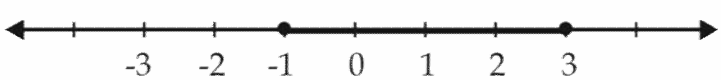
The above cases deal with the intervals where either both the end points are included or both are excluded. But there may be intervals where one of the the endpoints is included and the other is excluded.

Semi–Open/ closed interval: When one endpoint of an interval is included and the other is excluded, then the interval may have any one of the following forms:

(a, b]- 'a' excluded and 'b' included

Example: (-1,3] = {x: -1<x≤ 3}.

As a segment of the real number line, the representation is given by:



[a, b)- 'a' included and 'b' excluded

Example: [-1,3) = {x: -1 ≤ x< 3}.

As a segment of the real number line, the representation is given by:

That is to say,

(a, b)	**a < x < b**	**an open interval**
[a, b)	a ≤ x < b	closed on left, open on right
(a, b]	a < x ≤ b	open on left, closed on right
[a, b]	a ≤ x ≤ b	a closed interval

These are intervals of finite length which is given by 'b – a'.

We also have intervals of infinite length.

There are 4 possible "infinite ends":

Infinite intervals: Infinite intervals are those intervals that do not have an endpoint in either the negative or positive direction. These intervals are given in the table below:

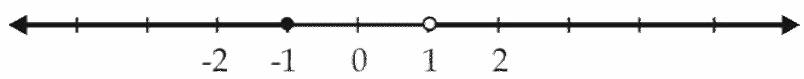
Interval	Inequality	
(a, +∞)	x > a	greater than a
[a, +∞)	x ≥ a	greater than or equal to a
(-∞, a)	x < a	less than a
(-∞, a]	x ≤ a	less than or equal to a

We could even show no limits by using this notation: (-∞, +∞)

We can also have two (or more) intervals together.

Example: $x \leq -1 \; or \; x > 1$

On the number line it may be represented as:



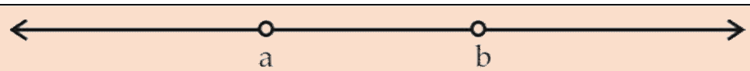
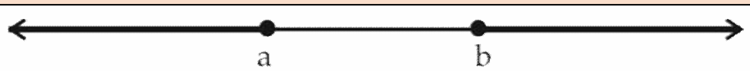
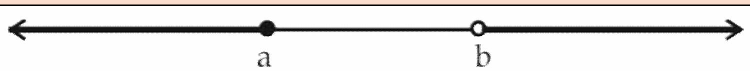
In the interval notation it is expressed as: $(-\infty, -\mathbf{1}] \cup (\mathbf{1}, \infty)$

NOTE: One must be very careful with the inequalities as above. The most common mistake committed by students is that they may combine the inequality into one as:

$-\mathbf{1} \geq \boldsymbol{x} \geq \mathbf{1}$ (✘ ***wrong***)

This is meaningless as '$\boldsymbol{x}$' cannot be less than -1 and greater than 1 at the same time.

BRIEF SUMMARY OF THE CONTEXT DISCUSSED ABOVE

INTERVAL NOTATION	NUMBER LINE	SET- BUILDER NOTATION
(a, b)		$\{x: a < x < b\}$
(a, b]		$\{x: a < x \leq b\}$
[a, b)		$\{x: a \leq x < b\}$
[a, b]		$\{x: a \leq x \leq b\}$
$(-\infty, \mathbf{a}) \cup$**(b, ∞)**		$\{x: x < a\} \cup \{x: x > b\}$
(-∞, a] ∪ **[b, ∞)**		$\{x: x \leq a\} \cup \{x: x \geq b\}$
(-∞, a) ∪ **[b, ∞)**		$\{x: x < a\} \cup \{x: x \geq b\}$
(-∞, a] ∪ **(b, ∞)**		$\{x: x \leq a\} \cup \{x: x > b\}$
(-∞,∞)		**R** (The set of Real numbers)

Absolute Value or Modulus of a Real Number

We would like to recall what you mean by the absolute value of a real number!

Absolute value of a real number describes the distance of the point from 0 on the real number line without considering the direction from zero on the number line.

The absolute value of a number a is denoted by |a| and is a non- negative number.. Example: |-54| = 54, |25|=25, |0|=0. The sign of the number depicts the direction of the number from 0 on the number line, that is to say that -54, 25 and 0 are placed at a distance of 54, 25 and 0 to the left, right and at 0 respectively on the number line.

Absolute Value Function

The absolute value function is the function f given by f($\boldsymbol{x}$) = | $\boldsymbol{x}$| and defined as:

$$f(x)=\begin{cases} x \text{ when } x > 0 \\ 0 \text{ when } x = 0 \\ -x \text{ when } x < 0 \end{cases}$$

That is to say that the absolute value of $\boldsymbol{x}$ equals:

$\boldsymbol{x}$: when $\boldsymbol{x}$ is greater than zero(e.g. |4| = 4 as 4> 0)

0: when $\boldsymbol{x}$ equals 0

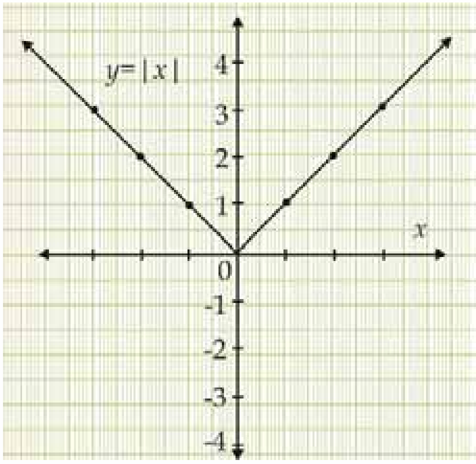
$-\boldsymbol{x}$: when $\boldsymbol{x}$is less than zero

(This "flips" the number back to positive, e.g. |-5| = -(-5) = 5, which is again positive)

Alternately, the modulus function can also be defined as: $\boldsymbol{f(x) = \sqrt{x^2}}$**,** where, x ∈ R, the set of real numbers and f(x) is the corresponding non-negative real number.

Note: $\boldsymbol{\sqrt{4} = 2}$ ***and not*** $\boldsymbol{\pm 2}$.

Graphically f(x) = |x| or y = |x| is represented as:



x	-3	-2	-1	0	1	2	3
f(x)	3	2	1	0	1	2	3

We observe that the graph is V-shaped.

(1) The vertex of the graph is (0, 0).

(2) The axis of symmetry is the line x=0 or y-axis. (Axis of symmetry is the line that divides the graph into two congruent halves)

(3) The domain of the absolute value function is the set of all real numbers.

(4) The range is the set of all non-negative real numbers (greater than or equal to 0).

(5) The x-intercept and the y-intercept are both 0, that is, the points where the graph cuts both the axes is origin.

Some useful properties of the modulus

Here are some properties of absolute values that can be useful:

i. $|a| \geq 0$

ii. $|a| = \sqrt{a^2}$

iii. $|a \times b| = |a| \times |b|$

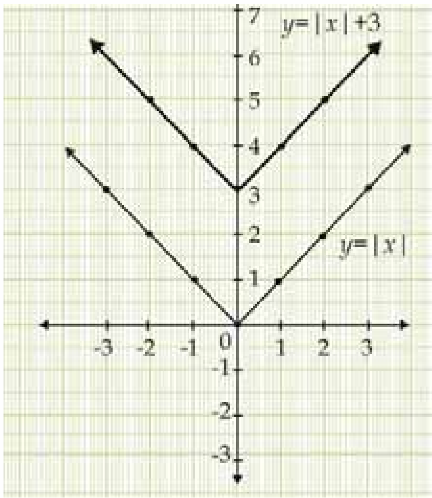
iv. $|u| = a$ is the same as $u = \pm a$ and vice versa; which is often the key to solving most absolute value questions, i.e. if $|u| = 4$, then it would mean $u = \pm 4$, as both $u = 4$ and $u = -4$ would give the absolute value as 4.

Vertical Shift

To obtain the graph of the function g(x)=f(x)+k, where f(x) = |x|, you can translate the graph of the absolute value function f(x) = |x| vertically.

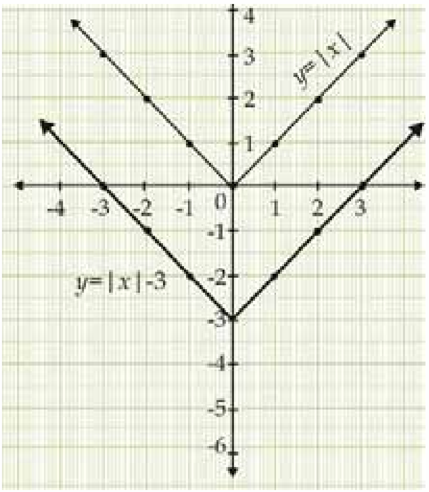
i. **When k>0, the graph of f(x) is translated k units up.**

Example: Plot the graph of the function g(x) = |x| + 3



ii. **When k<0, the graph of f(x) is translated |k| units down.**

Example: Plot the graph of the function g(x) = |x| - 3

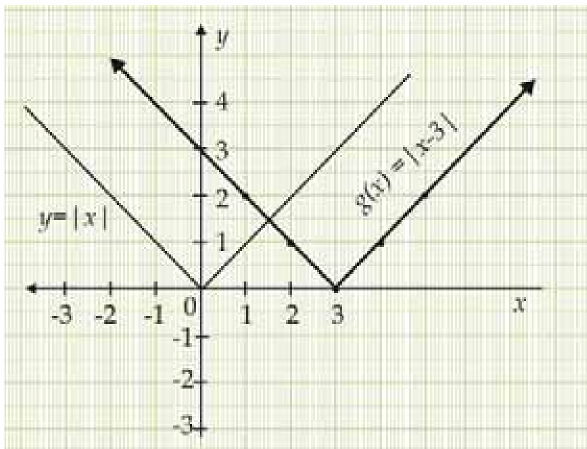


Horizontal Shift

To obtain the graph of g(x) = f (x−h), where f(x) = |x| translate the graph of the absolute value function f(x) = |x| horizontally.

i. **When h > 0, the graph of f(x) is translated h units to the right to get g(x)!**

Example: g (x) = |x-3|; the graph of |x| is translated 3 units to the right.

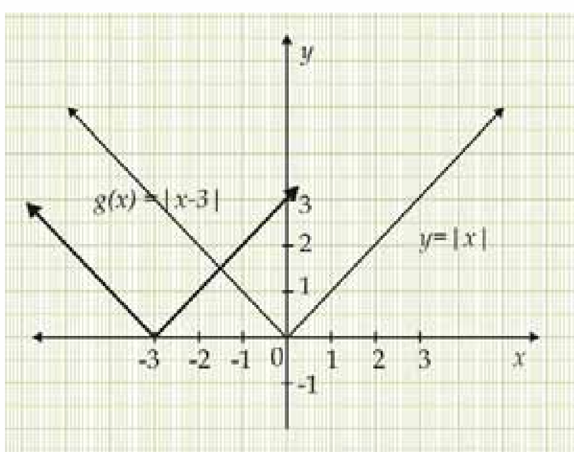


ii. **When h < 0, the graph of f(x) is translated |h| units to the left to get g(x)!**

Example: $g(x) = |x+3|$

$$=> g(x) = |x-(-3)|$$

The graph of |x| is translated 3 units to the left.



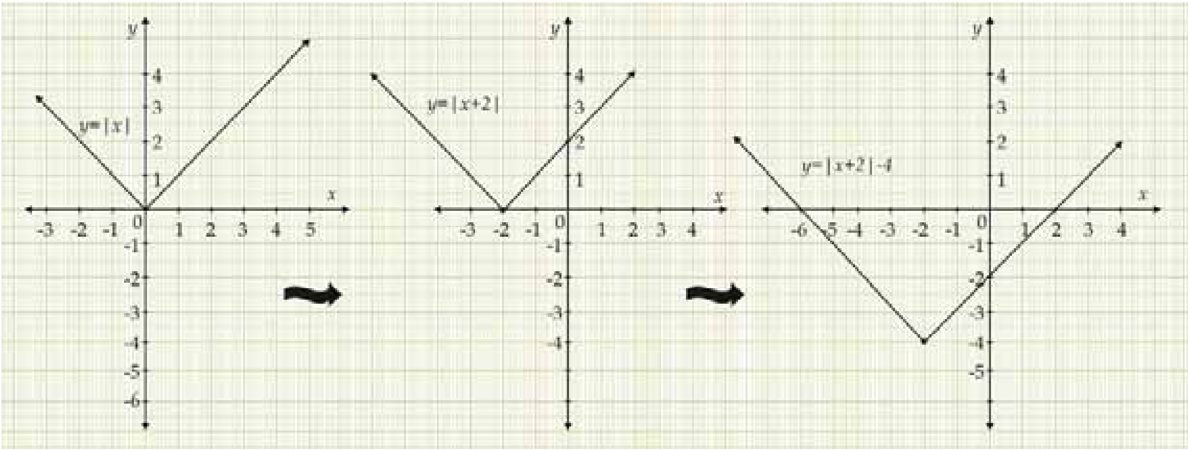
Let us consider the following example with both the vertical and horizontal shift:

Example: Solve $|x+2| = 4$ and graph $y = |x+2| - 4$

Solution: $|x+2| = 4$

- $x + 2 = \pm 4$
- $x + 2 = 4; x + 2 = -4$
- $x = 2 ; x = -6$
- The graph would intersect the x- axis at 2 and -6.

And here is the plot of $y=|x+2|-4$, but just for fun let's make the graph by shifting it around:



Start with $|x|$ Then shift it left to make it $|x+2|$ Then shift it down to make it $|x+2|-4$

Absolute Value Inequalities

Case I: Linear Inequalities Involving $<$, $\leq$

We start off with a simple inequality, say, solve: $|x| \leq 3$.

$|x| \leq 3$ means that x must have a distance of not more than 3 from the origin. This means that x must be somewhere in the range from -3 to 3, i.e.

$\lvert x \rvert \leq 3 \quad => \quad -3 \leq x \leq 3.$

Similarly with the inequality $' < '$ we get a similar result:

$\lvert x \rvert < 3 \quad => -3 < x < 3.$

Solve the following inequalities:

a) $|2x - 3| < 11$

- $-11 < 2x - 3 < 11$

Adding '3' throughout, we get

- $-8 < 2x < 14$
- $-4 < x < 7$

The interval notation for this solution - set is (-4, 7).

Graphically it may be plotted as

b) $|5 - 3y| \leq 7$

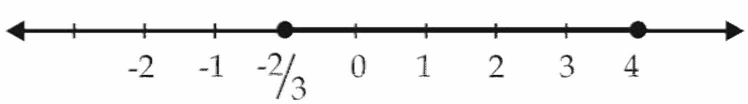
$-7 \leq 5 - 3y \leq 7$

- $-12 \leq -3y \leq 2$ (subtracting 5 throughout the equation)
- $-4 \leq -y \leq \frac{2}{3}$
- $4 \geq y \geq -\frac{2}{3}$

(Negative sign reverses the sign of inequality for e.g.3 $< 4 => -3 > -4$)

The interval notation for this solution - set is $[-\frac{2}{3}, 4]$.

Graphically it may be represented as:



Case II: Linear Inequalities Involving $>$, $\geq$

We start off with a simple inequality, say, solve: $|x| \geq 3$.

$|x| \geq 3$ means that x must be at least at a distance of 3 from the origin. This means that x must be somewhere in one of the following two ranges,

$$\boldsymbol{x \leq -3 \quad or \quad x \geq 3}$$

$\boldsymbol{|x| \geq 3 \quad implies \quad x \leq -3 \quad or \quad x \geq 3}$

Note: The most common mistake committed by the students in the problems related to this inequality in particular is that they combine these into a single double inequality as:$-3 \geq x \geq 3$, which isn't valid as 'x' cannot be simultaneously both less than -3 and greater than 3.

Similarly with the inequality $' > '$ we get a similar result:

$|\boldsymbol{x}| > 3 \quad implies \quad \boldsymbol{x} < -3 \quad or \quad \boldsymbol{x} > 3$

Solve the following inequalities:

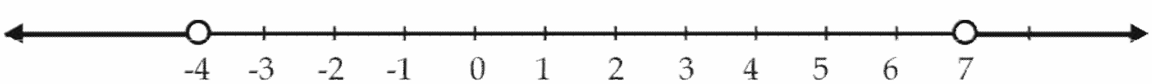
We may consider the above equations with different inequality:

a) $|2x - 3| > 11$

- $2x - 3 < -11 \quad or \quad 2x - 3 > 11$
- $2x < -8 \quad or \quad 2x > 14$
- $x < -4 \quad or \quad x > 7$

The interval notation for this is given by$(-\infty, -4) \cup (7, \infty)$.

The graphical representation is given by:

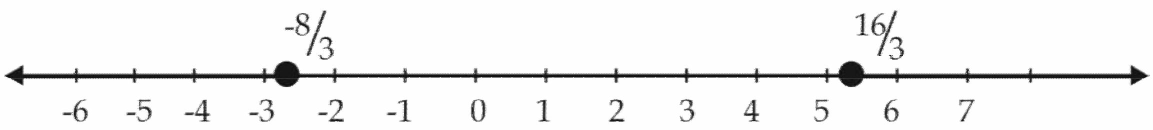


b) $|4 - 3t| \geq 12$

- $4 - 3t \leq -12 \quad or \quad 4 - 3t \geq 12$
- $-3t \leq -16 \quad or \quad -3t \geq 8$
- $t \geq \frac{16}{3} \quad or \quad t \leq -\frac{8}{3}$

In the interval notation it is given by $\quad , \frac{8}{3}$ or $\frac{16}{3}, \quad .$

Graphically it is represented as:



NOTE:

i) Solving the inequality | 2x + 5 | < -2 is not possible as the absolute value can never be negative or less than a negative number.

ii) On the other hand solving the inequality | x+5 | > -2 would always be true for all real numbers x as the absolute value is always a non-negative number, so is greater than every negative number.

Case III: Inequalities involving Poynomials of degree greater than 1:

Example: Solve $x^2 - 12 < x$.

Step 1: Get a zero on one side of the inequality, i.e.

$$x^2 - x - 12 < 0 \qquad \text{(a)}$$

Step 2: If possible factorise the polynomial, which might not always be possible, but that won't change things. (This step is only to simplify the process). We get

$$(x-4)(x+3) < 0 \qquad \text{(b)}$$

Step 3: Determine the points where the polynomial is zero.

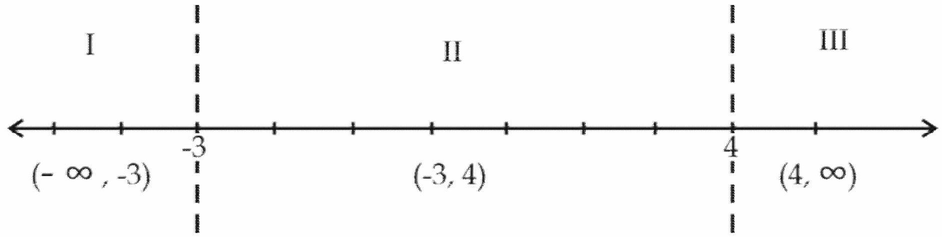
We get the points as -3 and 4.

Step 4: Mark these points on the number line. We observe that the line gets divided into three regions which are as follows:

i. $(-\infty, -3)$

ii. $(-3, 4)$

iii. $(4, \infty)$



Step 5: Now pick up a test point from each of these regions. We'll observe that the test points either satisfy the inequality or they don't. If the test point from a region satisfies the original inequality, then that region is a part of the solution.

If the test point doesn't satisfy the inequality then that region, to which this test point belongs to, is not the part of the solution.

NOTE 1: If a point in the region satisfies (or doesn't satisfy) the given inequality, then all the points in the region will satisfy (or not satisfy) the inequality.

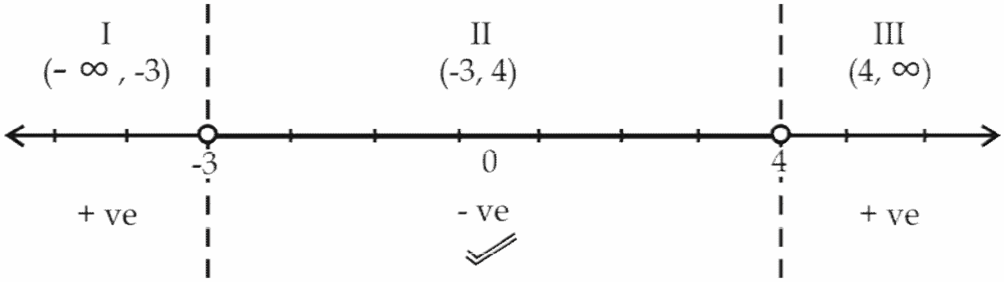
NOTE 2: When we pick a test point make sure that an easy number is picked to work with. So, don't choose large numbers or rational numbers other than integers unless the problem itself forces you to do so.

NOTE 3: We should not be bothered about the value of the polynomial at the test points; we just need to check the sign of the polynomial at the test points, e.g. the product of two negatives is a positive etc. That is to say,

Zeroes of the polynomial: -3, 4		Testing Point	Verification
$x < -3$	$(-\infty, -3)$	$x = -4$	positive
$-3 < x < 4$	$(-3, 4)$	$x = 0$	negative
$x > 4$	$(4, \infty)$	$x = 5$	positive

As we need the interval which gives the values of 'x' where the inequality is negative, therefore the solution of the above problem both in the interval notation form and graphically is given by:

Ans. $x \in (-3, 4)$



Example: Solve: $x^2 - 7x \geq -6$

Solution: $x^2 - 7x + 6 \geq 0$

- $(x - 1)(x - 6) \geq 0$

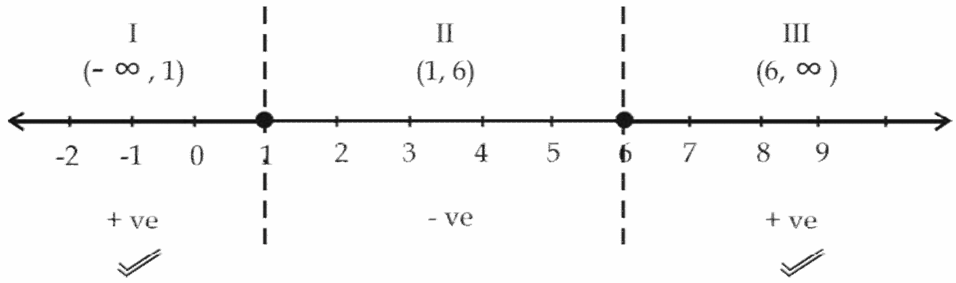
The polynomial will be zero at x=1 and x=6.

Zeroes of the polynomial: 1, 6		Testing Point	Verification
$x \leq 1$	$(-\infty, 1]$	$x = 0$	positive
$1 \leq x \leq 6$	$[1, 6]$	$x = 2$	negative
$x \geq 6$	$[6, \infty)$	$x = 7$	positive

Note: Choose the test point which belongs to the corresponding open interval. The test point must not be any of the end points of the corresponding interval.

As we need the interval which gives the values of 'x' where the inequality is non-negative, the solution of the above inequality both in the interval notation form and graphically is given by:

On the number line we may identify the solution to the above problem as:



In the interval notation the solution is: $(-\infty, \mathbf{1}] \cup [\mathbf{6}, \infty)$

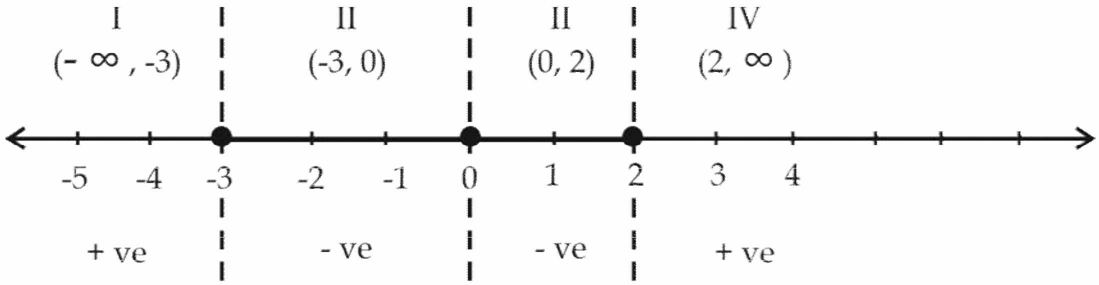
Example: Solve: $x^4 + x^3 - 6x^2 \leq 0$

Solution: $x^4 + x^3 - 6x^2 \leq 0$

- $x^2(x^2 + x - 6) \leq 0$
- $x^2(x-2)(x+3) \leq 0$

The polynomial is zero at x=-3, x=0 and x=2.

On the number line we may identify the solution to the above problem as:



In the interval notation the solution is: [-3, 2].

DO IT YOURSELF

Solve following inequalities and plot them graphically:

1) $|x + 3| > 4$ Ans. (-∞, -7) ∪ (1, ∞)

2) $|2x + 5| \leq 7$ Ans. [-6, 1]

3) $|5 - 3x| + 2 < 7$ Ans. $(0, \frac{10}{3})$

4) $x^2 - x - 2 \geq 0$ Ans. $(-\infty, -1) \cup (2, \infty)$

5) $2x^2 - x \leq 0$ Ans. $[0, \frac{1}{2}]$

REFERENCES

- https://en.wikipedia.org/wiki/Set_(mathematics)
- https://en.wikipedia.org/wiki/Absolute_value
- www.purplemath.com/modules/ineqsolv.htm
- https://www.purplemath.com/modules/absineq.htm

CHAPTER 4

WHAT IS FUNCTION?

Dear Reader

You desire to purchase school bags for winners of races held on the Annual Sports Day of school. Cost of each bag is Rs.250. Total amount Rs. a to be spent on purchase will depend on number of bags n required.

When n=1, a= 250

When n=2, a = 2(250) = 500

When n= 3, a= 3(250) = 750

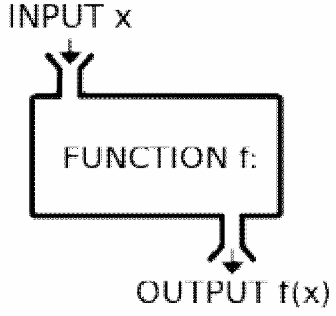
So we can say that

a = n (250), where Rs. a is the amount spent and 'n' is the number of bags purchased.

So the relationship between amount and number of bags indicate that if in a machine input of number of bags is processed it will come out as output in the form of amount to be spent.

This machine mathematically is termed as Function.

A function is a special relationship where each input has a unique output.



That means a Function is an association between members of two sets A and B so that for every member of set A there exists a unique member in set B.

A Function is generally represented as f(x) and we write y = f(x).

This means that for every value of x inserted in equation $y = f(x)$ we get a unique value of y.

For example

For $y = 3x + 1$

When $x = 0$, $y = 3(0) + 1 = 1$

When $x = 1$, $y = 3(1) + 1 = 4$

When $x = 2$, $y = 3(2) + 1 = 7$

When $x = 3$, $y = 3(3) + 1 = 10$

Thus we can see that

- x and y vary, so they are called variables.
- Value of y depends on the chosen value of x. So x is called independent variable and y is called dependent variable.
- Collection of all possible values of x (the independent variable) is called Domain.
- Collection of all values of y (the dependent variable) obtained is called Range.

In case of bag –amount function discussed above

- The Collection of all natural numbers 1, 2, 3, ..., m is the domain as bags can be counted as 1, 2, 3, ..., m
- The Collection of all values obtained as amount to be given on purchase of bags 250, 500, 750……..is the range.

  Domain = {1, 2, 3, ………, m}

  Range = {250, 500, 750, 1000, ….., 250 m}

Is function an equation?

A Function always represents the relation between two variables through sign of equality so we can say that function is an equation containing at least two variables with real coefficients.

Does every equation represent a function?

Observe the equation $x^2 + y^2 = 2$

For $x = 1$, $y = \pm\sqrt{(2 - x^2)} = \pm 1$

Here for one value of x we are getting two values of y i.e. +1 and -1. This violates the definition of a function. But, this equation represents two functions $y = \sqrt{(2-x^2)}$ and $y = -\sqrt{(2-x^2)}$ **Can we represent function graphically?**

For a function $y = f(x)$

For all values of x in the domain of function we can find the corresponding value of f(x).

So, a function can be thought of a set of elements in the form of ordered pairs.

In above example $y = 3x+1$

For $x = 0$, $y = 1$

For $x=1$, $y = 4$ and so on.

So above function can be represented as { (0,1), (1,4), (2,7)……..}

These ordered pairs can be represented on a Cartesian plane.

Why is the graphical representation of a function required?

With the help of graphical representation we can

- Locate all points satisfying the functional relationship.
- Identify the domain and range.
- Predict the nature and properties of function.

Before drawing the graph of a function it is important to understand some relevant terms.

1. **Line of symmetry**: line of symmetry divides the graph in two congruent parts. Portion on one side of line of symmetry looks the mirror image of the portion on the other side.

Observe the axis of symmetry in the following graphs.

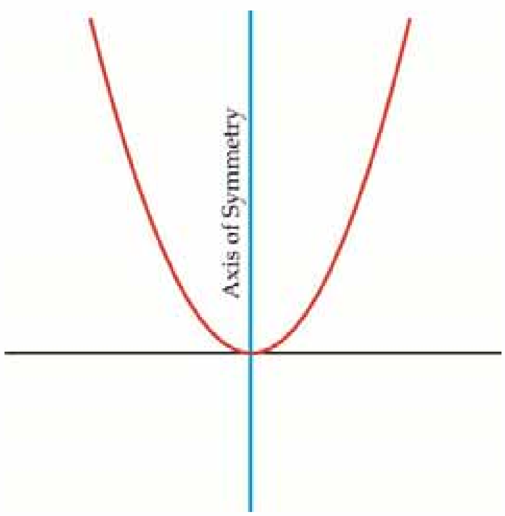


Figure 1

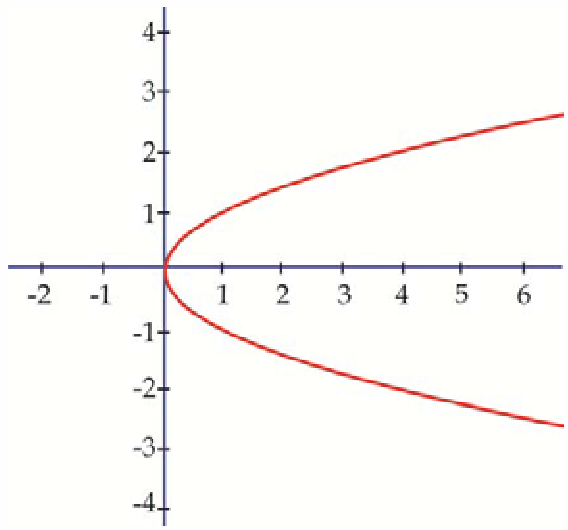


Figure 2

In figure 1 y-axis is the axis of symmetry while in figure 2 x-axis is the axis of symmetry.

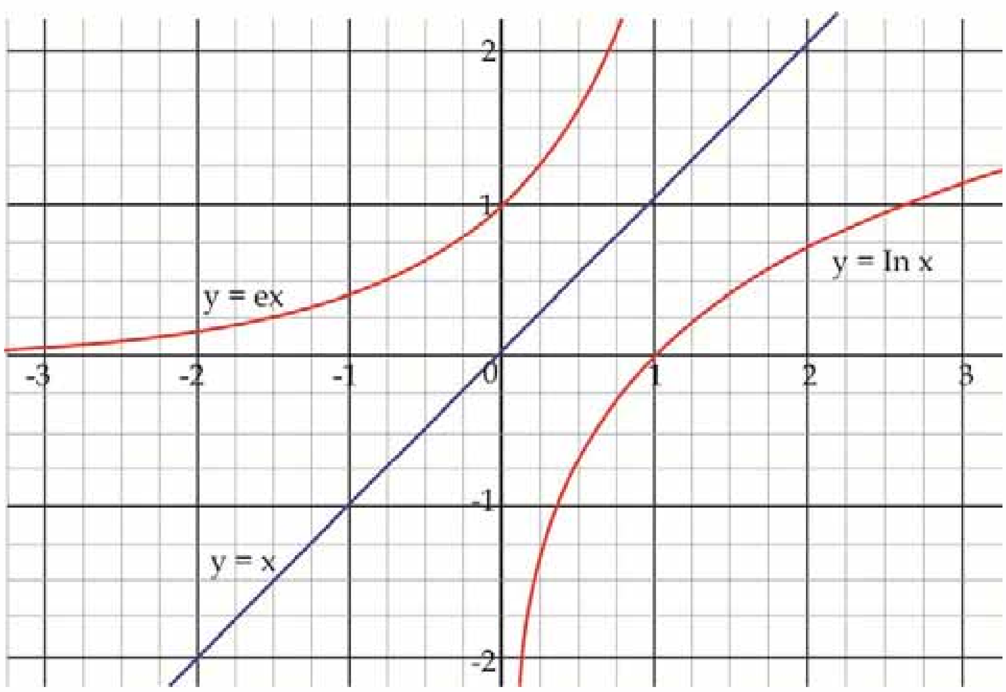


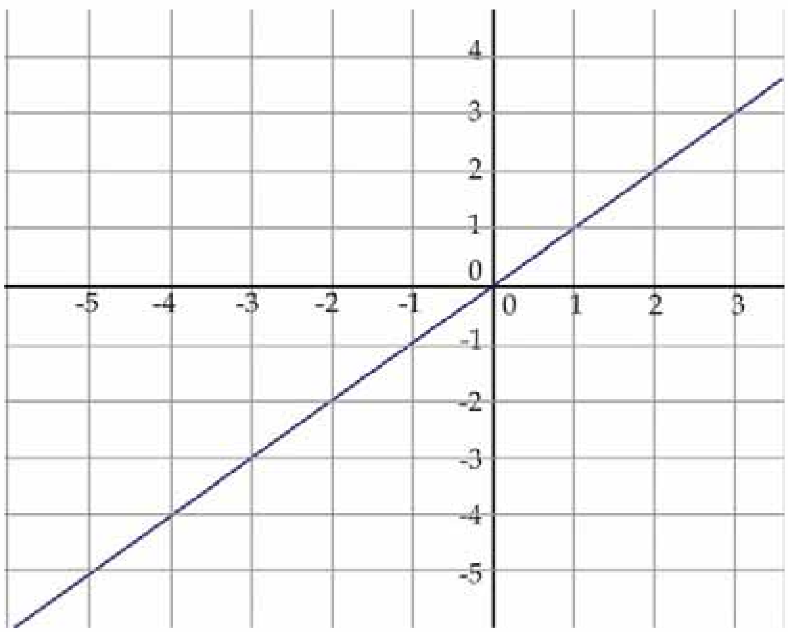
Figure 3

In figure 3 Graph is symmetrical about the line y=x.

Recall: In earlier classes you have learnt the drawing of graphs of linear equations.

For drawing graph you used to express linear equation ax+by +c =0 as $y = \frac{c-ax}{b}$.

Actually you were converting the equation in terms of function y = f(x). Review some of them.

<table>
<tr><th>Function</th><th>Graph</th><th>Remark</th></tr>
<tr><td>y = x</td><td>
</td><td>Graph is line passing through origin (0,0) and lies in I and III quadrant.
<table>
<tr><td>X</td><td>1</td><td>2</td><td>-3</td><td>0</td></tr>
<tr><td>y</td><td>1</td><td>2</td><td>-3</td><td>0</td></tr>
</table></td></tr>
</table>

DO IT YOURSELF

I. Draw the graph of following functions:

1. x = 3
2. y = 0
3. x = - 5
4. y = 7
5. y= -x
6. y = 2x

7. $y = -2x$ 8. $y = x/2$

9. $x = 3y$ 10. $Y = x+ 5$

11. $y = 2x+5$ 12. $Y = -2x +5$

II. Steps of drawing graph of a function y = f (x)

1. Find the x- intercept and y intercept of y= f(x) :

- x intercept can be determined by putting $y = 0$.
- y intercept can be determined by putting $x = 0$

For example for function $y = x^2 - 2$

For $x=0$, $y= -2$, so y intercept is -2 , which means that the curve passes through (0,-2)

For $y=0$, $x = \pm\sqrt{2}$, so x intercept is $\pm\sqrt{2}$, which means that the curve passes through the points ($\sqrt{2}$, 0) and ($-\sqrt{2}$,0)

2. Identify the Axis of Symmetry

- If the power of x is even then graph of function $y=f(x)$ is symmetrical about y-axis.

For example,

Consider the function $y= f(x) =3x^2 + 2$

When x is replaced by $-x$,

$f(-x)= 3(-x)^2 + 2= 3x^2 + 2 = f(x)$

This type of functions are called even functions.

We can say that

- Even functions are symmetric about y- axis.
- For even functions $f(x) = f(-x)$ for all values of x in the domain of function f(x)
- For even functions power of x is always even in expression of function.

- If the power of y is even then graph of function $y=f(x)$ is symmetrical about x-axis. For example the graph of $x= y^2 + 3$ is symmetrical about x–axis
- If in an equation representing the function $y=f(x)$, x is replaced by y and y is replaced by x and the resulting equation is same then graph of equation is symmetrical about the line $y= x$
- If in an equation representing the function x is replaced by -y and y is replaced by -x and the resulting equation is same then graph of equation is symmetrical about the line $y=- x$

5 CHAPTER

TRIGONOMETRY

Dear Reader

In your previous classes you have read about triangles and trigonometric ratios. A triangle is a polygon formed by joining least number of points i.e., three non-collinear points.

You also know that a triangle has six basic measures three sides and three angles. Based on the measure of sides and angles you can construct triangles using geometrical tools like foot rule, compass and protractor.

Also for the construction of a triangle all measures are not required. With the help of some of the measures the construction could be possible.

With time it was found that in a right angled triangle by defining the ratios of different sides of triangle a whole new branch of mathematics came into existence this branch is known as Trigonometry.

You have learned in class IX and X about trigonometric ratios, identities and its applications.

In class XI, we will now use the terms trigonometric functions and also extend our understanding of trigonometry from right angled triangles to any triangle.

In this chapter you will learn the following:

(i) Trigonometric functions as circular functions.

(ii) Difference between trigonometric ratios and trigonometric functions.

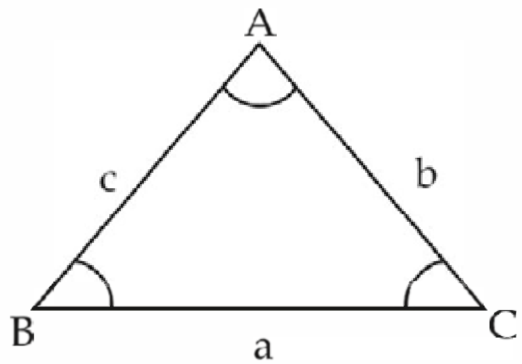
(iii) Drawing of graphs of sine and cosine functions.

(iv) Advance applications of Trigonometry.

Meaning

The word "Trigonometry" is derived from two Greek words "Trigon" meaning triangle, and "metron" meaning measure. Hence, trigonometry means measurement of triangles.

Note: A triangle has six measures: the sides of lengths a, b and c and the angles A, B, C in degrees or radians.



Question: Given some of these measures, find the others.

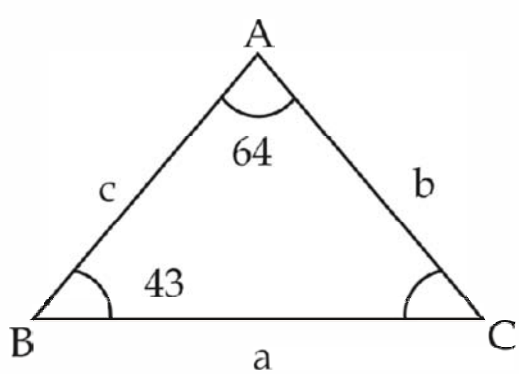
Method 1: Construct the triangle on the basis of the given measures (it is assumed that the given measures are enough for the purpose) and then find the other measures by using isometric tools foot rule, protractor D.

Method 2: Use trigonometry ---------- which is all about triangles

⟶ A big part of trigonometry consists of solving triangles, when we say solving measures of the triangle we mean finding the missing sides and angles of the triangle.

Look at the following example in triangle ABC, $\angle A = 64^\circ$ $\angle B = 43^\circ$, side c = 8 cm.

Whereas angle C can be found by using $\angle A + \angle B + \angle C = 180^\circ$ we need trigonometry to find the measures of sides AC and BC



Question: what is the minimum number of measures of a triangle that must be known to find the other measures of the triangle?

Three, however they should not be all angles

(Why? Hint: Think of similar triangles).

Trigonometric Identities

Equations that are true for all permissible values of the unknown/s are called **Trigonometric identities.**

Some of trigonometric identities are as follows:

$$\tan\theta = \frac{\sin\theta}{\cos\theta'}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin(-\theta) = -\sin\theta$$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \ \ etc.$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos c$$

or $$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

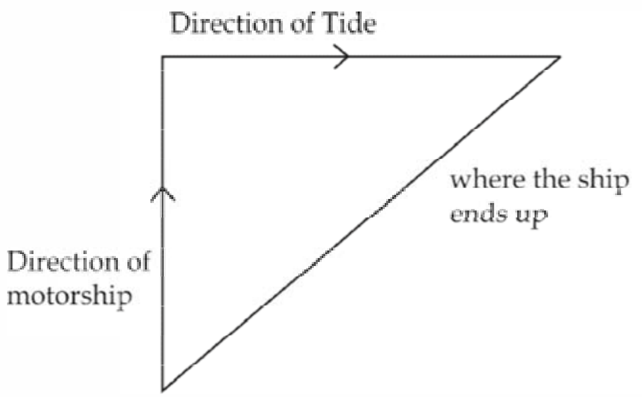
Law of Tangents

$$\frac{a+b}{a-b}=\frac{\tan\left(\frac{1}{2}\ (A+B)\right)}{\tan\left(\frac{1}{2}\ (A-B)\right)}$$

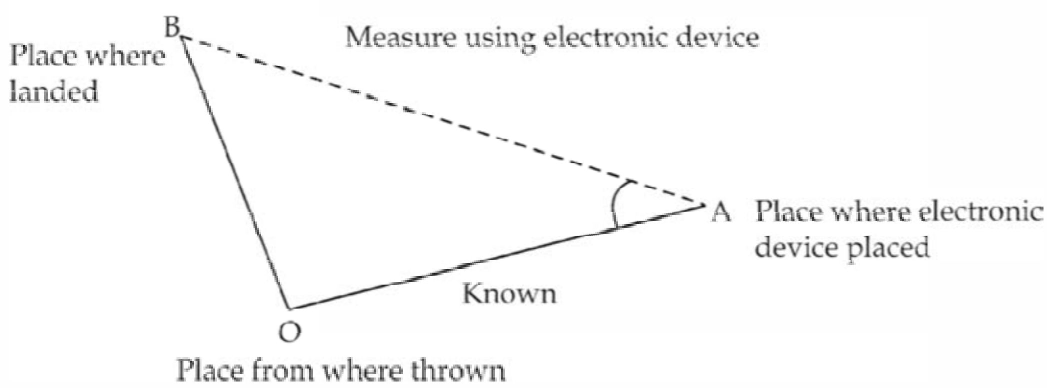
Note: Here a, b, c and A, B, C have their usual meaning as mentioned earlier.

Why Do We Need to Study Trigonometry?

Trigonometry finds its application in real life, most importantly, in navigation especially in sea for ships and boats.



Trigonometry is used in measuring distance of a shot put or spear throws by an athlete in a sports competition.



OA, AB and ∠A becomes known. Now use trigonometry to find OB. (using computers it becomes an instantaneous solution)

Advanced Use: Trigonometric ratios (sine, cosine, tangent) are used as functions and then manipulated using algebra.

Applications: Trigonometry is applied in Electronic circuits and mechanical engineering. Graphs are drawn and used to analyze things like motion and waves. For example, a radio or sound wave can be described by an equation of the type $Y = A \sin (Bx + C)$.

Circular functions: We begin with any circle and then introduce unit circle. The advantage with unit circle is that we can measure sine, cosine, and tangent directly; the radius being 1. As we move along the circle we get different angles and the process continues even after we complete one full round.

The values now start replicating to what we have already got, i.e. we get repeating pattern.

When we move anti-clock wise along the circle the angle is positive and as we move clock wise the angle measure is negative.

It θ is more than 360^0 (2π radian) we subtract 360^0 (or one full rotation) or as many full rotations as needed to make it less than 360^0(but more than 0^0). $\sin\theta$ will have the same value as sine of new angle thus obtained.

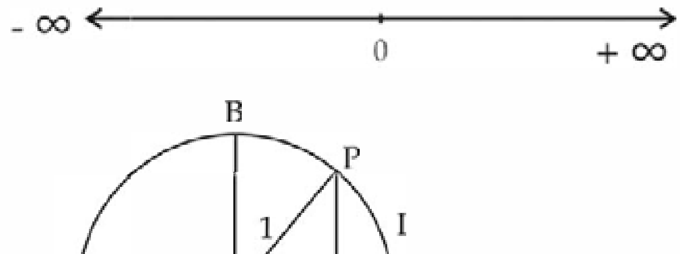
It θ, to begin with is negative we add as many full rotations needed to make it more than 0^0 (but less than 360^0) to calculate its sines and cosines.

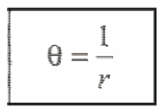
Graphs of sine and cosine functions

Let T be a unit circle i.e, a circle with radius 1 unit. Let R be the real line. Let us observe the following:

1 : The positive side of the real line from O to $+\infty$ can be wrapped around the unit circle in the anticlock wise direction infinite number of times with point O of the real line coinciding with point A of the circle.

2: The negative side of the real line from O to - ∞ can be wrapped around the unit circle in the clockwise direction with point O of the real line coinciding with point A of the unit circle.

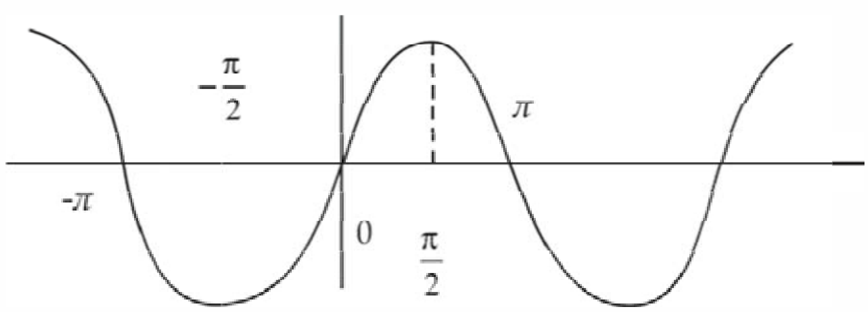




sin $(\pm 2n\pi + x)$ = sin x where n = 0, 1, 2, 3,……

Hence the value of sine of all numbers on the real line can be obtained this and its graph can be drawn.

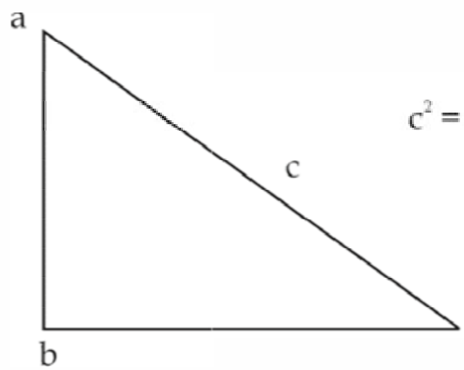
The graph looks like as shown



similarly the graphs of cosine and tangent functions can also be drawn.

Applications: Recall that we have used a right angled triangle (also called a right triangle) to define sine, cosine tangent etc. it is natural to ask, can we use these to find to side length of a right triangle?

In case we are given or we know two of the sides of right triangle we can use pythagoras' theorem to find the third side



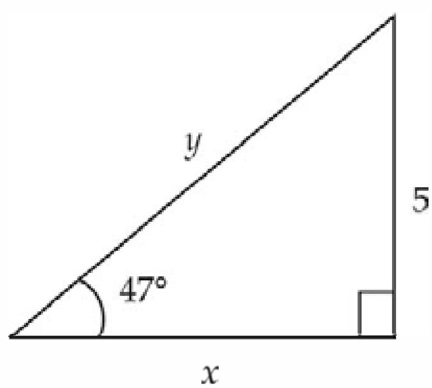
$c^2 = a^2 + b^2$ ______________ (1)

if we know two of these values a,b,c we can use (1) to find the third

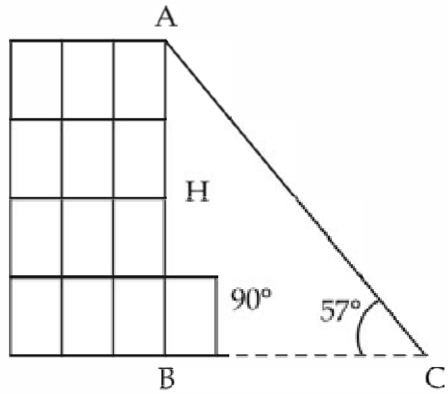
what happens in case we know only one of the sides and one of the angles (and hence all the angles) of a right triangle. We can't use the pythagoras' theorem now. However we can use trignomentric ratios to find the remaining sides.

For example if one side is 5 units and one angle is 47^0 as shown we can find x and y as follows:

$\sin 47^0 = 5/y$ so $y = 5/\sin 47^0$, But $\sin 47^0 = 0.73135$ approx hence $y = 5/\ 0.73135 = 6.83667$, we can now find x using $x/y = \cos 47^0$ or $5/x = \tan 47^0$

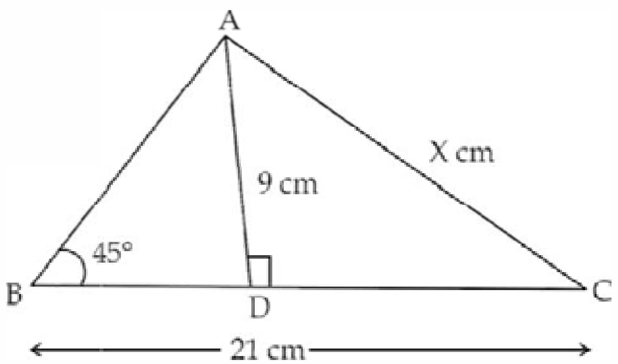


Example: If you are standing at a distance of 30 ft from the base of a building and if you have to look up at 57° (called the angle of elevation) to see the top of the building can you find the height of the building?



The answer is yes. we consider the right triangle ABC. Here h/30 = tan 57^0 = 1. 0723 approx. so h = 30 x 1.0723 so height of the building is 32.169 ft approximately.

Example: With dimensions as shown in the figure and with AD perpendicular to BC, find the value of *x*



Example: If the shadow of a building increases by 20 meters when the angle of elevation of the Sun rays decrease from 75^0 to 60^0. What is the height of the building?

Example: Pushpendra is frolicking in a hot air balloon. He can enjoy the ride for half an hour more if he can tell the distance between a flag pole and a car parked on the ground below. He asks the pilot to come directly over the flag pole and start rising vertically upward. He observes the following:

At some point during his ride the angle of depression of the car is 30^0. He allows the balloon to rise further 60 meters and observes that the angle of depression has now become 35^0. He thinks he has sufficient data to answer the question. Draw a diagram and solve the problem for pushpendra.

PHYSICS

Chapter 1 : Frames of Reference

Chapter 2 : Graphs: Their linkage with differential coefficient

Chapter 3 : Vectors

Chapter 4 : Understanding Differentiation

Chapter 5 : Understanding Integration

Chapter 6 : Introduction to Logarithms

Chapter 7 : Understanding Binomial Theorem

CHAPTER 1

FRAMES OF REFERENCE

Dear Reader

The concept of **frames of reference** is a fundamental and useful concept for our understanding of nature and natural phenomena. It is an extremely important concept for describing, and analyzing, the details of motion of different objects.

How do we (usually) define motion? We say that an object is in motion if its position, with respect to its immediate surroundings is changing with time. It is regarded to be "at rest" if there is no change in its position, "with respect to its immediate surroundings", over the span of time its position is being observed.

It is important to appreciate the need of using the words, "with respect to its immediate surroundings", in our definitions of the states of rest, or motion, of a given object. These words imply that we need to have "something to refer to", to decide, whether a given object, at a given time, is "at rest" or "moving". This "something to refer to" is, informally speaking a "frame of reference" for the object under consideration.

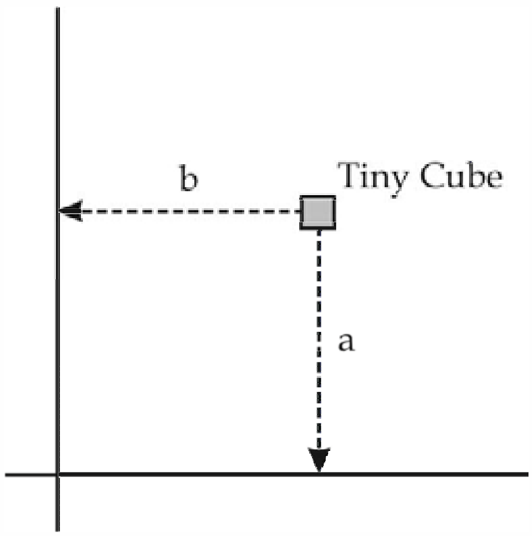
Understanding more about Frame of Reference

We give a formal meaning to the term "frame of reference", by using the concept of "co-ordinate axes" that we use in co-ordinate geometry.

Let us first consider the simplest case of locating, and specifying, the position of a point on a given straight line. For this purpose, we take some specific point, on the given straight line, as our reference point, or as the **origin**. We can then locate any point, on this line, by simply knowing its distance from the origin. The ambiguity, about whether this distance is to be measured to the right or the left of the origin, is removed by using the positive (+) and negative (-) signs. A positive sign, attached with the measure of the distance, implies that the point is located to the right of the origin. A negative sign, on the other hand, implies that the point is located to the left of the origin.

The measure of the distance, (along with its attached (+) or (–) sign), is said to define the one **co-ordinate**, needed to specify the location of a point, on a given line.

Let us next see how we specify the position of an object, say a tiny cube, on the floor. We can take two mutually perpendicular lines - like the edge lines of the floor with two adjacent walls - as our "reference lines", or as our **axes of co-ordinates**. We can then uniquely and unambiguously, specify the position of the tiny cube, on the floor, if we know its perpendicular distances (say a and b) from each of these two reference lines, or axes of co-ordinates. A different set of values, of these two perpendicular distances, would locate a different point on the floor.



We call the pair of values (like a and b), of these two perpendicular distances, as the **co-ordinates** of the point, being specified, and located, on the floor. These two **co-ordinates** are usually known as the 'x' and 'y' co-ordinates of the concerned point.

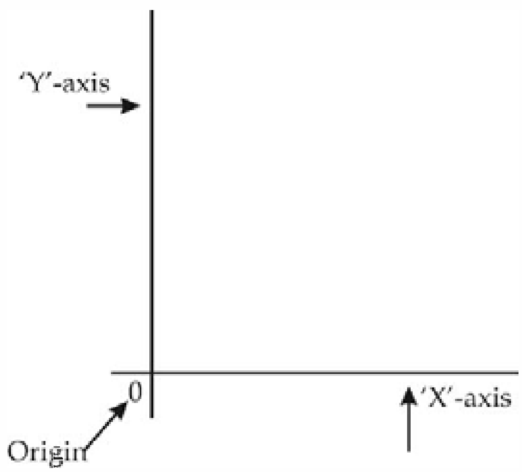
We thus realize that we can locate a point, on a plane, (like the floor) by:

(i) Taking a pair of mutually perpendicular, adjacent, lines as our co-ordinate axes or axes of co-ordinates.

(ii) Knowing the perpendicular distances, of the given point, from each of the two axes of co-ordinates.

It is useful and relevant to remember that here, the 'point of intersection', of the two 'co-ordinate axes', is taken as the origin.

The combination of the origin and the two co-ordinate axes, define a **frame of reference**, for a (given) plane. The diagram given below, shows such a 'frame of reference', in the X-Y plane.



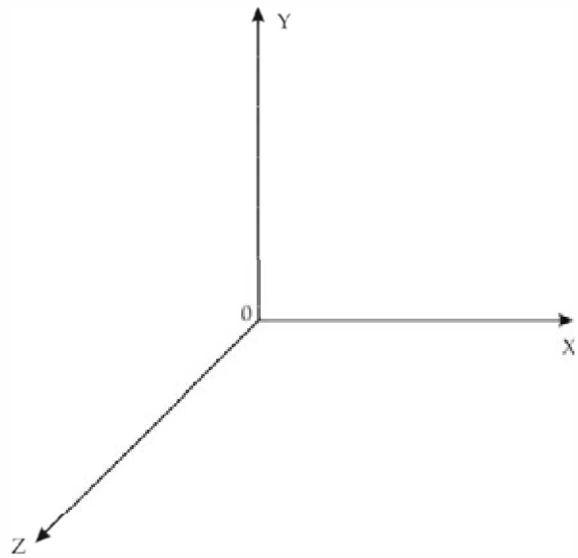
A Frame of Reference, in two dimensions, or, in a plane. (This plane is referred to as the X-Y plane).

The General Case

The general case, of course, would be to define the frame of reference for an object located anywhere in space. Consider, for example, the case of different objects in a room. To specify their location, we would, now need not only their two perpendicular distances (from two mutually adjacent perpendicular lines on the floor), but also their height above the floor. We thus need a set of three distances (say a, b, and c) to completely specify the location of a point, "in space". These three distances, are referred to as the three coordinates (the x, y and z coordinates) of the given point, in space.

It is easy to realize that (just as we need a set of two co-ordinates axes to specify the location of a point in a given plane), we would need a set of three co-ordinate axes (usually called the x, y and z axes) to specify the location of any point, in a given region of space. All these three co-ordinate axes are drawn from a common reference point, or a (given) origin of co-ordinates. The three co-ordinates (x, y, z), of a given point, are the three perpendicular distances, of the given point, from these three co-ordinate axes.

The set of three co-ordinate axes (having a common intersection point, or ORIGIN), chosen in a given region of space, defines the 'frame of reference' for that region of space. We can use this 'frame of reference' (i) to specify the location of different points in that region of space and (ii) to describe and analyse, the details of motion of a given object in that region of space. It is important, however, to remember that this description, and analysis, is only with respect to the chosen 'frame of reference'. It can change if a different 'Frame of reference' is chosen for this purpose, by a different observe.



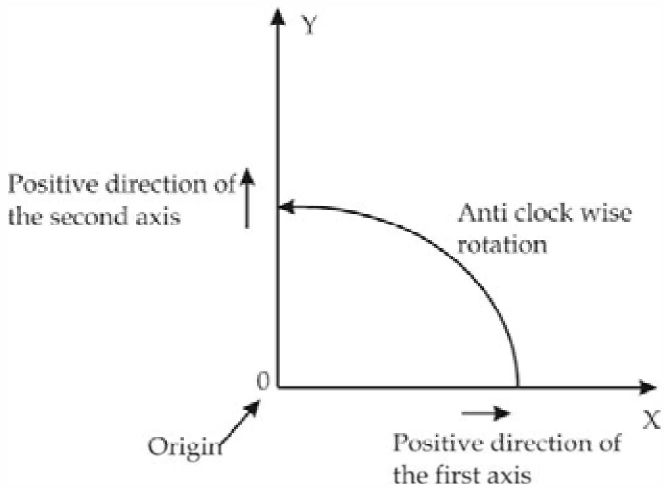
A set of three co-ordinate axes, (along with their origin point) defines a 'frame of reference' in space, or as we often say, 'THREE DIMENSIONS'.

The Right Hand Convention

We have noted, in the case of location of a point along a straight line, that we assign a (+) sign to distances to the right of origin, and a (–ve) sign to distances to the left of origin. We can easily realize that there is a need for a similar sign convention, for each of the three co-ordinate axes, for a frame of reference in three dimensions. There is also a need for a standard convention, for specifying the three co-ordinate axes in an unambiguous manner. For example, if we have specified the x and y axes on the floor, do we take the z-axis directed upward, above the floor, or directed downwards below the floor?

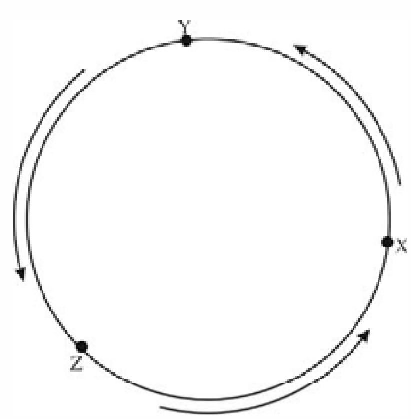
To remove such ambiguities and to have a standard way of specifying the three axes, it has been decided to have a standard convention has been adopted for this purpose. This standard convention, called the 'Right hand Convention' goes like this:

Draw a line on the 'floor' (i.e., a plane, suitable for the given situation) and mark a point on it as the origin. Next draw a line, perpendicular to this line, from the origin. The sense, of an anticlockwise rotation, from the right hand side of the first line, is taken as the positive direction of this second axis.



Next imagine that a screw is held in the right hand with its tip at the origin, and with its length perpendicular to the floor (i.e. the chosen suitable plane). Imagine this screw to be rotated, from the first to the second axis, i.e. the anticlockwise sense. The sense of advancement of this screw, is then taken as the positive direction of the third axis.

The three axes, drawn in the way described above, are known as the x, y and z axis respectively. The 'right hand rule' (or anticlockwise rotation rule), described above, can then be pictorially represented as shown below.



The set of x, y and z axes, drawn as per this right hand convention, is the appropriate, (three dimensional) (3D) frame of reference, for a given system, or situation, or 'set up'.

Selecting an appropriate "Zero" of Time

Suppose we have to describe / analyse the motion of a given object that has been at rest for quite a while. It is appropriate then to think, and take, about the motion of this particle only from the time if starts moving.

It is usual to take the starting instant, of observing the motion of some given object, as the "zero" of time for describing its motion. We can the observe / analyse the motion of the object at a time 5, 10, 15, ... etc. seconds later than its starting instant.

It is important to note that the object need not always be at rest at $t = 0$. For example, for a moving car, we may think of describing its motion from the time it has a speed of 40 km/hour to a time when it acquires a speed of 60 km/hour. The zero of time here is the time when its speed is 40 km/hour. Thus, for this car, speed, at $t = 0$, is 40 km/hour and not zero.

The choice, of the 'frame of reference' (a suitable set of x, y and z axes for the given system / object) and an appropriate zero of time, enables us to fully understand, and analyse, the motion of a given object, moving under its relevant conditions.

The complete **frame of reference**, therefore, may, therefore, be said to include:

(i) The three co-ordinate axes, chosen appropriately, as per the right hand convention.

(ii) An appropriate instant as the zero of time.

Once we have such a complete frame of reference, for a given object, we can describe and analyse, its motion in a systematic, and sequential, way.

DO IT YOURSELF

1. Give an example of a situation/'set-up' in which a given object is regarded (i) at rest (ii) in motion, by two different observers.
2. Imagine that you are standing on the foot path of a busy road. Give two examples each, of the objects around you, which you would regard as being (i) at rest, (ii) in motion. Justify your choices.

3. Draw a pair of co-ordinate axes on a graph paper. Take one small square, of the graph paper, to represent a unit distance along both these axes.
   (a) Mark points corresponding to the values (10, 23), (5, 17), (25, 37) as co-ordinates.
   (b) Mark any three points, arbitrarily, on your graph paper. Try to assign values, to the co-ordinates of the points, marked by you.

4. Using a similar graph paper, and adopting the same scale as used in the above question, mark any four points (on the graph paper) that have
   (a) Different values of their second (or "y") co-ordinate but have the same value for their first (or "x") co-ordinate.
   (b) Different values of their first (or "x") co-ordinate but have the same value for their second (or "y"-co-ordinate)

   Join the four points, marked by you, in each case. State your observation/s, if any, about the curves/lines, obtained by you by joining the four points.

5. Suggest a suitable frame of reference for:
   (a) Specifying the positions of different mile-stones on a long straight road.
   (b) Specifying the positions of different pieces of crockery on a dining table-top.
   (c) Specifying the position of different objects located between the floor and the ceiling of a room.

6. State why it advisable / necessary to select (along with the suitable number of co-ordinate axes) an appropriate zero of time for describing / analyzing, the motion of a given object.

2 CHAPTER

GRAPHS: THEIR LINKAGE WITH DIFFERENTIAL COEFFICIENT

Dear Reader

We very often come across pairs of physical quantities, in which one quantity depends on the other. For example

(i) The current flowing through a conductor depends on the potential difference maintained between its ends

(ii) The distance, covered by a particle, in a given time, depends on its average speed.

There are many ways in which this interdependence can be expressed. We may express the interdependence through appropriate data or through a mathematical relation between the two quantities. However, one of the most convenient ways of depicting this interdependence is by plotting graphs. Graphs are important because they communicate information visually. For this reason, graphs can help us in understanding the interdependence by getting the point across quickly and visually. **Graphs are beneficial because they summarize and display information in a manner that is easy to comprehend.** Graphs are used in many academic disciplines. A graph is a pictorial representation depicting the variation of a quantity with respect to some other quantity, Hence the kind of relationship the two quantities are sharing with each other becomes quite clear. When two quantities depend on each other, one of them can be made to change "at will" and is the **independent** variable .The other one, which varies as a result of this change, is the **dependent** variable.

We generally follow the following steps, while plotting a graph:

STEP 1-Procuring the data- Before plotting the graph between two variables, identify the independent and the dependent variables. In order to plot the graph we must have sufficient data, i.e. a sufficient set of values for the independent variables and the corresponding values of dependent variable. This data can be obtained theoretically or practically. (The theoretical data is obtained by putting the values of independent variable in the theoretical relationship and calculating the corresponding values of the dependent variable.)

STEP 2- Drawing the axes- Draw axes keeping in view the nature of values- their range and sign. If all the data to be plotted on the graph is positive, we draw two lines, one in the horizontal direction and the other in the vertical direction from the left hand bottom corner of the graph paper. The horizontal line is called x-axis and the vertical line is called y-axis. The point of intersection of x and y axes is called the origin. However if the data has both positive and negative values, the origin is shifted to the middle of the graph paper.

STEP 3- Choice of axes- The independent variable is plotted on the x-axis, on the y-axis the dependent variable is plotted.

STEP 4- Choosing the scale- The scale chosen should be such that the whole range of variation gets well spread out on the whole graph paper i.e., it should not be very small so that all the points of the data may get crowded in a very small portion of the graph. Also the scale should not be so large that the entire data may not get represented on the graph

STEP 5- Plotting of points on the graph- The points, plotted on the graph, are marked as sharp dots each enclosed in a circle, with its co-ordinates mentioned by its side.

STEP 6- Joining the points-

a) The plotted points may fall on a straight line. It is not necessary that all the points may fall exactly on the line. In that case, the best fit line is drawn i.e. the line having maximum number of points falling on it, or passing closest to the maximum number of points, which appear on either side of the line.

b) The points plotted may not fall on a straight line, they may be joined with free hand, resulting into a best fitting curve

Example 1

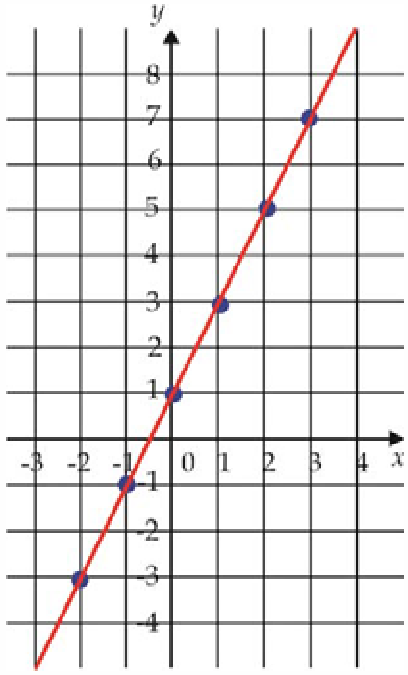
http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i14/s3eg4.gif
A line has the equation $\boldsymbol{y = 2x + 1}$.

Using x values from –2 to +3, plot the graph of this equation.

Solution: In this case x can be treated as independent variable and y as dependent variable. For different values of x ranging from –2 to +3, calculate, the values of $y = 2x + 1$ using the equation and write the values in tabular form

x	–2	–1	0	1	2	3
$y = 2x + 1$	–3	–1	1	3	5	7

Now the axes are plotted and suitable scales are chosen for both the axes. Next, each pair of x and y values can be plotted on the graph. In this case the co-ordinates are: (–2, –3), (–1, –1), (0, 1), (1, 3), (2, 5) and (3, 7). Finally the points are joined with a straight line



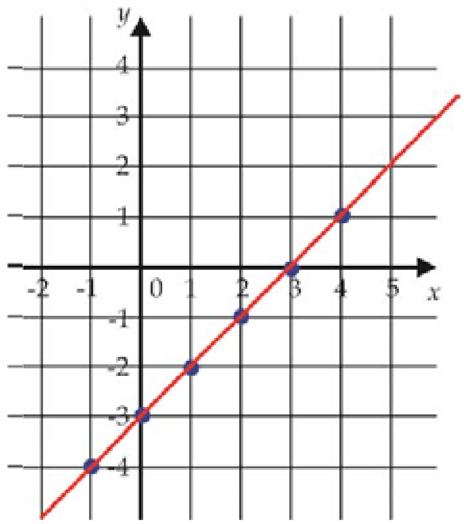
Example 2

(http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i14/s3ea4.gif)
For the equation $y = x - 3$, plot y vs. x graph

(a) Using x values from –1 to +4, compute the values of y and write them in the tabular form as shown in the table below.

x	−1	0	1	2	3	4
$y = x - 3$	−4	−3	−2	−1	0	1

(b) Draw x and y axes on the graph paper. Choose suitable scales for both the axes. Plot the points on the graph and join them.



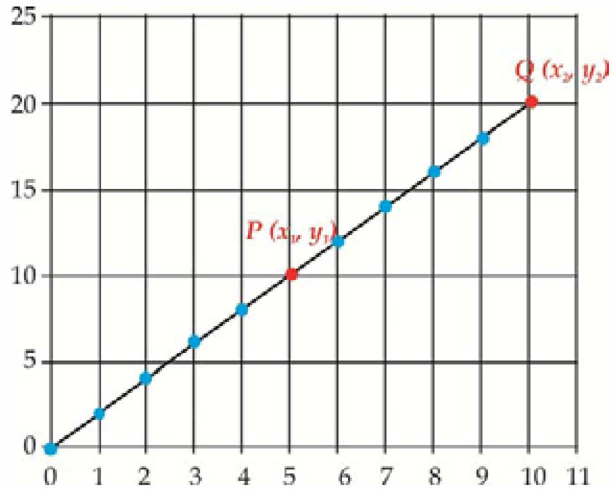
Slope of a graph: We can use the plotted graph (between the variables x and y) to obtain a lot of important and useful information. One of the characteristics of the graph - its slope - usually plays a very important role in obtaining such information. The **slope** of a graph indicates how steep the graph is. Mathematically it is computed in the following manner. Choose any two points (say P and Q), on the graph, having co-ordinates (x_1, y_1) and (x_2, y_2) respectively. The slope equals the ratio of the change in y co-ordinates i.e., (y_2-y_1), to the corresponding change in x co-ordinates, i.e., (x_2-x_1).

Slope of a graph equals

$$\frac{\text{change in } y}{\text{change in } x.} \quad \text{or} \quad \frac{\text{vertical change}}{\text{horizontal change}} \quad \text{or} \quad \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad \frac{\text{rise}}{\text{run}}$$

The slope may also be defined as the rate of change of y with respect to x.

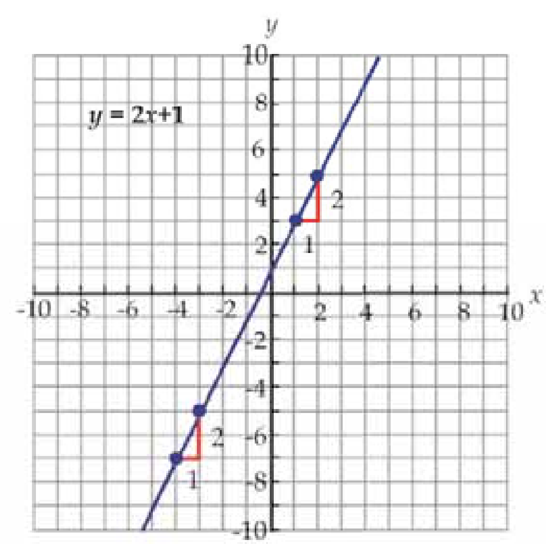
Let us calculate the slope of a line y=2x, whose graph is given below



Pick two points P and Q, that are **on the line.** We could take, point P, for which, $x_1 = 5$ and $y_1 = 10$, and point Q for which, $x_2 = 10$ and $y_2 = 20$.

We use the values of these two points, to calculate the slope:

$$\text{Slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(20-10)}{(10-5)} = \frac{10}{5} = 2$$

Consider the line $y = 2x + 1$, shown at the right. By how much has the value of y changed between the two points (-4,-7) and (-3,-5)? Answer: 2 units By how much has the value of x changed between the two points (-4,-7) and (-3,-5)? Answer: 1 unit The slope = $\frac{\text{vertical change}}{\text{horizontal chage}} = \frac{2}{1} = 2$	

It is easy to verify that this slope will be the same if any other pair of points, is used for calculating it. Hence, for straight lines, the slope, has a constant value.	

Additional Comments on the Slope

a) Straight line graph

As noted above, the slope of a graph has a constant value, if the graph is a straight line. This implies that: If a physical quantity (y) varies in proportion to another physical quantity (x), the rate of change of y with respect to x has a constant value.

The rate of change of y with respect to x, is also known as the differential coefficient of y with respect to x and is denoted by $\frac{dy}{dx}$. Thus for a straight line graph, between two quantities y and x, we can say that

$$\frac{dy}{dx} = \text{slope of a straight line graph between } y \text{ and } x = a \text{ constant}$$

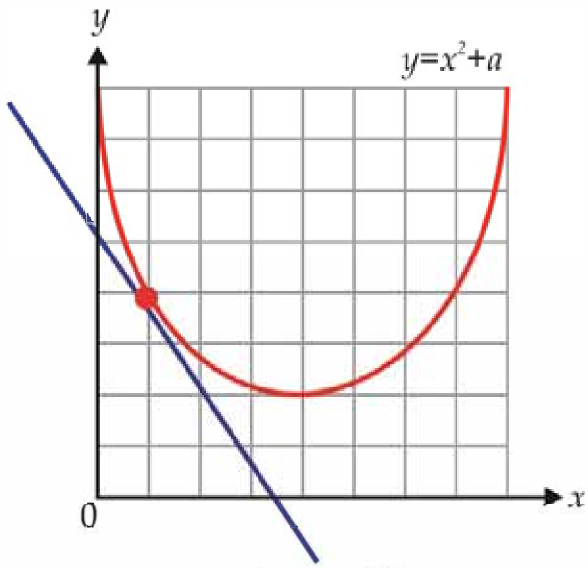
b) Non-Linear or Curved Graphs

In general, the variation of a physical quantity (say y') with respect to another physical quantity (say x'), is not a proportional variation. In such cases, the graph between y' and x' would not be a straight line. It would be a curve, whose exact shape and nature, will depend on the way in which y' varies with x'.

Can we define the slope of a non-linear, or curved, graph between two quantities? Yes, we can do so, but with a small, but significant difference. We now need to define this slope for each point on the curve. This slope, at different points, can have different values. This implies that the rate of change of y', with respect to, x', is not a constant and has different values at different points on the graph. Remembering that the 'rate of change' also equals the differential coefficient of y' with respect to x', we can say that:

If graph between y' and x' is not a straight line, $\frac{dy'}{dx'}$ is not a constant and can have different values at different points on the graph.

How do we calculate the slope of a non-linear, or curved, graph at a particular point? For this, we draw a tangent to the graph at that particular point and then calculate the slope of this tangent (a straight line) in exactly the same way as we calculated the slope of a straight line graph. The slope of this tangent line then gives the 'rate of change', or the value of $\frac{dy'}{dx'}$ at that particular point.



We thus realize that the slope of a graph, be it a straight line or a curved graph, is equal to the value of rate of change, or $\frac{dy'}{dx'}$. This value is a constant for a straight line graph but is not so for a curved graph.

The slope of a graph thus helps us to understand the geometrical, or pictorial, meaning of the differential coefficient of a variable (y) with respect to another variable (x).

CHAPTER 3

VECTORS

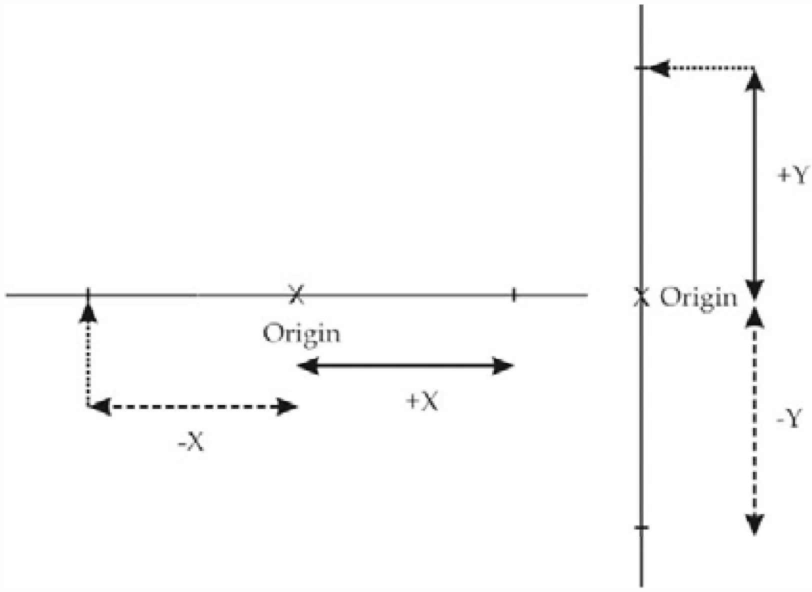
Dear Reader

Let a particle move along a straight line path (i.e. have a one-dimensional motion). We can easily specify its position, with respect to a reference point, by knowing its distance from the origin and by knowing which side of the origin (right or left; up or down), it has moved. We differentiate between these two possibilities by adopting the convention:

(i) A distance x, to the right (or upwards), from the origin, will be attached a (+) sign, i.e., it would be noted as +x.

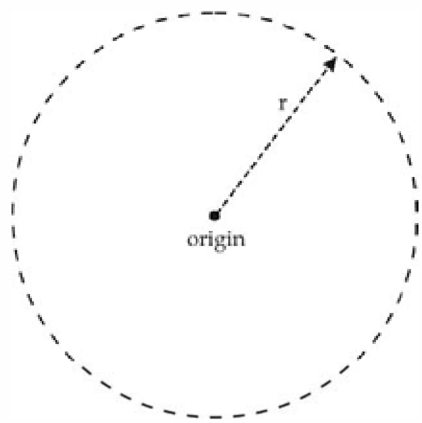
(ii) A distance x, to the left (or downward), from the origin, will be attached a (–) sign, i.e. it would be noted as –x.

A simple use of these two signs, as per the above convention, enables us to, unambiguously specify the position of the particle, with respect to the (chosen) origin, on its straight line path.



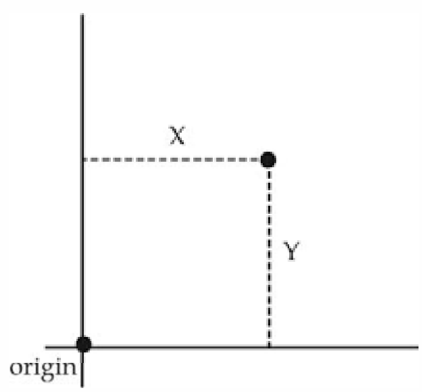
For a particle moving along a straight line path, we just need to use a (+) or (–) sign, along with its distance (from the origin), to completely specify its position.

Let us next consider the case of a particle moving in a plane, say the plane of this page. Let us choose a particular point, say O, as the 'origin', or the reference point. If we are now told that the particle is at a distance r from the origin, can we just use this information to specify its exact location?

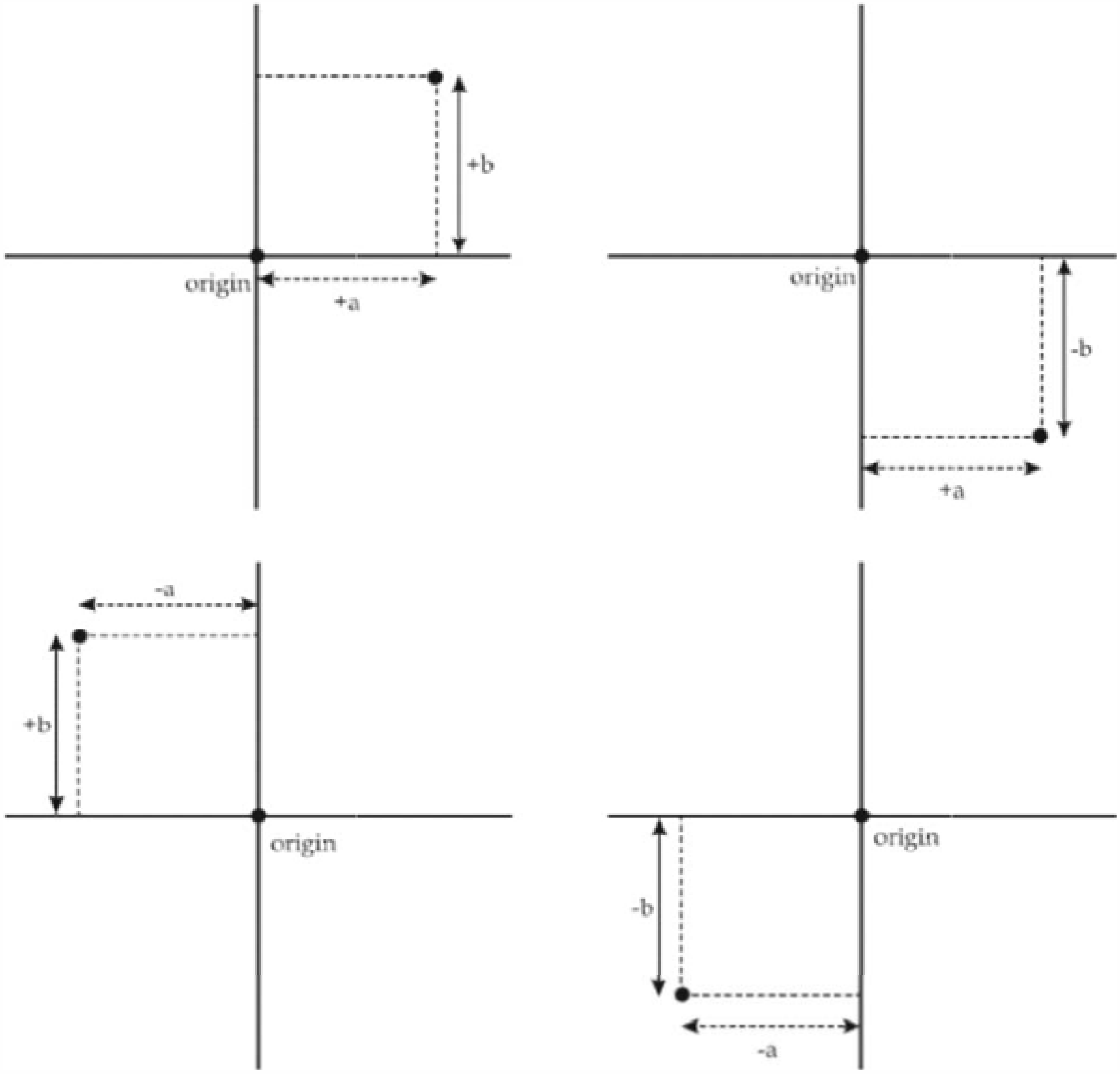


It is easy to realize that the answer to the above query is a big 'NO'. The particle can be anywhere on the circumference of a circle, having a radius r and centered at the origin. We now need a system in which the direction of the location of the particle, with respect to some standard line, (taken as the reference line), also gets specified. There are two simple ways of doing that:

(i) We take two mutually perpendicular lines, passing through the origin, as our two reference lines, (or axes of co-ordinates, as they are called). If we know the two perpendicular distances, of the particle from these two lines, we would have an unambiguous specification of its location.



We, however, need to remember that the convention of attaching (+) and (–) signs to these distances (as given above), will have to be followed here also. This is illustrated through the following diagrams:

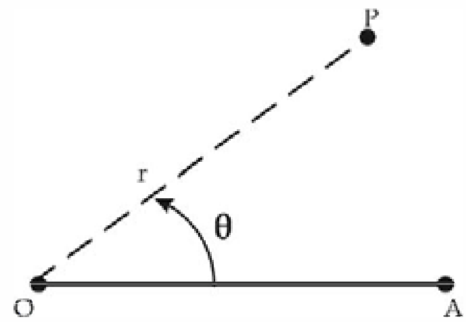


(ii) We can take only one line, passing through the origin, as our reference line. We now need to know:

(a) The distance r, of the particle P from the origin.

(b) The angle, θ made by the line OP, with the reference line, say line OA.

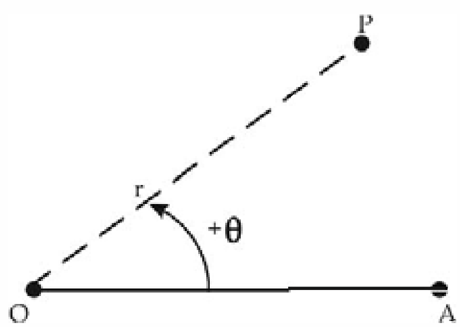
We however, need to remember that there is again a convention for knowing the value of θ here.

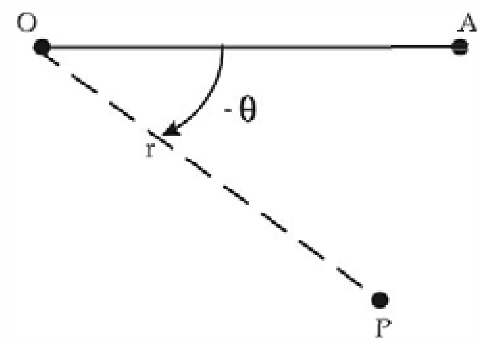


We take θ as the angle through which the reference line OA, must be made to rotate, in the anticlockwise sense, to coincide with the line OP. In other words, we attach a (+) sign only with an anticlockwise rotation.

It follows then we must take of a (–ve) sign, attached to an angle θ, as associated with a clockwise rotation.

The diagrams, given below, indicate how this convention is used to specify the location of a particle P when its distance from the origin is r but the angle, made by the line OP, with the reference line OA, equals (a) (+θ) and (b) (–θ).





Scalar and Vector Quantities

We thus realize that we need to know both; a magnitude, and a direction, to completely specify the location of a particle in a plane. A simple knowledge of a magnitude (the distance of the particle from the origin), is NOT sufficient here.

These are many other situations, in which the knowledge of the direction, along with the magnitude, is a must for understanding the relevant phenomenon. Consider a football that has been kicked off by a player, with a force of 100N, say. Is a mere knowledge, of the magnitude of the force applied, sufficient here to know its effect? The answer is again a NO. We must also know the direction, in which the force has been applied, to know its effect.

We thus realize that there are several physical quantities that need both a magnitude and a direction for their complete specification. There are, of course, several other physical quantities that can be completely specified by a (algebraic) magnitude only i.e. a magnitude to which a (+), or (–) sign has been attached, as per some (standard) convention.

We distinguish between these two kings of quantities by saying that:

(i) All physical quantities, that can be completely specified by an (algebraic) magnitude only, are known as SCALAR quantities.

(ii) All physical quantities, that need both a magnitude and a direction, for their complete specification, are known as VECTOR quantities.

Examples of Scales and Vector Quantities

Consider the physical quantity "temperature". It is easy to realize that we can assign a meaning to the temperature of a body (or to the change in temperature of a body), without any reference to the directions, in a plane, or in space. This meaning can be assigned through an algebraic magnitude. The temperature of a body has a (+) magnitude if it is above the zero of the chosen scale; it has a (–) magnitude if it is below the zero of the chosen scale. Similarly a change in temperature has a (+) / (–) magnitude if it corresponds to a rise / fall, in the temperature of the body.

The temperature of a body, is, therefore, a scalar quantity.

Like temperature, several other physical quantities - like mass, volume, density, energy, time etc., are also taken as scalar quantities. This is because they can be specified simply through a magnitude / algebraic magnitude, without any reference to the different directions; in a plane, or in space.

Consider next the effect of a physical quantity like force. Can we have the complete specification, of the effects of a force, simply by saying that this force has a magnitude of, say 10 newtons? The answer is clearly a NO. We also need to know the (specific) direction in which this force acts. A force of a given magnitude, acting to the right, or to the left, upwards or downwards, or inclined at some angle, to the horizontal, would produce an effect that would depend on its direction.

Force is thus a vector quantity; it needs both a magnitude, and a direction, for its complete specification.

Like force, several other physical quantities, like displacement, velocity, acceleration, momentum, electric field and gravitational field, are also taken as vector quantities. We need to specify both their magnitude and their direction to have a complete specification of such quantities themselves, or of the effects associated with them.

Representation of Vector Quantities

Scalar quantities, as we know, are represented through numbers; we also need to add the **unit** of the relevant physical quantity. Thus we may say. The maximum temperature, in Delhi on last Monday, was 37˚C; the mass of this ball is 75g; it takes me 40 minutes to reach my school, and so on. For the unknown magnitude of a scalar quantity, we may use algebraic symbols, we may say "Let us use the given data to find the unknown density (= x kg/m^3) of this oil".

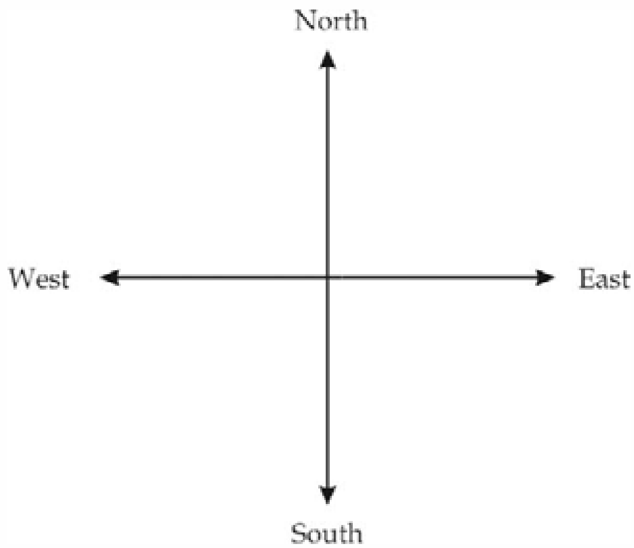
How do we represent vector quantities? Since vectors are associated with a magnitude as well as direction, we cannot use simple numbers to represent them. We use the concept of "directed line segments" to represent vector quantities. Here we draw a line, directed along the direction of the given vector quantity, and take its length corresponding to the magnitude of the vector quantity. To accommodate both large and small magnitudes of different vector quantities, we need to choose a suitable scale; we may say that our scale, for this vector (for example force), is 1 cm = 10 newton and so on.

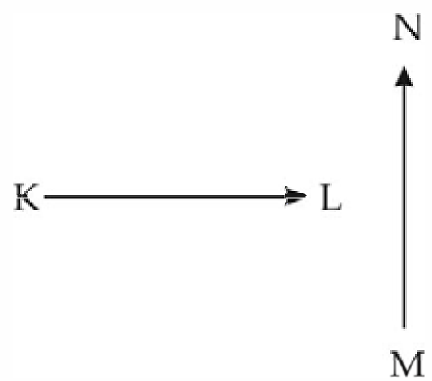
To understand this idea clearly, let us see how we really use it. Let the velocities of two particles A and B, at some given time, be 10 metre per second, towards the east, and 15 metre per second, towards the north, respectively.

We may say that our scale is

1 cm = 5 metre per second

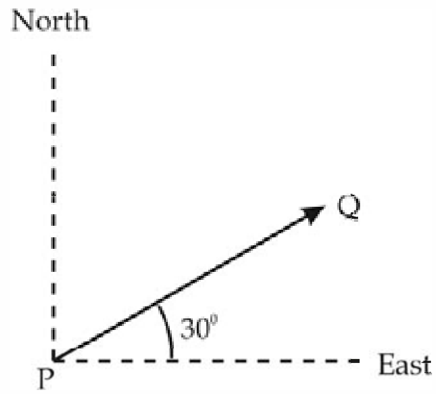
We then draw a line of length 2cm $\left(\frac{10}{5}=2\right)$ directed parallel to the east direction and another line of length 3cm $\left(\frac{15}{5}=3\right)$ directed parallel to the north direction. These two directed line segments, KL and MN, then, are the vector representations, of the velocities of the two particles, A and B respectively.

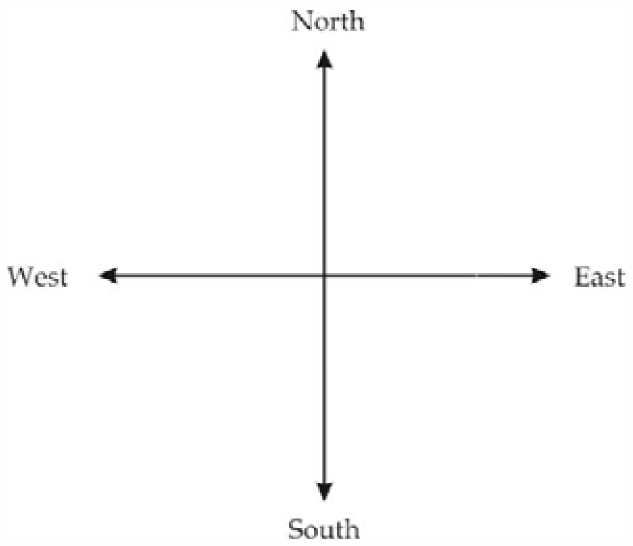




The same idea may be used to give a vector representation to any physical quantity. Remember: we have to do this representation by adopting a convention, like the one shown above, for representing the east, west, north and south directions.

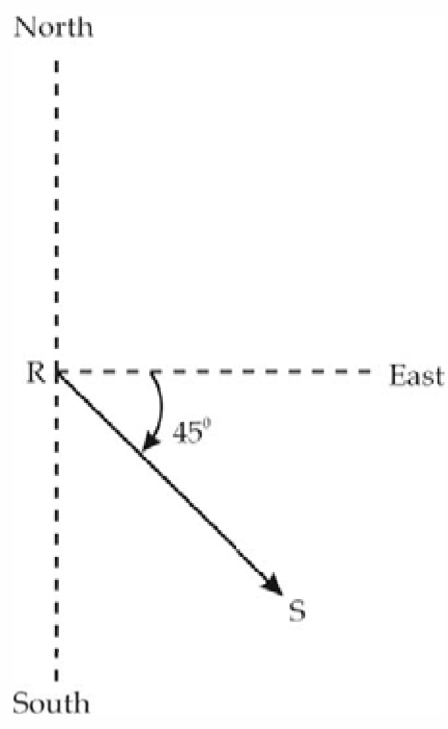
Let us now represent a force of magnitude 25 newton acting along a direction that is 30^0 north of east. We first choose a scale; say 1 cm = 10 N. We then draw a line that is inclined 30^0 to the east direction in the sense that takes us from east towards north. The directed line segment, PQ, of length 2.5 cm $\left(\frac{25}{10}=2.5\right)$ drawn in the sense, shown here, then gives a representation of the given vector quantity, i.e. a force of 25 newton acting along a direction that is 30° north of east.





The directed line segment, RS, again of length 2.5 cm, say, would represent a force of 25 newton, directed along a direction that is 45˚ south of east.

The dotted lines, corresponding to the different directions shown here, are just to help us understand and appreciate the basic ideas associated with the "directed line segment representation" of vector quantities. We need to draw only the directed line segments, PQ and RS, to represent the forces involved in the two cases.



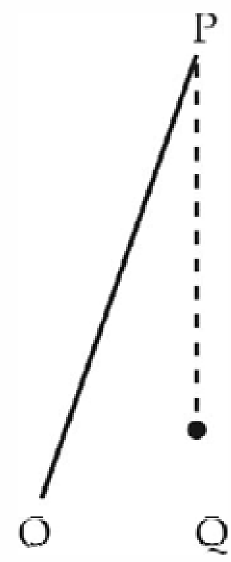
A symbolic representation of these two vectors is done through the symbols, $\overrightarrow{PQ}$ and $\overrightarrow{RS}$, respectively. We, similar use a symbol, along with an arrow line on top of it, to represent an unknown vector quantity. Thus the symbol, $\vec{r}$, may be used to represent the unknown displacement of a particle. An unknown velocity may be represented through the symbol $\vec{v}$; an unknown acceleration, through the symbol $\vec{a}$ an unknown force, through the symbol $\vec{F}$, and so on.

In the printed form – in books and journals etc. – the use of the arrow, over the symbol, is avoided by printing the symbol in BOLD print. Thus, in the printed form, the unknown displacement, velocity acceleration and force, referred to above, would be represented through the symbols, **r v, a** and **F** only. Remember, a bold symbol for an unknown physical quantity (in a printed page), implies that the unknown physical quantity is a vector quantity.

Two important Vectors

Two vectors that we need very often (to describe and analyse the motion of a particle) are the **position vector** and the **displacement vector**. Let us understand the meaning of these two terms one by one.

Position Vector



Let a particle be at some point, O, at the start of its motion. (We then, usually, take O as the origin, or reference point, for describing its motion). Let the particle move to some other point P, over a time t. The directed line segment, **OP**, is then said to represent the POSITION VECTOR of the particle, at the time instant t, with respect to the starting point, O, of its motion.

Remember that the position vector, of a particle, at any time instant, has to be always defined with respect to some reference point. This reference point is usually the point (O), taken as the ORIGIN for the description of its motion. However, this need not always be so. In the above example, let the particle be at some position Q, at a time say $\frac{t}{2}$, after the start of its motion. The directed line segment **QP**, then represents the position vector of the particle, at the time instant t, with respect to the point Q.

We may thus say:

(i) The position vector of a particle, at any time, has to be defined with respect to some (pre-decided) reference point. (Usually the reference point is the origin, for the description of motion).

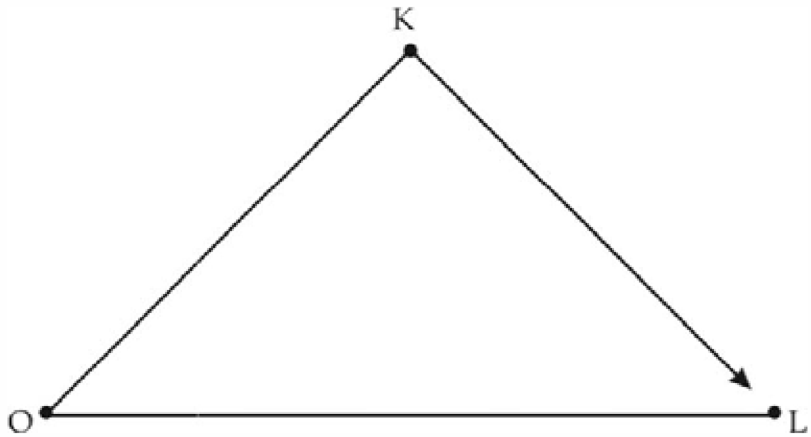
(ii) The position vector (with respect to the (pre-decided) reference point) is the directed line segment **OP**, joining the present position point (P) of the particle, with the (pre-decided) reference point (O). The direction of this position vector is along OP, i.e., its associated directed line segment has to be directed from the reference point (O) to the present position (P) of the particle.

Displacement Vector

The concept of **displacement vector** is closely linked with the concept of the position vector. We can understand this concept as follows:

Let O be the reference point, or the starting point, (or the origin), for the description of motion of a given particle. Let this particle be at point K at some time instant t_1 (=5 s, say) and at the point L, at some time instant t_2 (=10 s, say) ($t_2 > t_1$) after the start of its motion. The directed line segments, **OK** and **OL**, are then the position vectors of the particle, at the time instant t_1 and t_2, respectively.

If we now draw the directed live segment **KL**, we say that this represents the displacement vector of the particle, during the time interval from t_1 (= 5s) to t_2 (= 10s). Thus the **displacement vector**, of a particle, over a given time interval, is the directed line segment, joining its present position point (L), to its previous position point (K). Its direction is from the previous position point to the present position point.



It is now easy to realize that in the diagram, given for position vectors, the directed line segment:

(i) **QP** can also be viewed as the displacement vector, for displacement of the particle, from $\left(t = \frac{t}{2}\right)$ to (t = t).

(ii) **OP** can also be viewed as the displacement vector, for the displacement of the particle from (t = 0) to (t = t).

A little close thinking will reveal that these conclusions are consistent with the following definition of the displacement vector:

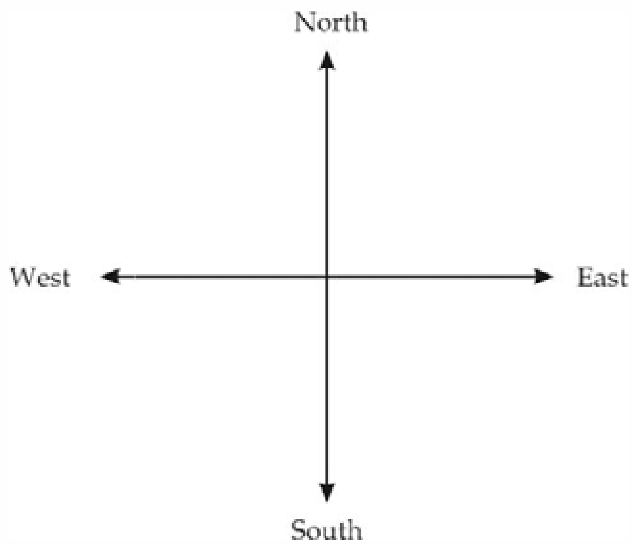
"The directed line segment, directed from the previous position point of the particle, to, its present position point."

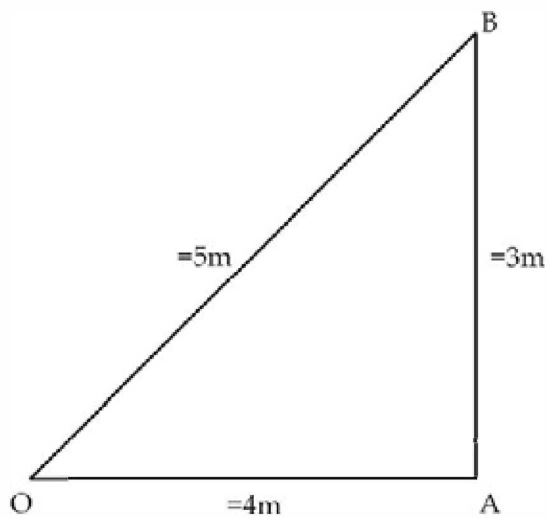
We can now appreciate the close link between the concepts of the position vector and the displacement vector. This link would become clearer after we understand the techniques and rules for addition and subtraction of vectors.

Addition of Vectors

We are all familiar with the rules and techniques for addition of simple numbers. These rules can be used as such, for addition of scalar quantities. Thus, if the mass of one ball is 75g and that of another is 125g, the total mass, of the two balls, is (75+125)g, i.e. 200g. Similarly, if the volume of one tank is 750 litre and that of another tank is 1 kilolitre, the total volume of the two tanks is (750+1000) litre, i.e., 1750 litre. And so on.

It turns out, however, that this simple and straightforward approach cannot be used for addition of two vector quantities. To understand this, consider a particle, initially at the origin. Let it get displaced through a distance of 4m along the east direction. Let it next get displaced, from its new position, through a distance of 3m, along the north direction. Can we now say that its total displacement, from the origin, is (4+3) m, i.e. 7m? A simple close look, at the meaning of vector quantities, will reveal that the above conclusion' is incorrect. If we draw a scaled diagram, we find (using a scale of 1cm = 1m), that the two displacement, of the particle, are represented by the directed line segments (or displacement vectors) **OA** (= 4cm length) and **AB** (= 3cm length), respectively. The net displacement of the particle, from the origin, (as a result of these two displacement) is now represented by the directed line segment (or displacement vector) OB. A careful measurement would reveal that the total displacement (= length, **OB**, as per the chosen scale) is only 5m. We are thus, in a way, finding that the addition of a displacement of magnitude 4m, to another displacement of magnitude 3m, can add up to a displacement of magnitude 5m only. It need not be 7m, as would be expected from the simple rules for addition of numbers.



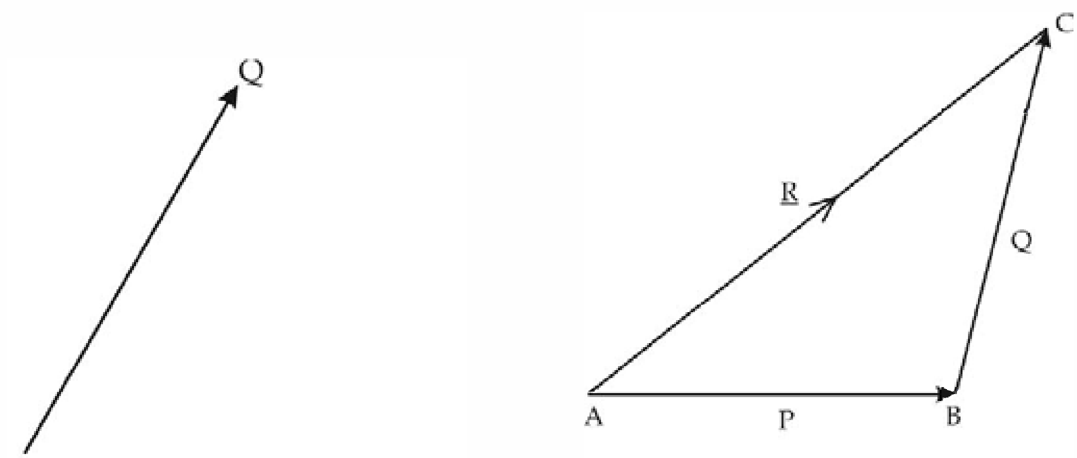


It is clear then that the rules, and ways, of adding two vector quantities can be quite different from the simple rules for addition of numbers, or scalar quantities. A close analysis, of the kind of diagram drawn here, has revealed that we can use either of the following two rules for addition of two vector quantities. These rules have been named as the (i) the triangle law and (ii) the parallelogram law for vector addition. We now talk about these two rules in some detail.

The Triangle Law of Vector Addition

The approach, needed for adding vector quantities, has been illustrated through the addition of two displacements (4 m towards east followed by 3 m towards north). This approach has been generalized to give us a general technique for vector addition. We refer to this as the 'triangle law of vector addition'.

This 'triangle law' is stated as follows:



(R = P + Q)

Let one line (AB), be drawn from a suitable point such that its length represents (on a suitable scale) the magnitude of one of the vectors (vector **P**) (to be added) and its direction is parallel to this vector. From the end point of this line (point B), let another line (BC) be drawn such that its length represents (on the same scale) the magnitude of the second vector (vector **Q**) (to be added), and its direction is parallel to this second vector. The closing side (side AC) of the triangle would then represent the magnitude of the sum (resultant) of the two vectors. The direction AC (that is the direction directed from the first, or starting point (A) to the third, or end point (C)) gives the direction of this resultant vector.

A more compact way of stating this triangle law, is as follows:

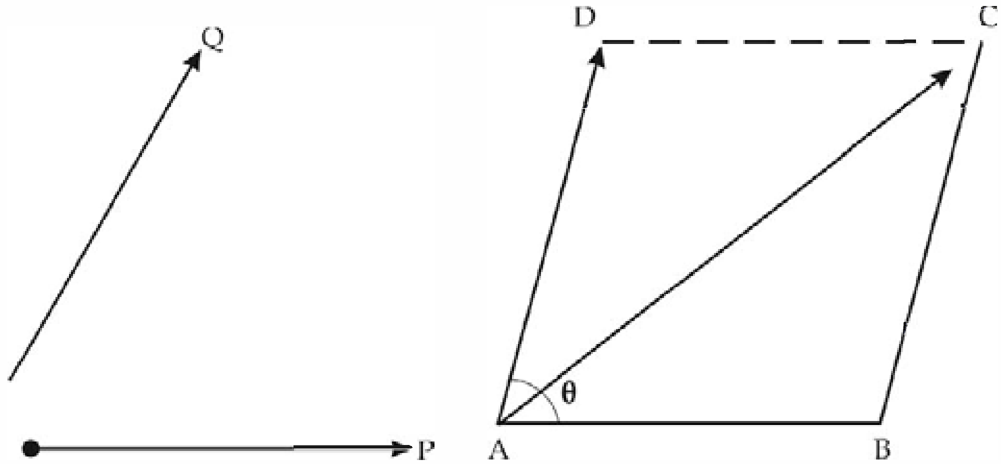
If the two sides of a triangle, taken in order, represent the magnitudes and directions, of the two vectors to be added, the third (or closing) side of this triangle, taken in opposite order, represents the magnitude and direction of the resultant (sum) of these two vectors.

The triangle law is a compact way of adding two vectors. As mentioned at the start it has been illustrated through the example of addition of two displacements.

The Parallelogram Law of Vector Addition

The parallelogram law of vector addition again enables us to find the sum, or resultant (as it is better called, for vectors), of any two vectors. This law is stated as follows.

Let two lines (say AB and AD) be drawn from a given point (say point A) such that their lengths represent (on a suitable chosen scale), the magnitudes of the two vectors, and their directions are parallel to the directions of the two vectors. If now these two sides be used to complete their associated parallelogram, the directed line segment AC, corresponding to the diagonal of this parallelogram, passing through the point A, represents the resultant (i.e., the sum) of the two given vectors.

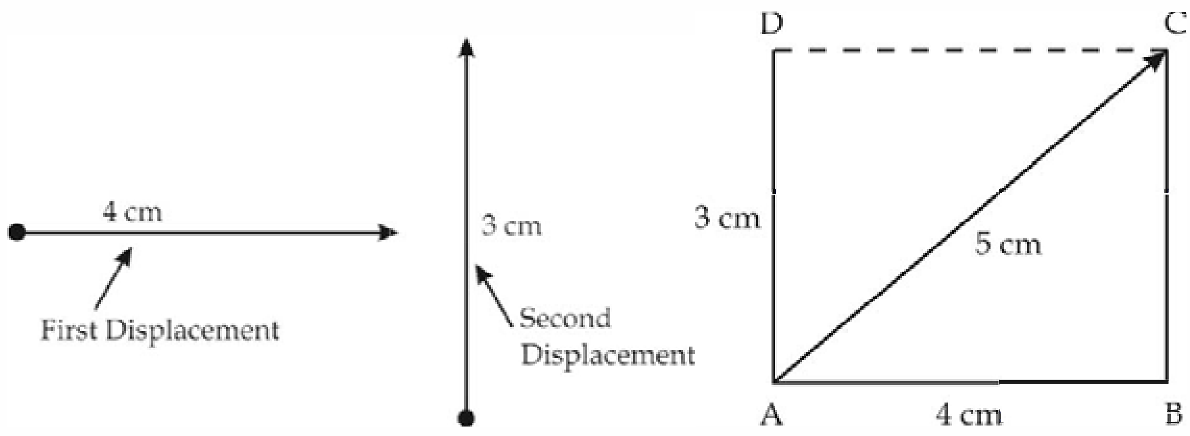


Thus let two vectors, **P** and **Q**, be represented by the two directed line segments, shown here. From a point A, we draw two lines, AB and AD, parallel to, and equal to, these two directed line segments. We then complete the parallelogram, having AB and AD, as its two sides. The diagonal, AC, of the parallelogram, passing through the point A, then represents the resultant (say **R**), of the two vectors **P** and **Q**, in both magnitude and direction.

This result is often represented through the equation:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$

Let us now see how the use of this parallelogram law gives us the correct result corresponding to the addition of the two (vector) displacements of 4m (along east) and 3m (along north). The parallelogram, in this case, is a rectangle having two sides of length 4cm and 3cm, respectively (on the scale 1 cm= 1 m). We now measure the length of the diagonal AC and find it to be 5cm, as per the result obtained earlier.



It can be proved mathematically that the magnitude, of the diagonal AC of a parallelogram, having two sides of magnitudes AB and AD, is given by

$$AC = \sqrt{AB^2 + AD^2 + 2AB.AD.\cos\theta}$$

where θ is the angle between the sides AB and AD. This result may also be expressed through the equation.

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

Here P and Q are the magnitudes of the two vectors **P** and **Q** and θ is the angle between their directions. The magnitude of their resultant (or sum) has been represented through the symbol R.

For the example of the two displacements given above,

P = 4 cm, Q = 3 cm and $\theta = 90^0$

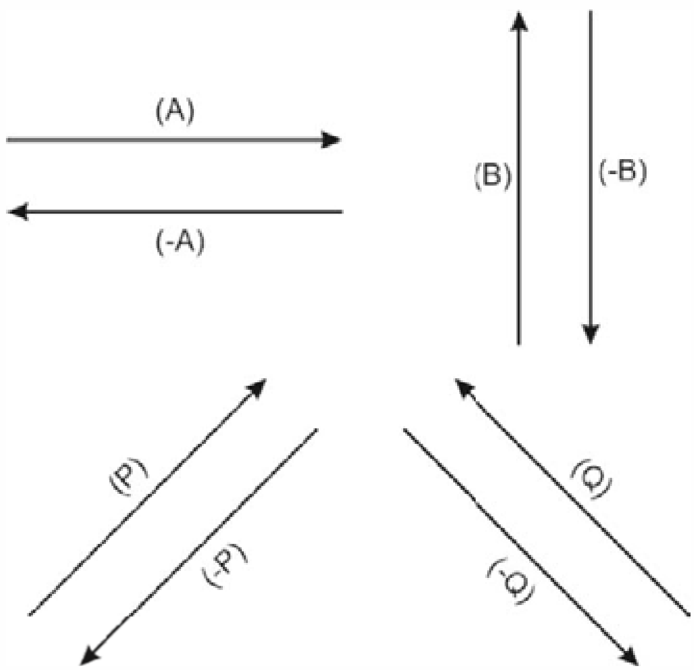
Hence, $R = \left[\sqrt{4^2 + 3^2 + 2x4x3x\cos 90^0}\right]$ cm

$= \sqrt{16+9}$ cm $= 5$ cm as obtained through a direct measurement (Remember: $\cos 90^0 = 0$).

The Opposite (or Negative) of a given Vector

We need the concept of the opposite (or negative) of a vector to understand the operation of subtracting one vector from another.

How do we define the opposite (or negative) of a vector? The vector (–**A**) [corresponding to a given vector (**A**)] is a vector whose magnitude equals that of the vector **A** and whose direction is exactly opposite to that of **A**. The diagrams given here show the pairs of directed line segments in which the second line, in each pair, represents the negative or opposite, of the vector represented by the first line of the pair.



It follows then that we just need to reverse the sense of the directed line segment (representing the given vector), to get a representation of the negative of that vector.

Subtracting One Vector from Another

How do we subtract a vector (say **B**) from another vector (say **A**)? We carry out this process as follows:

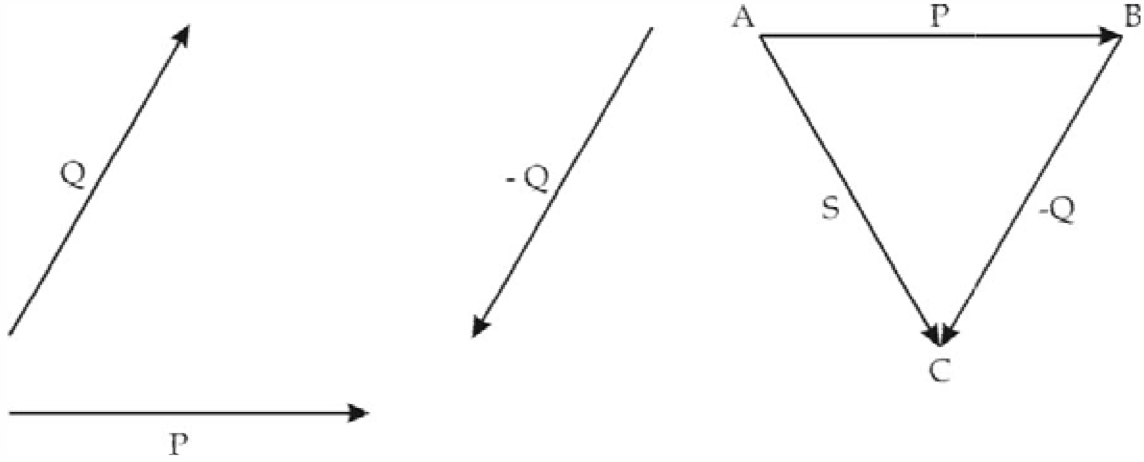
Let **C** = **A**+ (–**B**)

We can then write

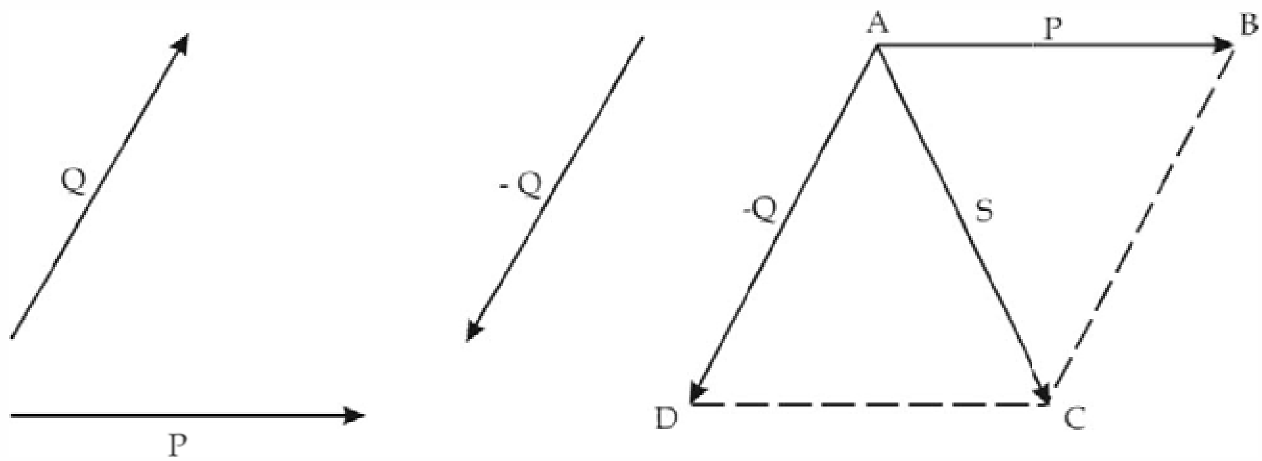
C = **A** – **B**

This shows that the required vector (vector **C**) can be viewed as the vector sum of the first vector (vector **A**) and the NEGATIVE of the second vector (vector **B**). We can then carry out the required process by using, either of the two laws of vector addition, i.e., the triangle law or the parallelogram law.

We illustrate the process of vector subtraction through the diagrams given below.



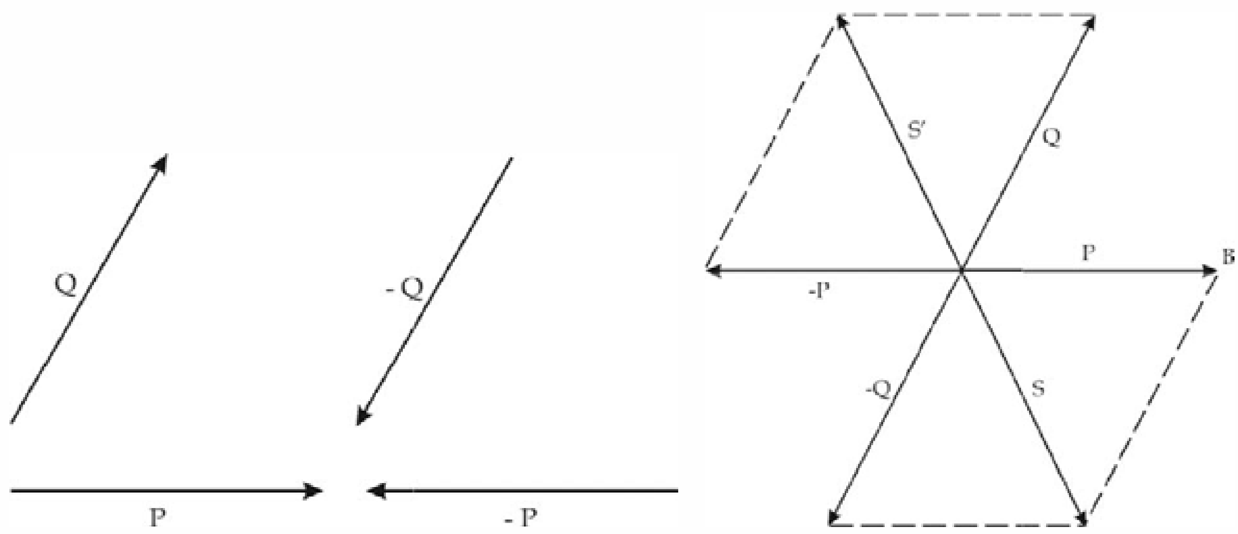
[Illustrating the sequence of steps, to be followed, for using the triangle law, to find the vector **S where S** = (**P** – **Q**)]



[Illustrating the sequence of steps, to be followed, for using the parallelogram law, to find the vector **S** Where **S** = (**P** – **Q**)]

The above illustrations show that the process of (vector) subtraction can be carried out by using the laws of (vector) addition. We need to remember that here we have to add the negative of the second vector to the first vector.

It is interesting here to realize that the reverse vector difference, i.e., (**Q** – **P**) (= vector **S**'), is again the negative of the vector **S**. The following diagrams illustrate this.



Here, **S** = (**P** – **Q**) and **S**' = (**Q** – **P**)

An Important Point

We have noted above that

(i) Physical quantities, which have only a magnitude associated with them, are known as scalar quantities.

(ii) Physical quantities, which have both a magnitude and a direction associated with them, are known as vector quantities.

It is important, however, to note that the requirement.

"Having both a magnitude and a direction" is a necessary, BUT NOT a sufficient, condition for classifying a quantity as a vector quantity. We can have quantities that have both a magnitude and a direction but are still not classified as vector quantities. 'Electric current' is a well known example of such a quantity. We often say that a current of (say) 1.2 A is flowing along the direction AB (say) in a branch of a given circuit. We, therefore, associated both a magnitude (= 1.2.A) and a direction (direction AB (say)) with an electric current. However, electric current is not regarded as a vector quantity; it taken as a scalar quantity.

What, then is the additional sufficiency condition that a physical quantity must follow so that if may be classified as a vector quantity?

We have noted above that the basic algebra of vectors is quite different from that of scalars. We have to use the triangle law, or the parallelogram law, to add, or subtract, two vector quantities. We, therefore, have an additional condition, namely, "A physical quantity is a vector quantity if two (or more) of such quantities have to be added by using the triangle law, or the parallelogram law, of vector addition."

We may now say that the necessary and sufficient conditions, for regarding a physical quantity as a vector quantity, are:

(i) The physical quantity must have both a magnitude and a direction associated with it.

(ii) Two such physical quantities have to be added (subtracted), only by using the triangle law / parallelogram law of vector addition.

The second of these conditions is often not given its due importance. However, we must notice that it is an essential part of the necessary, and sufficient, conditions, that a physical quantity must satisfy to be classified as a vector quantity. If a physical quantity satisfies only the first of these conditions - but does not satisfy the second - it would not be labelled as a vector quantity. It is for this reason that electric current is NOT regarded as a vector quantity. We can add / subtract two electric currents just by using the simple algebra of numbers. There is no need of using the triangle law (or the parallelogram law) for adding (subtracting) two electric currents.

Physical quantities like displacement, velocity, acceleration, force and momentum are regarded as (proper) vector quantities because they satisfy both the conditions listed above. They not only have a magnitude, as well as a direction, associated with them, but also need the triangle law (or the parallelogram law) for adding (subtracting) two of them.

This important point, about the basic and essential requirements for a vector quantity, must always be kept in mind. We must ensure that a physical quantity does satisfy both of the above requirements; it is only then that it would be regarded as a vector quantity.

Solved Examples

Example 1

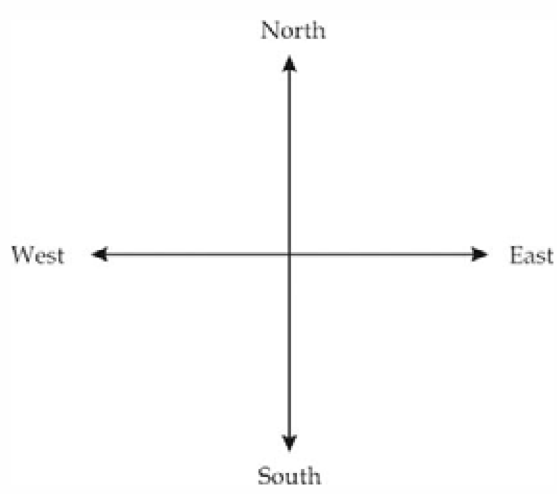
A particle is acted upon by (i) a force of 10 N directed west and (ii) a force of 20 N directed 30° east of north. Draw diagrams to show how (a) the parallelogram law and (b) the triangle law, may be used to find the resultant of these two forces.

Solution

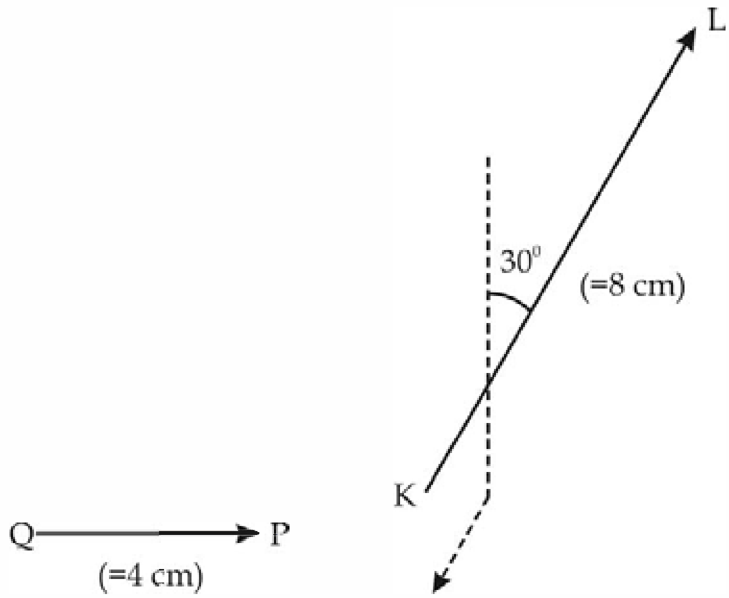
Let us choose the scale:

1 cm = 2.5 N

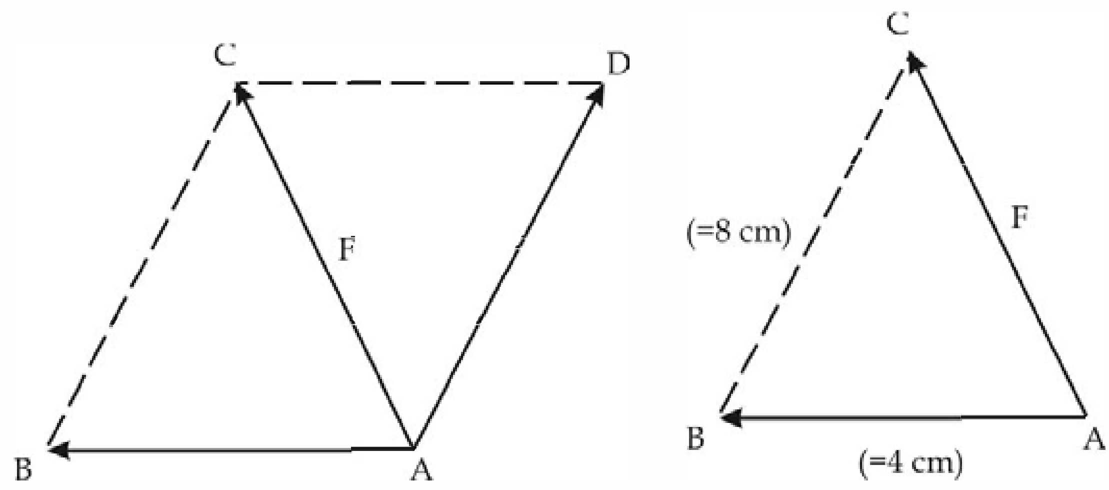
We can then represent the two forces by the directed line segments **PQ** and **KL**, respectively.



The parallelogram law may be used to find the resultant (F) of these two forces as shown.



The triangle law may be used to find the resultant (F) of these two forces as shown:



Example 2

It is required to subtract a vector **A** (of magnitude 10 units, directed north) from a vector **B** (of magnitude 6 units, directed 30° south of east).

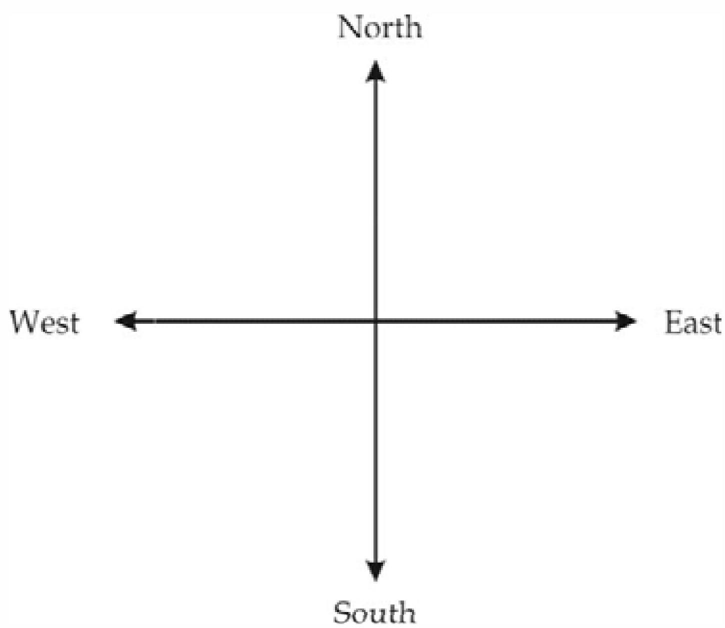
Draw diagrams to show how (i) the parallelogram law and (ii) triangle law may be used to carry out this subtraction.

Solution

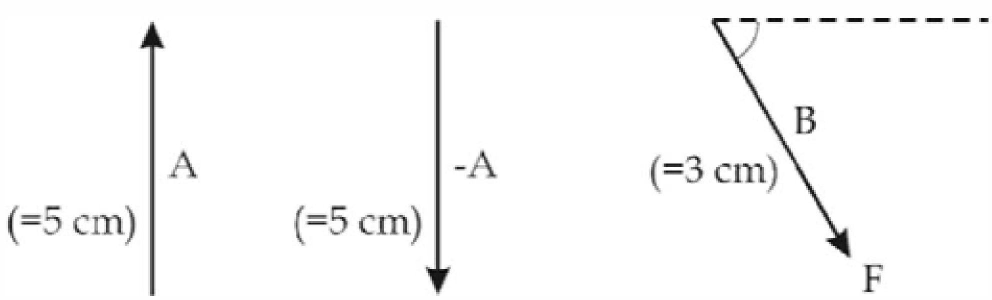
Let is choose the scale:

1 cm = 2 units

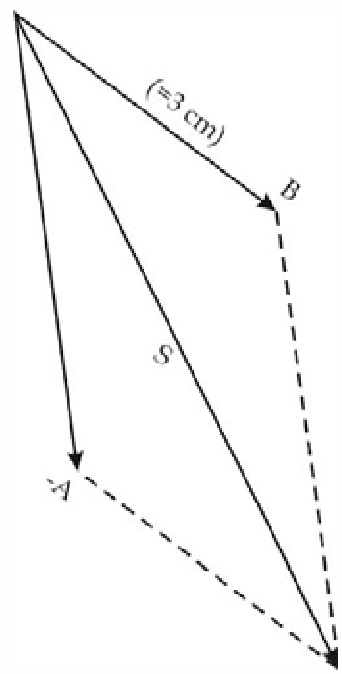
We can then represent the vectors **A** and **B**, as shown.



To subtract A from B, we need to add the vector –A to B. The vector (–A) is drawn as shown.

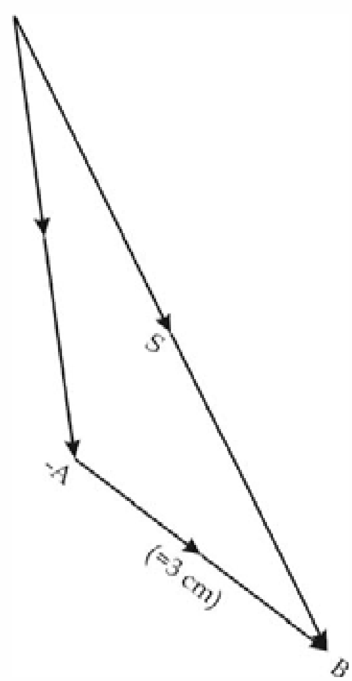


We may now use the parallelogram law to add the vector (–A) to the vector B.



The vector **5**, the sum of vectors (–**A**) and (**B**) equals **B** + (–**A**) = (**B** – **A**)

The triangle law may be used for the same purpose, as shown.



DO IT YOURSELF

1. Name four physical quantities that can be specified without bringing in the concept of direction.

2. Name four physical quantities that need both a magnitude and direction for their unambiguous specification.

3. How do we distinguish between the veloiuty of a particle, moving along a straight path towards the right, from that of another, moving towards the left?

4. Why do we say that a particle, located in a plane, can have an infinity of locations when only its distance, from a given reference point (origin) is known?

5. Using a graph paper, a scale and a 'D', draw directed line segments representing.

   (a) (i) A force of magnitude 10 N directed 30^0 north of east.

   (ii) A velocity of magnitude 15 m/s directed 30^0 west of south.

   (iii) An acceleration, of magnitude 2 m/s^2, directed 60^0 east of north.

   (b) (i) The position vector of a point distant 20 m from the origin and located exactly mid way between the north and west directions.

   (ii) The position vector of a particle, distant 30 m from the origin, and in a direction that is 25^0 south of west.

6. A particle is initially at a point A that is 25 m away from the origin, along the north direction. It then moves to another point B, that is 30 m away from the origin and is along a direction that is 60^0 south of east.

   Draw the position vectors of the particle, corresponding to its locations A and B. Also draw the displacement vector, corresponding to its displacement from point A to point B.

   Measure the length AB and find out its value in metres using the scale that you have used. Do you find it to be (nearly) equal to either (25+30)m (=55m) or to (30–25m), i.e. 5m. How do you explain your observation?

7. Draw diagrams showing how (i) the parallelogram law and (ii) the triangle law may be used to find the resultant vector corresponding to the addition of

   (a) A force of 10 N, directed towards the north, with a force of 20 N directed 30° south of west.

   (b) A velocity of 5 m/s, directed towards the east, with a velocity of 15 m/s, directed 30° west of north.

   (c) An acceleration of 2 m/s^2, directed towards the west, with an acceleration of 4 m/s^2 directed 30° east of north.

   (d) A momentum of 10 kg ms^{-1}, directed towards the south, with a momentum of 20 kg ms^{-1}, directed 45° west of north.

8. It is required to subtract

   (i) A vector of magnitude 10 units, directed west, from a vector of magnitude 20 units, directed 30° north of west.

   (ii) A vector of magnitude 15 units, directed north, from a vector, of magnitude 10 units directed west.

   Draw diagrams showing how (a) the parallelogram law and (b) the triangle law, may be used to carry out the required subtraction.

4 CHAPTER

UNDERSTANDING DIFFERENTIATION

Dear Reader

The mathematical concept of **differentiation** is intimately linked with the concept of **rate of change**. We all observe some change or the other, constantly taking place in different objects/situations around us. Let us take two familiar examples to understand this concept of rate of change.

1) Observe a car moving on a road. Its speed, in general, is constantly changing. Speed may change from 0 to, say, 20m/s in, say, 40s. The average rate of change of speed of the car is then ($\frac{20}{40}$m/s) per second, i.e. 0.5m/s^2.

2) Observe a plant growing in a pot. The height of its stem may increase from, say, 10cm to 16cm in 3 days. The average 'rate of change' of the height of the stem is then 2cm per day.

Let us now try to get an idea of the rate of change of speed of the car, say during the interval, from the 20th to the 22nd second during its motion. We would then need to know the change in its speed during this interval of 2 seconds. If the interval of time is progressively shortened, we would need to know the change in speed during such progressively small time intervals. Suppose we ask ourselves the question, "What is the rate of change of speed of car at, say, the 10th second of its motion?" we would then need to find the very small change in speed of the car in a very short time interval (say, 1/100th of a second), at t=10s. The rate of change of speed of the car at this instant would then need the division of a very small change in speed, by a, very small time interval. In the limit, this division would almost approach the division of '0' by '0'.

The mathematical technique of differentiation helps us to find the limiting values of such rates of change. We need to define, and understand, the concepts of **functions** and **limiting values** to understand the details of the technique. This is something you will do in your class XII Mathematics. However, we can easily understand the meaning of the concept of functions.

In the examples cited above the speed of the car, or the height of the plant, is changing with time. It can be said that the speed of the car is a function of time or the height of the plant is a function of time.

If the value of a quantity y depends on the value of another quantity x, then y is said to be a function of x .We can use the notation y=f(x) to represent this mathematically. In this notation, the quantity y is called the dependent variable, and the quantity x is called the independent variable.

The limiting value, of the rate of change of a quantity y (the dependent variable) with respect to the quantity x (the independent variable), is called the **differential coefficient** of y with respect to x. It is also known as the **derivative** of y with respect to x. It is denoted by the symbol $\frac{dy}{dx}$ or $\frac{d}{dx}(y)$. Remember $(\frac{d}{dx})$ is one complete symbol; we cannot cancel the 'd' in the numerator and denominator of this symbol .

The term $\frac{dy}{dx}$, implying the rate of change of y with respect to x, is calculated mathematically by finding the value of the ratio $(\frac{\Delta y}{\Delta x})$ (read as delta y by delta x). Δy represents the very very small change caused in the value of y, when x is changed by a very very small amount Δx.

In the example of moving car given above, Δx would correspond to a very very small time interval, in which a very very small change ($=\Delta y$) takes place in the speed of the car.

We give below a list of the differential coefficients of some of the commonly used functions and basic rules of differentiation for a combination (addition, multiplication etc.) of different functions. Derivation and explanation of the same will be done later in the class XII Mathematics syllabus.

Differential coefficients of some common Functions

f(x)	$\frac{d}{dx}[f(x)]=[f'(x)]$
x^n	$n\,x^{n-1}$
sin x	cos x
cos x	- sin x

tan x	$\sec^2 x$
cot x	$-\csc^2 x$
sec x	sec x tan x
csc x	- csc x cot x

The basic rules of differentiation

1. Derivative of a constant function

The derivative of f(x) = c, where c is a constant, is given by **f '(x) = 0**

Example

f(x) = 15 , then f '(x) = 0

2. Derivative of a power function (power rule)

The derivative of $f(x) = x^n$ where n is a constant real number is given by

$\mathbf{f'(x) = n\,x^{n-1}}$

Example

$f(x) = x^{-4}$, then $f'(x) = -4\,x^{-5} = -4 / x^5$

3. Derivative of a function multiplied by a constant

The derivative of f(x) = c u(x) is given by

f '(x) = c u '(x)

Example

$f(x) = 3x^4$,

Let c = 3 and $u(x) = x^4$, then f '(x) = c u '(x)

$= 3\,(4x^3) = 12\,x^3$

4. **Derivative of the sum of functions (sum rule)**

The derivative of $f(x) = u(x) + v(x)$ is given by

$\mathbf{f\,'(x) = u\,'(x) + v\,'(x)}$

Example

$f(x) = x^3 + x$

Let $u(x) = x^3$ and $v(x) = x$, then $f\,'(x) = u\,'(x) + v\,'(x) = 3x^2 + 1$

5. **Derivative of the difference of functions (difference rule)**

The derivative of $f(x) = u(x) - v(x)$ is given by

$\mathbf{f\,'(x) = u\,'(x) - v\,'(x)}$

Example

$f(x) = x^4 - x^2$

Let $u(x) = x^4$ and $v(x) = x^2$, then

$f\,'(x) = u\,'(x) - v\,'(x) = 4x^3 - (2x) = 4x^3 - 2x$

6. **Derivative of the product of two functions (product rule)**

The derivative of $f(x) = u(x)\,v(x)$ is given by

$\mathbf{f\,'(x) = u(x)\,v\,'(x) + v(x)\,u\,'(x)}$

Example

$f(x) = (x^2 - 2)(x - 2)$

Let $u(x) = (x^2 - 2)$ and $v(x) = (x - 2)$, then $u\,'(x) = 2x$ and $v\,'(x)=1$

Therefore $f\,'(x) = u(x)\,v\,'(x) + v(x)\,u\,'(x) = (x^2 - 2)(1) + (x - 2)(2x)$

$= x^2 - 2 + 2x^2 - 4x = 3x^2 - 4x - 2$

7. **Derivative of the quotient of two functions (quotient rule)**

The derivative of $f(x) = u(x) / v(x)$ is given by

$\mathbf{f\,'(x) = [\,v(x)\,u\,'(x) - u(x)\,v\,'(x)\,] / v(x)^2}$

Example

f(x) = (x - 2) / (x + 1)

Let u(x) = (x - 2) and v(x) = (x + 1), then u '(x)=1 and v '(x)=1

Therefore, f '(x) = { v(x) u '(x) - u(x) v '(x) } / v(x) ^{2}

= { (x + 1)(1) - (x - 2)(1) } / (x + 1) ^{2}

= 3 / (x + 1) ^{2}

8. **Chain Rule**

If we have

y=f (u) and u =g (x)

Then

$$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$$

Example

if y= sin u where u=2x, we would have dy/du=cosu, du/dx=2,

Therefore dy/dx= (dy/du) (du/dx)= 2cosu= 2cos2x

Linkage of Graph with Differentiation

We have already learnt that the mathematical relation y= f(x), showing the inter-relation between a variable y (the dependent variable) and a variable x (the independent variable), can be given a pictorial form through the GRAPH between y and x. We thus know that the graph of a linear function is a straight line and graph of a non-linear function is a curve.

We are already familiar with the concept of **slope** of a graph. It turns out that the slope of the graph between y and x, has a direct relation with the differential coefficient of 'y' with respect to 'x' i.e. , with $\frac{dy}{dx}$. (We also often use the notation y to represent $\frac{dy}{dx}$). This relation is expressed through the following statements.

"For a linear graph, slope of the graph is constant and is equal to differential coefficient of y with respect to x."

"For a non-linear graph, slope of the tangent to the graph, at any point, is equal to the rate of change of y with respect to x, at that point. This slope, of the tangent to the graph, thus corresponds to the value of $(\frac{dy}{dx})$ at that point."

We now give below some examples to show the relation between the slope of the graph (between variables y and x) and the differential coefficient of y with respect to x.

Example 1

a) Plot the graph for the function y=x

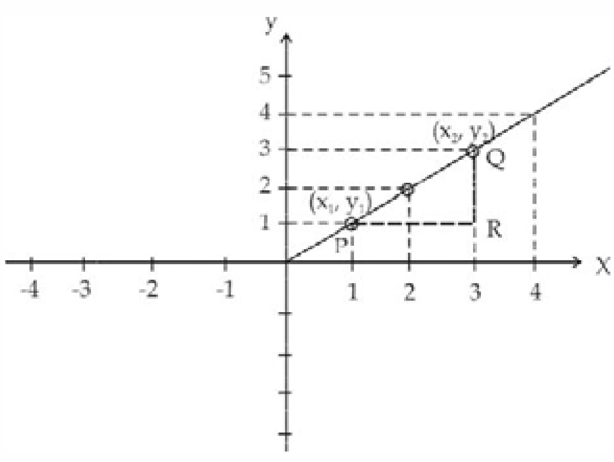
b) Calculate the slope of the line obtained after plotting the above graph

c) Compute $\frac{dy}{dx}$

d) Is the slope of the line equal to $\frac{dy}{dx}$?

Solution

a)



b) The slope of the line, represented by $y=x$, can be computed in the following manner. Choose any two points P & Q on the line (obtained after plotting the graph), having coordinates (x_1,y_1) and (x_2,y_2) respectively.

$$\text{Slope}=\frac{QR}{PR}=\frac{y_2-y_1}{x_2-x1}=\frac{3-1}{3-1}=\frac{2}{2}=1$$

c) $\frac{dy}{dx} = \frac{dx}{dx} = 1$ $(\because y = x)$

d) Yes, the slope of the graph is equal to the differential coefficient of y w.r.t x i.e.$\frac{dy}{dx}$

Example 2

a) Plot the graph for the function $y=x^2 +1$

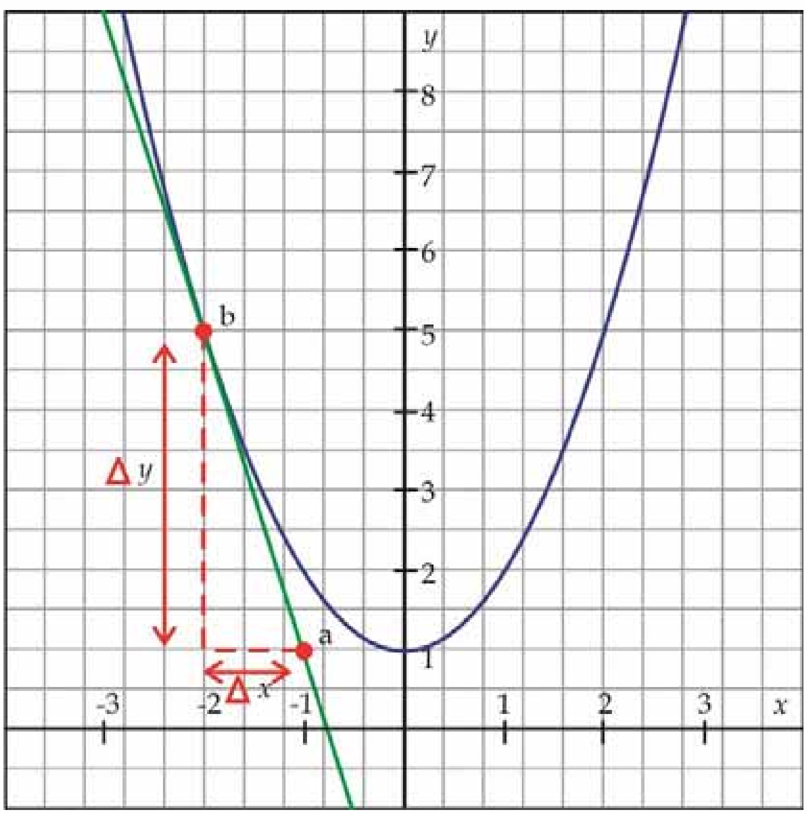
b) Calculate the slope of the tangent to the curve obtained after plotting the above graph at the point x=-2

c) Compute $\frac{dy}{dx}$ at x=-2

d) Is the slope of the tangent to the curve, at x=-2, equal to $\frac{dy}{dx}$ at x=-2?

Solution

a)



b) The slope of the tangent at x=-2 is calculated by A (-1, 1) & B (-2, 5) on the graph

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{-2-(-1)} = -4$$

c) We have $\frac{dy}{dx}$ =2x,

at x=-2, we get

$\frac{dy}{dx}$=-4

d) Yes, the slope of the tangent to the curve at x=-2 is equal to $\frac{dy}{dx}$ at x=-2

DO IT YOURSELF

1. Compute the derivative of the following functions w.r.t x (use the derivative rules):

1.1 $y = 2$

1.2 $y = x^3$

1.3 $y = \sqrt{x}$

1.4 $y = x^{-4}$

1.5 $y = \sqrt[3]{x}$

1.6 $y = 5x^3$

1.7 $y = \frac{8}{9}x^9$

1.8 $y = x^{-3} + x^3$

1.9 $y = 7x^3 + 6x$

1.10 $y = \frac{3}{4}x^4 - x^2$

1.11 $y = \frac{3}{4} - x^6$

1.12 $y = (x^6 - 5)(x - 1)$

1.13 $y = (x^2 - 2) / (x^3 + 1)$

1.14 $y = (\sqrt{x} - 4) / (x)$

2. Compute:

2.1 $y = 6x^3 - 9x + 4$

2.2 $y = -7 + 5x - \frac{5}{2}x^2 - 3x^4$

2.3 $y = \frac{6}{x} + \frac{7}{x^3} - \frac{1}{2x^5}$

2.4 $y = \frac{x+2}{x-2}$

3. Differentiate the following functions with respect to x:

3.1 sin 2x

3.2 x sin x

3.3 sin 3x

3.4 cos 3x

3.5 tan 4x

3.6 sin (ax + b)

3.7 cos (ax + b)

4. (a) Plot the graph for the function y=3x+1

(b) Calculate the slope of the graph

(c) Compute $\frac{dy}{dx}$

(d) Is the slope of the line equal to $\frac{dy}{dx}$?

ANSWERS

1.1 $y' = 0$

1.2 $y' = 3x^2$

1.3 $y' = \frac{1}{2\sqrt{x}}$

1.4 $y' = -4x^{-5}$

1.5 $y' = \frac{1}{3}x^{-2/3}$

1.6 $y' = 15x^2$

1.7 $y' = 8x^8$

1.8 $y' = -3x^{-4} + 3x^2$

1.9 $y' = 21x^2 + 6$

1.10 $y' = 3x^3 - 2x$

1.11 $y' = -6x^5$

1.12 $y' = 7x^6 - 6x^5 - 5$

1.13 $y' = \frac{-x^4 + 6x^2 + 2x}{\left(x^3 + 1\right)^2}$

1.14 $y' = \frac{4}{x^2} - \frac{1}{2x^{3/2}}$

2.1 $y' = 18x^2 - 9$

2.2

$$y = -7 + 5 \cdot x - \frac{5}{2} \cdot x^2 - 3 \cdot x^4$$

$$y' = 0 + 5 - 2 \cdot \frac{5}{2} \cdot x^{2-1} - 3 \cdot 4 \cdot x^{4-1}$$

$$= 5 - 5x - 12x^3$$

2.3

$$y = \frac{6}{x} + \frac{7}{x^3} - \frac{1}{2x^5}$$

$$y = 6x^1 + 7x^{-3} - \frac{1}{2}x^{-5}$$

$$y' = -1\left(6x^{-1-1}\right) - 3\left(7x^{-3-1}\right) - 5\left(-\frac{1}{2}\right)x^{-5-1}$$

$$= -6\left(x^{-2}\right) - 21\left(x^{-4}\right) + \frac{5}{2}\left(x^{-6}\right)$$

$$= -\frac{6}{x^2} - \frac{21}{x^4} + \frac{5}{2x^6}$$

2.4

$$y = \frac{x+2}{x-2}$$

Let $f(x) = x + 2$

$\therefore f'(x) = 1$

Let $g(x) = x - 2$

$\therefore g'(x) = 1$

$$g^2(x) = (x-2)^2$$

$$y' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

$$= \frac{1(x-2)-(x+2)1}{(x-2)^2} = \frac{-4}{(x-2)^2}$$

3.1 2 cos 2x

3.2 x cos x + sin x

3.3 3 cos 3x

3.4 -3 sin 3x

3.5 4 sec^2 4x

3.6 a cos (ax + b)

3.7 -a sin (ax + b)

4. (b) Slope = 3

(c) $\frac{dy}{dx} = 3$

(d) Yes

5 CHAPTER

UNDERSTANDING INTEGRATION

Dear Reader

The concept of **Integration**, mathematically speaking, is the "Inverse" of the concept of differentiation. This implies that if we first differentiate a function, say f(x), and get the result $f'(x)$, the integration of $f'(x)$, would give us back the function f(x). This, in a way, is similar to addition and subtraction or multiplication and division. Suppose we add 8 to 16, we get the result 24. If we now subtract 8 from 24, we get back the number 16. Similarly, if we multiply, say 9 by 6, we get 54. On dividing 54 by 6, we get back the number 9.

Let us first understand the physical meaning of the term integration. Consider the mass of a tiny droplet of water. It would be very small. Next, consider the mass of a collection of a very-very large number of such droplets. It would, as we know, be a reasonable and measurable quantity. We are, in a way, here adding a very-very large number of quantities, each of which is very-very small. **Integration** is the mathematical technique that enables us to find such sums- the sum of a very-very large number of quantities each of which is very-very small. We could in a rather crude way, think of it as finding the product of (0) with (infinity). Mathematicians use a special symbol for integration, this symbol is ∫. It is interesting to note that the symbol for integration, in a way, resembles the letter 'S' that has been stretched out both ways. This may be linked with the physical meaning of integration- sum of a very-very large number of very-very small terms. The very very small terms, that are added up through integration, are denoted, mathematically, by the symbol f(x) dx. Here f(x) could be any function of x. For example: we could have

$f(x)=x^2+1$

or $f(x)= a \sin x$

or $f(x)=x^2+\log x$

and so on….

The addition of such small terms is known as integration and is expressed through the complete symbol $\int f(x)dx$. The symbol then implies the addition of a very very large number of very very small terms.

Let $\int f(x)dx = f'(x)$, we would then have *d/dx* [*f*(*x*)] = $f'(x)$. This is because the process of integration, and differentiation, are the "inverse" of each other.

Mathematicians who developed the techniques of integration; i.e. finding such sums, for different kinds of mathematical functions, found that they could carry out the relevant mathematical steps by thinking of integration as the inverse of differentiation.

The integration of '0'

It is important to remember that if y=constant say (c), the rate of change of y, would have to be zero. We can, therefore, say that here $\frac{dy}{dx} = \frac{d(c)}{dx} = 0$.

Remembering that integration is the inverse of differentiation, this result implies $\int(0)dx$= c=a constant. We can thus say "The integration of '0', would give us a 'constant' as its result." This result will hold irrespective of the actual value of the constant (c). We, therefore say;

The integration of '0', gives an indeterminate constant as its answer. This result has an interesting implication. Suppose we integrate a function f(x) and get $f'(x)$as the answer.

we would then write

$\int f(x)dx = f'(x)$

However we can always write f(x) = f(x)+0

But $\int [\, f(x)+0 \,]\, dx = \int f(x)\, dx + \int (0)\, dx = f'(x)$ + constant .

This implies that instead of writing

$\int f(x)dx = f'(x)$

we should write

$\int f(x)dx = f'(x) + C$

Therefore, the result of integration, of any given function, does not give us a unique/definite answer. To the result $f'(x)$, we can always add a (arbitrary) constant.

We now quote the mathematical results for integration of some standard functions. Remember that the term, (+c), in these results, is due to the above stated, and discussed, property of the integration of zero

Integration of some common functions

Derivatives		**Integrals (Anti derivatives)**
(i)	$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right)=x^n$	$\int x^n dx=\frac{x^{n+1}}{n+1}+C,\ n\neq -1$
	Particularly, we note that	
	$\frac{d}{dx}(x)=1;$	$\int dx = x + C$
(ii)	$\frac{d}{dx}(\sin x)=\cos x;$	$\int \cos x\, dx=\sin x+C$
(iii)	$\frac{d}{dx}(-\cos x)=\sin x;$	$\int \sin x\, dx=-\cos x+C$
(iv)	$\frac{d}{dx}(\tan x)=\sec^2 x;$	$\int \sec^2 x\, dx=\tan x+C$
(v)	$\frac{d}{dx}(-\cot x)=\cos ec^2 x;$	$\int \cos ec^2 x\, dx=-\cot x+C$
(vi)	$\frac{d}{dx}(\sec x)=\sec x \tan x;$	$\int \sec x \tan x\, dx=\sec x+C$
(vii)	$\frac{d}{dx}(-\cos ec\, x)=\cos ec\, x \cot x;$	$\int \cos ec\, x \cot x\, dx=-\cos ec\, x+C$

[*For case (i) For n= -1, we have $\int x^n dx = \int \frac{1}{x}\,dx$ .This function gives '*ln x*' as its answer. Here (*ln x*) stands for logarithm of *x* to a special base, known as 'e'. It turns out that *ln x* = 2.303 log *x*, where log *x* represents the logarithm of *x* to the base 10.]

Rules for Integration

For the following statements and examples, the symbols, *a*, *b* and *c*, all refer to terms that are constants

1 $\int af(u)\,du = a\int f(u)\,du$

2 $\int [f(u)+g(u)\,du = \int f(u)\,du + \int g(u)\,du$

3 $\int [f(u)-g(u)\,du = \int f(u)\,du - \int g(u)\,du$

4 $\int [af(u)+bg(u)]\,du = a\int f(u)\,du + b\int g(u)\,du$

Example 1

$$\int \sqrt[5]{x^3}\,dx$$

$$= \int x^{\frac{3}{5}}\,dx$$

$$= \frac{x^{\frac{3}{5}+1}}{\left(\frac{3}{5}+1\right)} + C$$

$$= \frac{5}{8}x^{8/5} + C$$

Example 2

$$\int\left(4x^3+\frac{1}{x^3}-2x\right)dx$$

$$=\int\left[4x^3+x^{-3}-2x\right]dx$$

$$=\frac{4x^4}{4}+\frac{x^{-2}}{(-2)}-\frac{2x^2}{2}+C$$

$$=x^4-\frac{1}{2x^2}-x^2+C$$

Example 3

$$\int 5\sin x\,dx$$

$$=5\int \sin x\,dx$$

$$=-5\cos x+C$$

Example 4

$$\int 16\cos x\,dx$$

$$=16\int \cos x\,dx$$

$$=16\sin x+C$$

Example 5

$$\int 2\sec^2 x\,dx=2\int \sec^2 x\;dx$$

$$=2\tan x+C$$

Example 6

$$\int (5 + 4\sec x \tan x)\,dx$$

$$= \int 5d\,x + \int 4\sec x \tan x\,\mathrm{d}\,x$$

$$= 5\int dx + 4\int \sec x \tan x\,\mathrm{d}x$$

$$= 5\,x + 4\sec x + C$$

DO IT YOURSELF

Find the Following Integrals

(a) $\int x^5 dx$

(b) $\int (x^4 + 4)\,dx$

(c) $\int 3x^2 dx$

(d) $\int (3x^3 + 2x^2)\,dx$

(e) $\int (4x^{-3} + 2x^{-4})\,dx$

(f) $\int (\sqrt{x+1})\,dx$

(g) $\int (x^{3/2} + x^{\frac{1}{2}} + 1)\,dx$

(h) $\int y^2 \sqrt[3]{y}\,dy$

(i) $\int d\theta$

(j) $\int 8\sin(x)\,dx$

(k) $\int 9\cos\theta\,d\theta$

ANSWERS

1. (a) $\dfrac{x^6}{6} + C$

(b) $\dfrac{x^5}{5} + 4x + C$

(c) $x^3 + C$

(d) $\dfrac{3x^4}{4} + \dfrac{2x^3}{3} + x + C$

(e) $\frac{4x^{-2}}{-2}+\frac{2x^{-3}}{-3}+C$

$= -2x^{-2} - \frac{2}{3}x^{-3}+C$

$= \frac{-2}{x^{2}} - \frac{2}{3x^{3}}+C$

(f) $\frac{2x^{3/2}}{3}+x+C$

(g) $\frac{2}{5}x^{5/2} + \frac{2x^{3/2}}{3}+x+C$

(h) $\frac{3}{10}x^{10/3}+C$

(i) $\theta+C$

(j) $-8\cos x+C$

(k) $9\sin\theta+C$

6 CHAPTER

INTRODUCTION TO LOGARITHMS

Dear Reader

Logarithms are a tool originally designed to simplify complicated arithmetic calculations. They were extensively used before the advent of calculators. Logarithms transform multiplication and division processes to addition and subtraction processes which are much simpler.

As an illustration, you may try to multiply 98762578 and 45329576 without using a calculator. Does the idea frighten you? Addition of the above numbers is however a lot simpler. With the advent of calculating machines; the dependence on logarithms has been greatly reduced. However the logarithms have still not become obsolete.

They are still relevant, rather a very important tool, in fields like the study of radioactive decay rates in Physics and order of reactions in Chemistry. The study of logarithms, and their simple power property is of importance to us.

To begin with, we will define logarithms and use them to solve simple exponential equations. What does **logarithm** mean? This is the first question to which we would seek an answer. Logarithms have a precise mathematical definition as under:

For a positive number b (called the base);

if $b^p = n$; then

$\log_b n = p$

The above expression is read as: "logarithm (or simply log) of a given number (n), to a base (b), is the power (p), to which the base has to the raised, to get the number n".

Does the definition leave you confused?

Try this simple example as an illustration.

We have $10^3 = 1000$

So $\log_{10} 1000 = 3$

i.e. logarithm or log of the number (1000) to the base (10), is the power (=3) to which the base (10) has to be raised to get the number (1000).

Remember, logarithms will always be related to exponential equations. For a very clear understanding of logarithm, it is important that we learn how to convert an exponential equation to its logarithm form and also to convert logarithmic expression to exponential form.

Let us consider a few examples to develop clarity about the concept of logarithm.

Exponential to Logarithmic Form

Example 1: Write $3^4 = 81$ in logarithmic form.

Solution: Here number - 81; Base = 3; Exponent / Power = 4

$\therefore \log_3 81 = 4$; this is read as: "log of 81 to base 3 is 4".

Example 2: Write $8^2 = 64$ in logarithmic form.

Solution: $\log_8 64 = 2$; this is read as "log of 64, to base 8, is 2.

Example 3: Write $2^{-4} = \frac{1}{16}$ in logarithmic form.

Solution: log $\frac{1}{16}$ = -4; this is read as log of $\frac{1}{16}$ to the base 2 is – 4".

DO IT YOURSELF

Express the following in logarithmic form:

1. $2^7 = 128$
2. $7^0 = 1$
3. $10^{-3} = \frac{1}{1000}$

4. $(27)^{1/3} = 3$

5. $(16)^{1/2} = 4$

6. $64^{1/3} = 4$

ANSWERS

1. $\log_2 128 = 7$

2. $\log_7 1 = 0$

3. $\log_{10}\left(\frac{1}{1000}\right) = -3$

4. $\log_{27} 3 = \frac{1}{3}$

5. $\log_{16} 4 = \frac{1}{2}$

6. $\log_{64} 4 = \frac{1}{3}$

Expressing Logarithmic Expression in Exponential form

We have studied how to write exponential expressions in their equivalent logarithmic expressions. We will now take up some examples to express a given logarithmic expression in its equivalent exponential form.

Illustration

Example 1: $\log_9 81 = 2$, in logarithmic form, it can be written as $9^2 = 81$ in exponential form. This is just the reverse of what we have studied in the definition of logarithms.

Example 2: Write $\log_2 \frac{1}{16} = -4$ in exponential form.

Solution: $2^{-4} = \frac{1}{16}$

Example 3: Write $\log_{10} 10000 = 4$ in exponential form.

Solution: $10^4 = 10000$

DO IT YOURSELF

Express the following expressions in exponential form

1. $\log_5 125 = 3$

2. $\log_3 \frac{1}{27} = -3$

3. $\log_{36} 6 = \frac{1}{2}$

ANSWERS

1. $5^3 = 125$

2. $3^{-3} = \frac{1}{27}$

3. $36^{\frac{1}{2}} = 6$

Hence logarithms are related to exponential equations.

Solving exponential and logarithmic equations

Example 1: Calculate x if $\log_6 36 = x$

Solution: Rewriting the given equation in exponential form; we get

$6^x = 36$

Also $6^2 = 36$

Comparing; we get $x = 2$

Example 2: Solve for x: $\log_8 x = 3$

Solution: We can write $8^3 = x$

$\therefore$ x = 8 x 8 x 8 = 512

Example 3: Solve for x: $\log_2 \frac{1}{64} = x$

Solution: We have $\frac{1}{64} = 2^x$

Also $\frac{1}{64} = \frac{1}{2^6}$

$\therefore 2^x = \frac{1}{2^6} = 2^{-6} \Rightarrow x = -6$

Example 4: Evaluate $\log_4 64$

Solution: Suppose $\log_4 64 = x$

Then $4^x = 64 = 4^3$

$\therefore x = 3$

Example 5: Evaluate $\log_8 8^3$

Solution: Let $\log_8 8^3$ be y. Then $\log_8 8^3 = y$

In exponential form $8^y = 8^3 \Rightarrow y = 3$

Example 6: Evaluate $3\log_3 9$

Suppose $3\log_3 9 = y$

Then $\log_3 9 = y / 3$

$3^{y/3} = 9 = 3^2 \Rightarrow y = 6$

Equality of Logarithmic Functions

For $b>0$ and $b \neq 1$

$\log_b x = \log_b y$ if and only if, x = y

i.e. if logs of two numbers, to a given positive base b are equal, the numbers are also equal.

We can use this equality to solve the following types of equation:

Example 1: Solve for x: $\log_2(2x+9) = \log_2(x+5)$

Solution: As the bases on the two sides are equal, we have

$2x+9 = x+5$

$\therefore$ x = –4

Example 2: Solve for x: $\log_7(x^2+6) = \log_7 7x$

Solution: As the bases on the two sides are equal, and the logs are given to be equal, we have

$x^2+6=7x$

or $x^2-7x+6=0$

Factorizing (x–6) (x–1) = 0

$\therefore x=6$ or $x=1$

Example 3: Solve for y: $\log_3(y^2-40) = \log_3 3y$

Solution: As the bases on the two sides are the same, and the logs are equal, we have

$y^2 - 40 = 3y$

or $y^2 - 3y - 40 = 0$

or $(y-8)(y+5) = 0$

$\Rightarrow$ y = 8 or y = –5

The equation appears to have two solutions.

However, logarithm of a negative number is not defined. As bases are positive; the arguments would also by essentially positive.

$\therefore$ y = –5 is not a solution.

Hence y = 8.

Note: $\log_3(-15)$ is not defined; because 3^y can never be negative. Hence $\log_3(-15)$ is not defined.

DO IT YOURSELF

1. $\log_2(5x+4) = \log_2(3x+8)$,

   find x.

2. $\log_7(y^2+8) = \log_7(6y)$,

   find y.

3. $\log_3(x^2-35) = \log_3 2x$,

   find x.

ANSWERS

1. 2
2. 4; 2
3. 7

Fundamental Laws of Logarithms

(i) Law of Product

$$\log_a p.q = \log_a p + \log_a q$$

If $\log_a p = x$ and $\log_b q = y$

Then $a^x = p$ and $a^y = q$

$\therefore$ $p.q = a^x . a^y = a^{x+y}$

which implies $\log_a pq = x + y = \log_a p + \log_a q$

The law of product, stated above, can be extended to any number of quantities.

i.e. $\log_a p.q.r.s = \log_a p + \log_a q + \log_a r + \log_a s$

(ii) Law of Quotient

$$\log_a \frac{p}{q} = \log_a p - \log_a q$$

If $\log_a p = x$ and $\log_a q = y$

we have $a^x = p$ and $a^y = q$

$$\therefore \frac{p}{q} = \frac{a^x}{a^y} = a^{x-y}$$

which implies

$$\log_a \frac{p}{q} = x - y = \log_a p - \log_a q$$

(iii) Law of Power

$$\log_a p^q = q \log_a p$$

The law of power is an extension of the law of product.

$$\log_a p^q = \log_a p.p.p.....q \ times = \log_a p + \log_a p + ... q \ times$$

$$= q.\log_a p$$

Other laws of logarithms

1. $\log_a 1 = 0 \because a^0 = 1$

2. $\log_a a = 1 \because a^1 = a$

3. $\log_a p = \dfrac{\log_b p}{\log_a b}$ (Base change formula)

Example: If $a^x = b;\ b^y = c;\ c^z = a;$ evaluate xyz.

Solution: As $a^x = b$

Taking logs; $\log a^x = \log b$

$\therefore x \log a = \log b \Rightarrow x = \dfrac{\log b}{\log a}$

Similarly from $b^y = c$; we get $y = \dfrac{\log c}{\log b}$

and $z = \dfrac{\log a}{\log c}$

$\therefore x.y.z. = \dfrac{\log b}{\log a} \cdot \dfrac{\log a}{\log b} \cdot \dfrac{\log a}{\log c} = 1$

Systems of Logarithms

Common Logarithms: (Base 10). Common logarithms use base 10. The usual logarithmic tables use base 10.

Natural Logarithms: Natural logarithms use base e. It is also denoted as $\ell n\ x$ read as natural log of x. e is an irrational number given by the (inifinte) series

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots.. \frac{1}{n!} + \ldots..$$

A rough value of is (nearly) 2.718.

Using Common Logarithms for Calculation / Computation

As stated earlier logarithms are used to simplify calculations. From the definition of the logarithms, it is easy to realize that the logarithm of any given number, to a given base (= 10, for common logarithms), would not be always an integral number. It would, in general, have an integral as well as a fractional part. The logarithm of a number, therefore consists of the following two parts:

(a) **Characteristic:** It is the integral part of the logarithm.

(b) **Mantissa:** It is the fractional, or decimal part, of the logarithm.

If $\log_{10} N = 2.2352$, the characteristic is 2 and mantissa is 0.2352

Remember

1. The characteristic, the integral part of the logarithm, may be positive, zero or negative.
2. The decimal part, or the mantissa, is always taken as positive.
3. In case the logarithm of a number is negative, the characteristic and mantissa are rearranged, to make the mantissa positive. This is discussed in the following examples.

Example

Suppose $\log_{10} N = -2.2325$

We write $\log_{10} N = -2.2325$

= (–2) + (–0.2325)

= (–2–1) + (–0.2325+1) [Add and subtract 1]

= –3 + (0.7675)

The characteristics becomes –3 and mantissa becomes +0.7675.

The negative characteristic is represented by putting a bar on the number.

We, therefore, write

$\log_{10} N = \bar{3}.7675$

Here $\bar{3}$ implies that the characteristics is –3.

Using Logarithms

We fight below the TABLE (to the base 10)

(i) The Logarithms (common)

(ii) The Antilogarithms (common)

It is these tables that are used for detailed calculation using logarithms. We now discuss the ways and means of using these tables.

Log tables are the standard tables, available for to use for calculations. In general, these are four digit tables. Logarithmic tables, to the base 10, are the tables that are (almost) always used in practice. It may, therefore, be understood that the base of the logarithms, used in all our subsequent discussion, is the base 10 (unless mentioned otherwise). As discussed above, the log of a number has two parts: the characteristic and the mantissa.

Finding Characteristic: In order to find the characteristic part, it is convenient to express the given number in its standard form, i.e., the product of a number between 1 and 10, and a suitable power of 10. The power of 10, in the standard form of the number, gives the characteristic of the logarithm of the number. For numbers greater than 1, the characteristic is 0 or positive. For numbers less than 1, the characteristic is negative. The logarithms of negative numbers are not defined.

The standard form of a number and the characteristic can be computed as under:

Example 1: $1297.3 = 1.2973 \times 10^3$, [A number between 1 and 10 x Power of ten]

$\therefore$ Characteristic of log 1297.3 = 3

Example 2: $15.29 = 1.529 \times 10^1$, $\therefore$ Characteristic = 1

Example 3: $2.352 = 2.352 \times 1 = 2.352 \times 10^0$, $\therefore$ Characteristic = 0

Example 4: $257325000 = 2.57325000 \times 10^8$, $\therefore$ Characteristic = 8

Note that in all the above four examples; the number is greater than one and hence the characteristic is zero or positive.

For numbers less than 1, expressed in standard form, the power of 10 will always the negative and hence the characteristic will also be negative.

Example 1: $0.7829 = 7.829 \times 10^{-1}$, $\therefore$ Characteristic = -1 or $\bar{1}$

Example 2: $0.06253 = 6.253 \times 10^{-2}$, $\therefore$ Characteristic of log (0.06253) = -2 or $\bar{2}$

Example 3: $0.00002775 = 2.775 \times 10^{-5}$, $\therefore$ Characteristic of log (0.00002775) = -5 or $\bar{5}$.

Mantissa

It is the decimal / fractional part of the log of a given number. The mantissa is read off from the log tables. It is always positive.

For a given number N, we express the number in standard form the find the characteristic as detailed above.

To find the mantissa, the decimal point, the zeros in the beginning, and at the end of the number, are ignored

(i) The number is rounded off to the fourth place (say 1237).

(ii) Take the first two digit, i.e. 12, and locate the same in the first column of the log table.

(iii) Follow the horizontal row beginning with the first two digits (i.e. 12) and look for the column under the third digit (3) of the four figure log table and record number (see figure 1). [0.0899]

LOGARITHMS

N	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						5	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	16	20	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	29	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1293	1271						3	6	10	13	16	19	23	26	29
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1466	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	14	17	20	23	26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	9	11	14	17	20	23	26
											3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	6	8	11	14	16	19	22	24
											3	5	8	10	13	16	18	21	23

Figure 1

(iv) Continue in the same horizontal row and record the mean difference under the fourth digit. [Mean difference = 24 for 7]

Add the mean difference, recorded in (iv), to the number in (iii).

$\therefore$ The mantissa is 0.0899+0.0024 = 0.0923

$\therefore$ log (1237) = (Ch) + (Mantissa)

= 3.0923

Example 1: Find (1) log (0.056) (2) log (129.7)

Solution: (1) 0.056 = 5.6×10^{-2} (in standard form)

$\therefore$ Characteristic = –2 = $\bar{2}$

To find the mantissa, ignore the decimal point and add two more zeros at the end to make 56 a four digit number, i.e. 5600

Locate 56 (the first two digits) in the first vertical column and read the same horizontal line under 0 as shown. There is no mean difference as the fourth digit is zero.

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7769	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6

Figure 2

$\therefore$ log 0.056 = $\bar{2}$. 7482

(2) 129.7 = 1.297 x 10^2 (Standard form)

$\therefore$ Characteristic = 2

For the mantissa; see the table. We get: mantissa= 0.1106 + 0.0024 = 0.1130

$\therefore$ Log 129.7 = 2.1130

DO IT YOURSELF

Find logs of following

1. 2925   2. 2775300   3. 2.3723

ANSWERS

1. 3.4661   2. 6.4433   3. 0.3751

Using the (common) Antilogarithm table

These tables are used to find the number whose logarithm (to the box) has a known value

The number N, whose logarithm is L, is called the antilogarithm of L.

$\therefore$ If log N = L, we have N = Antilog L

We have (i) Log 0.056 = $\bar{2}$.7482 [Example above]

$\therefore$ Antilog $\bar{2}$.7482 = 0.056

(ii) We have: log 129.7 = 2.1130;

: Antilog 2.1130 = 129.7

Finding Antilogarithms

Let us now understand the procedure to be followed for finding antilogarithms from standard antilog tables that are available for computations. The following steps are followed to get the antilogarithm of a given number.

1. To read antilogarithm table; the characteristic is ignored. The tables are read only for the mantissa i.e. the decimal part.

   To get antilog of $\bar{1}$.3478; we use only 3478 to read the antilog tables.

2. Take the first two digits i.e. 34 and locate in their position the first vertical column of the four figure antilog table.

3. Go through the horizontal row beginning with 34, and look up the value under the column headed by the third digit (7 in 34$\underline{7}$8). The number, from the tables, 2223.

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
-00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
-01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
-02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
-03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
-04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
-05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
-06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2

-07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
-08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
-09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
-10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
-11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
-12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
-13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
-14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
-15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
-16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
-17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
-18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
-19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
-20	1585	1289	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
-21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
-22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
-23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
-24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
-25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
-26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
-27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
-28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
-29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
-30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
-31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
-32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
-33	2138	2143	2418	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
-34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
-35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
-36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
-37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
-38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	5	5
-39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
-40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
-41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
-42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
-43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
-44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
-45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
-46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
-47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
-48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
-49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Figure 3

4. In the same horizontal row, see the mean difference under the fourth digit (8 in 347<u>8</u>) and add it to 2223. We get (2223 + 4 = 2227).

   Write this number in standard form (= 2.227) and multiply by (10) raised to a power equal to the characteristic part we than get the antilog of the given log value

   $\therefore$ Antilog $\bar{1}.3478 = 2.227 \times 10^{-1}$

   We will illustrate it by another example.

   Find antilog 4.8897

   Take the decimal part i.e. .8897

   Locate 88 in the first column of the antilog table and read the horizontal line in the column under 9. The number is 7745. Add mean different under the fourth digit 7 (889<u>7</u>) i.e. 12 to get (7745 + 12) = 7757.

   Write it is standard form 7.757 and multiply by $10^{\text{characteristic}}$ ($=10^4$)

   $\therefore$ Antilog (4.8897) = 7.757×10^4

Use of Logarithms

Example 1: Calculate $22.89 \times 7454 \times 0.005324$

Solution: Suppose $x = 22.89 \times 7454 \times 0.005324$

$\therefore$ Log x = log 22.89 + log 7454 + log 0.005324

$= \log (2.289 \times 10^1) + \log (7.454 \times 10^3) + \log (5.324 \times 10^{-3})$

$= 1.3594 + 3.8665 + \bar{3}.7262$

$= 5.2261 + \bar{3}.7262 = 5.2261 + (-3 + 0.7262)$

$= 2.9523$

$\therefore$ x = Antilog of 2.9523

Example 2: Evaluate $\frac{7245}{9798}$

Solution: Let x be $\frac{7245}{9798}$

Taking logs; we get

$$\log x = \log\left(\frac{7245}{9798}\right)$$

$= \log 7245 - \log 9798$

$= \log (7.245\text{x}10^3) - \log (9.798\text{x}10^3)$

$= 3.8600 - 3.9912$

$= \bar{1}.8688$

$\therefore x = \text{Antilog } (\bar{1}.8688)$

$= 7.392\text{x}10^{-1} = 0.7322$

Example 3: Evaluate $(4327)^7$

Solution: Suppose $x = (4327)^7$

$\therefore \log x = 7 \log (4327)$

$= 7 \text{ x } \log (4.327 \text{ x } 10^3)$

$= 7 \text{ x } 3.6362 = 25.4534$

$\therefore x = \text{Antilog } (25.4534)$

$= 2.841 \text{ x } 10^{25}$

Example 4: Evaluate $(0.00195)^{1/5}$

Solution: Suppose $x = (0.00294)^{1/5}$

$$\log x = \frac{1}{5}\log(0.00295) = \frac{1}{5}\log\left(2.95\text{x}10^{-3}\right)$$

$$= \frac{1}{5}\left(\bar{3}.4698\right) = \frac{1}{5}(-3 + 0.4698)$$

$$= \frac{1}{5} \quad (-5+2+0.4698) = \frac{1}{5} \quad (-5+2.4698)$$

$$= (-1+0.49396) = \bar{1}.4940$$

$\therefore$ x = antilog ($\bar{1}$.4940) = 3.119 x 10^{-1} = 0.3119

Exmaple 5: Evaluate $(0.06424)^{1/5}$

Solution: Suppose $x = (0.06424)^{1/5}$

Then $\log x = \frac{1}{5}\log(0.04624)$

$$= \frac{1}{5}\left[\log\left(4.624\text{x}10^{-2}\right)\right]$$

$$= \frac{1}{5}\left(\bar{2}.6650\right)$$

Negative characteristic should be made multiple of denominator (5), before dividing.

$$= \frac{1}{5}(-2 + 0.6650)$$

$= \frac{1}{5}(-5 + 3.6650)$ [Add and subtract 3]

$$= (-1 + 0.7330) = \bar{1}.7330$$

$\therefore$ x = Antilog ($\bar{1}$.7330) = 5.408 x 10^{-1}

= 0.5408

DO IT YOURSELF

1. Evaluate the following:

   $(0.05246)^{\frac{1}{8}}$ Y8

2. Find the seventh root of 0.5504

3. The radius of a given sphere is 27.53 cm. Calculate its area. [Use area $A = 4\pi r^2$]

4. A cube of mass 42.95 g, has each edge of length 9.32cm. Calculate the density of the cube. [Density $\rho = \frac{M}{V} = \frac{M}{\ell^3}$]

5. The radius, of a 19.27 cm long cylinder, is 2.573 cm. Calculate the volume of the cylinder. [Use $V = \pi r^2 h$]

ANSWERS

1. 0.6918

2. 0.9182

3. 947.5 cm^2

4. 0.5307 g cm^{-3}

5. 2806 cm^3

CHAPTER 7

UNDERSTANDING BINOMIAL THEOREM

Dear Reader

A **binomial** is a polynomial having two terms e.g. $(3x+5)$, $(x^3\text{-}y^3)$, $(2x+3)y$, $t^5 - \frac{7}{9}t^4$ etc.

The two terms are separated by + or (–) signs. The number of variables in a binomial, may differ. The first binomial has one variable (x), while the second has two variables, x and y.

The binomial theorem provides a useful method to raise any binomial to a non-negative integral power.

Consider the expansion of $(x+y)^n$

We have $(x+y)^0 = 1$ (No. of terms is 1)

$(x+y)^1 = x+y$ (No. of terms is 2)

$(x+y)^2 = x^2+2xy+y^2$ (No. of terms is 3)

$(x+y)^3 = x^3+3x^2y+3xy^2+y^3$ (No. of terms is 4)

$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$ (No. of terms is 5) and so on.

It is important to note that

(a) Each s/c (n+1) terms i.e. one more than the integral power. Hence $(x+y)^{11}$ will have (11+1) or 12 terms.

(b) The exponents, of x, decrease from n to 0, while that, of y, increase from 0 to n.

(c) The degree of each term (the sum of the exponents of x and y) is n.

The coefficients, in a binomial expansion are called **binomial coefficients**. The coefficients (in the last of the above examples) are 1; 4; 6; 4; 1. The coefficients, of the first and the last terms are 1 each and that of the second term, and the second last term are 4 each (=n).

In the last expansion, the degree of the fourth term, $(4xy^3)$ is $(1+3) = 4$. The 'degree' has the same value (=4) for all the terms in the last of the above examples.

Binomial Expansion for Positive Index

If 'n' is a positive integer, x and y are real numbers, we have

$$(x+y)^{n} = x^{n} + \frac{n}{1}\left(x^{n-1}\right)y + \left(\frac{n(x-1)}{1.2}\right)x^{n-2}y^{2} + \frac{n(n-1)(n-2)...(n-r+1)}{1.2.3........r}x^{n-2}a^{r} + ... + a^{n}$$

The expansion has (n+1) terms.

Binomial Theorem (For any index)

If n is any number, positive, negative or fractional, and x is any real number between –1 and +1; we have

$$(1+x)^{n} = 1 + nx + \frac{x(n-1)}{1.2}x^{2} + \frac{x(n-1)(n-2)}{1.2.3}x^{3} + \frac{x(n-1)(n-2)...(n-r+1)}{1.2.3.......r}x^{n-2} + ...\infty\,\text{terms}$$

Hence, for positive integral values of n, the number of terms is (n+1). For negative and fractional values of n; the number of terms is infinity, i.e., there is, then, no last term.

For $|x|<<1$, i.e., for very small values of x; only the first two terms are (usually) significant. The higher order terms, having being very very small values, cam be neglected.

$$\text{We get,}\quad \left.\begin{array}{l} (1+x)^{n} = 1+n^{x} \\ (1+x)^{-n} = 1-n^{x} \\ (1-x)^{n} = 1-n^{x} \\ (1-x)^{-n} = 1+n^{x} \end{array}\right\} \text{for } |x|<<|$$

Example 1: Write down the first four terms in the expression of $(1+x)^{-2}$

Solution: $(1+x)^{-2} = 1+(-2)x + \frac{(-2)(-2-1)}{1.2}x^{2} + \frac{(-2)(-2-1)(-2-2)}{1.2.3}x^{3}$

$$= 1-2x+3x^{2}-4x^{3}+....$$

Example 2: For $x << 1$; Show that $\frac{1-x}{1+x} = 1-2x$

Solution: We have $\frac{1-x}{1+x} = (1-x)(1+x)^{-1}$

= $(1-x)(1-x)$

= $1+x^2-2x$

= $1-2x$ [Neglecting x^2 for $x << 1$].

Example 3: The acceleration due to gravity g_h, at a height h, above the surface of the earth, is given by $g_h = \frac{gR^2}{(R+h)^2}$

where 'g' is the acceleration due to gravity on the earth and R is the radius of the earth.

If h << R, show that

$$g_h = g\left(1-\frac{2h}{R}\right)$$

Solution: $g_h = \frac{gR^2}{(R+h)^2} = \frac{gR^2}{R^2\left(1+\frac{h}{R}\right)^2} = g\left(1+\frac{h}{R}\right)^{-2}$

$= g\left[1+(-2)\frac{h}{R}\right]$ (neglecting higher order terminates

$= g\left(1-\frac{2h}{R}\right)$

CHEMISTRY

Chapter 1 : Chemical Bonding

Chapter 2 : Mole Concept and Stoichiometry

Chapter 3 : Thermodynamics

Chapter 4 : Equilibrium

Chapter 5 : Redox Reactions

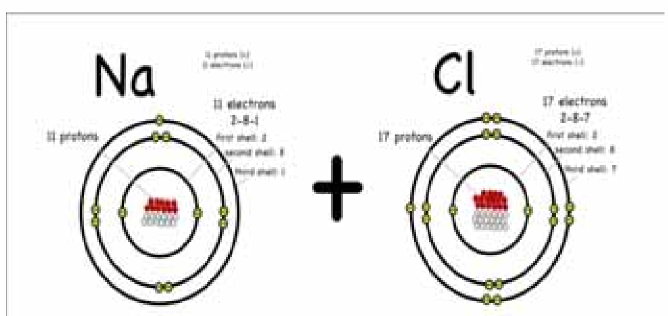
Chapter 6 : Electrochemistry

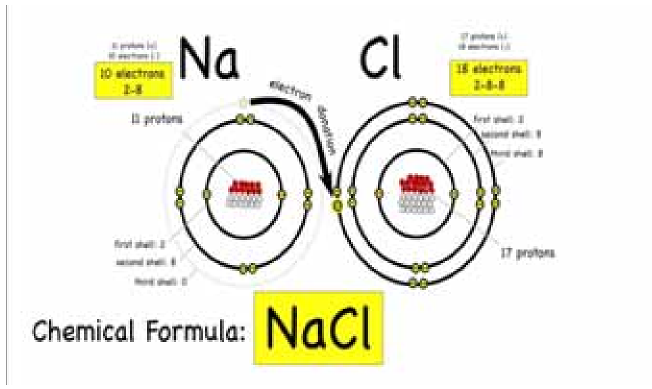
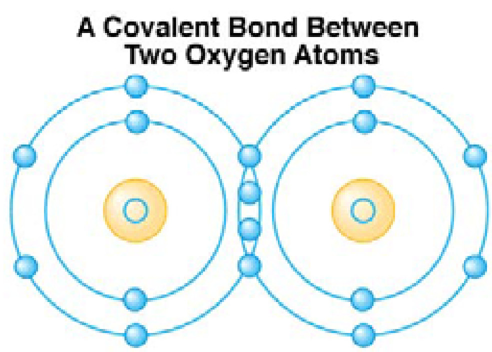
CHAPTER 1

CHEMICAL BONDING

Dear Reader

As you have already studied that the substances exist as discrete units called molecules. These molecules are formed by the combination of atoms. When these atoms combine, it is said that a chemical bond has been formed between the atoms. The reason that why do atoms combine to form molecules is explained on the basis of their electronic configurations. Atoms of different elements have different electronic configurations. It has also been observed that the elements of 18 group are quite unreactive and they contain two electrons (eg. He) or eight electrons (Ne, Ar, Kr, Xe and Rn) in their outermost shell. This electronic arrangement was therefore, regarded as a stable configuration. So it was said that all atoms have the tendency to attain this stability by completing octet (or doublet) in their outer most shell. Atoms generally do so by losing or gaining electrons when they form ions and oppositely charged ions are held together to form ionic or electrovalent bond. The second way by which these atoms complete their octets by sharing of electrons and the bond formed is known as covalent bond. Formation of these two types of bonds is briefly given below.

Bonding	Definition	Representation
Ionic Bonds or Electrovalent Bonds	• Formed between a metal and a non-metal. • Complete transfer of electrons from one atom to another to attain duplet or octet configuration. • Formation of	Formation of NaCl:  Sodium (Na) donates its outer-shell electron to chlorine (Cl)

	positive ions by loss of electrons and negative ions by gain of electrons. • The oppositively charged ions are held together by electrostatic forces of attraction. • Non directional in nature.	
Covalent Bonds	• Formed between non-metals. • Mutual sharing of electrons between the combining atoms of the same or different elements. • Directional in nature.	

Valency

- Electrovalency: the number of electrons lost or gained during the formation of electrovalent bond. e.g. sodium loses one electron to form Na+ ion , so its valency (electrovalency) is one.

- Covalency: The number of electrons contributed by each atom. e.g. covalency of chlorine is one.
- Variable valency: an atom can have different valencies and still be stable.

List of some cations

Charge 1+

CATION	NAME
H^{+}	Hydrogen ion
Li^{+}	Lithium ion
Na^{+}	Sodium ion
K^{+}	Potassium ion
Ag^{+}	Silver ion

Charge 2+

CATION	NAME
Mg^{2+}	Magnesium ion
Ca^{2+}	Calcium ion
Ba^{2+}	Barium ion
Zn^{2+}	Zinc ion
Cu^{2+}	Copper ion
Fe^{2+}	Ferrous ion

Charge 3+

CATION	NAME
Al^{3+}	Aluminium ion
Cr^{3+}	Chromium ion
Bi^{3+}	Bismuth ion
Sb^{3+}	Antimony ion
Fe^{3+}	Ferric ion

Polyatomic cation: NH_4^+ (Ammonium ion)

H_3O^+ (Hydronium ion)

List of some anions

Charge 1- **Charge 2-** **Charge 3-**

ANION	NAME	ANION	NAME	ANION	NAME
H^-	Hydride	O^{2-}	Oxide	N^{3-}	Nitride
F^-	Fluoride	S^{2-}	Sulphide	P^{3-}	Phosphide
Cl^-	Chloride				
Br^-	Bromide				
I^-	Iodide				

List of some polyatomic anions

ANION	NAME	ANION	NAME	ANION	NAME
OH^-	Hydroxide	CO_3^{2-}	Carbonate	PO_4^{3-}	Phosphate
NO_2^-	Nitrite	SO_4^{2-}	Sulphate	BO_3^{3-}	Borate
NO_3^-	Nitrate	$C_2O_4^{2-}$	Oxalate		
MnO_4^-	Permanganate	MnO_4^{2-}	Manganate		
HCO_3^-	Bicarbonate				

Writing the formulae of Molecular Compounds

Knowing the valences of the atoms or ions present in a compound, its formula can be written as explained in the following steps.

(i) The symbol of the constituting atoms or ions is written side by side.

(ii) For ionic compounds first positively charged ion (cation) is written and then negatively charged ion (anion) is written.

(iii) The valency of each atom or ion is written under it.

(iv) These valencies are then criss-crossed and written on the bottom right of the other species.

(v) If needed, these valencies are divided by a common factor to get small whole number.

Some examples are given below:

Formula of Hydrogen sulphide

Symbols:	H S	
Valencies:	1 2	
Formula	H_2S_1 or H_2S	

Writing the formulae of Ionic Compounds

Formula of Sodium oxide

Symbols:	NaO	
Valencies:	1+ 2-	
Formula	Na_2O_1 or Na_2O	

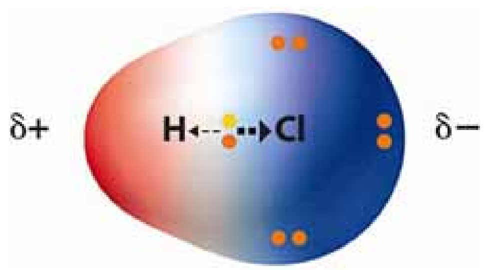
Polar Covalent Bond

In molecules like chlorine ($C\ell_2$), the electron pair is shared between two chlorine atoms because both have the same electronegativity. Such a bond is known as non-polar covalent bond.

Let us now consider a heteronuclear diatomic molecule like hydrogen chloride ($HC\ell$). Here, we find that chlorine atom has higher electronegativity than the hydrogen atom. So the shared pair of electrons is attracted more towards the chlorine atom. Due to this unequal distribution of electrons, the chlorine end of the molecule acquires slight negative charge and hydrogen end becomes slightly positive. Such molecules having two oppositely charged poles are called polar molecules and the bond is said to be a polar covalent bond. Such molecules are generally represented as shown below:

S + S-

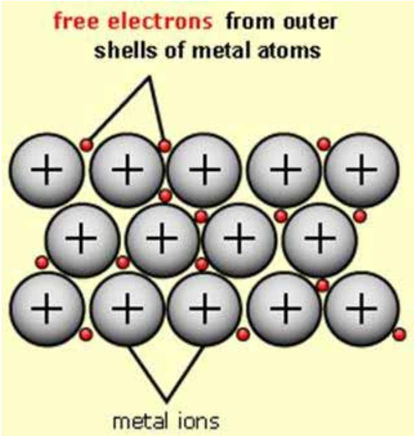
$H - C\ell$



Metallic Bonding

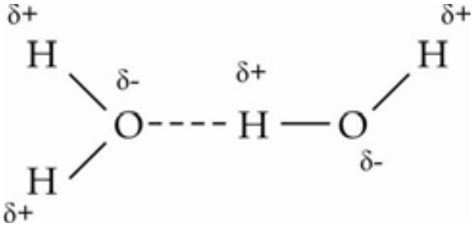
Metals consist of atoms only, then how they are held together? This is explained by a different type of bonding called metallic bonding.

In the atoms of metals, the electrons in the outermost orbit are loosely held and they are also called free electrons. The remainder portion of the atom is known as Kernal which is positively charged. These positively charged spheres are packed in a regular fashion. The free electrons are mobile in nature and move from one kernal to another throughout the metal. The metal crystal may be pictured as an arrangement of positive ions immersed in a sea of electrons. Hence a metallic bond may be defined as the bond formed as a result of simultaneous attraction of an electron by two or more than two positive ions of the metal.



Hydrogen Bond

Whenever a substance contains a hydrogen atom linked to a highly electronegative atom like N, O or F, then this becomes a polar covalent bond. In this the hydrogen atom becomes slightly (+)ve and the other atom becomes slightly (-)ve. The negative end of one molecule attracts the positively charged hydrogen of the other molecule and in this way, a bond is formed which is called hydrogen bond and is represented by a dotted line. For example, the hydrogen bond in water can be shown as below:



In this type of bonding, the hydrogen atom is attached simultaneously to two electronegative atoms, and it acts as bridge between two molecules. This type of hydrogen bonding is more specifically known as intermolecular hydrogen bonding.

Such a bond may occur between two functional groups present in the same molecule then it is known as intramolecular hydrogen bonding.

Both the types of hydrogen bonds affect the physical properties of various substances.

SUGGESTED VIDEO LINKS

- https://youtu.be/d9Xpt5Xh_D4
- https://youtu.be/5IJqPU11ngY
- https://youtu.be/X9FbSsO_beg
- https://youtu.be/Bi0rUNV8mEw

DO IT YOURSELF

I. Multiple choice questions:

1. Ionic compounds conduct electricity :
   a. In molten form as there are free electrons.
   b. In solid form as there are free ions.
   c. In molten form as there are free ions.
   d. In solid form as there are free electrons.

2. Which of the following represents a molecular substance?
   a. NaCl
   b. CO_2
   c. $MgCl_2$
   d. Al_2O_3

3. The type of bonding existing in C_2H_5OH is:
   a. Covalent bond.
   b. Ionic bond.
   c. Polar covalent bond.
   d. Polar covalent bond and hydrogen bonding

4. The formula of a compound is X_2Y_3. The valencies of elements X and Y will be respectively:

a. 2 and 3

b. 3 and 2

c. 1 and 3

d. 3 and 1

5. Silicon belongs to the 14th group. The formula of its oxide is:

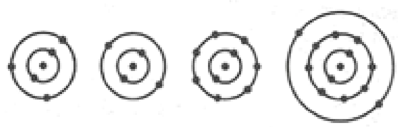
a. SiO_2

b. SiO

c. Si_2O

d. SiO_4

6. Given below is the arrangement of electrons for the elements P, Q, R, S and T respectively. Identify the elements which have the same valency?



P Q R S

a. P and Q

b. P and R

c. P, R and S

d. Q and S

II. Fill in the blanks:

1. Group of atoms carrying a positive charge ______________.

2. More than one valency shown by an element is called ______________ valency.

3. Ionic compounds are generally soluble in ____________.

4. Ionic compounds are good conductor of electricity in the ________________ or in their ________________.
5. The _____________ compounds exist as molecules.
6. $KMnO_4$ is an example of a compound having both _____________ and ________ bonds.

III. Short answer questions:

1. Define chemical bond.
2. Write two main causes for the formation of a chemical bond.
3. HCl is a covalent compound but it ionizes in the solution. Why?
4. Metals conduct electricity. Why?
5. The formula of the compound is A_2B_3.What are the number of electrons in the outermost orbit of A and B respectively?

IV. Answer the following:

The electronic configuration of four elements A, B, C and D are as follows:

	K	L	M
A	2	8	
B	2	6	
C	2	8	2
D	2	8	7

1. The most stable form of A is ________.
2. The formula of compound formed between C and D is _________.
3. The formula of compound formed between B and D is __________.
4. The type of compound formed between B and D is __________.

V. Complete the following table:

Chemical compound	Cation	Anion	Chemical formula
Calcium bicarbonate	Ca^{2+}	1.______	2.________
Calcium oxalate	Ca^{2+}	3._______	4._______
5._______	6._______.	7._______	NH_4NO_3
Chromium sulphate	8._______.	9._______.	10.______.

ANSWERS

I. Multiple choice questions:

1. c
2. b
3. d
4. b
5. a
6. d

II. Fill in the blanks:

1. Polyatomic cation
2. Variable
3. Water
4. Solution, molten states
5. Covalent
6. Ionic, covalent

III. Short answers:

1. The attractive force which holds together the constituent particles in a chemical species.
2. The two main causes are:
   - Tendency to acquire noble gas configuration
   - Tendency to acquire minimum energy
3. HCl is a polar covalent compound.
4. Metals conduct electricity because their electrons are mobile.
5. 3 and 6.

IV. Answer the following:

1. A
2. CD_2
3. BD_4
4. Covalent

V. Complete the following table:

1. HCO_3^-
2. $Ca(HCO_3)_2$
3. $C_2O_4^{2-}$
4. CaC_2O_4
5. Ammonium nitrate
6. NH_4^+
7. NO_3^-
8. Cr^{3+}
9. SO_4^{2-}
10. $Cr_2(SO_4)_3$

CHAPTER 2

MOLE CONCEPT AND STOICHIOMETRY

Dear Reader

You have studied about the term 'mole' in your previous class. It is defined as the amount of a substance containing as many constituting particles (atoms, molecules or ions) as there are carbon atoms in 12g of carbon-12 ($^{12}_{6}C$ isotope). 12g carbon contains 6.022×10^{23} atoms and this number is called Avogadro's number. Thus the collection of 6.022×10^{23} atoms of an atomic substance is one mole. It was also observed that the mass of one mole of an atomic substance is equal to its atomic mass in grams. For example, the atomic mass of sodium is 23u, so 23g of sodium constitute one mole and this amount will contain 6.022×10^{23} atoms of sodium.

Similarly the mass of one mole of a molecular substance will be equal to its molecular mass in grams which is also known as molar mass. For example a mole of CH_4 will be 12 + 4 = 16 grams or in other words we can say that the mass of 6.022×10^{23} molecules of methane would be 16g or it is said that the molar mass of methane is 16g mol^{-1}.

$$no.of\ moles = \frac{given\ mass\ of\ substance(g)}{Molar\ mass\ of\ substance(g\ mol^{-1})}$$

$$No.of\ moles = \frac{Given\ number\ of\ particles}{Avogadro\ number}$$

Mole can be related to other quantities in the following manner:

Molar mass of a substance in g/mol ← One Mole of a substance → Avogadro number 6.022×10^{23} of entities (atoms/ions/molecules/electrons

One Mole of a substance ↓ 22.4 L of a gas at STP (standard temp 0^oC/273K and standard pressure 1 atm)

Let us recall that a chemical reaction can be represented by a **chemical equation** written in terms of the symbols and formulae of the reactants and products involved. The equation can be made more meaningful by

(a) **Balancing** the number of atoms of each element of the reactants and products. (to satisfy the law of conservation of mass)

(b) Writing the **physical states** of the reactants and products. (s- solid, l-liquid, g-gas, aq-aqueous)

(c) By mentioning the amount of **heat evolved or absorbed** during a chemical reaction.

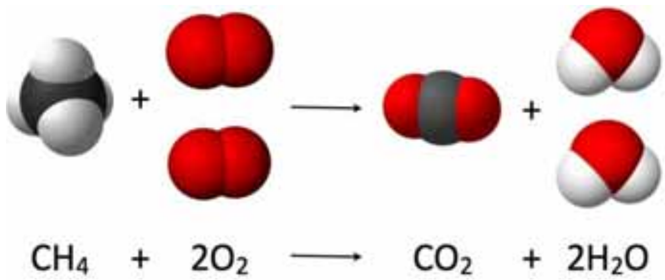
(d) For example the reaction taking place in **Haber's process** for manufacture of ammonia gas can be represented in the form of chemical equation as: $3H_2(g)+N_2(g)\rightarrow 2NH_3(g)+92.4kJ$

Let us now learn about the **Stoichiometry of Chemical reactions**

One of the most important aspects of a chemical equation is that when it is written in the balanced form, it gives quantitative relationships between the various reactants and products in terms of moles, masses, molecules and volumes. This is called **stoichiometry** (Greek word, meaning to 'measure an element'). The coefficients of the balanced equation are called **stochiometric coefficients.**

Consider the reaction;

$CH_4(g)+2O_2(g)\rightarrow CO_2(g)+2H_2O(g)$



We can interpret the equation in the following manner

(i) 1 mol of Methane gas react with 2 mol oxygen gas to form 1 mol carbon dioxide and 2 mol of H_2O (g).

(ii) 16 g of Methane gas reacts with (2×32g) = 64 g oxygen gas to form 44 g carbon dioxide and (2×18) = 36 g of H_2O (g).

(iii) 1 volume (22.4 L) of methane gas react with 2 volumes (44.8 L) of oxygen gas to form 1 volume(22.4L) of CO_2(g) and 2 volumes (44.8 L) of H_2O (g) under STP conditions.

To solve the numerical problems, following steps have to be followed:

(i) Write the balanced chemical equation in molecular form.

(ii) Select the symbols and formulae of the species (atoms and molecules) whose weights/ volumes are either given or to be calculated.

(iii) Write down the molar masses/ moles/ molar volumes of the selected species involved in the calculations.

(iv) Calculate the quantity of the desired substance by simple mathematical calculations (unitary method)

Example 1 (Involving mass-mass relationship)

What mass of copper oxide will be obtained by heating 12.35g of copper carbonate? (Atomic mass of Cu = 63.5 u)

$$CuCO_3(s) \xrightarrow{heat} CuO(s) + CO_2$$

63.5+12+48	63.5+16
=123.5g	=79.5g

123.5g of copper carbonate upon heating give CuO = 79.5 g

1.0g of copper carbonate upon heating gives CuO = $\frac{79.5}{123.5} g$

$\therefore$ 12.35g of copper carbonate upon heating give CuO = $\frac{79.5}{123.5} \times 12.35 = 7.95 g$

Example 2 (Involving mass-volume relationship)

$KC\ell O_3$ on heating decomposes to give $KC\ell$ and O_2. What is the volume of O_2 at STP liberated by 0.2 mol of $KClO_3$?

$$2KC\ell O_3(s) \xrightarrow{heat} KC\ell(s) + 3O_2(g)$$

2 mol $\qquad\qquad$ 3×22.4 L = $67.2\,\ell$

2 mol of $KC\ell O_3$ evolve O_2 at STP = $67.2\ \ell$

1 mol of $KC\ell O_3$ evolves O_2 at STP = $\frac{67.2}{2}\ell$

0.2 mol of $KC\ell O_3$ evolve O_2 at STP = $\frac{67.2}{2} \times 0.2\ell = 6.72\ell$

Example 3 (Involving volume-volume relationship)

What volume of oxygen at STP is required to affect complete combustion of 300 cm^3 of acetylene?

Combustion of Acetylene:

$$2C_2H_2(g) + 5O_2(g) \rightarrow 4CO_2(g) + 2H_2O(g)$$

2 Vol. $\qquad$ 5 Vol. $\qquad$ 4 Vol.

2 Vol. of C_2H_2 require O_2 for complete combustion = 5 Vol.

$\therefore$ 300 cm^3 of C_2H_2 require O_2 for complete combustion = $\frac{5}{2} \times 300 = 750 cm^3$

SUGGESTED VIDEO LINKS

- http://www.learnerstv.com/video/Free-video-Lecture-9206-Chemistry.htm
- http://www.learnerstv.com/video/Free-video-Lecture-9208-Chemistry.htm

DO IT YOURSELF

I. Multiple choice Questions (with one or more than one correct answer/s) :

1. 27 g of Al (at. Mass 27 u) will react with oxygen equal to

(a) 24 g (c) 8 g

(b) 40 g (d) 10 g

2. 10 g $CaCO_3$ on reaction with HCl acid will produce CO_2

(a) 1120 cm^3 (c) 2240 cm^3

(b) 112 cm^3 (d) 224 cm^3

3. 8 g O_2 has the same number of molecules as

(a) 7 g CO (c) 14 g N_2

(b) 11 g CO_2 (d) 16 g SO_2

4. 1 g Mg was burnt in a closed vessel containing 2 g of oxygen. The incorrect statements are:

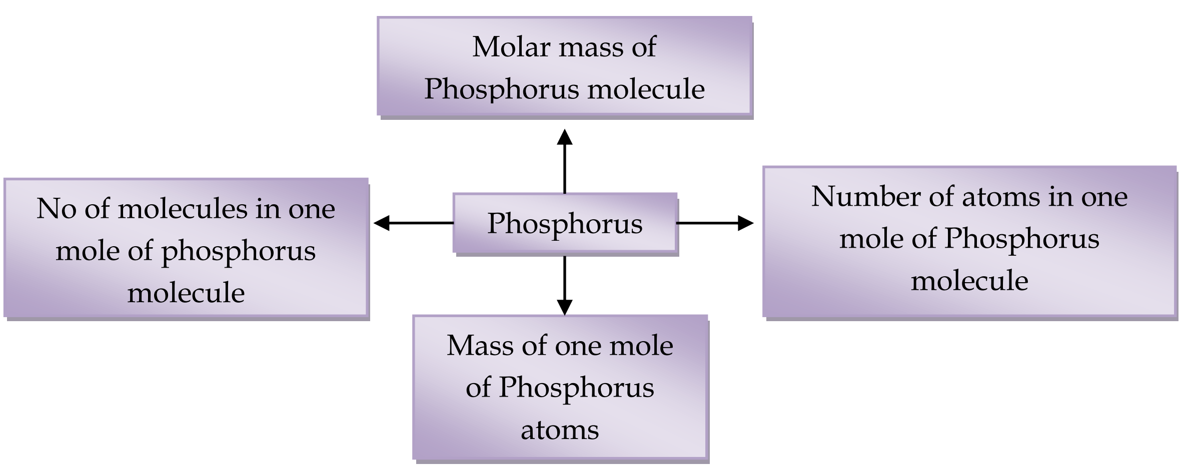
(a) 0.25 g of Mg will be left unburnt.

(b) 1.33 g of O_2 will be left unreacted.

(c) 2.5 g of MgO will be formed

(d) The mixture at the end will weigh 3 g.

5. One atom of Phosphorus has a relative atomic mass 24 u. Its molecule has four phosphorus atoms. Use this information to fill in the following boxes.



6. Match the entries of column I with appropriate entries of column II

	Column I (No. of moles)		Column II (Amount of substance)
A	0.1 mol	P	4480 mL of CO_2 at STP
B	0.2 mol	Q	0.1 g atom of Fe
C	0.25 mol	R	1.5×10^{23} molecules of oxygen gas
D	0.5 mol	S	9 mL of water
		T	200 mg of hydrogen gas

II. Fill in the blanks:

1. The number of g –atom of oxygen in 6.022×10^{22} CO molecules is -------.

2. In the reaction: $2Al(s)+6HCl(aq)\rightarrow 2Al^{3+}(aq)+6Cl^{-}(aq)+3H_2(g)$, The volume of $H_2(g)$ at STP produced for every mole of HCl(aq) consumed is --------.

3. If ten volumes of dihydrogen gas reacts with five volumes of dioxygen gas, -------- volumes of water vapour are produced.

4. Mass of one ^{12}C atom in grams = -------

ANSWERS

I. Multiple choice questions:

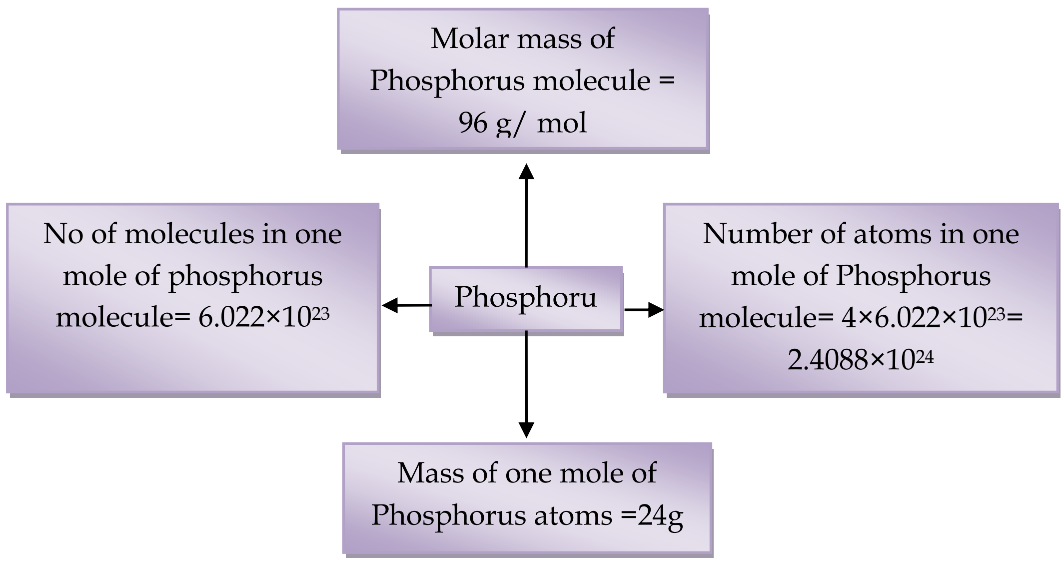
1. (a)

2. (c)

3. (a), (b), (d)

4. (a), (c)

5.



6. A – Q, T B – P C – R D – S

II. Fill in the blanks:

1. 0.1
2. 11.2 L
3. 10 volumes
4. 1.9927×10^{-23} g

3 CHAPTER

THERMODYNAMICS

Dear Reader

You are familiar with many chemical reactions which are accompanied by the absorption or release of energy. For example, when coal burns in air, a lot of heat is given out. Another example is the decomposition of water into oxygen and hydrogen when electricity is passed through it. Both heat and electricity are different forms of energy.

The branch of Science that deals with quantitative aspects of interconversion of various forms of energy and conversion of energy into work or vice-versa is called Thermodynamics. Thermodynamics of chemical processes is studied under Thermochemistry.

The two most important terms used in thermodynamics are system and surroundings.

A thermodynamic system is that part of the universe which is selected for investigation. Everything that is not a part of the system, constitutes the surroundings.

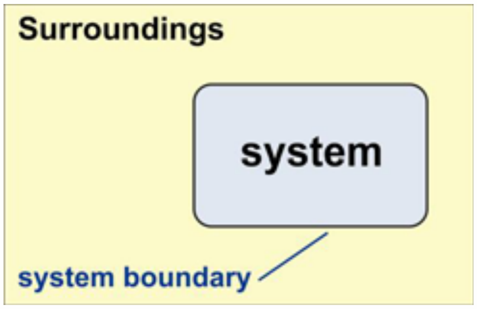


Figure 1

- The system and surroundings are separated by a boundary.
- If our system is a gas in a container, then the boundary is simply the inner wall of the container itself. The boundary need not be a physical barrier.
- The single property that the boundary must have is that it should be clearly defined, so we can unambiguously say whether a given part of the world is in our system or in the surroundings.

Properties and the state of a system

- The properties of a system are those quantities such as the **pressure**, **volume**, **temperature**, and its **composition**, which are measurable and capable of assuming definite values.
- These have special significance because they determine the values of all the other properties; they are therefore known as ***state properties*** because if their values are known then the system is in a definite ***state***.

Change of state: the meaning of Δ

- In dealing with thermodynamics, we must be able to unambiguously define the change in the state of a system when it undergoes to some process. This is done by specifying changes in the values of the different state properties using the symbol Δ (delta) as illustrated here for a change in the volume:

  $\Delta V = V_{final} - V_{initial}$
- You can compute similar delta-values for changes in P, T, ni (the number of moles of component i)

Let us discuss the units of Energy

- Energy is measured in terms of its ability to perform work or to transfer heat. Mechanical work is done when a force F displaces an object by a distance d:

  Work (W) $= F \times d$.
- The basic unit of energy is Joule.
- One joule is the amount of work done when a force of 1 Newton acts over a distance of 1 m;

  Thus 1J = 1 Nm.
- The Newton is the amount of force required to accelerate a 1kg mass by 1 m/sec^2, so the basic dimensions of the joule are kg m^2 s^{-2}.

Now we take two more familiar terms: Heat and work

Heat (q) and work (w) are both measured in energy units, so they must both represent energy. The term "heat" has a special meaning in thermodynamics; it is a process in which a body acquires or loses energy as a direct consequence of its having a different temperature than its surroundings (the rest of the world).

When two bodies are placed in thermal contact and energy flows from the warmer body to the cooler one, we call the process transfer of "heat".

- A transfer of energy to or from a system by any means other than heat is called **"work".**
- Work refers to the transfer of energy by some means that does not depend on temperature difference. Work, like heat, exists only when energy is being transferred.
- Work, like energy, can take various forms, the most familiar being mechanical and electrical.

Let us know about Inter-convertibility of heat and work

Work can be completely converted into heat (e.g. by friction), but heat can only be partially converted into work.

Sign Conversations for Heat and Work

Heat is given positive sign (q = (+)ve) if it is absorbed by the system (endothermic process) and is given negative sign (q = (-)ve) when it is lost by the system (exothermic process).

Work is considered as (+)ve when it is done on the system and is (-)ve when it is done by the system.

Law of Conservation of Energy

This law can be stated in any of the following ways:

- Energy can neither be created nor be destroyed, although it may be converted from one form to another.

- Energy of an isolated system remains constant. A system is said to be isolated if it can not exchange energy or work with the surroundings.
- The energy of universe is constant.

Internal Energy (U)

Internal Energy is the total energy possessed by a system. It includes the entire energy of its constituent atoms and molecules, all forms of kinetic energy (translational, rotational and vibrational) as well as the potential energy due to all types of interactions between molecules, atoms and sub-atomic particles.

Since the absolute value of internal energy cannot be determined. The change in internal energy during any process is given by

$$\Delta U = U_{final} - U_{initial}$$

Heat changes at constant pressure: the Enthalpy

- For a chemical reaction that performs no work on the surroundings, the heat absorbed is the same as the change in internal energy: **$q = \Delta U$.** But many chemical processes do involve work in one form or another.
- A thermodynamic function of a system, equivalent to the sum of the internal energy (U) of the system and the product of its volume multiplied by the pressure exerted on it (PV)by its surroundings is called **Enthalpy(H)**. H is also called heat content, total heat and can be represented by the equation:

$$\mathbf{H=U+PV}$$

Let us discuss the heat changes in different forms of Reactions:

- On the basis of heat changes reactions can be of two types:

  1. Exothermic Reactions
  2. Endothermic Reactions

Exothermic Reactions

- An exothermic chemical reaction transfers energy to the surroundings.

  Reactants → products + energy

  If the products contain less energy than the reactants, heat is released or given out to the surroundings and the change is called an **exothermic reaction** (exothermic energy transfer, exothermic energy change of the system).

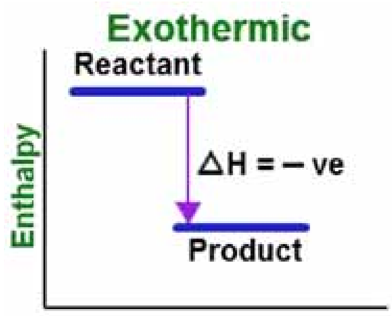


Figure 2

Endothermic Reactions

- An endothermic chemical reaction absorbs energy from the surroundings.

  reactants + energy → products

  If the products contain more energy than the reactants, heat is taken in or absorbed from the surroundings and the change is called an endothermic reaction (endothermic energy transfer, endothermic energy change of the system).

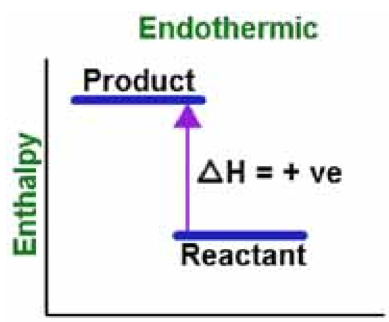


Figure 3

Difference between Endothermic and Exothermic Reactions:

Exothermic	Endothermic
System loses energy	System gains energy
Surroundings gain energy	Surroundings lose energy
Surroundings become warmer	Surroundings become cooler
Value of H is negative	Value of H is positive
Energy adds on right side of the equation (with products)	Energy adds on left side of the equation (with reactants)

SUGGESTED VIDEO LINKS

- https://www.youtube.com/watch?v=rxwmn0NH4s8
- https://www.youtube.com/watch?v=TJpZpUdMVxc
- https://www.youtube.com/watch?v=pTujFtkucwA
- https://www.youtube.com/watch?v=fsHc7B9dgwI
- https://www.youtube.com/watch?v=VGqFnMG2eO8
- https://www.youtube.com/watch?v=ubteilhOilc
- https://www.youtube.com/watch?v=YC72Tn4SEjg

DO IT YOURSELF

Fill in the blanks:

1. Thermo-chemistry deals with _______________. (Thermal Chemistry, Mechanical Energy, Potential Energy)

2. The factor E + PV is known as _________. (Heat content, Change in Enthalpy, Work done)

3. Anything under examination in the Laboratory is called __________. (Reactant, System, Electrolyte)

4. The environment in which the system is studied in the laboratory is called __________. (Conditions, Surroundings, State)

5. An endothermic reaction is one, which occurs__________. (With evolution of heat, With absorption of Heat, In forward Direction)

6. An exothermic reaction is one during which __________. (Heat is liberated, Heat is absorbed, no change of heat occurs)

7. The equation $C + O_2 \rightarrow CO_2$ $\Delta H = -408KJ$ represents ________________ reaction. (Endothermic, Exothermic, Reversible)

8. The equation $N_2 + O_2 \rightarrow 2NO$ $\Delta H = 180KJ$ represents ________________ reaction. (Endothermic, Exothermic, Irreversible)

9. Enthalpy is __________. (Heat content, Internal energy, Potential Energy)

10. When the bonds being broken are more than those being formed in a chemical reaction, then ΔH will be __________. (Positive, Negative, Zero)

11. When the bonds being formed are more than those being broken in a chemical reaction, then the ΔH will be __________. (Positive, Negative, Zero)

12. The enthalpy change when a reaction is completed in single step will be __________ as compared to that when it is completed in more than one step. (Equal to, Partially different from, Entirely different from)

13. The enthalpy of a system is represented by __________. (H, ΔH, ΔE)

14. The total heat change in a reaction is the ___________whether it takes place in one or several steps. (same, different, varies)

15. If the enthalpy is negative the reaction is __________.

16. If the enthalpy of a reaction is positive, the reaction is___________.

17. If the products have greater total bond energy than the reactants the reaction will be___________.

18. If the products have less total bond energy than the reactants the reaction will be___________.

19. Write in detail:

    What is the information conveyed by chemical equations-

    1. --
    2. --
    3. --
    4. --

ANSWERS

1.	Thermal Chemistry	2.	Change in Enthalpy
3.	System	4.	Surroundings
5.	With absorption of Heat	6.	Heat is liberated
7.	Exothermic	8.	Endothermic
9.	Heat content	10.	Negative
11.	Positive	12.	Equal to
13.	ΔH	14.	Same
15.	Exothermic	16.	Endothermic
17.	Exothermic	18.	Endothermic
19.	(Write all possible information)		

4 CHAPTER

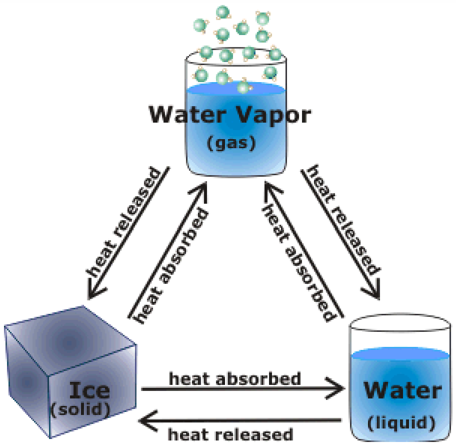
EQUILIBRIUM

Dear Reader

Let us recall the **phase transformations** studied in class IX.

We learnt that water and ice coexist at 0ºC and atmospheric pressure. Similarly at 100ºC, water and water vapour coexist.

The above facts can be explained on the basis of the term **Equilibrium**.



Phase transformation

Equilibrium represents the state of a process in which the properties like temperature, pressure, concentration of the system do not show any change with the passage of time.

In all processes which attain equilibrium, two opposing processes are involved. When the rates of the two opposing processes become equal, it is said that the state of equilibrium has been attained.

If the opposing processes involve only physical changes, the equilibrium is called physical equilibrium.

The symbol $\rightleftharpoons$ represents reversibility or equilibrium.

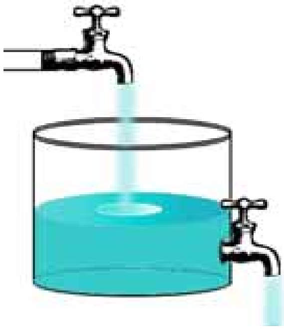
The state of equilibrium when two opposite forces balance each other and no change occurs at all in the system is said to be in a state of **static equilibrium.** eg. When two well matched people sit on the opposite sides of a see saw.

Static equilibrium

When at equilibrium it appears that no change is taking place in the system, but actually both the processes are taking place at equal rates in the forward and backward direction, it is called **Dynamic nature** of equilibrium.

While static equilibrium results from 'no change' occurring in the system, dynamic equilibrium results from 'no net change' due to two equal but opposite changes occurring simultaneously.

Consider the example of a bucket which is fitted with a tap at the bottom. If we adjust the tap so that the water flows out of the bucket at the same rate as the rate of filling, the level of water in the bucket becomes stationary. This is the state of **dynamic equilibrium**



Dynamic equilibrium

Equilibria in Physical Processes

The most common equilibria involving physical processes are those which involve phase transformation. Besides these, there are equilibria involving dissolution of solids or gases in liquids. They have been discussed below.

Some Features of Physical equilibria

Process with example	Rates becoming Equal	Property becoming constant at equilibrium
Solid $\rightleftharpoons$ Liquid (Melting of ice) $H_2O(s) \rightleftharpoons H_2O(l)$	Rate of melting of ice= rate of freezing of water.	Melting point at constant pressure
Liquid $\rightleftharpoons$ Gas (Evaporation of water in closed vessel) $H_2O(l) \rightleftharpoons H_2O(g)$	Rate of evaporation = Rate of condensation	Vapour pressure at constant temperature. (The pressure exerted by the vapours in equilibrium with the liquid at that temperature is called **vapour pressure of the liquid**)
Solute (s) $\rightleftharpoons$ Solute(solution) Dissolution of sugar in water. Sugar(s) $\rightleftharpoons$ Sugar(solution)	Rate of dissolution = Rate of precipitation	Concentration of the solute in the solution at given temperature.
Gas(g) $\rightleftharpoons$ Gas (aq) Dissolution of a gas in a	Mass of the gas dissolved is constant for constant	$\frac{[Gas(aq)]}{[Gas(g)]}$ =Constant at a

liquid under pressure in a closed vessel) $CO_2(g) \rightleftharpoons CO_2(aq)$	equilibrium pressure at constant temperature.	given temperature. (Square bracket represents molar concentration)

General Characteristics of Equilibria involving Physical Processes

(1) At equilibrium, some **observable property** of the system becomes **constant**.

(2) Equilibria involving gases can be attained only in **closed vessels**. (Otherwise the gas will escape and there will be no equilibrium.

(3) Equilibrium is **dynamic in nature** i.e. there are two opposing processes taking place at equal rates)

(4) At equilibrium, the **concentration** of the different substances **becomes constant** at constant temperature.

(5) At equilibrium, there exists an expression involving the concentration of the substances which is constant at constant temperature.

eg. , for $CO_2\,(g) \rightleftharpoons CO_2\,(aq)$

$\frac{CO_2(aq)}{CO_2(g)}$ =constant at constant temperature. This constant is called **equilibrium constant**.

(6) The **magnitude** of the equilibrium constant represents the extent to which the reaction proceeds before equilibrium is attained.

Chemical Equilibrium

When equilibrium is reached in a chemical process it is called **chemical equilibrium.**

Recall the difference between reversible and irreversible reaction

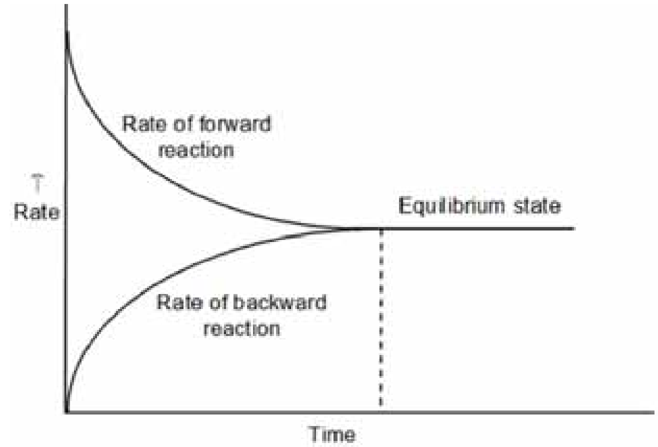
Reversible reaction	Irreversible reaction
If a reaction can take place not only in the forward direction but also in the backward direction under the same conditions	If a reaction cannot take place in the reverse direction i.e. the products formed do not react to give back the reactants under the same condition.
It is represented by a double arrow ($\rightleftharpoons$)	It is represented by a single arrow ($\rightarrow$)

A chemical equilibrium **results from a reversible reaction** occurring at equal rates in both the directions.

Example 1:

$H_2(g) + I_2(g) \rightleftharpoons 2\,HI(g)$

If H_2 and I_2 are enclosed in a glass bulb at equimolar ratio, the colour is deep purple in the beginning, due to the presence of I_2. As time passes, the intensity of the colour decreases due to the formation of colourless HI. After some time, the **colour becomes constant** indicating the state of equilibrium.

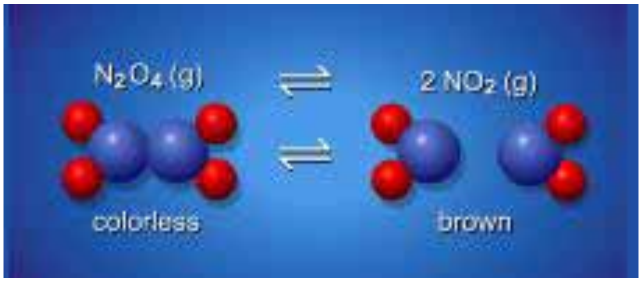


(Variation in reaction rates for forward and backward reaction with time)

Example 2: Decomposition of N_2O_4 in a closed vessel

N_2O_4 (g)   2 NO_2 (g)

If N_2O_4 is taken in a sealed glass bulb and placed in ice, it is almost colourless. Now, if the sealed bulb is shifted to a vessel containing water at 25°C, the bulb starts acquiring brown colour which first deepens and then becomes constant. This indicates a state of equilibrium in which the concentrations of N_2O_4 and NO_2 have become constant.



Characteristics of chemical equilibrium

(i) At equilibrium, the concentration of each of the reactants and products becomes constant.

(ii) At equilibrium, the rate of forward reaction becomes equal to the rate of backward direction and hence equilibrium is dynamic in nature.

(iii) A chemical equilibrium is established if none of the products is allowed to escape or separate out as solid.

(iv) Chemical equilibrium can be attained from either direction.

(v) A catalyst does not alter the state of equilibrium.

Detailed study of Equilibrium will be dealt with in Class XI.

SUGGESTED VIDEO LINKS

- http://www.learnerstv.com/video/Free-video-Lecture-9195-Chemistry.htm
- http://learn.mindset.co.za/resources/physical-sciences/grade-12/chemical-equilibrium/01-introduction-chemical-equilibrium

DO IT YOURSELF

I. Multiple choice questions:

1. Identify whether the given system is at equilibrium or not?

(a) A burning candle

(b) Water and ice in a beaker at -5^{o}C.

(c) Undissolved sugar in a test tube containing its saturated solution.

(d) A sealed bottle containing alcohol.

2. Identify whether the equilibrium in the following systems is static or dynamic:

(a) Common salt in its own saturated solution in water.

(b) Sugar kept in a closed glass bottle.

(c) Carbon dioxide in a sealed bottle of a soft drink

(d) Iodine crystals kept in a closed vessel.

3. Which of the following is not a general characteristic of equilibria involving physical processes?

(a) Equilibrium is possible only in a closed system at a given temperature.

(b) All measurable properties of a system are constant at equilibrium.

(c) All the physical processes stop at equilibrium.

(d) The opposing processes occur at the same rate and there is a dynamic but stable condition.

4. At a particular temperature and atmospheric pressure, the solid and liquid phases can exist in equilibrium. Which of the following terms define(s) this temperature?

(i) Normal melting point

(ii) Equilibrium temperature

(iii) Boiling point

(iv) Freezing point

(a) (i) and (ii) (b) (i) and (iv)

(c) (i) only (d) (iv) only

5. **When a volatile liquid is introduced into an evacuated closed vessel at a particular temperature, both evaporation and condensation takes place simultaneously. The system reaches equilibrium state when**

(a) the liquid is completely transformed into the corresponding vapour.

(b) equal amounts of liquid and vapour are present in the system.

(c) the rate of evaporation becomes equal to rate of condensation .

(d) liquid cannot be converted into vapour and vice versa.

6. **For a specific reaction, which of the following statements can be made about K, the equilibrium constant?**

(a) It always remains the same at different reaction conditions

(b) It increases if the concentration of one of the products is increased.

(c) It changes with change in temperature.

(d) It may be changed by addition of a catalyst.

7. **When the system A + B $\rightleftharpoons$ C + D is at equilibrium**

(a) The sum of the concentrations of A and B must be equal to the sum of the concentrations of C and D

(b) Both the forward and the reverse reactions have stopped.

(c) The reverse reaction has stopped

(d) The reactions take place at equal rates in the forward and the reverse direction

8. **Assertion (A): For any chemical reaction at a particular temperature, the equilibrium constant is fixed and is a characteristic property.**

Reason (R): Equilibrium constant is independent of temperature.

(a) true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false.

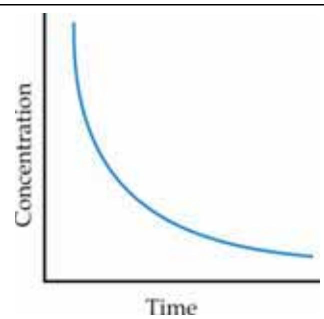
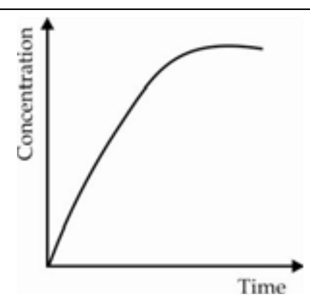
(d) Both A and R is false.

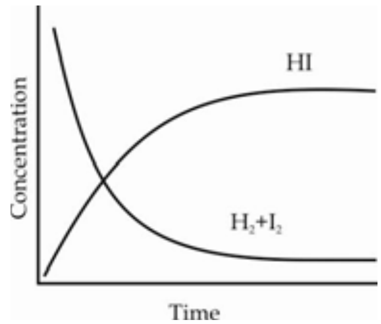
II. Match the following:

1. Match the following equilibrium with the corresponding condition:

(1) Liquid $\rightleftharpoons$ Vapour	(a) Saturated solution
(2) Solid $\rightleftharpoons$ Liquid	(b) Boiling point
(3) Solid $\rightleftharpoons$ Vapour	(c) Sublimation point
(4) Solute(s) $\rightleftharpoons$ Solute(solution)	(d) Melting pont
	(e) Unsaturated solution

2. Match the following graphical variation (concentration vs. time) with their description:

 (i)	(a) Variation in product concentration with time
 (ii)	(b) Reaction at equilibrium

 (iii)	(c) Variation in reactant concentration with time.

ANSWERS

I. Multiple choice questions:

1. (a) No (b) No
   (c) Yes (d) Yes
2. (a) Dynamic (b) Static
   (c) Dynamic (d) Dynamic
3. (c)
4. (b)
5. (c)
6. (c)
7. (d)
8. (c)

II. Match the following:

1. 1; (b) 2; (d)
   3; (c) 4;(a)
2. (i) c (ii) (a) (iii) (b)

5 CHAPTER

REDOX REACTIONS

Dear Reader

You have studied about different types of reactions in your previous class. One of those reactions are oxidation-reduction reactions. In order to understand these reactions, let us first learn about the process of oxidation and reduction with the help of following examples.

When charcoal burns in air, it form carbon monoxide and carbon dioxide depending upon the availability of oxygen.

$$2C(s) + O_2(g) \rightarrow 2CO(g)$$

$$C(s) + O_2(g) \rightarrow CO(g)$$

A piece of magnesium burns in air to form magnesium oxide

$$2Mg(s) + O_2(g) \rightarrow 2MgO(s)$$

In the above reactions, an element combines with oxygen to form an oxide. This process of combining an element with oxygen is known as **oxidation**. Reverse of this process, that is when a compound loses oxygen, is termed as **reduction**.

Now look at the following reactions.

$$F_2(g) + H_2(g) \rightarrow 2HF(l)$$

$$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$$

We find that in these reactions hydrogen has added to fluorine or nitrogen. The processes of oxidation or reduction are also defined on the basis of reaction with hydrogen. A reaction in which hydrogen is added , is known as **reduction** and the process which involves the loss of hydrogen is called **oxidation**.

Let's investigate the following reactions

$$Fe_2O_3(s) + 2\,Al(s) \rightarrow Al_2O_3(s) + 2\,Fe(s)$$

$$2\,NH_3(g) + 3\,Cl_2(g) \rightarrow N_2(g) + 6\,HCl(g)$$

In the first reaction , we find that ferric oxide has lost oxygen , hence it is reduced and that oxygen has been taken up by aluminium, so aluminium is oxidised. In the second reaction ammonia has lost hydrogen, so it is oxidised and chlorine is reduced because it has taken that hydrogen. So we observe that in the above reactions , the process of oxidation and reduction is occurring simultaneously. Such reactions are termed as **redox** reactions.

Oxidation and reduction in terms of electron transfer

Let us look again at the reaction of magnesium with oxygen in a way as you have studied in bond formation. We split the reaction in two half reactions as given below.

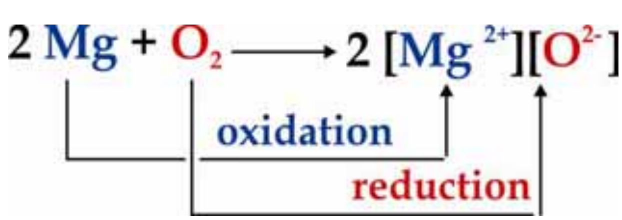
$Mg \rightarrow Mg^{2+} + 2\ e^-$

$O_2 + 4\ e^- \rightarrow 2\ O^{2-}$

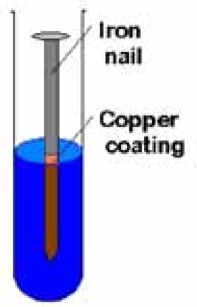
In the first half reaction, each magnesium atom loses two electrons to form an Mg^{2+} ion and in the second half reaction ,each O_2 molecule gains four electrons to form two O^{2-} ions. These ions exist together as magnesium oxide. The complete reaction can also be written as given below

$2\ Mg + O_2 \rightarrow 2\ [Mg^{2+}][O^{2-}]$

As we have discussed, magnesium is said to be oxidised in this reaction because it has combined with oxygen but in this process it has lost electrons. So the process of oxidation can be defined in terms of electrons as the process involving loss of electrons. The reverse process where the electrons are gained is termed as reduction. In the above reaction , magnesium is oxidised and oxygen is reduced. The processes of oxidation and reduction always occur together because if one species loses the electrons ,there must be another species to accept these electrons. It is impossible to have one without the other, as shown in the figure below.



Another type of reactions involving displacement of metals, with which you are familiar, can also be classified as redox reactions. If a nail of iron is placed in a solution of copper sulphate , metallic copper is deposited on the iron nail and some iron passes into the solution as Fe^{2+} ions and forming ferrous sulphate.



$$Fe(s) + CuSO_4(aq) \rightarrow Cu(s) + FeSO_4(aq)$$

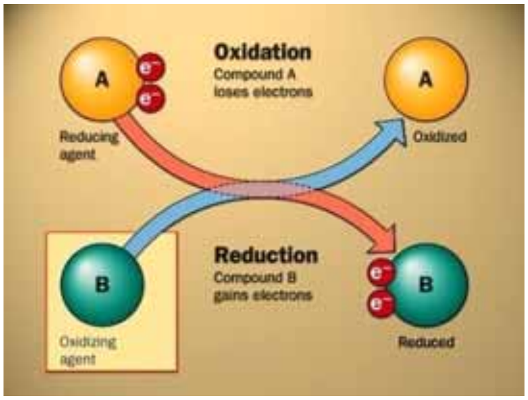
In terms of electrons, this reaction can be divided into oxidation and reduction halves.

$$Fe(s) \rightarrow Fe^{2+}(aq) + 2e^- \quad \text{(Oxidation)}$$

$$Cu^{2+}(aq) + 2e^- \rightarrow Cu(s) \quad \text{(Reduction)}$$

The substance which accepts the electrons and gets reduced is known as **oxidising agent** because it has oxidised the other species. For example in the above reaction copper ions are oxidising agent. The species which loses electrons , gets oxidised in the process and reduces the other species, hence is called **reducing agent**. So in the above reaction Fe is a reducing agent.

The relationship between oxidising and reducing agents can be clearly understood with the help of the following diagram.



The process of oxidation and reduction can also be looked into by using the concept of oxidation number which you will study in class XI.

SUGGESTED VIDEO LINKS

- http://learn.mindset.co.za/resources/physical-sciences/grade-11/chemical-change-types-reactions-redox-reactions/01-introduction-redox-reactions
- http://www.learnerstv.com/video/Free-video-Lecture-9210-Chemistry.htm
- https://www.youtube.com/watch?v=KmhD8BmEFIo
- https://www.youtube.com/watch?v=OV0PK-vMcGg

DO IT YOURSELF

1. Fill in the blanks with appropriate words:

(i) The process of oxidation involves .... of oxygen.

(ii) The process involving gain of electrons is called ....

(iii) A substance that .... electrons is known as oxidising agent.

(iv) A reducing agent itself gets .... during the reaction.

(v) A .... reaction can be written as two half reactions.

2. In the reaction:

$$Fe(s) + CuSO_4(aq) \rightarrow Cu(s) + FeSO_4(aq)$$

identify the species

(i) which is oxidized.

(ii) which is reduced.

(iii) which acts as reducing agent.

3. In terms of electrons, oxidation involves:

(a) gain of electrons

(b) loss of electrons

(c) exchange of electrons

(d) conversion of a metal ion into the metal atom.

4. In a reaction between zinc and iodine, zinc iodide is formed. What is being oxidized?

(a) Zinc ions

(b) Iodide ions

(c) Zinc atom

(d) Iodine

5. Conversion of $PbSO_4$ into PbS is-

(a) reduction reaction

(b) neutralization reaction

(c) oxidation reaction

(d) dissociation reaction

6. Which of the following is a redox reaction?

(a) reaction between NaOH and H_2SO_4

(b) O_3 from O_2 by lightening in atmosphere

(c) Nitrogen oxides from nitrogen and oxygen by lightening in atmosphere.

(d) Evaporation of water

7. The number of electrons lost or gained during the reaction:

$$3Fe + 4\,H_2O \rightarrow Fe_3O_4 + 4H_2$$

(a) 2

(b) 4

(c) 6

(d) 8

8. **In the following equation:**

$$3Br_2 + 6CO_3^{2-} + 3H_2O \rightarrow 5Br^- + BrO_3^- + 6HCO_3^-$$

(a) Bromine is oxidized and carbonate is reduced

(b) Bromine is reduced and carbonate is oxidized

(c) Bromine is neither reduced nor oxidized

(d) Bromine is reduced as well as oxidized

ANSWERS

1. (i) gain (ii) reduction
   (iii) loses (iv) oxidized
   (v) Redox
2. (i) Fe(s)
   (ii) Cu^{2+}(aq)
   (iii) Fe(s)
3. (b)
4. (c)
5. (a)
6. (c)
7. (d)
8. (d)

6 CHAPTER

ELECTROCHEMISTRY

Dear Reader

You know that electricity is one of the various forms of energy and is considered as flow of electrons in one direction. When electricity is passed through the chemical substances, some changes occur. On the other hand, electricity may be produced during some chemical reactions. All such changes or reactions are studied under electrochemistry.

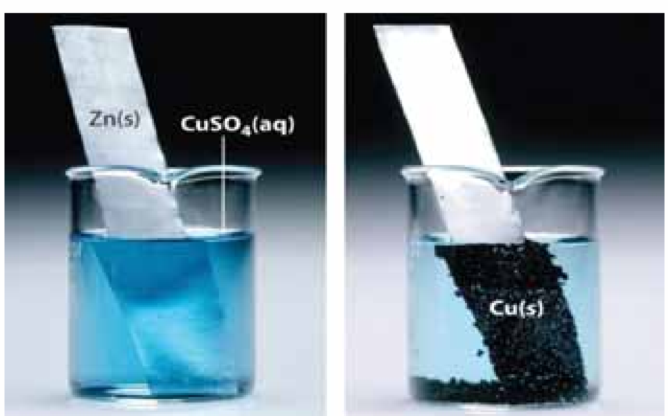
We have discussed that all redox processes involve gain or loss of electrons. Consider the following redox reaction.

$$Zn(s) + CuSO_4(aq) \longrightarrow Cu(s) + ZnSO_4(aq)$$

Here sulphate ions do not participate in the reaction and the ionic reaction may be written as

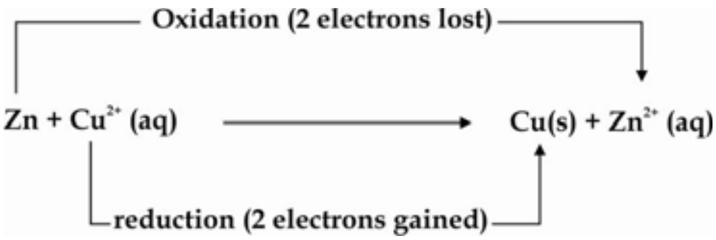
$$Zn(s) + Cu^{2+}(aq) \longrightarrow Cu(s) + Zn^{2+}(aq)$$

We observe that in this reaction Zn(s) is oxidised to $Zn^{2+}(aq)$ by losing two electrons and $Cu^{2+}(aq)$ ions are reduced to Cu(s) by gaining these two electrons. Thus , during this reaction , there is a net transfer of two electrons from a zinc atom to a copper ion. This reaction can be carried out in a beaker in which a solution of copper sulphate is taken and a strip of metallic zinc is dipped as shown below.

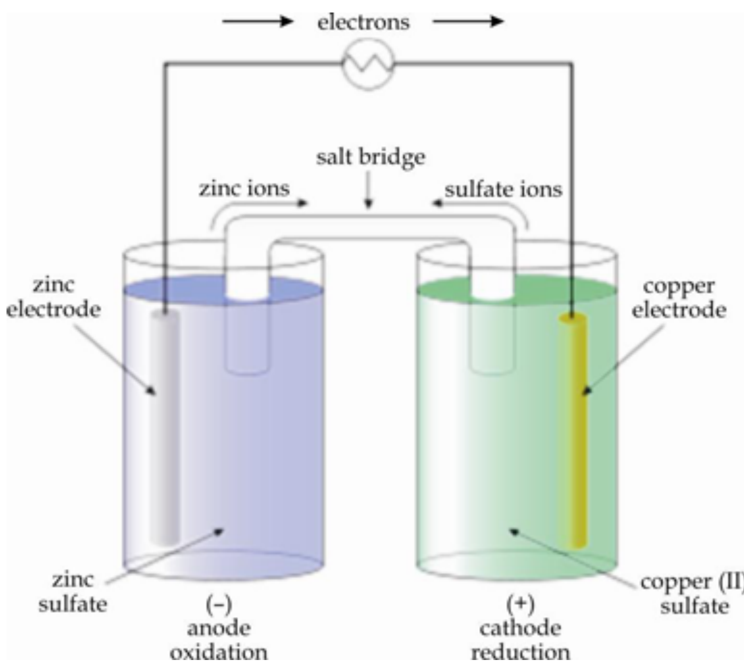


$$Zn\ \ s + Cu^{2}\ \ aq \rightarrow Zn^{2+}\ \ aq + Cu\ \ s$$

This reaction can be written in two halves as given below.



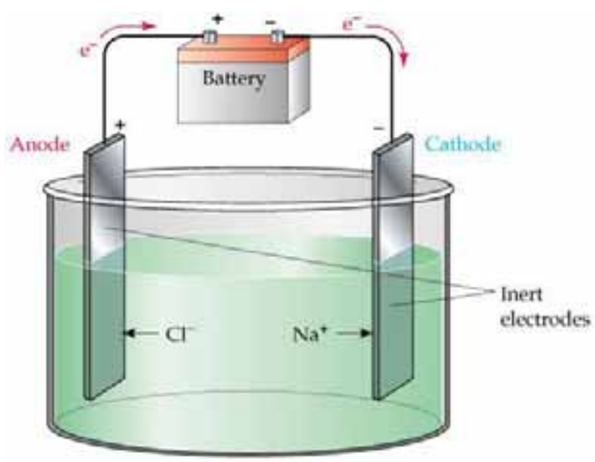
We observe that in this reaction there is a movement of electrons from zinc metal to copper ions by direct contact. If somehow this movement of electrons can be made through a wire , this would be generation of electricity. This can be achieved by the following type of setup where a rod of zinc metal is dipped in zinc sulphate solution and a rod of copper metal is dipped in copper sulphate solution in separate beakers.



In this way the electrons lost by zinc atoms must travel through the wire to the copper rod where they are transferred to copper ions which are reduced to copper atoms. Such a device is called **electrochemical cell** and the two portions where half of the redox reaction occurs are known as **electrodes** . However electrical contact between the two half cells is established with the help of a salt bridge containing KCl.

These electrodes are given the names as anode or cathode and are represented with + or – sign on the basis of half cell reaction that occurs on them. The electrode where oxidation process occurs is called **anode** and the electrode where process of reduction occurs is called **cathode**. As in the above cell, the zinc metal loses two electrons means this electrode involves the process of oxidation so this acts as anode. These electrons are taken up by copper ions and they get reduced so the process of reduction is occurring at copper electrode , hence it acts as cathode. The zinc electrode is given the negative polarity as electrons are moving away from it and the copper electrode is assigned the positive polarity as the electrons are moving towards it.

There are another type of cells possible , where electricity is used up and some chemical reaction occurs. Such type of cells are called **electrolytic cells**. Let us consider the example, when electricity is passed through molten sodium chloride by dipping two inert electrodes as shown below.



The electrode which is connected to the (+)ve terminal of the battery acquires positive charge so the negative ions (chloride ions) move towards this electrode. Here they lose electrons , the process of oxidation occurs hence this electrode is called anode . The other electrode which is connected to the (–)ve terminal of the battery, so positive ions (sodiumions) move towards this electrode. Here they get electrons and are converted into sodium metal and the process of reduction occurs, hence this electrode is called cathode. The half cell reactions occurring at two electrodes may be written as follows:

$Na^+(l) + e^- \rightarrow Na(s)$

$2\,Cl^- (l) \rightarrow Cl_2 (g) + 2\,e^-$

Note carefully that in the electrochemical cells and the electrolytic cells , the electrodes have been assigned opposite charges. In an electrochemical cell anode is (–)vely charged whereas in electrolytic cell it is (+)vely charged. There is no error in this. Actually electrodes are given the names on the basis of process which occurs at that electrode whereas the charge depends upon the deficiency or excess of electrons on a particular electrode.

You will study different types of cells and batteries in class XII.

SUGGESTED VIDEO LINKS

- http://learn.mindset.co.za/resources/physical-sciences/grade-12/electrochemical-reactions/02-types-electrochemical-cells
- http://learn.mindset.co.za/resources/physical-sciences/grade-12/electrochemical-reactions/08-electrolytic-cells
- http://www.learnerstv.com/video/Free-video-Lecture-9216-Chemistry.htm
- http://www.learnerstv.com/video/Free-video-Lecture-9217-Chemistry.htm

DO IT YOURSELF

1. Fill in the blanks with appropriate words:

(i) The reaction that occurs at anode is ....

(ii) The .... is assigned the negative charge in an electrolytic cell.

(iii) The charge on cathode in an electrochemical cell is ....

(iv) In an electrochemical cell, the electrons move from .... to...........

(v) Electrochemical cells are used to convert chemical energy into .... energy.

2. Which species undergoes oxidation at anode in an electrochemical cell with the following reaction?

$$Zn(s) + CuSO_4(aq) \longrightarrow Cu(s) + ZnSO_4(aq)$$

3. **Which of the following lose charge at cathode?**

   (a) ions

   (b) anions

   (c) cations

   (d) both cations and anions.

4. **During electrolysis of molten NaOH, which of the following ions move towards anode?**

   (a) Na^+

   (b) OH^-

   (c) H^+

   (d) Both, Na^+ and H^+

5. **The reaction at cathode during the electrolysis molten NaCl is -**

   (a) $Cl^- \rightarrow Cl$

   (b) $Cl \rightarrow Cl^-$

   (c) $Na \rightarrow Na^+$

   (d) $Na^+ \rightarrow Na$

6. **In the electrolysis of $CuSO_4$, the reaction:**

   $Cu^{2+} + 2e^- \rightarrow Cu$ occurs at -

   (a) Anode

   (b) Cathode

   (c) In solution

   (d) In salt bridge

7. **The reactions which occur at anode and cathode in an electrochemical cell are respectively:**

   (a) oxidation, reduction

   (b) reduction, oxidation

   (c) reduction, cation exchange

   (d) anion exchange, oxidation

8. **In an electrochemical cell energy changes as:**

   (a) chemical energy into electrical energy

   (b) electrical energy into chemical energy

   (c) potential energy into kinetic energy

   (d) internal energy into electrical energy

ANSWERS

1. (i) oxidation (ii) cathode

   (iii) positive (iv) anode, cathode

   (v) electrical

2. Zn(s)

3. (c)

4. (b)

5. (d)

6. (b)

7. (a)

8. (a)

BIOLOGY

Chapter 1 : How Life Begins?

Chapter 2 : Cell Division - An Introduction

Chapter 3 : Introduction to Heredity and Variation

Chapter 4 : Biomolecules

Chapter 5 : Respiration

Chapter 6 : Plant Anatomy

Chapter 7 : Active and Passive Transport

CHAPTER 1

HOW LIFE BEGINS?

Dear Reader

You see a diverse variety of living forms around you. An estimated 10 to 14 million species are found on earth, of which only 1.2 million have been documented and over 86% are yet to be described. Have you ever wondered how this diverse array of life forms originated? The origin of life is a scientific problem which is not yet fully solved. Plenty of ideas have been given by different scientists regarding origin and evolution of life; in this chapter we will share a few of them.

This information will further help you to understand the first Unit - Diversity of Living Organisms, particularly the basis of classification of animals which you will study in Chapter 4 - The Animal Kingdom in Class XI and Chapter 7 - Evolution (Unit VII) in Class XII.

Origin of Life

The beginning of the story of origin of life is not recent. It dates back to billions of years. The story of origin of life is a conjectured but interesting story. It is believed that earth was formed around 4.5 billion years ago, and life appeared on the earth around 3.5 billion years ago. The atmospheric conditions on the earth at that time were quite different from the present day conditions. The chemicals from which life is thought to have formed on sea coast are:

- Hydrogen(H_2)
- Methane (CH_4),
- Ammonia (NH_3),
- Water (H_2O) and
- Carbon dioxide (CO_2) or carbon mono oxide (CO)

Molecular oxygen (O_2) and ozone (O_3) were either rare or absent.

There were three main stages in origin of life from chemicals:

Stage 1: The origin of monomers or molecules of low molecular weight

Stage 2: Formation of polymers by joining of monomers

Stage 3: Evolution of cells from monomers and polymers

Video link: https://www.youtube.com/watch?v=1prZPo4OCL0

Ontogeny and Phylogeny

Present day organisms such as fishes, amphibians, reptiles, birds and mammals belong to diverse groups. Do you notice any fundamental similarity in them? Let us do an activity to understand the basis of this similarity.

Try to recollect the class to which following animals belong and fill in the blanks:

Fish: _______________. Salamander: _________.

Tortoise: ___________. Chick: ________________.

Hog (Pig): ______________. Calf: _________________.

Rabbit: ________________. Human: __________.

Compare your answers with the answers given below:

Fish: Pisces. Salamander: Amphibia.

Tortoise: Reptilia. Chick: Aves.

Activity:

The figure given below shows two different stages of embryonic development of eight chordate animals. Observe and compare the embryos of these animals at both the stages and then try to identify the animal whose embryo has been drawn at number 1 to 8 out of the names given below: Hog, Calf, Rabbit, Fish, Salamander, Tortoise, Chick and Human

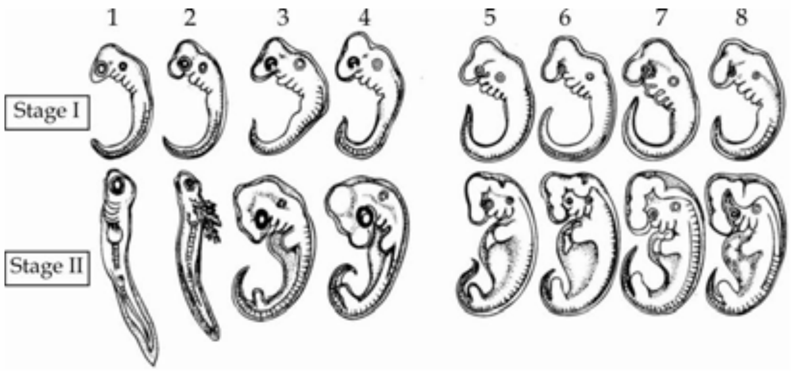


Figure 1: Stages in the embryonic development of chordate animals

Do you notice the similarity in the early embryonic development of all chordates irrespective of the class to which they belong?

What conclusions can you draw from your observations?

Now look at the figure given below to verify your answer.

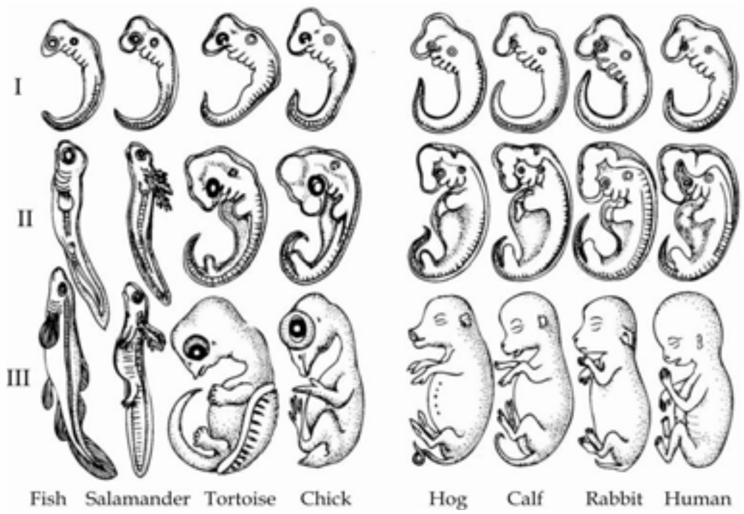


Figure 2: Stages in the embryonic development of chordate animals

Haeckel observed that if early stages of embryonic development of some chordate animals such as fish, salamander, tortoise, chick, rabbit and human are observed they look very similar.

On the basis of his observations, in 1866 Ernst Haeckel proposed that the embryonic development of an individual organism (its ontogeny) follows the same path as the evolutionary history of the species (its phylogeny). It is also called the **biogenetic law** or **recapitulation theory** which states that **ontogeny recapitulates phylogeny**.

This theory states that before the organisms are born, they pass through developmental stages that look like developmental stages of animals of their ancestral species, in roughly the same order in which they were formed during evolution. Thus, similarity in embryonic development of chordates points towards their evolution from a common ancestor.

Embryonic Development in Animals

You have learnt about the origin and evolution of life on earth, now let's talk about how life begins in an organism. All organisms begin their life from a single celled stage. In higher organisms, i.e. both plants and animals, this stage is a zygote which is formed by fertilisation of a male and a female gamete. The zygote then undergoes embryonic development or embryogenesis by cell division and cell differentiation to form a baby organism. You have read in Class X that the process of embryogenesis is different in plants and animals.

Let us now focus on embryogenesis in animals as this will help you to understand the basis for classification of animals.

A. Formation of Morula

In many species of animals particularly the vertebrates, the zygote undergoes a special type of equational cell division called **cleavage.** Each cleavage division produces two daughter cells. At least four initial cell divisions take place resulting in the formation of a dense ball of cells, having sixteen cells. This solid ball of cells appears like mulberry and is hence called **morula** (Latin, *morus*: mulberry).

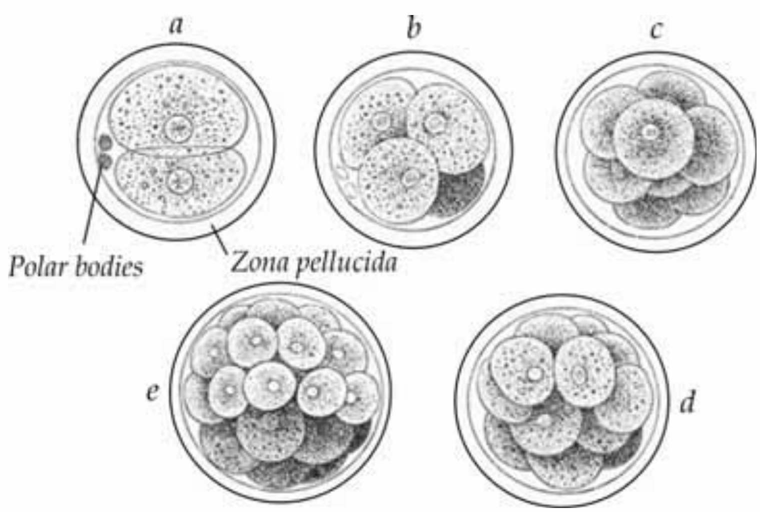


Figure 3: First stages of division of mammalian embryo

a. Two-cell stage. b. Four-cell stage. c. Eight-cell stage. d, e. Morula stage.

For growth and repair, cells divide by an equational cell division called mitosis, whereas, a zygote for embryonic development undergoes a special type of cell division called cleavage. Let us try to find how is mitosis different from cleavage?

In mitosis, the cell division is followed by growth so the daughter cells will finally have the same size as the parent cell. Cleavage, results in increase in the number of cells without increasing the mass as there is no significant growth.

B. Formation of Blastula

After the 6^{th}/7th cleavage has produced 64/128 cells, the embryo is called blastula. The blastula is usually a spherical layer of cells (the blastoderm) surrounding a fluid-filled or yolk-filled cavity (the blastocoel).

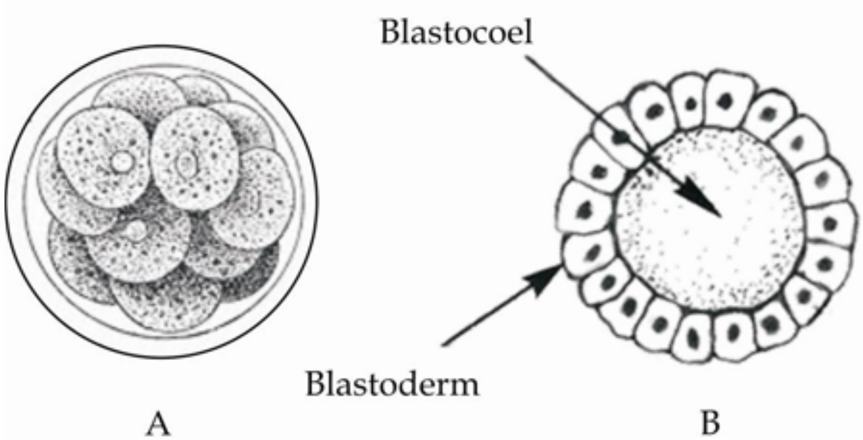


Figure 4: Formation of blastula (B) from morula(A)

In mammals, the cells of the morula are at first closely aggregated, but soon they become arranged into an outer or peripheral layer, the **trophoblast** and an **inner cell mass.** In mammals, blastula at this stage is also called as blastocyst. Each cell of the morula and blastula is called **blastomere**.

Trophoblast does not contribute to the formation of the embryo proper. It plays an important part in the formation of ectoderm, implantation of blastocyst in the endometrium of uterus and development of the placenta. The **inner cell mass** develops into embryo. Fluid collects between the trophoblast and the greater part of the inner cell-mass, and thus the morula is converted into a vesicle, called the **blastocyst.**

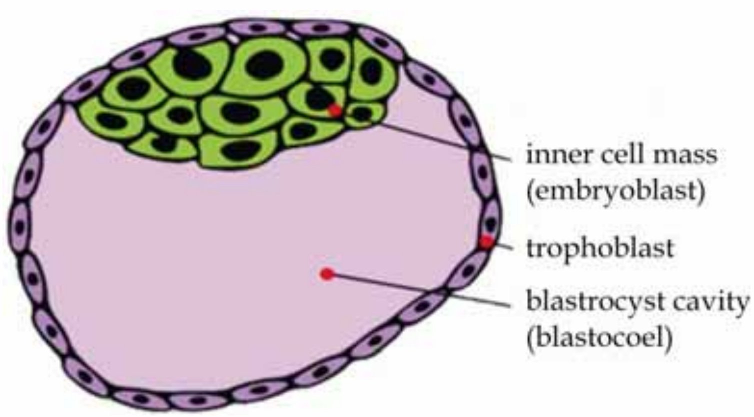


Figure 5: A human blastocyst

C. Formation of Gastrula

The single-layered blastula now undergoes gastrulation. During gastrulation, blastula enlarges and folds inward as its cells migrate to the interior of the blastula. This results in the formation of two or three layers of cells called the germ layers. Each germ layer gives rise to specific tissues and organs in the developing embryo. Depending upon whether the blastula organises into two germ layers or three germ layers, the animals are classified as **diploblastic** (having two germ layers) and **triploblastic** (having three germ layers). The embryo during this stage is called a gastrula.

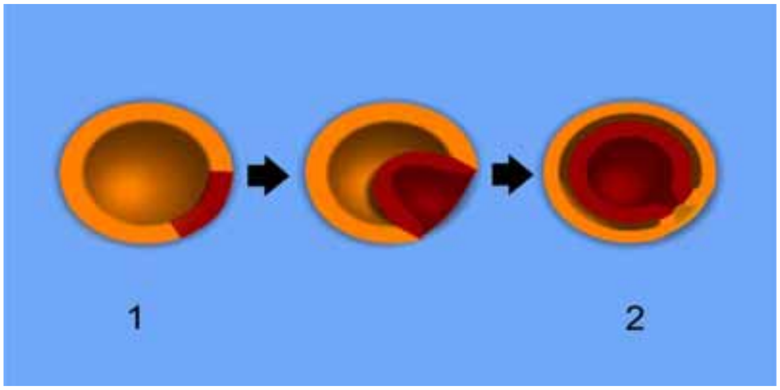


Figure 6: Diagrammatic representation of gastrulation in animals

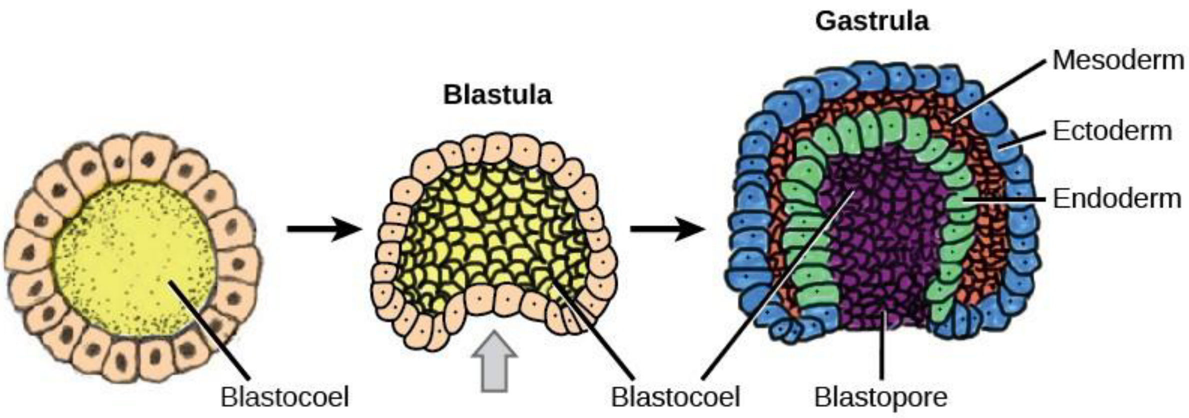


Figure 7: Diagrammatic representation of gastrulation in animals

In triploblastic animals, the germ layers are referred to as: the ectoderm (outer), mesoderm (middle) and endoderm (inner). In diploblastic animals two germ layers-ectoderm and endoderm are present. These have gelatinous non-cellular mesoglea present between ectoderm and endoderm. You will learn more about this concept in Chapter 4: Animal Kingdom in Class XI.

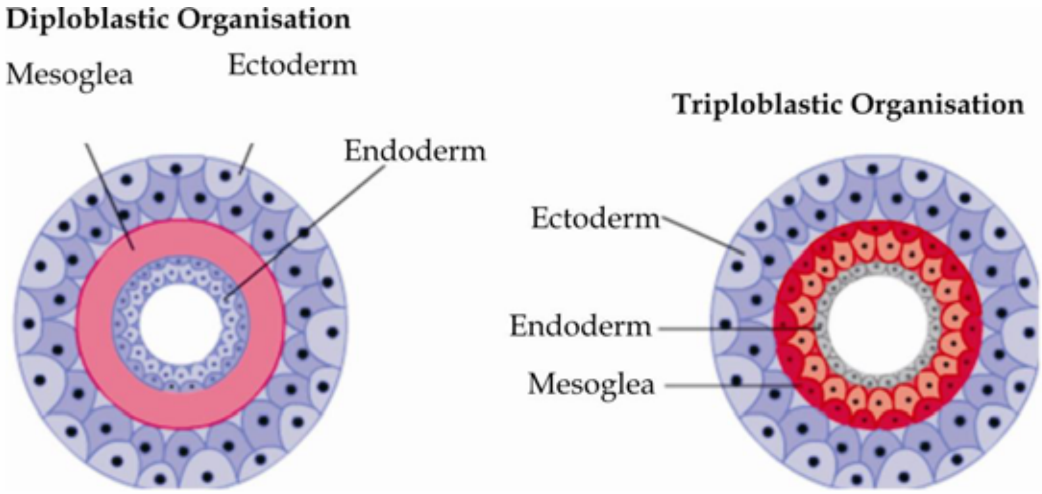


Figure 8: Gastrula showing germinal layers in diploblastic and triploblastic animals

DO IT YOURSELF

1. Try to develop a glossary using the following terms:
   a) Biogenetic law
   b) Cleavage
   c) Morula
   d) Blastula
   e) Gastrula
   f) Germ layers
2. What was the composition of early atmosphere of earth at the time of origin of life?
3. Differentiate between cleavage and mitosis.

4. Differentiate between diploblastic animals and triploblastic animals.

5. Complete the given flow chart that shows various stages of embryonic development in humans:

Zygote → .... → Blastula → ....

REFERENCES

- https://www.google.co.in/webhp?sourceid=chrome-instant&ion=1&espv=2&ie=UTF-8#q=embryonic%20development

2 CHAPTER

CELL DIVISION - AN INTRODUCTION

Dear Reader

In the previous chapter you have read about the diversity in the living world. One of the fundamental feature of all living organisms is **reproduction**. Reproduction as you are aware is the process by which parents can produce young ones of the similar kind. You are also familiar with asexual and sexual reproduction. Try to recall the differences between the two.

> Asexual Reproduction- only single parent is involved and genetically identical copies are produced.
>
> Sexual Reproduction- requires fertilization of male and female gametes; results in variation.

Let us try to recall your early childhood (or of some other child you know). What is the approximate height of a child who is:

- just born
- one-month old
- one-year old
- five-year old
- ten-year old?

Compare it with your present height. I am sure you agree that there is an increase in height at each stage as compared to the previous one.

You might have noticed increase in height of a plant as it grows from a seedling into an adult plant.

Growth is another fundamental feature of all living organisms.

Did you ever try to find out the mechanism of growth in living organisms? Is growth the result of an increase in cell size or an increase in cell number?

One of the changes that is associated with the growth of an organism is increase in the number of cells. How do cells increase in number?

Cells divide to increase their numbers. Try to recall the process of binary fission in *Amoeba* and budding in yeast. You are also aware about the fact that higher plants and animals cannot divide by binary fission and budding.

Cell division is the key phenomenon for both growth and reproduction. Cell division in higher organisms is mainly of two types: mitosis and meiosis. Mitosis is an equational division and meiosis is a reductional division. In this chapter we will study some basic concepts related with these two types of cell division.

Chromosomes

In eukaryotes, the nucleus of all cells contains thread - like structures called chromosomes. Each chromosome is further made up of helically coiled DNA. DNA is the hereditary material in all organisms except some viruses. During cell division, DNA is transmitted from parent cells to the daughter cells, thus DNA is the unit of inheritance. You will learn more about the structure of chromosome in Chapter 8- Cell: The Unit of Life in class XI.

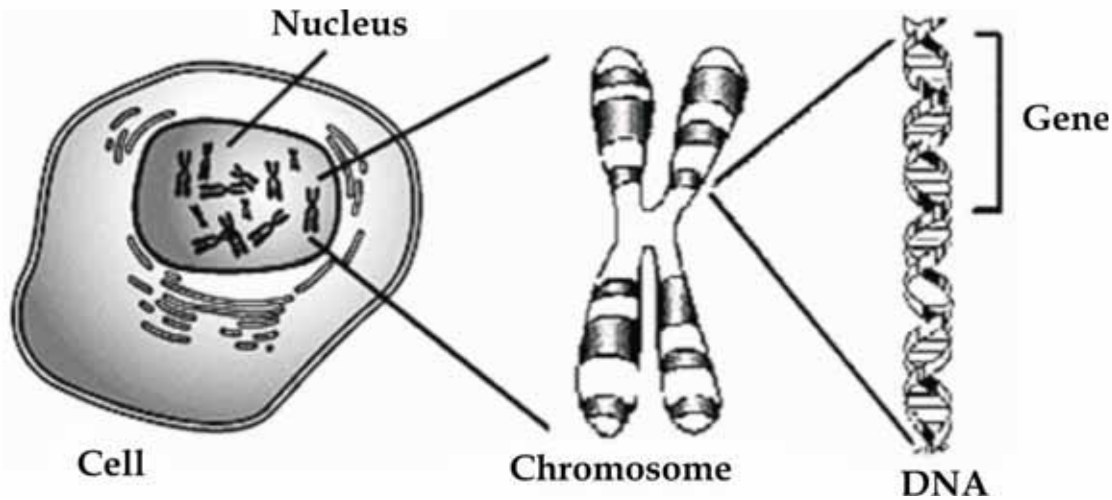


Figure 1: Relationship between Nuclear, Chromosomes, DNA and Genes

Haploid and Diploid Cells

Try to recall the number of chromosomes in human sperm and ovum.

The number of sets of chromosomes in a cell is called its **ploidy**. A cell with one set of chromosomes is called **haploid (n),** whereas a cell with two sets of chromosomes is called **diploid (2n)**. Gametes such as sperm and ovum have only one set of chromosomes, so they are haploid (n). Somatic cells have two sets of chromosomes, so they are diploid (2n).

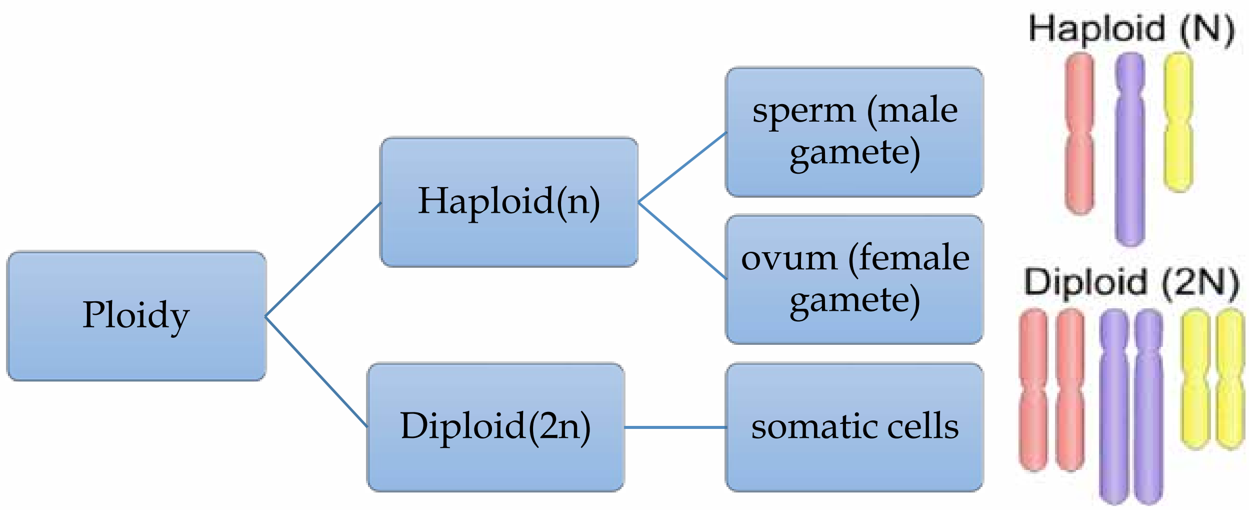


Figure 2: Ploidy of cells

Video link: http://www.britannica.com/science/diploid-phase/images-videos/The-terms-haploid-and-diploid-refer-to-the-number-of/68422

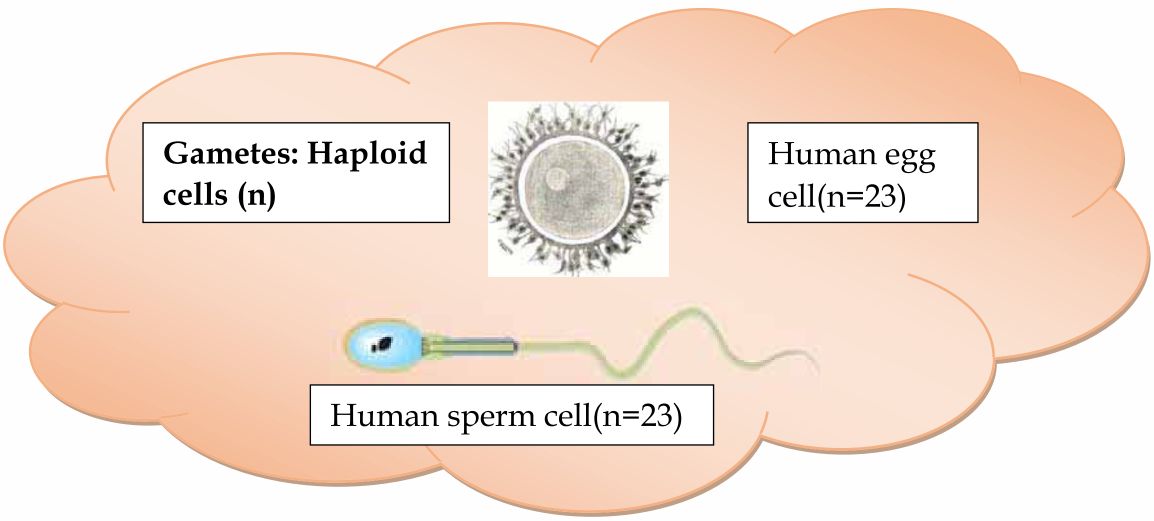
Let us know more about somatic cells and gametes present in higher animals.

Somatic Cells and Reproductive Cells

Somatic cells or vegetal cells make up all tissues and organs of the body such as skin, stomach, intestine, lungs, bones and blood. In a multicellular organism, any cell other than gametes, germ cells or stem cells is a somatic cell. Somatic cells are diploid and divide by mitosis or equational division to form more somatic cells which are also diploid. What is the number of chromosomes in human somatic cells?

Stem cells (2n) are undifferentiated cells normally present in the bone marrow of an adult. These cells divide by mitosis to form more cells of the same type from which some cells can differentiate into specialised cells.

In animals, the egg cell and sperms are the **gametes.** They are also called **reproductive cells,** as they participate in the process of sexual reproduction. Each mature reproductive cell is haploid. Haploid gametes are formed from diploid germ cells by a type of cell division called meiosis or reductional division.



Somatic cells: Diploid (2n)
Chromosome number in somatic cells of human being is
$2n = 2 \times 23 = 46$

Here we have taken the example of animals particularly humans to understand the basics of cell division, the process is similar in plants also. In higher plants, meristematic cells divide by mitosis to form simple and complex permanent tissues. Microspore mother cells in the anther and megaspore mother cell present in the ovule of a flower divide by meiosis to form haploid microspore and megaspore, respectively. Further development inside the microspores and megaspore leads to formation of male & female gametes respectively.

Homologous Chromosomes

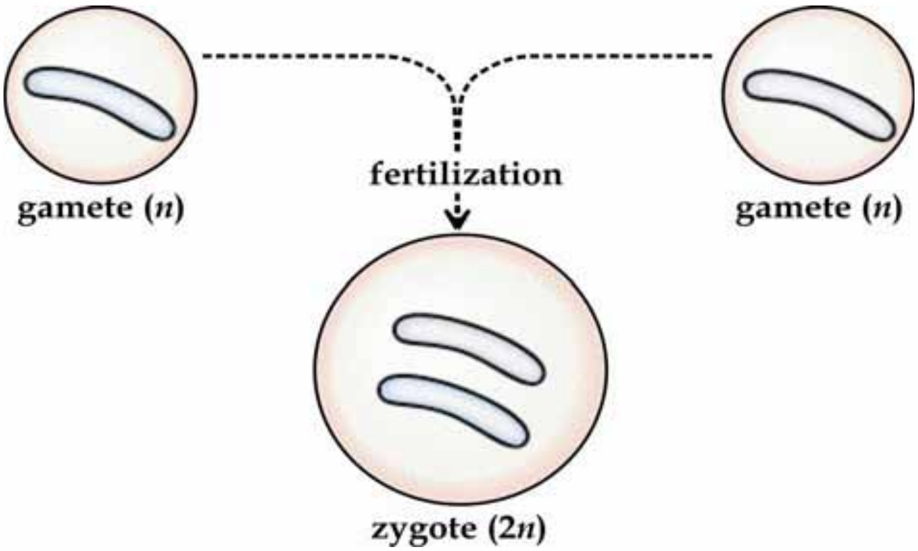


Figure 3: Fertilisation of a haploid male and a female gamete helps to restore diploidy in the zygote

When a haploid sperm (n) fertilises with a haploid ovum(n), it results in the formation of a diploid zygote(2n). The zygote thus receives one set of paternal(n) and one set of maternal chromosomes(n). These chromosomes are similar but not truly identical. Thus, for each paternal chromosome there is a homolog or similar maternal chromosome. Each of such a pair of similar chromosomes is called **homologous chromosome**. Each chromosome of a homologous pair carries same genes in the same order although their expression may vary. You will study in Chapter 10- Cell Cycle and Cell Division in class XI that in germ cells, homologous chromosomes pair up at the time of meiotic cell division.

Zygote undergoes a special type of mitotic cell division resulting in the formation of diploid daughter cells. Try to recall the term used for this type of cell division from chapter 1 - How life begins?

Mitosis and Meiosis

Each human somatic cell contains 23 pairs of chromosomes (2n) whereas each gamete in humans contains only 23 chromosomes (n).

Try to analyse the fact that although "all the cells in an organism are formed from the same zygote but still the somatic cells and gametes vary in the number of the sets of chromosomes possessed by them".

Actually, it happens because of the two different types of cell division taking place in them.

Somatic cells(2n) divide by the process of mitosis. Mitosis generally takes place when plants and animals need to make more cells for growth and repair. Mitosis is an equational division, resulting in the formation of two daughter cells(2n) which are genetically identical to the parent cell. The chromosome number in each daughter cell is equal to the number of chromosomes in the parent cell (2n).The daughter cells grow and behave as parent cell and enter another round of equational cell division and the process continues as the organism continues to grow.

Thus, a human somatic cell (2n = 2 x 23) will divide to produce two daughter cells each having 23 pairs of chromosomes.

Video link for mitosis - https://www.youtube.com/watch?v=DwAFZb8juMQ

Diploid (2n) germ cells (also known as meiocytes)present in the testes of a male and ovary of a female animal divide by the process of meiosis to form haploid male and female gamete respectively (n).For example, in humans, germ cells in the testes of a male and germ cells in ovary of a female, each having 23 pairs of chromosomes will divide by meiosis to form sperm and ovum respectively, each having only 23 chromosomes respectively (n). Thus, meiosis is a reductional division. It has two main phases - Meiosis I and II. Meiosis also introduces variations in the next generation. Reductional division during gametogenesis is essential to restore diploid chromosome number in the next generation.

What will happen if sperm and ovum are also diploid?

Video link – meiosis- https://www.youtube.com/watch?v=nMEyeKQClqI,

https://www.youtube.com/watch?v=D1_-mQS_FZ0

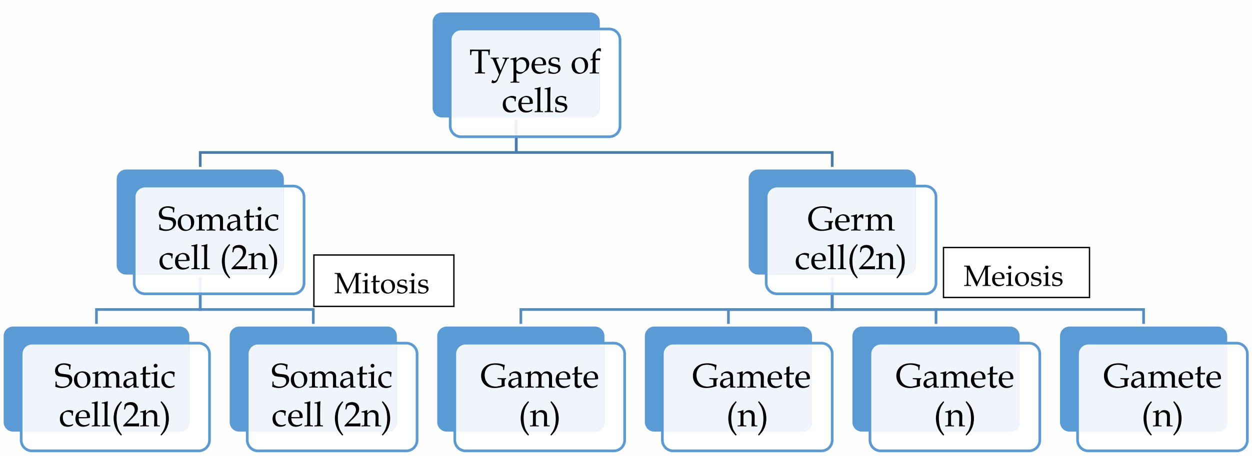
Video link - mitosis vs. meiosis - https://www.youtube.com/watch?v=Ba9LXKH2ztU

Do you know that chromosome number is constant for a species?
The number of chromosomes does not correlate with the apparent complexity of an organism.

Chromosome number (2n) in some common organisms-

Human (*Homo sapiens*) = 46, *Gorilla gorilla*= 48,

Pea (*Pisum sativum*) = 14, Onion (*Allium cepa*) = 16



You will study the details of mitosis and meiosis in Chapter 10 - Cell cycle and Cell Division in Class XI.

DO IT YOURSELF

1. **Identify the ploidy of the following cells:**
   a. Zygote: __________
   b. Intestinal cells: _________
   c. Pollen mother cells: ________
   d. Ovum: _____________

2. **Which type of cell division will take place in the following cells:**
   a. Pollen mother cells: _____________
   b. Skin cells: ______________________
   c. Megaspore mother cells: _____________
   d. Meristematic cells: _____________

3. **Write the chromosome number in the following cells:**

S. No.	Name of organism	Chromosome number in meiocyte	Chromosome number in gamete
A.	Onion (*Allium cepa*)	16	………..
B.	Human being (*Homo sapiens*)	46	………..
C.	Fruitfly (*Drosophila melanogaster*)	………..	4
D.	Rice (*Oryza sativa*)	………..	12

REFERENCES

- http://evolution.about.com/od/Overview/a/Asexual-Vs-Sexual-Reproduction.htm

3 CHAPTER

INTRODUCTION TO HEREDITY AND VARIATION

Dear Reader

You are aware that nucleus of each cell contains thread - like structures called chromosomes. Each **chromosome** is further made up of helically coiled **DNA**. DNA is the hereditary material in higher organisms. You will learn about the structure of DNA in Class XI and XII. (Refer page 208, figure 1)

A **gene** is a small segment of DNA that codes for a functional RNA or a protein product.

A gene is thus the unit of inheritance that determines the transmission of a trait from parents to offspring.

Now you will wonder about the term trait. Genetically determined characteristics in living organisms are called **traits**. Shape of the seeds, colour of flower or height of plant in a pea plant are some examples of traits. Eye colour and skin colour in human beings are some more examples of traits.

A trait can have more than one expression in different organisms. For example, the shape of seeds in pea plant can be round in some plants but wrinkled in others. Similarly, the colour of a flower can be purple or white and the height of a pea plant can be long or short. These different expressions of a trait are controlled by the variant forms of the same gene located on the same position or **locus** on a pair of homologous chromosomes. A variant or alternative form of a gene present on the same locus on a set of homologous chromosomes is called **allele**.

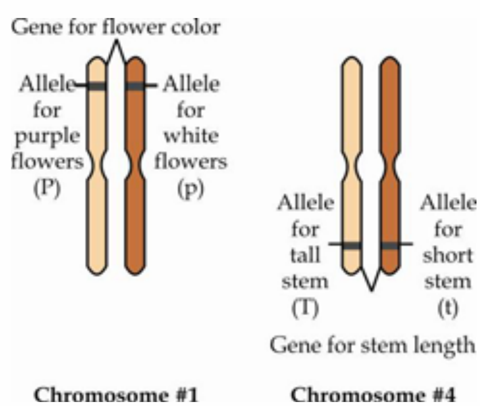


Figure 1: Relationship between alleles and genes

In higher organisms, chromosomes occur in pairs (recall diploid cells). Thus, in all diploid cells, each gene has two alternative expressions or alleles – one on each chromosome in the pair of homologous chromosomes. Since one member of a homologous pair of chromosome is inherited from one parent and the other from the other parent so an organism inherits one allele of each pair from the mother and the other allele from the father.

Both the alleles of a gene on a pair of homologous chromosome can be identical or different. When both the alleles of a gene controlling inheritance of a trait are identical, the organism is called **homozygous**. For example, in a pea plant when each chromosome of a pair of homologous chromosomes has the allele for purple flower (P), such an organism is said to be homozygous (PP) for this trait. Another example is presence of allele for tall height (T) on both the chromosomes of a pair of homologous chromosome, TT.

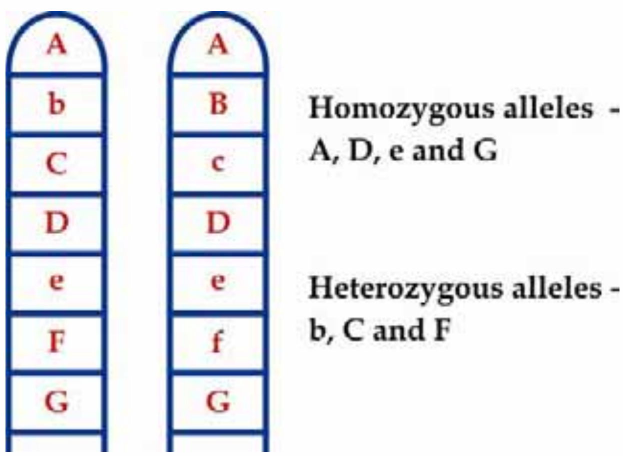


Figure 2: Homozygous and heterozygous alleles on a pair of homologous chromosomes

If two alleles for a gene (controlling the inheritance of a trait) on a pair of homologous chromosomes are different, then the organism is said to be **hybrid** or **heterozygous**. For example, in a pea plant when one chromosome of a pair of homologous chromosome has the allele for purple flower(P) and the other chromosome has the allele for white flower (p), such a plant is called hybrid or heterozygous (Pp). Now the obvious question that comes to the mind is what will be the colour of flowers in a hybrid(Pp) pea plant: purple or white?

Video link - https://www.youtube.com/watch?v=tHwiIVhU3LA

Let us try to recollect Mendel's experiment. When Mendel crossed homozygous pea plants with purple flowers (PP) with homozygous pea plants with white flowers (pp), he was able to get all plants in F1 generation with purple flowers. On doing self-pollination in F1 plants, he was able to obtain purple and white flowers in the ratio of 3:1 respectively. In order to understand this,let us study the genotype and phenotype of organisms in the following cross.

Genotype is the set of genes in the DNA of an organism responsible for a particular trait.

Phenotype is the physical expression or the characteristics of the trait.

In this cross, allele for purple flower has been expressed as P and for white colour of flower has been expressed as p. Both P and p are thus two alleles for the gene of flower colour. The genotype of purple flower in the parent is PP (homozygous). Similarly, the other parent with white flowers (phenotype)has both the identical alleles pp (genotype), so it is homozygous for alleles for white flower colour.

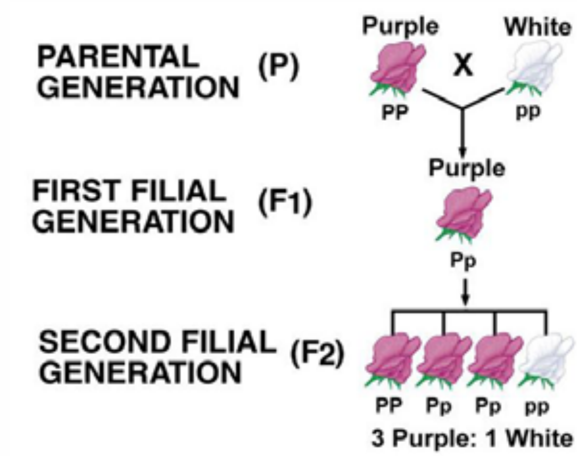


Figure 3: Mendel's monohybrid cross showing inheritance of flower colour in Pea plant, Pisum sativum

The genotype of F1 purple flowers is Pp, so it is hybrid or heterozygous as it has one allele (P) for purple colour whereas the other allele(p) is for white colour. Did you notice that in spite of presence of gene for white colour in hybrid condition it has not been able to express itself?

Similarly, in F2 generation, the plant with homozygous genotype for allele P (PP)as well as the plant with heterozygous genotype (Pp)have same phenotypic expression i.e. purple. Thus, in heterozygous condition (Pp), one allele (here P) masks or hides the expression of the other allele (in this case p).

The allele that is able to express itself in a heterozygous condition is said to be **dominant** and the allele whose expression is masked in heterozygous condition but can express only in homozygous condition is called **recessive**. Here, allele for purple flower colour(P) is dominant over the allele for white flower colour(p) in pea plant.

Video link- Dominant and recessive alleles

- https://www.youtube.com/watch?v=tHwiIVhU3LA
- https://www.youtube.com/watch?v=bjkB5cqXdlY
- https://www.youtube.com/watch?v=Zf2hnFhyJFI

Let us do an activity on the concept of dominant and recessive alleles.

Activity

Study the Figure given below and try to find out the dominant and recessive alleles for all the seven pairs of contrasting traits studied by Mendel in pea plant *Pisum sativum.*

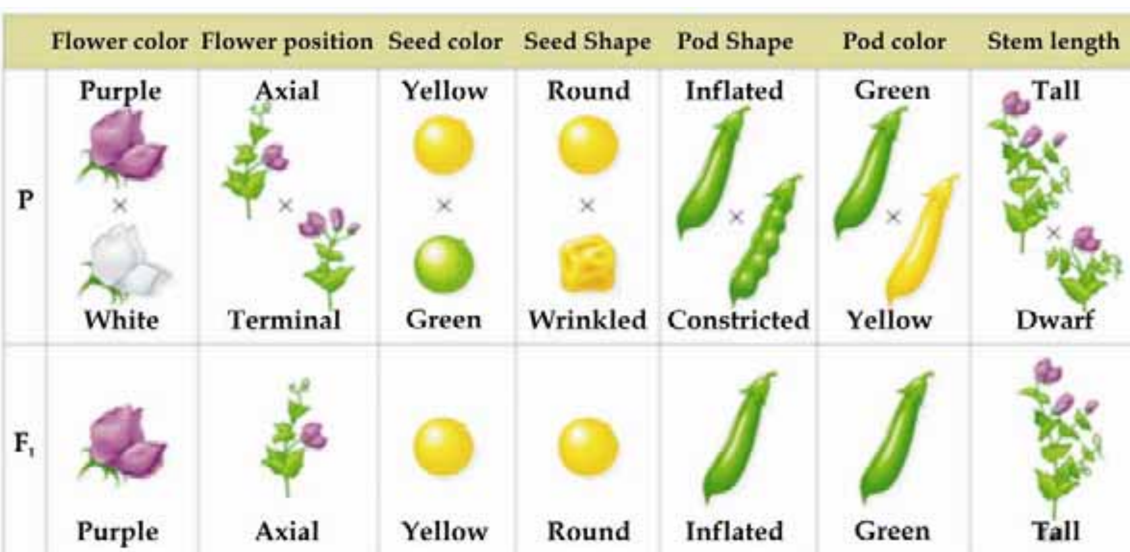


Figure 4: Inheritance of seven pairs of traits studied by Mendel

(Clue: Look at the phenotype in F1 generation)

Here, the example of inheritance of a single trait is being considered, in each experiment. The breeding experiment involving inheritance of only single trait from parents to next generation is called a **mono-hybrid cross**, for example, inheritance of shape of seeds in pea plant.

The breeding experiment involving the inheritance of two traits simultaneously is called a **di-hybrid cross**, for example, inheritance of shape and colour of seeds in a pea plant.

You will learn more about such crosses in Chapter 5- Principles of Inheritance and Variation in Class XII.

DO IT YOURSELF

1. Differentiate between the following terms with the help of examples:
   a Gene and allele
   b Homozygous and heterozygous
   c Dominant and recessive
   d Monohybrid cross and di-hybrid cross
2. A homozygous tall pea plant is crossed with a homozygous short pea plant. What proportion of phenotype in F2 generation can be expected to be tall?
3. What will be the percentage of homozygous and heterozygous offspring in F2 generation in a typical monohybrid cross?
4. Write the genotypic and phenotypic ratio in F2 generation in a typical monohybrid cross showing the inheritance of height in a pea plant.
5. On self- pollination, in purple flowered hybrids of a pea plant, the offspring produced 450 purple and 150 white flowers. Write the genotype and phenotype of all possible offspring in F1 generation.

REFERENCES

- http://www.genomeatlantic.ca/UserFiles/ChromosomeFundamentals.pdf
- http://www.bbc.co.uk/schools/gcsebitesize/science/add_aqa_pre_2011/celldivision/celldivision1.shtml
- http://www.diffen.com/difference/Allele_vs_Gene
- http://www.differencebetween.net/science/difference-between-gene-and-allele/
- https://www.google.co.in/webhp?sourceid=chrome-instant&ion=1&espv=2&ie=UTF-8#q=dominant%20and%20recessive%20alleles
- https://www.google.co.in/webhp?sourceid=chrome-nstant&ion=1&espv=2&ie=UTF-8#q=allele

4 CHAPTER

BIOMOLECULES

Dear Reader

In the previous chapter you have read about DNA present in the chromosomes. It is one of the many organic chemical compounds present in all living organisms. The organic compounds involved in the metabolic processes of living organisms are called **biomolecules or biological molecules**. These are primarily made up of elements carbon, hydrogen, oxygen and nitrogen.

Try to recall life processes such as photosynthesis, digestion and respiration. Prepare a list of biomolecules synthesised during photosynthesis or broken down during digestion and respiration and complete the following table.

Biomolecules present in:	Photosynthesis	Digestion	Respiration
Raw material	CO_2 and H_2O (inorganic molecules)	……………., proteins and ……………….	………………….
Product	……………………...	Glucose, ……………., fatty acids and glycerol, respectively.	CO_2 and H_2O (inorganic molecules)

Thus, you see a variety of biomolecules present in all living organisms which may be small or large. The four major classes of large or macro-biomolecules are carbohydrates, proteins, fats and nucleic acids. There are some other biomolecules too which may be small - primary metabolites, secondary metabolites or natural products about which you will study in Chapter - 9, Biomolecules in class XI. This chapter, will introduce you with some of the major biomolecules.

Four main types of macromolecules

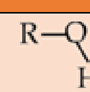
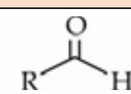
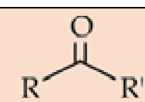
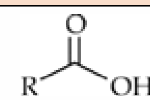
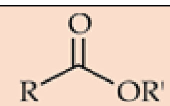
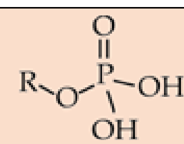
- Macromolecules
  - Carbohydrates
  - Proteins
  - Fats
  - Nucleic acids

Video link – Introduction- https://www.youtube.com/watch?v=YO244P1e9QM

Biomolecules (Details) -- https://www.youtube.com/watch?v=f4Gicf7ONGA

Functional groups

Try to recall the functional groups such as aldehyde, ketone, hydroxyl group and carboxylic acids that you have read in Chapter 4 - Carbon and its compounds, in class X Science. This information will help you to understand the structure and composition of biomolecules.

Chemical class	Functional Group	Formula	Structural Formula
Alcohol	Hydroxyl	ROH	
Aldehyde	Aldehyde	RCHO	
Ketone	Carbonyl	RCOR′	
Carboxylic acid	Carboxyl	RCOOH	
Ester	Ester	RCOOR′	
Amines	Primary amine	RNH_2	
Phosphate	Phosphate	$ROP(=O)(OH_2)$	

Carbohydrates- the source of energy

Try to recall the molecule that is synthesised by the process of photosynthesis in green plants. Relate it with the molecule that is oxidised in cells of all living organisms to provide energy. Both of them are carbohydrates. Carbohydrates are the main source of energy in all living organisms.

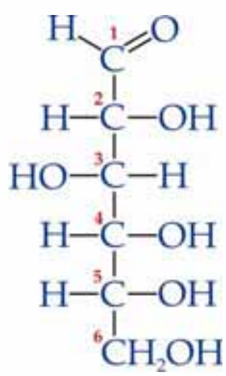
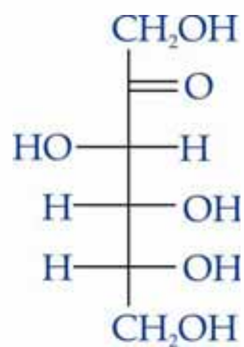
Figure 1: Grain products- rich sources of carbohydrates

Carbohydrates are also known as saccharides. These are aldehyde or ketone derivatives of polyhydroxy alcohols. On the basis of number of sugar molecules present, carbohydrates are classified into three main categories.

Carbohydrates

- Monosaccharides (One sugar molecule, sweet to taste, soluble in water at room temperature. e.g. Glucose, Fructose.)
- Disaccharides (contain two sugar molecules, sweet to taste and soluble in water at room temperature. e.g. Sucrose, Lactose)
- Polysaccharides (contain many sugar molecules, tasteless and insoluble in water at room temperature. e.g. Starch, Glycogen and Cellulose)

Monosaccharides have general formula: $C_nH_{2n}O_n$. Most common monosaccharide in living organisms is glucose. Fructose or fruit sugar is also a monosaccharide. Both have same molecular formula $C_6H_{12}O_6$, but different structural formula. Glucose has an aldehyde group at carbon 1, whereas fructose has ketone as the functional group at carbon 2. On the basis of functional group present, monosaccharides can be aldoses or ketoses.

Aldoses	**Ketoses**
Functional group: Aldehyde	**Functional group: Ketone**
 **D- Glucose** **$C_6H_{12}O_6$**	 **D- Fructose** **$C_6H_{12}O_6$**

Now, let us learn about other types of common carbohydrates present in living organisms.

Do you know that sucrose (common sugar) that you dissolve in the milk and lactose which is naturally present in milk are **disaccharides**? Both have the molecular formula $C_{12}H_{22}O_{11}$. A sucrose molecule is made up of a glucose and a fructose molecule joined together by a glycosidic bond. Find out the constituents of lactose.

Try to recall the nutrient stored in potato, wheat and rice. Starch, a storage **polysaccharide** in plants, is also a carbohydrate. It is formed by large number of glucose molecules joined together by glycosidic bonds. In which form is the food stored in the liver of animals?

You will learn more about carbohydrates in Chapter 9, Biomolecules, in class XI.

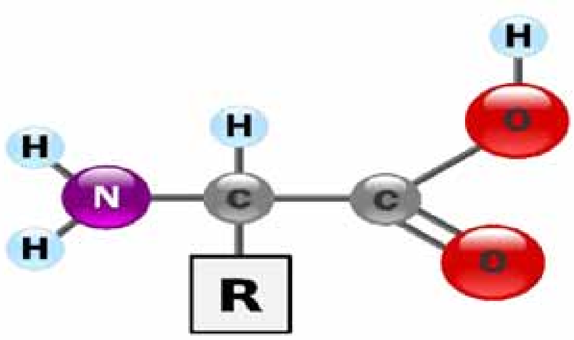
As you are aware that one of the important characteristic of all living organisms is growth. Try to recall the name of biomolecule which helps them in growth and development.

Proteins - the building blocks of life

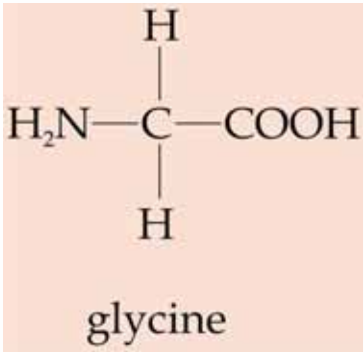
Besides grains, pulses and milk are also important components of our diet. Why? Pulses, milk, egg white and fish are all rich in proteins.

Proteins play a very important role in the body of living organisms. These help in growth, development and repair of damaged cells of the body. All enzymes and some hormones are also proteinaceous in nature. Look at Figure 3 to learn some more functions of proteins.

What is the chemical composition of proteins? Proteins are made of long chains of amino acids. Now, you must be wondering, what are amino acids? Amino acids are organic compounds containing at least one carboxyl (-COOH) and one amino group ($-NH_2$). Only 20 amino acids are present in living organisms. Just like 26 alphabets make infinite number of words in English language, these 20 amino acids by different combinations form infinite number of proteins that are present in living organisms. Glycine is the simplest amino acid. (Figure 2).



(a)



(b)

Figure 2: (a) General formula of amino acids (b) glycine

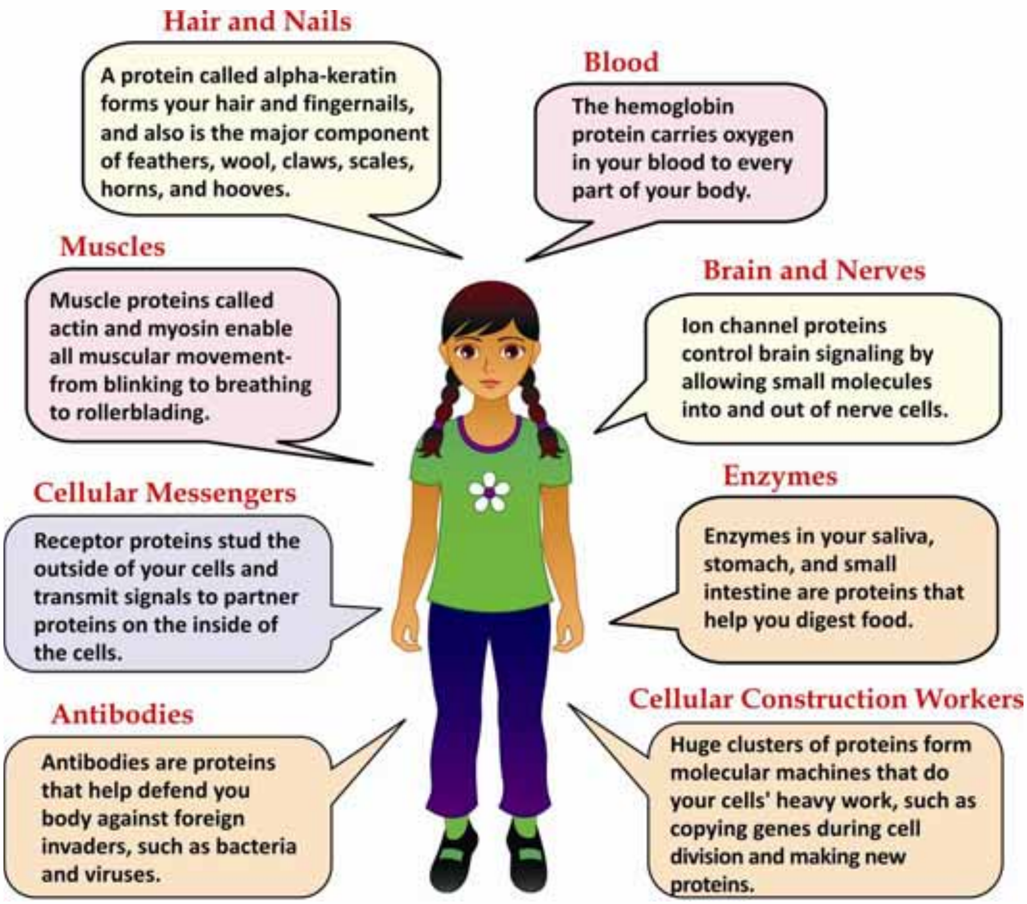


Figure 3: Functions of Proteins

As you will study more about the functions of proteins you will understand why protein deficiency in humans causes stunted growth and other deficiency diseases. It is for this reason that growing children and lactating mothers need to be given protein rich diet.

Lipids

Besides proteins, our diet also includes some oil. Try to recall some sources of edible oils. How is butter different from oils? Generally speaking, oils and fats have similar chemical composition. These are esters of fatty acids and an alcohol called glycerol (commonly known as glycerine). These belong to the category of biomolecules called Lipids. Lipids obtained from plants are called oils and lipids derived from animal sources are generally called fats. Oils are generally liquid and fats are semi-solid or solid at room temperature; this is because of the unsaturated and saturated fatty acids which constitute them, respectively. Some amount of lipids is also essential for a balanced diet.

Lipids are important constituents of cell membranes, along with the proteins. They form bilayers in cell membrane. (Figure 4)

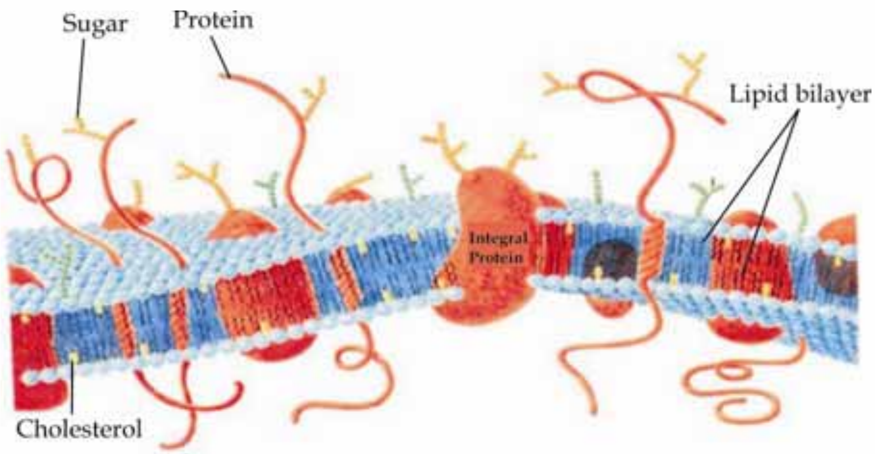


Figure 4: Fluid mosaic model of plasma membrane

Lipids are a group of naturally occurring molecules like fats, waxes and sterols. These are generally insoluble in water and soluble in nonpolar organic solvents like alcohol.

- **Lipids**
  - Fats
    - Fatty acids
    - Glycerol
  - Sterols (having 4 rings of C atoms) e.g. cholesterol
  - Waxes
    - Fatty acids
    - Long chain alcohols

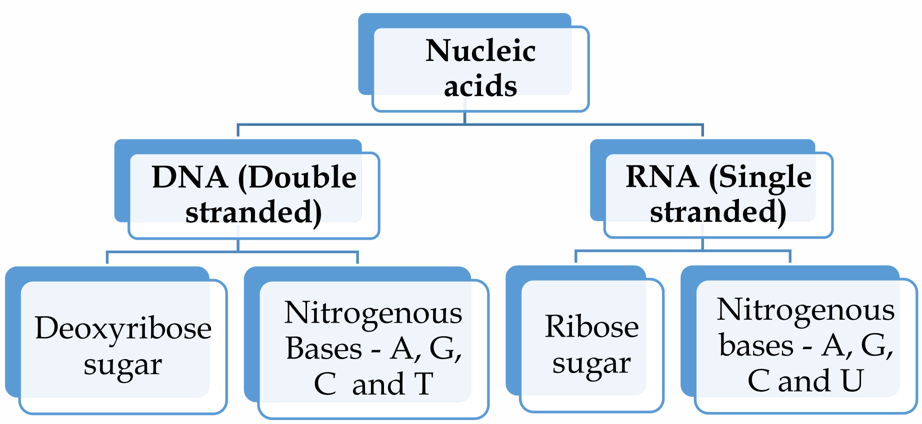
Both fats and waxes are esters of fatty acids and an alcohol. The alcohol in fats is glycerol whereas in waxes it is a long chain alcohol. The bee wax synthesised by honey bee in honey comb mainly consists of esters of fatty acids and various long chain alcohols.

Now let us have some idea about the chemical composition of the molecule that is the hereditary material in all living organisms.

Nucleic acids- the hereditary material

Recall that DNA is the hereditary material in all living organisms except some viruses such as HIV and Tobacco mosaic virus (TMV). RNA is the hereditary material in these viruses.

Deoxyribonucleic acid (DNA) and Ribonucleic acid (RNA) belong to a category of molecules called nucleic acids. Nucleic acids are of two types – DNA and RNA. These are made of long chains of repeating units called nucleotides. Each nucleotide consists of a sugar, a nitrogenous base and phosphate group.



You will learn more about nitrogenous bases and other biomolecules in Chapter 9, Biomolecules, in Class XI. You will also learn in class XII, that DNA undergoes a process called transcription to form RNA, in turn RNA undergoes translation to form proteins. Thus, the type of proteins being formed in any organism are regulated by its DNA.

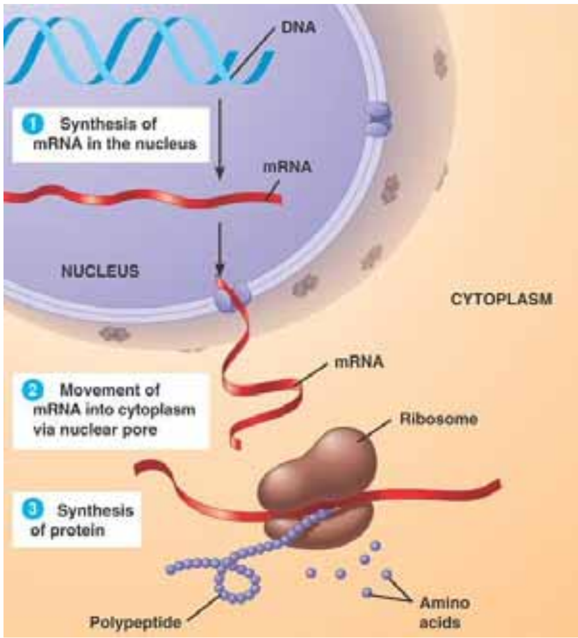


Figure 5: Transcription and translation

DO IT YOURSELF

1. Identify the type of biomolecule present in the following:
   a. Cholesterol …………………………………...……
   b. Hereditary material in humans ……………………
   c. Nutrient stored in pulses ……………………………..
   d. Sugar added for making lemonade ………………..
   e. Bilayer in the cell membrane …………………………
2. Fats and waxes differ with reference to which chemical constituent?
3. The sugar present in DNA is ……………………………………., whereas in RNA is……………………………………….
4. The different functional groups attached to the second carbon in an amino acid are ---------------, ----------------, ------------------ and ------------------.
5. Most common monosaccharide present in living organisms is……………….......

CHAPTER 5

RESPIRATION

Dear Reader

Respiration is a life process that occurs in all living organisms. The energy required by the unicellular or multicellular organism to do their work is obtained by break down and utilisation of food. The energy is obtained by oxidation of glucose or other nutrients. The energy molecule formed in the process is ATP.

Respiration may take place in microbes, plant or animals – either aerobically or anaerobically.

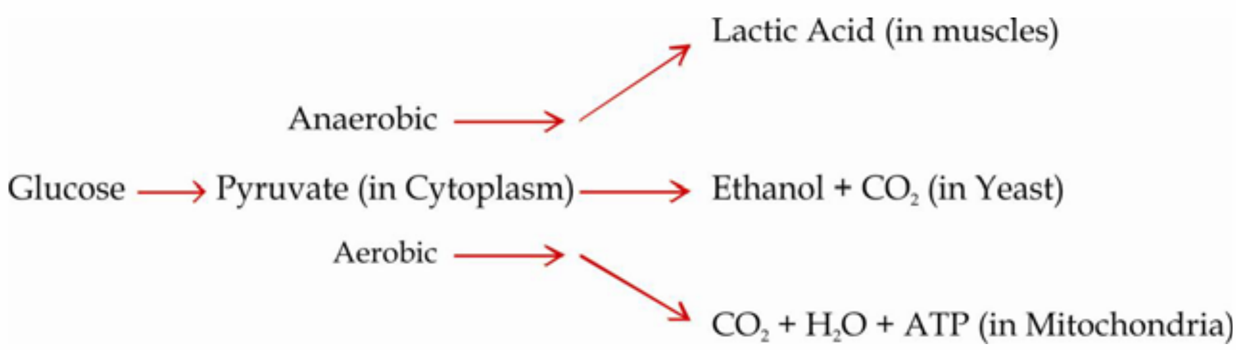


Figure 1: Aerobic and Anaerobic Respiration

Complete the Table given below:

S. No.	Aerobic Respiration	Anaerobic Respiration
1.	38 ATPs	2ATPs
2.		O_2 is not required
3.	Cytoplasm, Mitochondria	
4.		Lactic Acid/ Ethanol, CO_2

Respiration is a mechanical and biochemical process. It involves exchange of gases as well as cellular respiration- in both plants and animals.

There are different kinds of nutrient molecules (respiratory substrates) that can be oxidized to produce energy, e.g. glucose, proteins, fats and so on.

Equation of Respiration

$C_6H_{12}O_6 + 6O_2 \dashrightarrow 6CO_2 + 6H_2O + ATP$

The above equation has been explained in the figure given below:

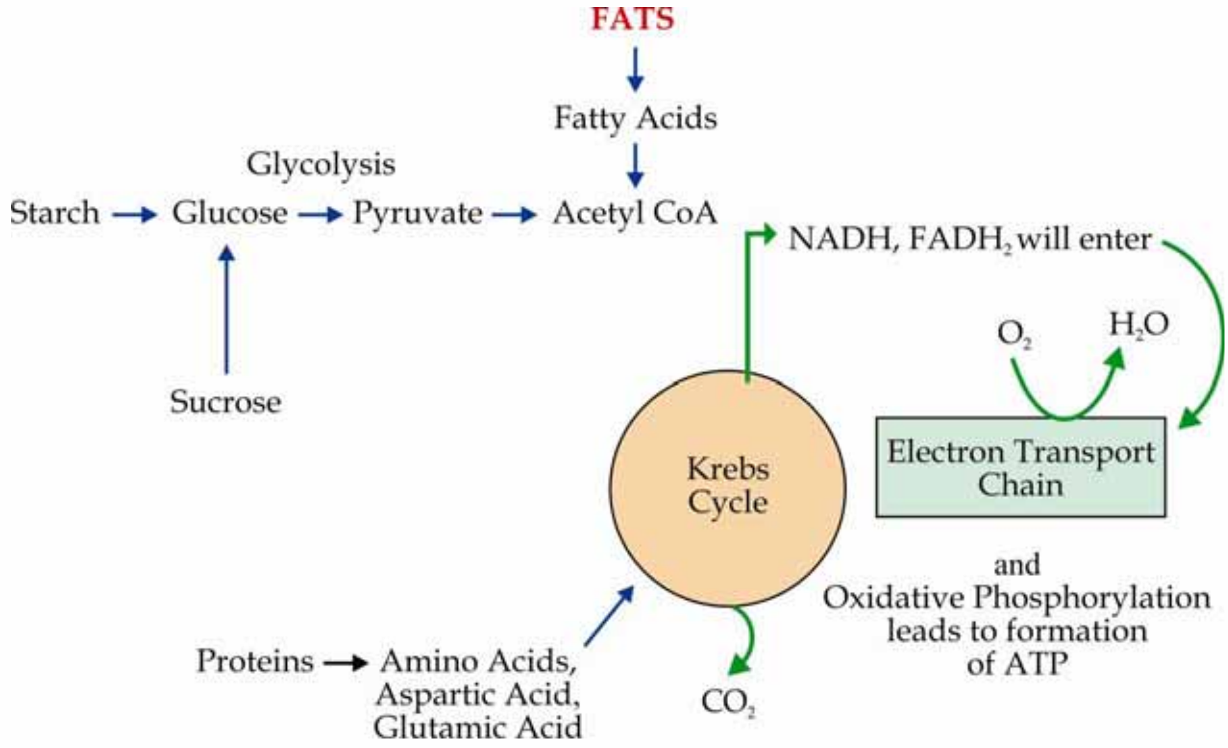


Figure 2: Generation of ATP from different Respiratory Substrates

NADH, $FADH_2$ and CoA are organic molecules

NADH – Nicotinamide Adenine Dinucleotide (Reduced)

$FADH_2$– Flavin Adenine Dinucleotide (Reduced)

CoA – Coenzyme A

Figure 3: Diagrammatic Representation showing Sites and Processes of Complete Oxidation of Glucose

DO IT YOURSELF

1. State true or false:

   a. Glycolysis takes place in Yeast

   b. Glycolysis takes place in our muscles

   c. Lactic Acid can be formed in human body

   d. Aerobic Respiration takes place in the RBCs

2. Make a Venn diagram to show common points between 'Aerobic Respiration in animal cell and Anaerobic Respiration in yeast.'

3. Find out where and how Pyruvate changes into Acetyl CoA .

4. A simple fermentation reaction is the basis of two major industries . Name these.

5. Is Krebs Cycle a common pathway, for metabolism of the three main nutrients? Explain.

6. What is the number of ATPs formed during-

   a) Glycolysis

   b) Lactic Acid fermentation

6 CHAPTER

PLANT ANATOMY

Dear Reader

You know that different systems of classification are based on morphology and anatomy. The artificial system of classification is based on study of morphological features, whereas, the natural system of classification is based on anatomical features as well as other internal characters.

Though it is easy to recognise a given plant by studying its external features, understanding the functional aspects is best facilitated by studying its internal structure.

Cell is the structural and funtional unit of life. Several cells combine to form a tissue. Different tissues combine to form the organs of plants, namely, flower, buds, stem, root, leaf and so on.

Plant anatomy can be studied by cutting sections of different kinds of stems, roots, leaves.

Morphology- Branch of Biology that generally deals with external structure of an organism. (Figure 1)

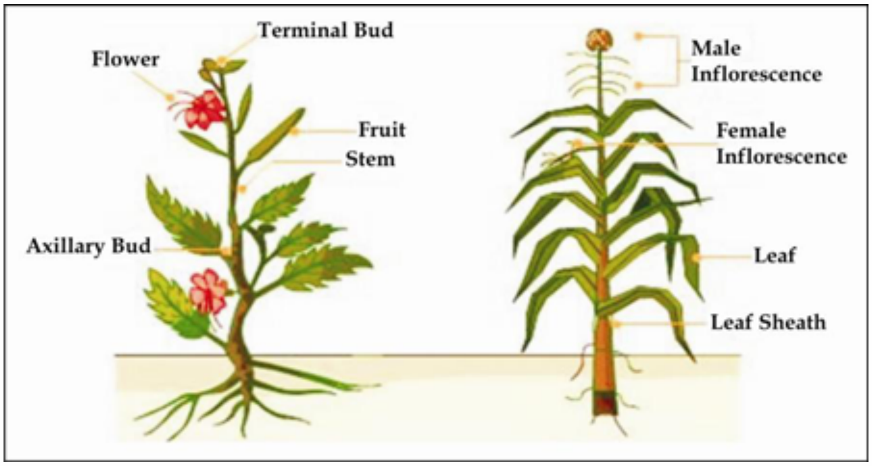


Figure 1: Morphological Features of Dicot and Monocot Plant

Anatomy- Branch of Biology that deals with internal structure of an organism.

Kinds of tissues present in a plant:- Meristematic and Permanent

Meristematic tissues have continuously dividing cells, with prominent nucleus, dense cytoplasm, thin cell wall and no intercellular spaces or vacuoles. Good examples are shot apex and root apex.

Kinds of Permanent Tissues:

Simple and Complex

Simple permanent tissue in flowering plants - parenchyma, collenchyma, sclerenchyma

Complex permanent tissue in flowering plants - xylem, phloem

Xylem comprises of vessels, tracheids, xylem fibres and xylem parenchyma

Phloem comprises of sieve tube elements, companion cells, phloem parenchyma and phloem fibres

Activity 1

- To observe carefully the diagrammatic and detailed structure of dicot stem given below
- To mark the specific tissue against the parts labelled in the diagram showing detailed structure of dicot stem
- To complete Table 2.1

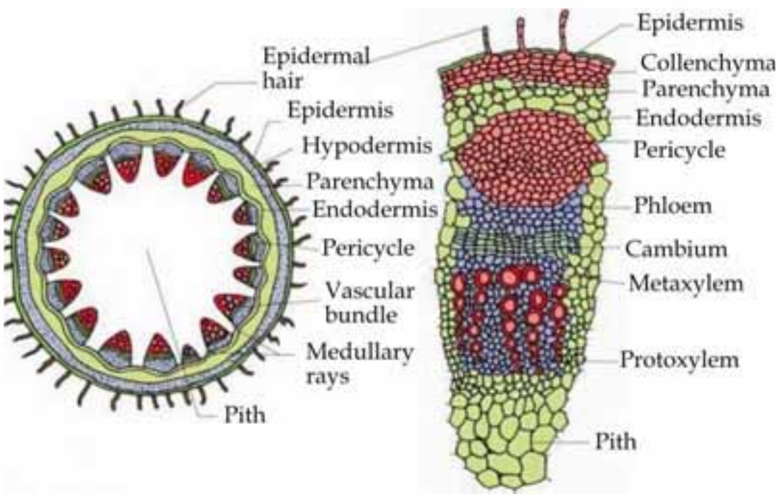


Figure 2: T. S. of dicot stem

TABLE 6.1

Parts of Dicot Stem	Tissue
Epidermis	
Hypodermis	
Endodermis	Parenchyma
Pericycle	Parenchyma
Medullary Rays	Parenchyma
Phloem	Complex Tissue
Cambium	Meristematic Tissue
Xylem	
Pith	

Activity 2

Observe the diagrams of Dicot and Monocot Stem given below and name-

(i) the tissues common to both dicot and monocot stem

(ii) the parts exclusively present in dicot stem

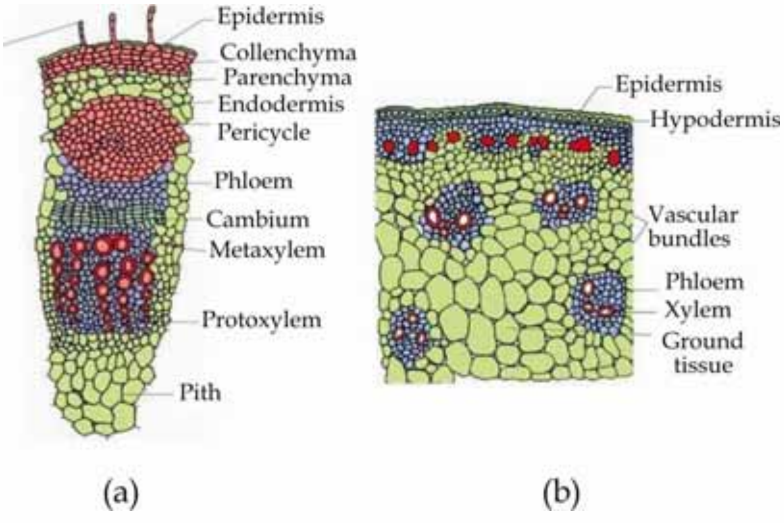


Figure 3: (a) Dicot Stem (b) Monocot Stem

Activity 3

Observe the figures given below and complete the table that follows:

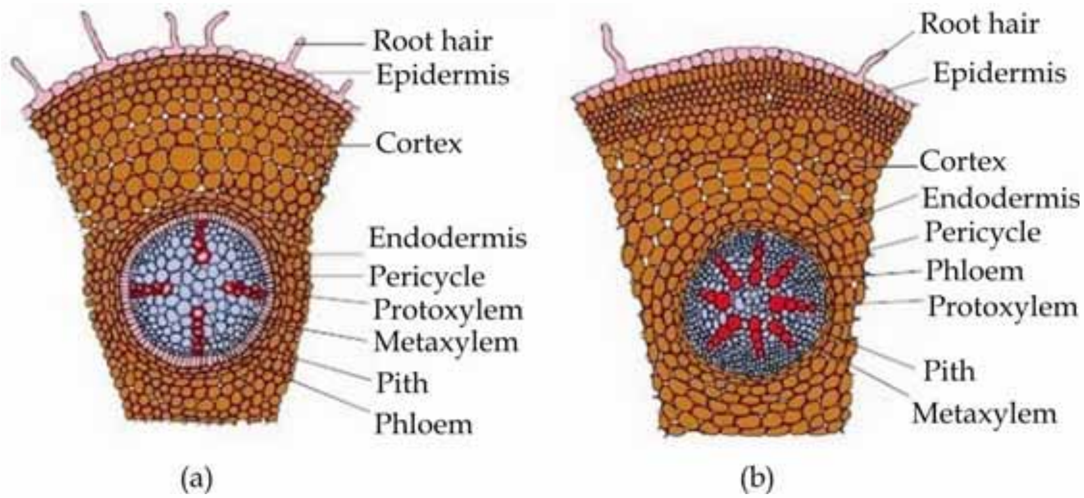


Figure 4: T.S. (a) Dicot root (Primary) : (b) Monocot root

TABLE 2.2

	DICOT STEM (Primary)	MONOCOT STEM	DICOT ROOT (Primary)	MONOCOT ROOT
Epidermis (Protective Layer)	√	√	√	√
Hypodermis				
Cortex				
Ground Tissue				
Endodermis	√	x	√	√
Pericycle				
Bundle Sheath				
Vascular	√	x	x	x

Cambium (divides to form Xylem & Phloem)				
Xylem (conducts water & minerals)				
Phloem (conducts prepared food)	√	√	√	√
Pith	Present	Small/Absent	Small/Absent	Present

DO IT YOURSELF

1. Make Venn diagrams to compare the tissues present in
   (i) dicot and monocot root
   (ii) dicot and monocot stem
2. Metaxylem is later formed xylem, it is bigger in size and with well developed vessels. What is (i) Protoxylem (ii) Metaphloem
3. Why is parenchyma categorized under simple tissues, and xylem under complex tissues ?
4. Draw a diagrammatic sketch of Dicot root.
5. Describe the properties of the cells that constitute vascular cambium.
6. Which tissue forms a barrier between the plant and the environment ? How thick is this layer?

CHAPTER 7

ACTIVE AND PASSIVE TRANSPORT

Dear Reader

An organism or cell is said to be living as it shows different kinds of movements. Movement can be at the molecular level too.

Movement of molecules, circulation of materials and translocation of water and nutrients is governed by different means of transport, namely, active and passive.

Passive Transport

Movement of molecules along the concentration gradient, without expenditure of energy. The process may or may not require membrane proteins.

Passive Transport includes:

- Diffusion
- Facilitated Diffusion
- Osmosis
- Ion Channels

Active Transport

Movement of molecules against the concentration gradient, with expenditure of energy. The process involves membrane proteins.

Active Transport includes:

- Sodium Potassium Pump
- Proton Pump

Passive Transport

Diffusion- the movement of molecules, from a region of high concentration to a region of low concentration e.g. diffusion of gases across the membrane in *Amoeba,* diffusion of gases in plants.

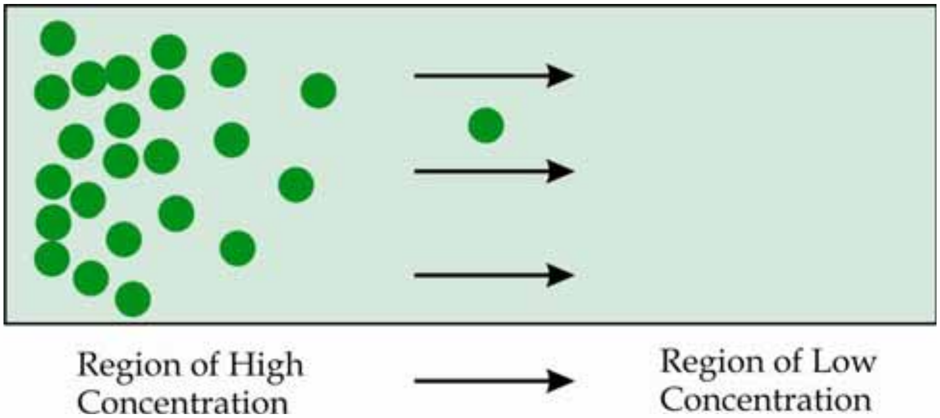


Figure 1: Diffusion

Osmosis- movement of water from a region of its high concentration to a region of its low concentration, across a semi-permeable membrane, e.g. movement of water into the cell, upon placing it in a hypotonic solution (endosmosis).

Semi-permeable Membrane

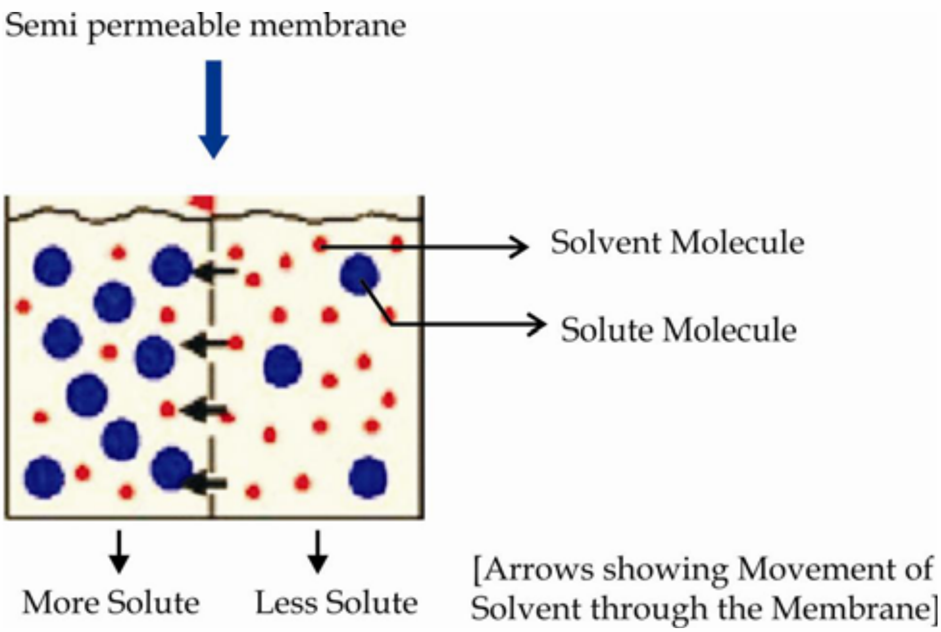


Figure 2: Osmosis

Facilitated Diffusion- the movement of molecules, from a region of its high concentration to a region of its low concentration, through membrane proteins, e,g. movement of fructose across the membrane, via the membrane protein.

Ion Channels- Specific membrane proteins (ion channels) allow movement of particular ions e.g. Na^+ channels in nerve fibre, water channels.

IONS

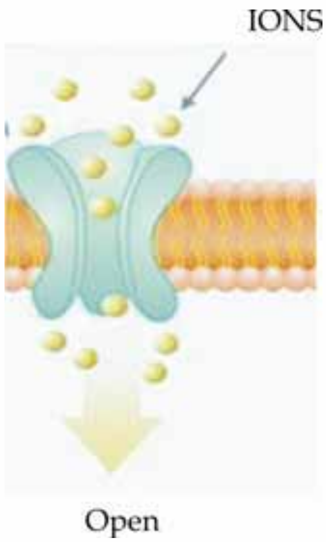


Figure 3: Ion Channel

Active Transport

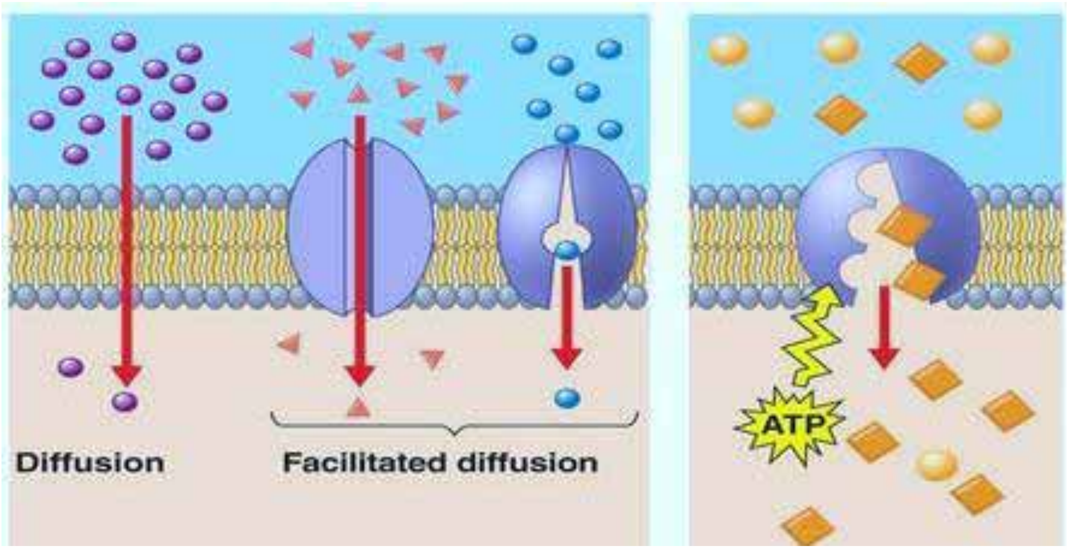
Sodium-Potassium Pump- a membrane protein that plays a role in transporting 3 Na^+ outside and 2 K^+ inside as in axon, while utilizing ATP.

https://sp.yimg.com/xj/th?id=OIP.Mf69dc0c6bdeb6cbb5fb11e057650443co0&pid=15.1&P=0&w=288&h=163

Proton Pump- during photosynthesis, a proton gradient is established due to the passage of protons into the thylakoid (of chloroplast) using proton pump

Absorption of digested products is carried out by both passive and active mechanisms e.g. amino acids are initially absorbed by facilitated diffusion and then, also by active transport when their concentration is low in the small intestine.

Reabsorption by renal tubules of the kidney is either by active or passive mechanisms e.g. glucose and Na^+ are reabsorbed actively, whereas Urea is absorbed passively.



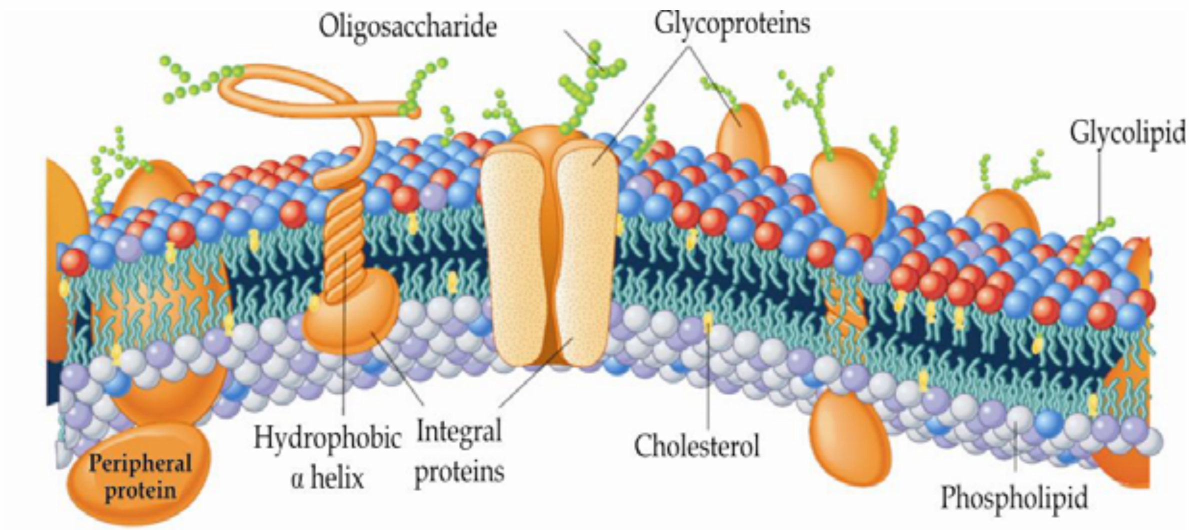
Passive Transport   Active Transport

Figure 4: Absorption and Reabsorption through Active and Passive Transport

DO IT YOURSELF

1. Make a table to compare active and passive transport.
2. (i) What are K^+ channels ?

   (ii) Aquaporins is the term used for denoting ---------------------- . (ion channels/ proton channels/ water channels)
3. You are familiar with the term osmosis. What is reverse osmosis ? Do you have any equipment at home that works on the principle of reverse osmosis ? Name it.
4. Suggest an experiment that would demonstrate exosmosis.
5. In the given diagram, several trans-membrane proteins are shown. Which kinds of transport are possible across these proteins ?


Oligosaccharide
Glycoproteins
Glycolipid
Peripheral protein
Hydrophobic α helix
Integral proteins
Cholesterol
Phospholipid

CBSE

CONTINUOUS AND COMPREHENSIVE EVALUATION

KNOW - AS YOU GROW

केन्द्रीय माध्यमिक शिक्षा बोर्ड
दिल्ली
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