Central Board of Secondary Education

UNDERSTANDING COMPETENCY BASED LEARNING FOR MATHEMATICAL LITERACY

Sri Aurobindo Society
Creating the Next Future

ACER

Foreword

The Government of India unveiled the National Education Policy 2020 with a vision to transform school education for equipping students with 21st century skills. The focus is to move from rote learning to Competency Based Learning. The Central Board of Secondary Education (CBSE), working towards this vision, has taken several steps. Aligned to its commitment of making education meaningful for learners, CBSE entered into an MOU with Sri Aurobindo Society in November, 2019. As a part of this initiative, Sri Aurobindo Society has collaborated with Australian Council for Educational Research (ACER) as its knowledge partner.

Through this collaborative effort, resource books have been prepared for English Reading, Scientific and Mathematical literacies. Each one is aligned to the broad competencies identified by the Organisation for Economic Co-operation and Development (OECD) for the Programe for International Student Assessment (PISA) frameworks. The documents introduce the concept of competency based education, PISA framework for the respective subject and strategies to foster and assess these in the classroom. Besides this, the mapping of the NCERT Learning Outcomes to the content areas (as defined in CBSE-TERM document) and the PISA competencies have been included. Practical examples for teaching and assessing across various content area and competencies are further inbuilt to enrich the teacher's repertoire for classroom transactions.

This document, *Understanding Competency Based Learning for Mathematical Literacy* prepared by Sri Aurobindo Society in collaboration with ACER for CBSE is a rich resource that can be used by schools and teachers for supporting students to acquire mathematical literacy and problem solving competencies. The resource samples the teaching pedagogy aligned with educational content that would benefit practioners in the field.

About CBSE

The Central Board of Secondary Education (CBSE) is a national Board under the Ministry of Education, Govt. of India. The Board has more than 25,000 schools affiliated to it in India and overseas, in 25 countries. These include the Kendriya Vidyalayas, the Jawahar Navodaya Vidyalayas, Central Tibetian Schools, Schools run/aided by the State Governments and Private Schools. The Board's main objective is to encourage quality education focussed on holistic development of learners. It motivates schools and teachers to adopt learner centric enquiry based pedagogies and use innovative methods to achieve academic excellence. The Board is committed to providing a stress-free learning environment that will develop competent, confident and enterprising citizens who will promote harmony and peace in the world.

About SAS

Sri Aurobindo Society (SAS) is an international, spiritual, and cultural, not-for-profit NGO. SAS has been recognised by the Government of India as a Charitable Organisation, a Research Institute and an Institute of National Importance. Sri Aurobindo Society has more than 300 centres and branches across the country, with its head office in Puducherry. SAS is setting up models, centers of excellence and training institutions that are Sustainable, Scalable and Replicable in the country. Rupantar, a dynamic and multidimensional program of Sri Aurobindo Society is dedicated to transform education in schools by harnessing the power of existing resources.

About ACER

Australian Council for Educational Research (ACER) is a leading and pioneer international organization working in the field of Competency Based Learning and has led a consortium of international organizations for the implementation of the PISA (Programme for International Students Assessment) survey in 2000, 2003, 2006, 2009 and 2012.

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ABBREVIATIONS AND ACRONYMS

ACER	Australian Council for Educational Research
CBSE	Central Board of Secondary Education
IEA	International Energy Agency
MHRD	Ministry of Human Resource and Development
NCTM	National Council of Teachers of Mathematics
NEP	National Educational Policy
NEQMAP	The Network on Education Quality Monitoring in the Asia-Pacific
NRC	National Research Council
OECD	Organization for Economic Cooperation and Development
PIAAC	Programme for the International Assessment of Adult Competencies
PIRLS	Progress in International Reading Literacy Study
PISA	Programme for International Student Assessment
TIMSS	Trends in International Mathematics and Science Study
UNESCO	United Nations Educational, Scientific and Cultural Organization

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EXECUTIVE SUMMARY

Fuelled by advances in technologies, especially education technologies, competency-based education has gained popularity from the education community across the world. It emphasises personalised learning of students following a well-designed learning pathway at their own pace. It is more important for students to demonstrate proficiency by applying their knowledge and skills to solve real-world problems than to spend specific amounts of time receiving instruction in a course. This learning framework is an effort to introduce competency-based learning to classroom teaching and learning in India in order to prepare Indian students for the rapidly changing future.

This framework spans the entire spectrum of the teaching learning process. Initially, mathematics domain is defined and the competencies that comprise the domain are presented. Then, practical techniques of teaching learning and methods of implementation are discussed. For a competency-based learning system to be effective, an effective system of assessment that is integral to the teaching-learning process must be adopted. Traditional approaches to assessment, with their emphasis on marks or grades, are not effective at describing student proficiency levels substantively. Therefore, a new approach to assessment that makes use of learning progressions that contain descriptions of proficiency levels at various stages is recommended in the framework. Finally, a learning progression is reviewed in detail to help teachers understand and implement the process.

A chapter-wise summary is provided below.

Chapter 1 introduces competency-based education in the 21st century. Fuelled by advances in technologies, especially education technologies, competency-based education has gained popularity from the education community across the world. It emphasises personalised learning of students following a well-designed learning pathway at their own pace. It is more important for students to demonstrate proficiency by applying their knowledge and skills to solve real-world problems than to spend specific amounts of time receiving instruction in a course. As a best practice example of competency-based assessment, the chapter introduces the Programme for International Student Assessment (PISA) of OECD. PISA is different from other large-scale learning assessments in that it measures knowledge and skills of 15-year olds and monitors trends in student performances in three main cognitive domains every three years. India's decision to participate in PISA 2021 is a big step forward toward promoting competency-based learning and to shift the focus of education from rote learning in order to prepare the new generation of learners for the rapidly changing future.

Chapter 2 analyses the importance of mathematical literacy and what the characteristics of a mathematical literate person are in society. It outlines how the objectives of mathematics education will change with the implementation of the National Educational Policy 2020 in India. It introduces the definition of mathematical literacy used by the PISA 2021 mathematical literacy framework. Mathematical literacy will be the major domain of PISA 2021 cycle. The definition emphasises how mathematical literacy will help individuals to appreciate the role that mathematics plays in the world and to make well-founded judgments and decisions needed by constructive, engaged, and reflective 21st-century citizens. The chapter concludes with a close examination of the status of mathematics instruction in Indian classrooms and the usefulness of mathematical literacy in initiating a paradigm shift in mathematics pedagogy in Indian classrooms.

After a brief overview of the PISA mathematical literacy in Chapter 2.

Chapter 3 tries to give more insight into the competencies assessed by PISA mathematics assessment. PISA assesses the competencies in the context of problem-solving. It considers formulating situations mathematically, employing mathematical concepts, facts, procedures and reasoning and interpreting, applying and evaluating mathematical outcomes, in terms of real-life situations as fundamental capabilities of mathematical literacy. Chapter 3 emphasises that mathematical reasoning is the core metacognitive skill behind all mathematical competencies.

Chapter 4 constitutes the core of the document. This chapter lays a solid groundwork for a competency-based teaching paradigm in mathematics classrooms. It emphasises the importance of the activation of metacognitive competencies to enhance mathematics capabilities of students. The chapter suggests research based on practical strategies to enhance reasoning and problem-solving capabilities in students.

Chapter 5 and 6 are dedicated to competency-based assessments. Chapter 5 gives further insight into mathematical proficiency with the help of PISA proficiency levels. The chapter suggests using learning progressions to elicit evidence of mathematical proficiencies in mathematics education. It associates learning progression in problem-solving with the PISA proficiency scale. The chapter also includes some examples of released PISA items to illustrate the association between PISA competencies and the proficiency scale.

Chapter 6 considers the limitations of using advanced statistical methods in classroom assessment and suggests that teachers use learning rubrics to collect information on students' mathematical proficiency. The chapter also includes guidelines on developing such rubrics.

Chapter 7 provides the linkage between NCERT textbooks from grade 6 to 10 and PISA competencies at a granular level. Subtopics in each chapter is aligned with learning outcomes, measurable learning outcomes, learning objectives and PISA competencies. Along with it, some exemplar teaching strategies are mentioned for a few topics.

Recommendation- the framework also provides recommendations for competency-based learning and assessment. For competency-based learning, it is recommended that teaching strategies for activation of metacognitive skills are used. Metacognitive strategies enable learners to think about their thinking, resulting in better control of the learning. They help students' master mathematics by connecting mathematical facts, procedures, and ideas.

One of the key recommendations of the framework is to use learning progressions to bridge the gap between student knowledge and curriculum expectations. Learning progressions map curriculum with development in students' mathematics learnings. They help in integrating learning outcomes, curriculum and proficiencies based assessments.

The framework recommends the use of learning rubrics in classroom instructions. Rubrics can provide feedback on instructional practices implemented in the classroom. It is suggested that teachers develop rubrics based on learning outcomes rather than specific tasks as it will help them to synchronise teaching-learning and assessment with the desired outcomes.

1. INTRODUCTION

We are living in a highly technology-driven world in which both promises and challenges coexist. It is estimated that around 50 percent of current work activities are technically automatable (McKinsey Global Institute, 2017). There have been discussions on changes in employment types due to automation. Experts expect that workforce transformation is inevitable and will take place in two ways (World Economic Forum, 2018):

- Large-scale decline in some roles as tasks within roles become automated or redundant
- Large-scale growth in new products and services – and associated new tasks and jobs – generated by the adoption of new technologies and other socio-economic developments

Most of the new tasks and jobs in demand due to these changes are different from traditional ones, including Social Media Specialists, Innovation Managers, AI and Machine Learning Specialists, Human-Machine Interaction Designers, and so on. This rapid transformation in workforce calls for prompt changes in the way we teach students and help ready them for the future. This is more important for India whose youth population is the world's largest (United Nations Population Fund, 2014).

In this challenging world, it is inevitable that our expectation from education and the way we prepare students for the future change. The international education community is moving toward preparing the new generation of learners for the era of technology when critical thinking, practical application of principles, problem solving, and creativity are more crucial than rote learning. In this regard, it is a promising move that India recognizes quality education for all and has introduced competency-based learning in its National Education Policy 2020 (MHRD, 2020).

Competency-based learning, unlike curriculum-based learning, emphasizes a student-centred approach to education. It is heavily driven by experience and discovery rather than lecture and memorization. The focus of competency-based learning is on what students understand and how they apply what they learn in real-life situations rather than what they remember. This learning framework is an effort to introduce competency-based learning to classroom teaching and learning in India in order to prepare Indian students for the rapidly changing future.

The Australian Council for Educational Research (ACER) has been a long-time proponent of competency-based education. The framework will be a meaningful contribution of ACER (India), partnering with Sri Aurobindo Society, to Indian education to promote competency-based teaching, learning, and assessment in classrooms.

1.1. Competency-based teaching and learning

Competency-based education initially gained popularity among researchers in the 1970s spurred by Benjamin Bloom's research (Steele et al., 2014). His approach to education focuses on attention to individual students, student-centred instruction and flexible pacing in teaching and learning. Bloom (1956) contributed to the body of educational research by defining the hierarchy of cognitive processes, which places learner's ability to apply, analyse, evaluate, and synthesise, higher than that of remembering and comprehending. However, this approach was not an entirely new idea. John Dewey's experiential learning emphasised that students constructed their own learning through real-world experience and engagement (Steele et al., 2014). Competency-based learning has regained its popularity recently with information technology supporting learners for self-motivated and personalised learning.

Traditional teacher-centric approach to education assumes that an educated man is equivalent to a competent man. In this approach, students are expected to acquire the knowledge and skills by

receiving lectures and learning the curriculum. On the contrary, competency-based education recognises that real learning does not take place automatically by taking lectures and memorising a body of facts. In a competency-based system, it is believed that mastery in learning comes from application of knowledge and skills in real-world situations. In this sense, competency-based learning is similar to experiential learning, proficiency learning, project-based learning, and flipped learning.

Competency is a general term that describes the desired knowledge, skills, and behaviours of a student graduating from a programme (or completing a course), whereas a learning outcome is more granular and specific. Competencies are commonly defined as the applied skills and knowledge that enable people to successfully perform in professional, educational, and other life contexts.

Competency-based education is in favour of reducing disparities in students' performance outcomes and bridging achievement gaps. It allows students to build academic foundations by mastering the current concepts or skills before they move on to new ones. In order to achieve success in competency-based teaching and learning, it requires unique features (Steele et al., 2014):

- Defined progression toward mastery – this means that learning progressions are clearly defined and students see a clear learning pathway for improvement.
- Flexible pacing – this means that each student progresses through content at his or her own pace, and potentially at different paces in different content areas. It is more important to set a student's starting point at the current level of competency in learning progression rather than his or her placement in an age-based grade level.
- Personalised learning – it is important to recognise that learners learn differently, have different skill levels, and encounter different opportunities to learn outside of school. As students mature, competency-based education provides opportunities to make more choices regarding how to acquire skills and knowledge, as well as how to provide evidence. This personalisation can lead to increased engagement.
- Anytime/anywhere learning – this means that access to out-of-school learning opportunities. This is catalysed by the advances in educational technology, enabling instructional system that incorporates both online and teacher-led instruction.
- Credit for mastery – students demonstrate proficiency and earn credit by applying knowledge and skills. Academic credit is based on evidence of learning, not only completion of a certain number of days or hours of instruction in a course.

1.2. Competency-based assessment

Assessment plays a critical role in competency-based learning. The fundamental purpose of assessment in education is to establish and understand where learners are in an aspect of their learning at the time of assessment. This usually means establishing what they know, understand, and can do. The traditional approach of curriculum-based assessment focuses on whether the skills have been attained and the content has been learned. On the other hand, competency-based assessment focuses on whether the learner can apply the skills and concepts that he or she has acquired and learned. In competency-based assessment, it is important to build a system of capturing the student's competencies such as knowledge, skills and attitudes in an area and their application to real-world problems.

Competency-based assessment focuses on students' mastery of skills, feedback to students, and improvement in performance rather than test scores or grades. Thus, an assessment is carried

out as a process in order to collect evidence about the performance and knowledge in a competency standard. A few assessment methods are emphasised in competency-based education, including performance-based, portfolio-based, and rubric-based evaluation (Steele et al., 2014). Evaluation against well-designed rubrics provides students with an indication of how well they performed on a task. At the same time, it guides students on what they need to do to improve and master the skill required. Therefore, development of well-defined learning outcomes and objectives aligned with a pathway demonstrating progression in student learning is a must in implementing a competency-based assessment. This allows students to expect what they should know and be able to do at each level of proficiency.

Many international learning assessments have established well-specified learning standards along with the hierarchy of proficiencies or benchmarks in core test domains, while developing tests designed to measure students' mastery of related knowledge and skills. Numerous governments around the world have adopted or have been setting quantifiable standards for successful learning. It is, however, more important for schools and teachers to wisely adopt the learning standards for classroom teaching to evaluate students' mastery by applying competency-based education pedagogy.

Competency-based assessment has gained recognition by the international education community. The Organisation for Economic Co-operation and Development (OECD) has long been supporting the competency-based approach. This is well reflected in its large scale educational assessments such as the Programme for International Student Assessment and the Programme for the International Assessment of Adult Competencies (PIAAC). As is expressed in the NEQMAP Strategy 2020-2024, the United Nations Educational, Scientific and Cultural Organization (UNESCO) adopts and recommends competency-based approach for educational assessments (NEQMAP, 2020).

1.3. PISA – an international competency-based assessment

Large-scale educational assessments are becoming more and more popular in the world, including OECD's Programme for International Student Assessment (PISA), IEA's Trends in International Mathematics and Science Study (TIMSS), Progress in International Reading Literacy Study (PIRLS), and India's National Achievement Survey. International large-scale assessments allow comparison of the strengths and weaknesses of different education systems and learning from other education systems around the world. The findings of the scientific educational assessments are an important indicator of the quality of the education systems in participating countries and economies. The assessments enable governments to effectively monitor their education systems and design policy interventions for improvement and reform.

Among the various international educational assessments, PISA is unique in that it assesses knowledge and skills of 15-year-olds and monitors trends in student performances in three cognitive domains. PISA is designed to gauge what is important for citizens to know and be able to do. In order to achieve this, PISA takes the approach of competency-based assessment rather than a curriculum-based one. It measures students' capacity to apply knowledge and skills, and to analyse, reason and communicate effectively as they identify, interpret and solve problems in real-life situations (OECD, 2019). This approach reflects the fact that modern economies reward individuals not for what they know, but for what can do with what they know. Thus, the focus of PISA is on students' mastery and readiness to apply knowledge gained from school to real-life situations.

In addition, PISA tests 15-year-olds in a country or economy rather than students at a specific academic grade level unlike other international assessments. This is to assess future

preparedness of 15-year-old youths who are nearing the end of their compulsory education in most countries and economies. On the contrary, most of the other large-scale learning assessments assess students studying at particular grade level(s) in order to measure the effectiveness of their schooling. For example, TIMSS and PIRLS survey students in Grade 4 and Grade 8.

Lastly, PISA is the world's largest international learning assessment thanks to the high degree of validity and reliability gained from the above mentioned points. In PISA 2018, a total of 79 countries and economies participated, including all 37 OECD countries and 42 partner countries and economies. A total of 87 countries and economies have signed up to participate in PISA 2021, which is delayed to 2022 due to the COVID-19 pandemic crisis, in order to gain important data on learning outcomes of their youths.

The advanced techniques applied to PISA to make the tests valid and reliable are based on rigorous scientific research from various disciplines including psychology, statistics, psychometrics, and education. OECD maintains strict implementation rules for all participating countries and economies in order to ensure comparability of the assessment results of the countries and economies. Thus, PISA results help governments around the world understand the strengths and weaknesses of their education system and to use them for evidence-based policy making.

It is promising that India recognises high quality education for all by adopting the National Educational Policy 2020 and has re-joined the next cycle of PISA 2021 to monitor its education system. Promotion of competency-based learning and assessment through PISA will facilitate improvement of the quality of education in India and preparation of the new generation of learners for the era of technology when practical application of principles, problem solving, and creativity are more critical than rote learning. This learning framework aims to help teachers understand competency-based education and apply its various pedagogical strategies to implement competency-based teaching in the classroom.

1.4. Prerequisites for competency-based learning

It is to be noted that competency-based education is for all levels of education – from early childhood education to higher and adult education. The key belief in a competency-based system is that students learn more effectively when they progress at their own pace through a series of personalized learning experiences. Therefore, not all students are expected to achieve the same level of proficiency at the same time. In this sense, it is immensely important for teachers to ensure and support students build foundational skills from an early stage of their education, since students who are weak in foundational skills have great difficulty in coping with the next level of proficiency in learning.

It is praiseworthy that the Government of India recognized the importance of early childhood education as the foundation of learning in the National Education Policy 2020. Competency-based education can be applied to earlier stages of education as effectively as to later stages of education in order to ensure students' development of foundational skills. It is not recommended to suddenly introduce the competency-based learning approach to students at any one stage of education. It is ideal to adopt the competency-based education for all levels of education.

This learning framework is developed to target teaching students in secondary education, from Class 6 to Class 10. The pedagogical strategies mentioned and examples shared in the framework are to help secondary education teachers and students. These may not be directly related to building foundational skills. Therefore, the framework is developed, assuming that students at

the secondary level must have acquired knowledge and skills expected at early learning and primary education levels.

In the following chapters, the details of competency-based learning will be discussed. In chapter 2, the subject domain is defined. It introduces the concept of the literacy approach and domain literacy and highlights importance of the domain. Chapter 3 is the introduction to domain competencies based on the PISA competencies. There will be a discussion on various pedagogical strategies in chapter 4 to promote competency-based learning in classroom teaching. Chapter 5 and chapter 6 are about how to assess the competencies. The key components of competency-based assessment, including the proficiency levels defined by PISA, are introduced in Chapter 5. In chapter 6, the discussion on how to link assessment to teaching is elaborated. Chapter 7 provides mapping of the NCERT textbooks with the PISA competencies.

2. DEFINING MATHEMATICAL LITERACY

2.1. Introduction to mathematical literacy

The world is changing rapidly with significant advancements in scientific and technological knowledge. Owing to digitalisation, enormous amount of data is collected and analysed for decision-making in almost every field of life and the increasing application of machine learning and artificial intelligence has reduced the need for human involvement. These advancements are also reducing the demand for unskilled jobs worldwide. Basic knowledge of the subject domain may not be sufficient for the kind of mathematical acumen required for future jobs. With machines taking over the tasks that were once performed by humans, we are faced with the unique challenge of taking on more dynamic and challenging roles requiring newer skills. Education systems across the world are faced with the hefty responsibility of preparing the future workforce to meet the evolving needs of the job market and daily life.

According to the National Education Policy 2020 (NEP), one of the fundamental principles that will guide both the education system and individual institutions within India is fostering creativity and critical thinking to encourage logical decision-making and innovation. The emphasis will be on real-world application of mathematical thinking from the beginning of school education. Thus, mathematics education must move towards acquisition of higher-order cognitive skills including the ability to think critically and solve problems (MHRD, 2020).

By the end of school education, a learner is expected to develop skills that help understand and address real-life situations, including in professional contexts. Mathematical skills learned during school years can serve as a critical tool for young people. Several studies have shown that mathematical skills learned in school years have a profound impact on young people's attitude and mind-set towards problem-solving in real-life situations (Asari, 2014). In a competency-based system, learners are expected to demonstrate their knowledge and skills. . According to the PISA mathematics framework, it is reasonable to ask 15-year-old students, “What is important for citizens to know and be able to do in situations that involve mathematics?” (OECD, 2018, 6). In other words, PISA seeks to assess the mathematical literacy of students.

2.2. Definition of mathematical literacy

In the most basic terms, mathematical literacy can be defined as real-world conversation ingrained in mathematical thinking. According to the Victoria Board of Education, Australia, mathematical literacy has two components; development of mathematical understanding and communication of mathematical reasoning (Victoria State Government, 2019).

A person can be considered mathematically literate when he/she can

- use mathematical facts and algorithms in both familiar and unfamiliar situations
- select and combine mathematical strategies for problem-solving
- communicate real-world scenario and problem-solving procedures mathematically
- reflect on the techniques used in problem-solving
- effectively use the latest technology in math problem solving
- appreciate the role of mathematics in real-life

(The Knowledge Network for Innovations in Learning and Teaching, 2012)

These skills can be broadly categorised under three major mathematical literacy domains, i.e., Reading, Interpreting, and Expressing. When a person acquires these three modes of thinking and communicating about mathematics, we can say that he/she is mathematically literate.

PISA 2021 mathematics framework defines mathematical literacy as

Mathematical literacy is an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged, and reflective 21st century citizens. (OECD, 2018, 7)

The definition focuses on core areas around content, competencies, applications and implications. The table 2.1 shows the four components of literacy.

Table 2.1: Four components of PISA mathematical literacy

Content (mathematical content knowledge)	Competencies	Applications	Implications
• Concepts • Procedures • Facts • Tools	• Reason mathematically (reasoning) • Formulate, employ, interpret (problem solving) • Describe, explain and predict phenomena (communication – mathematical and literary) • Well-founded judgements and decisions (logical reasoning to take a particular approach)	• Variety of real-world contexts (broader than the domain of mathematics)	• Know the role that mathematics plays in world (appreciation of mathematics) • Constructive, engaged and reflective 21st century citizen (social obligations)

Source 2-1: OECD (2018). PISA Mathematics Framework. OECD (2018). https://pisa2021-maths.oecd.org/files/PISA%202021%20Mathematics%20Framework%20Draft.pdf

For analysing the PISA definition we need to focus on three underlying aspects:

- Mathematical reasoning and problem solving
- Mathematical content learned during the school years
- Contexts coupled with selected 21st century skills

The first step in understanding the PISA definition can be to decode the skills required to solve problems. Problem-solving can be considered a systematic approach of comprehending a given problem, selecting and implementing appropriate strategies, and evaluating the answer and implemented strategies (Allen & Graden, 2002).

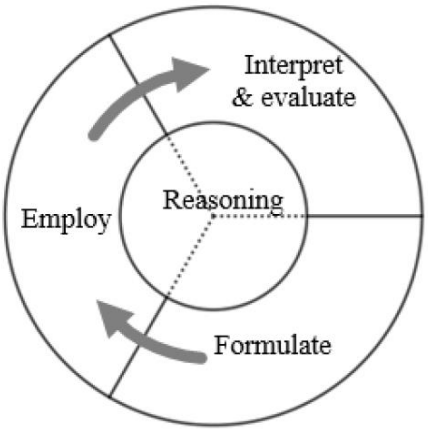
Mathematical reasoning is required to perform the above processes. Reasoning is an integral part of:

- comprehending a real-world scenario into a mathematical model (formulate)

- selecting and applying the previously acquired mathematical knowledge of facts and procedures (employ)
- interpreting/evaluating mathematical solution for its validity in real-life (interpret)

Thus, mathematical reasoning can be considered as a fundamental skill required to be mathematically literate. PISA's definition of mathematical literacy considers reasoning as a central component of mathematical competencies (OECD, 2018).

Figure 2.1: Mathematical literacy: the relationship between mathematical reasoning and the problem solving (modelling cycle)



Source 2-2: OECD (2018). PISA Mathematics Framework. OECD (2018). https://pisa2021-maths.oecd.org/files/PISA%202021%20Mathematics%20Framework%20Draft.pdf

Another fundamental aspect of mathematics literacy is language. The reasoning involved in mathematics should include mathematics terminology, mathematical facts, tools, logic, and modelling to describe, explain, and predict phenomena. A mathematically literate person can express themselves in oral, written, and other visual forms and can understand and appreciate the work of others.

2.3. Importance of the domain

Education systems worldwide are adopting teaching and learning of 21st century competencies, including critical thinking, collaboration, creativity, and problem-solving. These competencies cannot be understood in isolation. They require more emphasis on content and a broad liberal arts curriculum (Ananiadou & Claro, 2009). Mathematical literacy is one of the fundamental competencies needed to learn 21st century skills (Julie, Sanjaya, & Anggoro, 2017). The new generation of learners may find it challenging to apply 21st century skills in their professional lives if they lack foundational knowledge.

To function effectively at work and as citizens, factual knowledge and higher-order skills are required (Dede, 2007; Kalantzis & Cope, 2008). The PISA definition of mathematical literacy emphasises two critical considerations of the NEP; first, it clearly states that mathematical literacy is a multidisciplinary approach taking place in real-world contexts. Secondly, the definition emphasises on conceptual understanding of mathematics so that it can be used in problem-solving (OECD, 2021). Also, the PISA definition helps in laying the foundation of critical thinking to encourage logical decision-making. Mathematical literacy helps individuals know the role that mathematics plays in the world and make well-founded judgments.

Mathematical literacy is one of the challenges in mathematics education (Murtiyasa, 2016). Despite the various programs run by the Government of India, results are not as desired. According to the National Achievement Survey 2017-18 results for mathematics in CBSE affiliated schools for Class X level, only 48% students were able to correctly attempt the questions involving "carrying out or using a procedure through executing or implementing" (CBSE, n.d). Most Indian students are unable to perform higher-order thinking skills like critical and creative thinking in mathematics outlined in NEP 2020. As the Indian education system transitions from the teaching of disciplinary facts and procedures to the literacy model of pedagogy, the role of mathematical literacy has increased manifold.

Another important goal highlighted in NEP 2020 is the creation of life-long learners. Mathematical literacy ensures that a person is prepared for his role as a subject who studies independently for his or her whole life (Trilling & Fadel, 2009). Knowledge gained during the school years should not be limiting; instead, it should open a person's mind to experiment and have courage backed by expertise to solve unseen problems.

It is evident from the discussion that mathematical literacy enables a person to live a fulfilling life and become a creative and contributing citizen. Thus, every effort should be made to ensure that young people are mathematically literate, and that as a community, we foster the efforts towards this.

3. COMPETENCIES IN MATHEMATICS

3.1. Introduction to PISA competencies

The foundation of a knowledge-based economy lies in its human capital - skills, competencies, and expertise (Barman & Konwar, 2011). Two main aims of reforms in mathematics education are to replace superficial learning of mathematics with learning that imparts in-depth understanding and knowledge, and to equip young people with new skills and competencies in order to contribute actively to the new social and economic norms. The new skills and competencies required may be termed as 21st century skills and competencies (Ananiadou & Claro, 2009). Domain related skills and competencies acquired during compulsory school years are the core of these new skills and competencies.

Competency is more critical than knowledge or skills in a domain. It can meet complex demands by using psychosocial resources (Rychen & Salganik, 2003). Skills and knowledge give the ability to perform tasks and solve problems. On the other hand, competencies are related to applying skills and knowledge effectively in broader educational, professional, and personal life (Cedefop, 2008). Thus a competency may be considered as an adequate ability to use cognitive elements (factual knowledge) along with functional (application skills) and behavioural elements – for example, social and organizational skills, and ethical values. Niss (2003) defined the volume of competencies (metaphorically) as the 'product' of the degree of domain knowledge, the radius of action, and the technical level. According to him, mathematical competencies are activated in situations where there is a potential for mathematical challenges (Niss, 2003).

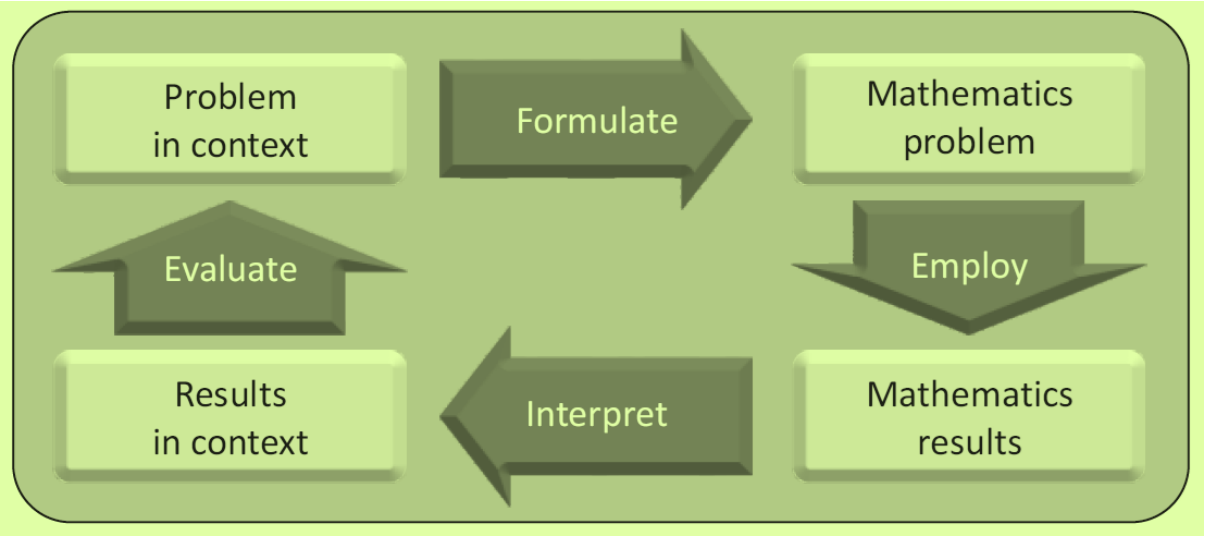
Real-life problems are much more complicated than the problems young students encounter while studying mathematics in schools. After completing school, they are expected to acquire the competency to solve real-life problems which can broadly be defined as "ability of an individual to find meaningful solutions to solve problems using effective and timely strategies" (Karabacak, Nalbant, & Topçuoglu, 2015, 174).

PISA assesses the competencies of 15-year-old students who are nearing the completion of compulsory education in most countries and economies. It defines mathematical competencies in the context of problem-solving. The three mathematics competencies assessed in PISA 2021 are:

- Formulating situations mathematically
- Employing mathematical concepts, facts, procedures and reasoning
- Interpreting, applying and evaluating mathematical outcomes.

In general, problem-solving is a systematic process involving five stages, namely, identifying a problem, defining and representing the problem, exploring possible strategies, acting on strategies, and looking back and evaluating the effect of chosen strategies. The first three stages fall under the formulation category, where a solver identifies, defines, and mathematises a given situation. The exploration and action stages are for 'employing.' A solver reckons his mathematical concepts, facts, and procedures to find a solution. The process of looking back and evaluating the results of the chosen strategy can be categorised as the interpretation of the solution in a real-life context. In the process, a learner verifies the results in terms of feasibility and appropriateness.

Figure 3.1: Relation between different components of problem-solving



Source 3-1: CBSE (n.d.). Teachers' Handbook Volume 1: Mathematical Literacy. http://cbseacademic.nic.in/web_material/Manuals/Teachers_handbook-Maths.pdf

3.2. Formulating situations mathematically

When encountered with a problem situation, a successful solver should have the ability to identify and organise information efficiently. It presupposes that the solver has understood the problem and clearly identified the mathematics involved in it. He then efficiently translates the information, using mathematics terminology, facts, and relations, to which mathematical strategies are applicable. The process involves translating a problem from a real-world setting to the mathematics domain with the help of mathematical structures, representations, and specificity. While formulating a situation mathematically, all constraints and assumptions of the problem should be reasoned. Table 3.1 illustrates activities which comes under formulating situations mathematically.

Table 3.1: Activities involving formulating situations mathematically

Activities involving formulating situations mathematically
• Selecting an appropriate model from a list; • Identifying the mathematical aspects of a problem situated in a real-world context and identifying the significant variables; • Recognising mathematical structure (including regularities, relationships, and patterns) in problems or situations; • Simplifying a situation or problem in order to make it amenable to mathematical analysis (for example by decomposing); • Identifying constraints and assumptions behind any mathematical modelling and simplifications gleaned from the context; • Representing a situation mathematically, using appropriate variables, symbols, diagrams, and standard models; • Representing a problem in a different way, including organising it according to mathematical concepts and making appropriate assumptions; • Understanding and explaining the relationships between the context-specific language of a problem and the symbolic and formal language needed to represent it mathematically; • Translating a problem into mathematical language or a representation; • Recognising aspects of a problem that correspond with known problems or mathematical concepts, facts or procedures; • Choosing among an array of and employing the most effective computing tool to portray a mathematical relationship inherent in a contextualised problem; and • Creating an ordered series of (step-by-step) instructions for solving problems.

Source 3-2: OECD (2018). PISA Mathematics Framework. OECD (2018). https://pisa2021-maths.oecd.org/files/PISA%202021%20Mathematics%20Framework%20Draft.pdf

3.3. Employing mathematical concepts, facts, procedures and reasoning

Once a problem is translated from the real world to the mathematics world using mathematical representations, symbols, and relations, the next task is to identify and develop the most appropriate strategy and find a solution. This process involves integrating existing mathematical knowledge (mathematical concepts, facts, procedures, and reasoning) to find the given problem's appropriate mathematical solution. The process of employing strategies requires mastery over mathematical procedures, including arithmetic computations, solving algebraic equations, inductive and deductive reasoning, drawing geometrical figures, graphs, interpreting, and predicting for data. Table 3.2 illustrates activities which comes under employing mathematical concepts, facts, procedures and reasoning.

Table 3.2: Activities involving the process of employing mathematical concepts, facts, procedures and reasoning

Activities involving the process of employing mathematical concepts, facts, procedures and reasoning
• Performing a simple calculation; • Drawing a simple conclusion; • Selecting an appropriate strategy from a list; • Devising and implementing strategies for finding mathematical solutions;

- Using mathematical tools, including technology, to help find exact or approximate solutions;
- Applying mathematical facts, rules, algorithms, and structures when finding solutions;
- Manipulating numbers, graphical and statistical data and information, algebraic expressions and equations, and geometric representations;
- Making mathematical diagrams, graphs, simulations, and constructions and extracting mathematical information from them;
- Using and switching between different representations in the process of finding solutions;
- Making generalisations and conjectures based on the results of applying mathematical procedures to find solutions;
- Reflecting on mathematical arguments and explaining and justifying mathematical results; and
- Evaluating the significance of observed (or proposed) patterns and regularities in data.

Source 3-3: OECD (2018). PISA Mathematics Framework (8). OECD (2018). https://pisa2021-maths.oecd.org/files/PISA%202021%20Mathematics%20Framework%20Draft.pdf

3.4. Interpreting, applying and evaluating mathematical outcomes

The problem-solving loop closes with the interpretation and evaluation of mathematical solutions when applied to real-life problems. A problem solver needs to analyse, critically evaluate, reflect, and present information and ideas to others. Once the early stages of problem-solving are complete, sharing and transmitting the solutions to the real world is critical for a successful problem-solver. It also involves the communication of the solutions to others. Practical skills are needed to communicate effectively, including the use of appropriate language and terminology. For example, once a real-life situation is translated into a simultaneous linear equation and the solution is worked out, it is vital to analyse and evaluate the solution in the real world and communicate using the right language.

Individuals engaged in this process may need to construct and communicate explanations and arguments in the context of the problem, reflecting on both the modelling process and its results. Table 3.2 illustrates activities which comes under interpreting, applying and evaluating mathematical outcomes.

Table 3.3: Activities involving the process of interpreting, applying and evaluating mathematical outcomes

Activities involving the process of interpreting, applying and evaluating mathematical outcomes
• Interpreting information presented in graphical form and/or diagrams; • Evaluating a mathematical outcome in terms of the context; • Interpreting a mathematical result back into the real-world context; • Evaluating the reasonableness of a mathematical solution in the context of a real-world problem; • Understanding how the real world impacts the outcomes and calculations of a mathematical procedure or model in order to make contextual judgments about how the results should be adjusted or applied; • Explaining why a mathematical result or conclusion does, or does not, make sense given the context of a problem;

- Understanding the extent and limits of mathematical concepts and mathematical solutions;
- Critiquing and identifying the limits of the model used to solve a problem; and
- Using mathematical thinking and computational thinking to make predictions, to provide evidence for arguments, to test and compare proposed solutions.

Source 3-4: OECD (2018). PISA Mathematics Framework (8). OECD (2018). https://pisa2021-maths.oecd.org/files/PISA%202021%20Mathematics%20Framework%20Draft.pdf

3.5. Mathematical literacy content

The literacy aspect of mathematics does not in discount the importance of the curricular concepts of mathematics and that the two functional and curricular aspects of mathematics cannot be delineated of the other.

To solve problems in real world setting a student must also know the procedures to employ to effectively solve the mathematical problems, they must have a repertoire of mathematical tools acquired from curricular mathematics that empowers them to use knowledge, concepts, and procedures effectively.

Mathematics curriculum if usually organised by topics and places high importance on procedures and concepts associated with the topic being discussed there by delinking the topic with the larger picture of the functional use of mathematics. While this approach it is important to teach or learn the concept it sometimes makes it difficult for the learners to visualise the functional connections with real world situation.

In the real world, the distinction between traditional branches of mathematics (for example, number and algebra, geometry, measurement) is not rigidly defined. However, solving challenges in the real world requires knowledge of a variety of mathematical concepts, procedures, facts and tools at an appropriate level of depth and sophistication. For assessing mathematical literacy of 15 years olds PISA includes a range of mathematical content structures around different phenomena that describe mathematics concepts, structures and ideas. In PISA the phenomena are overarching ideas of are; Change and relationships; Space and shape; Quantity; Uncertainty and data.

Change and relationship manifests itself in nature and its relation to traditional mathematics curriculum covers functions, relations, patterns and could to also cover data to describe change or draw interpretations

Space and shape has deep linkages with traditional geometry and we see shapes as patterns manifest itself around us in the form of man-made objects, natural shapes. Idea of space is closely related to sense of space where relative positions of objects or points help visualise the world we live in. In traditional mathematics this is associated with geometry and measurement

Quantity in PISA is focussed on numbers, their magnitude, operations, estimations, It involves conversion from one form to another and also visualising the identical approach or tools that can be used for solving seemingly different problems or identical underlying mathematics to problems set in different contexts. This overarching idea relates to the number system and operations with numbers in a traditional sense.

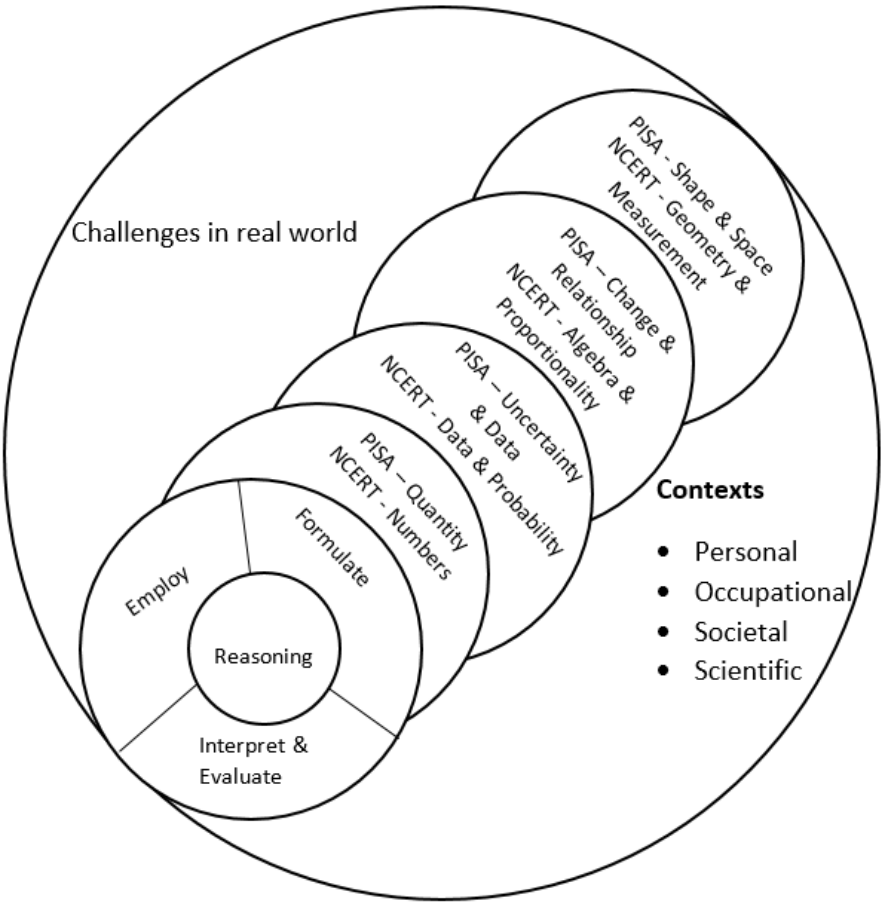
Uncertainty and data covers two related topics of probability and statistics. PISA views data as numbers in context and uncertainty has its relation with randomness and probability. It focusses on randomness as uncertain individual outcomes but having some pattern over many repetitions.

This in traditional sense is related to statistics and probability.

While these overarching ideas have underpinnings with mathematical content, they could also cover multiple traditional topics of mathematics such *change and relationships* could be linked to algebra, data, measurement and number, *space and shape* could be more aligned with geometry, measurement and number, whereas, *quantity* can be more associated with numbers and *uncertainty and data* has a close relationship with statistics and probability.

However, it should be noted that these relationships associated with traditional mathematics should not be the only relationships explored or limit the scope of the overarching idea.

Figure 3.2: Relationship between the mathematical competencies, the problem solving (modelling) cycle, mathematical contents (PISA and NCERT), and contexts.



Source 3-5: Adapted from OECD (2018). PISA Mathematics Framework. OECD (2018). https://pisa2021-maths.oecd.org/files/PISA%202021%20Mathematics%20Framework%20Draft.pdf

Figure 3.2 illustrate that mathematical literacy in PISA assesses problem-solving capabilities through the challenges set in the real world. The challenges or problems or test items are based on mathematical content students are expected to learn in school mathematics and require problem-solving competencies. The figure also relates the mathematical literacy content to traditional curriculum content. The contexts in which assessment items are set are discussed in the next section.

3.6. Contexts to assess competencies

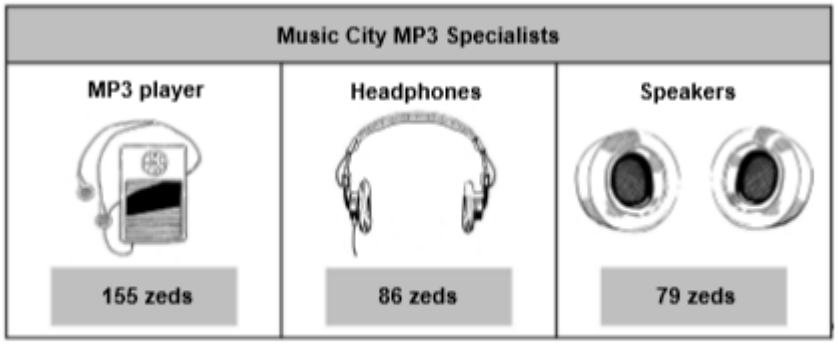
The competencies in PISA are assessed in problem contexts. Contexts help in making the problems authentic and put additional demands on students. For a balanced assessment like PISA survey, a broadest possible range of contexts appropriate for 15-years old are included. The contexts are further divided into four categories.

- Personal
- Occupational
- Societal
- Scientific

Personal – In this category, contexts include activities performed by individuals or their close associates like family, friends or peers. Examples of personal contexts include (not limited to) travel, sports or food choice, shopping, transportation or contexts related to personal health or finance.

Example of a problem set in personal context.

MP3 PLAYERS



Translation Note: The use of zeds is important to the unit, so please do not adapt "zed" into an existing currency.

Question 2: MP3 PLAYERS PM904Q02

Olivia added the prices for the MP3 player, the headphones and the speakers on her calculator.

The answer she got was 248.

Olivia's answer is incorrect. She made one of the following errors. Which error did she make?

A She added one of the prices in twice.
B She forgot to include one of the three prices.
C She left off the last digit in one of the prices.
D She subtracted one of the prices instead of adding it.

Source 3-6: OECD (2013). PISA 2012 RELEASED MATHEMATICS ITEMS. https://www.oecd.org/pisa/pisaproducts/pisa2012-2006-rel-items-maths-ENG.pdf

Occupational – In this category, contexts include activities focused on the world of work. Examples of occupational contexts include (not limited to) measuring, costing and ordering materials for building, payroll/accounting, quality control, scheduling/inventory, design/architecture and job-related decision making.

Example of a problem set in occupational context.

FAULTY PLAYERS

The *Electrix Company* makes two types of electronic equipment: video and audio players. At the end of the daily production, the players are tested and those with faults are removed and sent for repair.

The following table shows the average number of players of each type that are made per day, and the average percentage of faulty players per day.

Player type	Average number of players made per day	Average percentage of faulty players per day
Video players	2000	5%
Audio players	6000	3%

Question 1: FAULTY PLAYERS PM00EQ

Below are three statements about the daily production at *Electrix Company*. Are the statements correct?

Circle "Yes" or "No" for each statement.

Statement	Is the statement correct?
One third of the players produced daily are video players.	Yes / No
In each batch of 100 video players made, exactly 5 will be faulty.	Yes / No
If an audio player is chosen at random from the daily production for testing, the probability that it will need to be repaired is 0.03.	Yes / No

Source 3-7: OECD (2013). PISA 2012 RELEASED MATHEMATICS ITEMS. https://www.oecd.org/pisa/pisaproducts/pisa2012-2006-rel-items-maths-ENG.pdf

Societal – In this category, contexts include activities focused on one's community, at a local, national and global level. An individual is considered as a part of the community, and contexts are set around collective experiences. Examples of societal contexts include (not limited to) voting systems, public transport, government, public policies, demographics, advertising, national statistics and economics.

Example of a problem set in societal context.

CABLE TELEVISION

The table below shows data about household ownership of televisions (TVs) for five countries.

It also shows the percentage of those households that own TVs and also subscribe to cable TV.

Country	Number of households that **own TVs**	Percentage of households that **own TVs** compared to **all households**	Percentage of households that **subscribe to cable television** compared to households that **own TVs**
Japan	48.0 million	99.8%	51.4%
France	24.5 million	97.0%	15.4%
Belgium	4.4 million	99.0%	91.7%
Switzerland	2.8 million	85.8%	98.0%
Norway	2.0 million	97.2%	42.7%

Source: ITU, World Telecommunication Indicators 2004/2005
ITU, World Telecommunication/ICT Development Report 2006

Question 1: CABLE TELEVISION PM978Q01

The table shows that in Switzerland 85.8% of all households own TVs.

Based on the information in the table, what is the closest estimate of the total number of households in Switzerland?

A 2.4 million
B 2.9 million
C 3.3 million
D 3.8 million

Source 3-8: OECD (2013). PISA 2012 RELEASED MATHEMATICS ITEMS. https://www.oecd.org/pisa/pisaproducts/pisa2012-2006-rel-items-maths-ENG.pdf

Scientific – The focus of these contexts are on the application of mathematics to the real world. Issues and topics related to science and technology require mathematical skills. Examples of scientific contexts include (but are not limited to) areas such as weather or climate, ecology, medicine, space science, genetics, measurement and the world of mathematics itself.

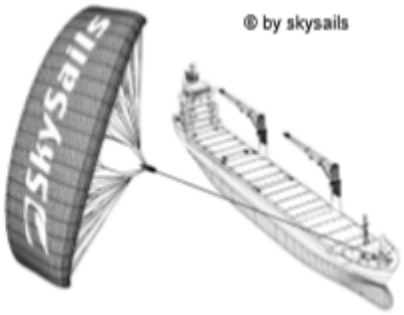
Example of a problem set in scientific context.

SAILING SHIPS

Ninety-five percent of world trade is moved by sea, by roughly 50 000 tankers, bulk carriers and container ships. Most of these ships use diesel fuel.

Engineers are planning to develop wind power support for ships. Their proposal is to attach kite sails to ships and use the wind's power to help reduce diesel consumption and the fuel's impact on the environment.

Translation Note: "© by skysails": Do not adapt skysails as this is a registered label.



Question 1: SAILING SHIPS

PM923Q01

One advantage of using a kite sail is that it flies at a height of 150 m. There, the wind speed is approximately 25% higher than down on the deck of the ship.

At what approximate speed does the wind blow into a kite sail when a wind speed of 24 km/h is measured on the deck of the ship?

A 6 km/h
B 18 km/h
C 25 km/h
D 30 km/h
E 49 km/h

Source 3-9: OECD (2013). PISA 2012 RELEASED MATHEMATICS ITEMS. https://www.oecd.org/pisa/pisaproducts/pisa2012-2006-rel-items-maths-ENG.pdf

The critical point to be noted in above examples is that contexts are made as real as possible and relevant for 15 year old students' interest, activities and capabilities. The objective of an assessment should be kept in mind while developing/selecting contexts.

4. APPLYING THE COMPETENCIES IN TEACHING

4.1. Introduction to applications of the competencies

In chapters 2 and 3, we have discussed why mathematical literacy and its competencies are essential. Mathematical literacy focuses on the literacy aspect of mathematics, which is closely related to the practical application of mathematics in everyday life. The knowledge of mathematical terms, facts, procedures, and applications gained in classrooms may not be enough to prepare young people to apply mathematics in their personal and professional lives. An in-depth understanding of mathematics and mathematical competencies is required to be successful in life. The focus of this chapter is on introducing teaching strategies necessary for fostering these competencies.

4.2. Why teach these competencies

The purpose of the Indian education system, as defined by NEP 2020, is to develop good human beings who have scientific temper and creative imagination, are capable of making rational decisions, and possessing compassion and empathy, courage, and resilience. An educated person should be engaged, productive, and a contributing citizen towards building an equitable, inclusive, and plural society (MHRD, 2020).

Mathematical literacy can be helpful in achieving this purpose. It helps individuals know the role that mathematics plays in the world and make well-founded judgments and decisions expected of constructive, engaged, and reflective 21st-century citizens (OECD, 2018). For being mathematically literate, individuals should have the capacity to reason mathematically and formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. The PISA competencies formulate, employ, and interpret are required in problem-solving tasks. Problem-solving skill is critical to becoming mathematically literate and is an integral part of mathematics education (NCTM, 2020). In mathematics, problem-solving refers to mathematical tasks that can provide intellectual challenges for enhancing students' mathematical understanding and development (NCTM, 2020).

Problem-solving is necessary for in-depth learning of mathematical facts and procedures. A problem solver needs metacognitive ability to flexibly use previously acquired knowledge when the solution is not immediately known (Costa, 2000). Metacognition is characterized by a superordinate ability to direct and regulate cognitive, motivational, and problem-solving processes to achieve desired goals. The essential problem-solving techniques like prediction, planning, monitoring, and evaluating require metacognitive skills. Efforts to increase metacognition skills enhance a person's problem-solving ability (Mulyono & Hadiyanti, 2018). In PISA, cognitive activation is one of the most effective strategies that support the development of mathematical literacy (OECD, 2013).

The communication of mathematical reasoning is another vital aspect of mathematical literacy (Victoria State, 2019). In real-life challenges, profound logic enables young people to evaluate situations, select problem-solving strategies, and draw logical conclusions. Reasoning is critical in describing solutions and recognising the application of solutions in real-life (Rosenstein, Caldwell, & Crown, 1996). Reasoning ability is considered the core of the mathematical literacy competencies (formulating, employing, interpreting, and evaluating) in the PISA 2021 framework. Reasoning is crucial in the formulation and justification of convincing mathematical arguments (Mueller, Yankelewitz, & Maher, 2014). Mathematical logic is an essential skill on

which a person's use of mathematics is founded, and in the absence of reasoning, mathematical understanding is meaningless (Ball & Bass, 2003a).

Mere exposure of students to mathematical topics, procedures, and facts may not be sufficient to equip them with mathematical competencies. Students must learn to reason and develop critical thinking skills to succeed in mathematics (NCTM, 2010). Research in mathematics education shows that the lessons that impacted children's mathematical understanding provided rich opportunities for children to reason (Ofsted, 2008; 2012). Reasoning is a critical skill that enables a student to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied (Rosenstein, Caldwell, & Crown, 1996).

4.3. How to foster these competencies in classroom

Mathematics is fundamentally a school subject; a significant level of mathematical competency can only be achieved in a school setting (OECD, 2010). Quality teaching is required to foster mathematical competencies like problem-solving and reasoning in a school environment. Quality teaching involves classroom management, classroom environment, and cognitive activation strategies. Precise well-structured classroom management ensures that physical resources necessary for the activation of mathematical competencies are available. A supportive and student-centred classroom environment ensures that students are free to investigate, learn, and experiment with mathematics (Klieme, Pauli, & Reusser, 2009). The most critical of the three factors of quality teaching is teachers' cognitive activation strategies. Research on student achievements has proved that instructional processes involving cognitive strategies that require students to summarise, question, clarify and predict result in higher student achievement (Burge, Lenkeit, & Sizmur, 2015).

Teaching strategies are a wide range of processes, including lesson-by-lesson activities used by a teacher to facilitate learning (OECD, 2010). Student learning strategies refer to cognitive and metacognitive processes employed by students as they attempt to learn. Learning strategies are the behaviours and thoughts students use as they try to complete various tasks associated with learning a new concept or acquiring, storing, retrieving, and using information. Various teaching and learning strategies which are helpful in activating cognitive skills are discussed in the sections below. The strategies suggested are based on scientific evidence. However, adaptation may be required as teaching and learning practices vary across schools within an education system. During mathematics instruction, the following considerations should be taken into account:

- A teacher should be aware of the potential of a mathematical task to facilitate learning. The tasks and activities selected by a teacher determine learning opportunities and instructions requirement. The teacher should be aware of the level of challenge and level of understanding that can be attained through those tasks and activities (Corte, Greer, & Verschaffel, 1996; Hiebert et al., 2005).
- Learning happens in relation to students' existing beliefs and prior knowledge. Teachers' ability to diagnose students' misconceptions, typical errors and frequently used strategies can help them to decide the appropriate cognitive activation strategies (Vosniadou & Vamvakoussi, 2005; Vosniadou & Verschaffel, 2004).
- Knowledge acquisition is an active process. Students' learning of mathematics and its deep understanding is dynamic. Students need guidance and support during the entire process (Mayer, 2004; Sfard, 2003). Teachers can use multiple strategies in the process.

Table 4.1 shows the indicators of the uses of these considerations in a classroom.

Table 4.1: Principles of teaching and learning

Principles of teaching and learning	
Students learn best when	**In learning environments that reflect this principle the teacher**
The learning environment is supportive and productive	• Builds positive relationships through knowing and valuing each student • Promotes a culture of value and respect for individuals and their communities • Uses strategies that promote students' self-confidence and willingness to take risks with their learning • Ensures each student experiences success through structured support, the valuing of effort, and recognition of their work.
The learning environment promotes independence, interdependence and self-motivation.	• Encourages and supports students to take responsibility for their learning • Uses strategies that build skills of productive collaboration.
Students' needs, backgrounds, perspectives and interests are reflected in the learning program.	• Uses strategies that are flexible and responsive to the values, needs and interests of individual students • Uses a range of strategies that support the different ways of thinking and learning • Builds on students' prior experiences, knowledge and skills • Capitalises on students' experience of a technology rich world.
Students are challenged and supported to develop deep levels of knowledge and application.	• Plans sequences to promote sustained learning that builds over time and emphasises connections between ideas • Promotes substantive discussion of ideas • Emphasises the quality of learning with high expectations of achievement • Uses strategies that challenge and support students to question and reflect • Uses strategies to develop investigating and problem solving skills • Uses strategies to foster imagination and creativity.

Principles of teaching and learning	
Students learn best when	**In learning environments that reflect this principle the teacher**
Assessment practices are an integral part of teaching and learning.	• Designs assessment practices that reflect the full range of learning program objectives • Ensures that students receive frequent constructive feedback that supports further learning • Makes assessment criteria explicit • Uses assessment practices that encourage reflection and self-assessment • Uses evidence from assessment to inform planning and teaching.
Learning connects strongly with communities and practice beyond the classroom	• Supports students to engage with contemporary knowledge and practice • Plans for students to interact with local and broader communities • Uses technologies in ways that reflect professional and community practices.

Source 4-1: *http://www.whitehorseps.vic.edu.au/app/webroot/uploaded_files/media/poltsummary.pdf*

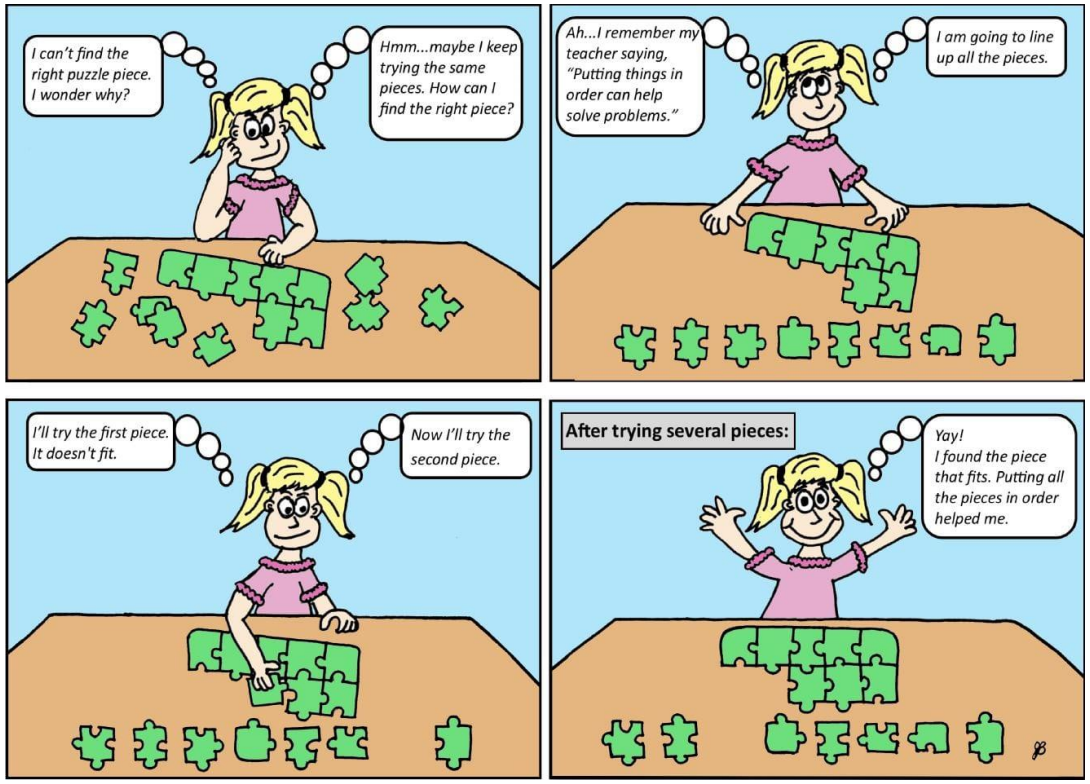
Teaching Strategies - Metacognitive Strategies

Knowledge acquisition is an active process; student learning of mathematics and its deep understanding is dynamic. Students need guidance and support during the entire process (Mayer, 2004; Sfard, 2003). Teachers can use multiple strategies in the process. A metacognitive strategy focuses on student thinking. They empower students to think about their own thinking. Students who become part of the learning process thus have better control of their learning, the capacity to self-regulate, and self-motivate. Cognitive activation strategies can be used in solving mathematical problems by summarizing, questioning, and predicting the problem and its solution. This strategy encourages students to:

- Focus on the method they use, rather than the answer of the problem
- Make connections among mathematical facts, procedures, and ideas

Table 4.2 illustrates examples metacognitive strategies.

Table 4.2: Examples of Metacognitive Strategies

Examples of Metacognitive Strategies		
Strategies	**Action**	**Example**
• Present students with problems in different contexts and ask them to apply what they have learned to new contexts • Give students problems with no immediately obvious method of solution or multiple solutions • Encourage students to reflect on problems • Give students maths problems that require them to think for an extended period of time • Ask students to use their own procedures for solving complex problems • Create a learning community where students are able to learn from mistakes • Ask students to explain how they solved a problem and why they chose that method	• Teaching problem- solving • Teaching study skills • Promoting self-questioning • Encouraging classroom discussions • Using concept mapping	 https://numeracyguidedet.global2.vic.edu.au/hits-birth-to-level-2/#Metacognitive%20Strategies

4.3.1. Teaching Strategies - Reasoning

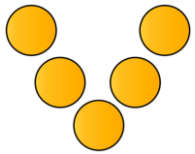
The core of all mathematical competencies is the ability of a student to reason. Thus, every mathematics class should focus on helping students to reason and make sense of mathematics for themselves (NCTM, 2020).

To inculcate reasoning skills among students, the teacher should:

- Select worthwhile tasks that engage and develop students' mathematical understanding, skills, and reasoning
- Create a classroom environment in which serious engagement in mathematical thinking is the norm
- Effectively present purposeful discourse aimed at encouraging students to reason and make sense of what they are doing
- Use a range of assessments to monitor and promote reasoning both in identifying student progress and in making instructional decisions
- Continually reflecting on teaching practice

Table 4.3 shows examples of reasoning strategies.

Table 4.3: Examples of reasoning strategies

Examples of Reasoning Strategies		
Strategy	**Action**	**Example**
Multiple mathematical representations of same concept	Select a mathematical concept where multiple representations are possible. • Let students explore a concept using manipulatives • Ask students to perform structured tasks • Help in generalisation of the mathematical concept	Ask students to construct quadrilateral using manipulatives (you may use geoboard, straws etc.) Give specific conditions like four sides should be equal and angles should be 90°. Help them to generalise the properties of a quadrilateral.
Open ended activities	Select an activity with • multiple entry points • multiple options for solution strategies • multiple representations • solutions not readily available • more than one solution	Put the nine numbers below onto a 3 by 3 grid so that each number occupies one space, and the product of every row, column and diagonal is equal to 1. 1 2 3 6 $\frac{1}{6}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{2}$ Source: https://nrich.maths.org/7189
Mathematical Dialogues	• Select a task in which collaborative work is required • Group students into small, heterogeneous groups • Encourage students to test conjectures and hear the justification of peers • Encourage students to develop discursive and convincing arguments for themselves • Use language structures in the form of sentence starters like, I think this	Place each of the numbers 1 to 5 in the V shape so that the two arms of the V have the same total.  How many different possibilities are there? Can you convince someone that you have all the solutions? What happens if we use the

Examples of Reasoning Strategies		
Strategy	**Action**	**Example**
	because… , The pattern looks like…, This can't be because… • Give students time to articulate and reflect on their ideas.	numbers from 2 to 6? From 12 to 16? From 37 to 41? From 103 to 107? Investigate the same problem with a V that has arms of length 4

Source 4-2: *https://nrich.maths.org/content/id/6814/NRICH-poster_MagicV.pdf*

4.3.2. Teaching Strategies – Problem Solving

Problem-solving can be a useful tool for teaching mathematics. Mathematics education is shifting from teaching problem solving to teaching through problem-solving (Taplin, n.d.). Teaching through problem-solving helps in-depth mathematics learning (Cai & Lester, 2010). This approach is in line with the nature of mathematics education recommended by NEP 2020. Students understand mathematical ideas and processes in the problem-solving approach by conjecturing, exploring, testing, and verifying. Learning occurs during the process of solving a problem in which relevant mathematical concepts and skills are embedded (Lester & Charles, 2003; Schoen & Charles, 2003). Thus, the selection of a problem or task which can create learning opportunities is critical. The problem should provide some degree of challenge, address essential mathematical ideas, and foster communication and reasoning. It is expected that during the process, students will develop and discover their own problem-solving strategies and become adept at using them for problem-solving. The competencies will help students tackling problem-solving tasks in any situation and enhance their reasoning skills. Problem-solving skills consist of five steps:

- Identify the problem
- Define and represent the problem
- Explore possible strategies
- Act on the strategies
- Look back and evaluate the process and solution

These steps may be combined to form the three major categories of competency- formulate, employ, and interpret and evaluate.

4.3.3.a. Problem solving strategies -Formulate

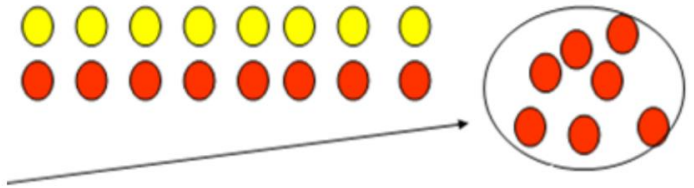
To enhance the ability to formulate, teachers can ask students questions similar to the ones listed below:

- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or a diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?

Table 4.4 shows examples of formulating problem solving strategies.

Table 4.4: Examples of problem solving strategies - Formulating

Examples of problem solving strategies - Formulating		
Strategy	**Action**	**Example**
Paraphrasing Strategy	To help students restate the math problem in their own words, ask students to • Read the problem • Underline or highlight key terms • Restate the problem in their own words • Write a numerical sentence	**Step 1**: Read the problem The middle school has 560 lockers available for the beginning of the school year. They have 729 students starting school. How many lockers will they be short of on the first day of school? **Step 2**: Underline or highlight key terms The middle school has 560 lockers available for the beginning of the school year. They have 729 students starting school. How many lockers will they be short of on the first day of school? **Step 3**: Restate the problem in your own words If there are 729 students and only 560 lockers, I need to know how much more 729 is than 560. **Step 4**: Write a numerical sentence 729 - 560 = 169 lockers are still needed.
Visualisation	To help students in visualizing and drawing the problem, ask students to • Read the problem • Underline important images in the problem • Draw a visual representation of the problem • Write a numerical sentence	**Step 1**: Read the problem Dave was hiking on a trail that took him to an altitude that was 15 miles below sea level. Susan hiked to an altitude that was 8 miles above Dave. What was the final altitude for Susan's hike? **Step 2**: Have the students underline important images in the problem

Examples of problem solving strategies - Formulating		
Strategy	**Action**	**Example**
	Note: Students' drawing should be a representation of the problem, not just pictures of the items mentioned in the problem.	Dave was hiking on a trail that took him to an altitude that was 15 miles below sea level. Susan hiked to an altitude that was 8 miles above Dave. What was the final altitude for Susan's hike? **Step 3:** Ask the students to draw a visual representation of the problem Dave is 15 miles below sea level and Susan is 8 miles above Dave. The final altitude for Susan can be represented by the expression –15 + 8.  **Step 4:** Write a numerical sentence -7 was left over after taking out zero pairs, so the final altitude for Susan's hike is 7 miles below sea level.

Examples of problem solving strategies - Formulating		
Strategy	**Action**	**Example**
Analyse the Information	To help students to analyse information in the problem ask students to • Analyse and review the information by making them work in groups/pairs • Complete the meta-cognitive checklist in pairs/groups	**Metacognitive Chart for Analyzing Data** Directions: *As you complete each phase of step1 in the problem solving process, check the appropriate box below.* 1. Read/Reread ☐ 2. Paraphrase (restate in your own words) ☐ 3. Visualize (Draw a picture to represent the problem) ☐ 4. Worked with a group to discuss the problem. ☐ *After you have check off all four steps, answer the following questions:* 1. What is the problem asking or what question am I trying to answer? 2. What information is still missing? 3. What type of mathematical computation will I need to use to solve the problem? 4. Does everyone in my group agree on these answers? If everyone does not agree, be sure to write down all extra responses in the space below (you may need them for step 2).

Source 4-3: Florida Department of Education. (2010).

http://floridarti.usf.edu/resources/topic/academic_support/kops/class_strategies.pdf

4.3.3.b. Problem solving strategies -Employ

Identifying or defining a problem is a significant part of solving it. When a student understands what is to be found or worked upon, he/she is ready for the next step. It is critical for teachers to ensure that students understand the given problem or task. It is essential to mention here that teaching strategies should be used as required. For example, if a student understands the meaning of all of the words mentioned in the problem, he/she should be able to restate the problem. If the student is unable to do so, the teacher should teach him/her how to paraphrase it.

After a student has understood a problem, the next step is devising and implementing the plan. He/she is required to find the connection between the available information and the unknown. At this step, existing knowledge and the ability to use it flexibly are needed. A student may have to recall formulae, procedures, or algorithms and adapt them to the problem at hand. Questions to ask students at this stage may include:

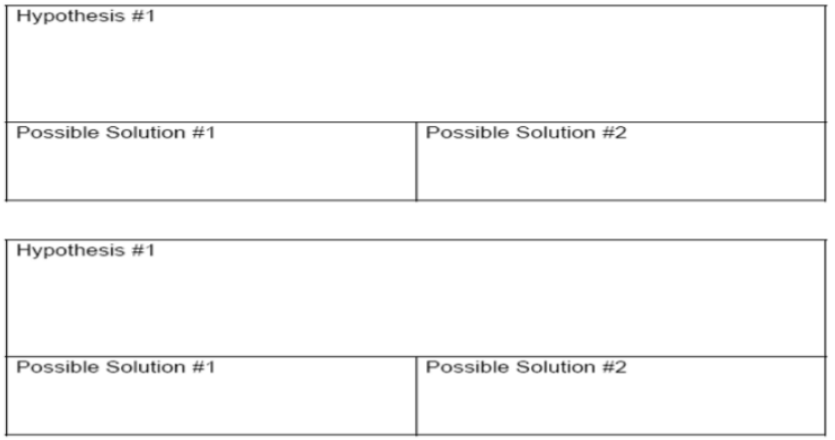
- Have you seen the same type of problem in a slightly different form?
- Try to think of a familiar problem having the same or a similar unknown.
- Could the problem be restated differently?
- Did you use all the data?

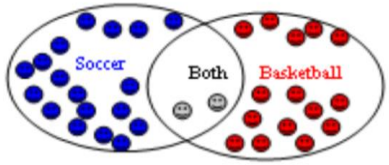
It is advisable to provide opportunities for students to approach the problems methodically and solve them. Students should be aware that different problems will need different problem-solving approaches. General computation methods include simplifying, solving equations, and completing a pattern, diagram, table, or chart or working backward. A problem may have multiple solutions; a student should look at each proposed solution's pros and cons and choose the most appropriate one. After students have decided to work on a specific plan, they should follow the steps outlined below (Florida Department for Education, 2010):

- Solve the problem using the chosen plan.
- If the plan is not working after a few attempts, try a different plan.
- Allow for mistakes (remember the plan may need some revision).
- Check the answer.

Table 4.5 shows examples of employing problem solving strategies.

Table 4.5: Examples of problem solving strategies - Employing

Examples of Problem Solving Strategies - Employing		
Strategy	**Action**	**Example**
Hypothesising	To teach hypothesizing, a teacher should • Model hypothesising for the students • Ask students to work on creating hypothesis in a group • Allow students to practice independently (quizzes, verbal responses, etc.)	Provide template to hypothesize as given below: 
Eliminating Possibilities	• Introduce a problem to students that will require them to eliminate possibilities in order to solve the problem. • Ask students to list all possible solutions. • Ask them to choose a solution in which all criteria are met.	In the game of football, a team can score either a touchdown for six points or field goal for three points. A team only scores touchdowns or field goals. The score was less than 30. It does not get any extra points. What **cannot** be possible team scores? • List the numbers 1 through 29. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 • Eliminate answers that are not possible solutions. Keep scoring criteria in mind. Any multiple of six would be a possible score of the game. If the team only scored touchdowns, they could score 6, 12, 18,

Examples of Problem Solving Strategies - Employing		
Strategy	**Action**	**Example**
		and 24 and so on. Therefore, all multiples of six should be eliminated. 1 2 3 4 5 * 7 8 9 10 11 * 13 14 15 16 17 * 19 20 21 22 23 * 25 26 27 28 29 • Any multiple of three would be a possible score of the game. If a team scored only field goals, they could score 3, 6, 9, and so on. Therefore, all multiples of three should be eliminated. • 1 2 * 4 5 * 7 8 * 10 11 * 13 14 * 16 17 * 19 20 * 22 23 * 25 26 * 28 29 • Therefore, team score cannot be • 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25, 26, 28, 29
Logical reasoning	Logical reasoning is a problem-solving strategy that involves the use of Venn diagrams or charts to help students use logic to solve a problem. • Ask students to read the problem and paraphrase if necessary • Students should ask themselves "What do I know about this problem?" • Ask them to draw the Venn diagram/chart.	In a sports event, 32 children have participated. 19 played soccer and 15 played basketball. How many children played both sports? Logic tells that at least some children played both sports, while others played only one. Put 19 counters inside the soccer circle and 15 counters inside the basketball circle. With only 32 counters, 2 of them must be placed inside both circles. The answer is 2 children played both sports. 

Examples of Problem Solving Strategies - Employing		
Strategy	**Action**	**Example**
Look for a Pattern	Ask students to • Read the problem • Make a table • Look for the pattern • Solve the problem	An overweight man joined a diet and exercise program to lose 36 kg. He loses 11 kg in the first week, 9 kg in the second week, and 7 kg in the third week. Continuing at the same rate, how long will it take for him to lose 36 kg? Each week, the man loses 2 kg less than he did the previous week. Extend the pattern or make a table for 7 weeks. It takes the man 7 weeks to lose 36 kg.

Source 4-4: Florida Department of Education. (2010).

http://floridarti.usf.edu/resources/topic/academic_support/kops/class_strategies.pdf

4.3.3.c. Problem solving strategies –Interpret and Evaluate

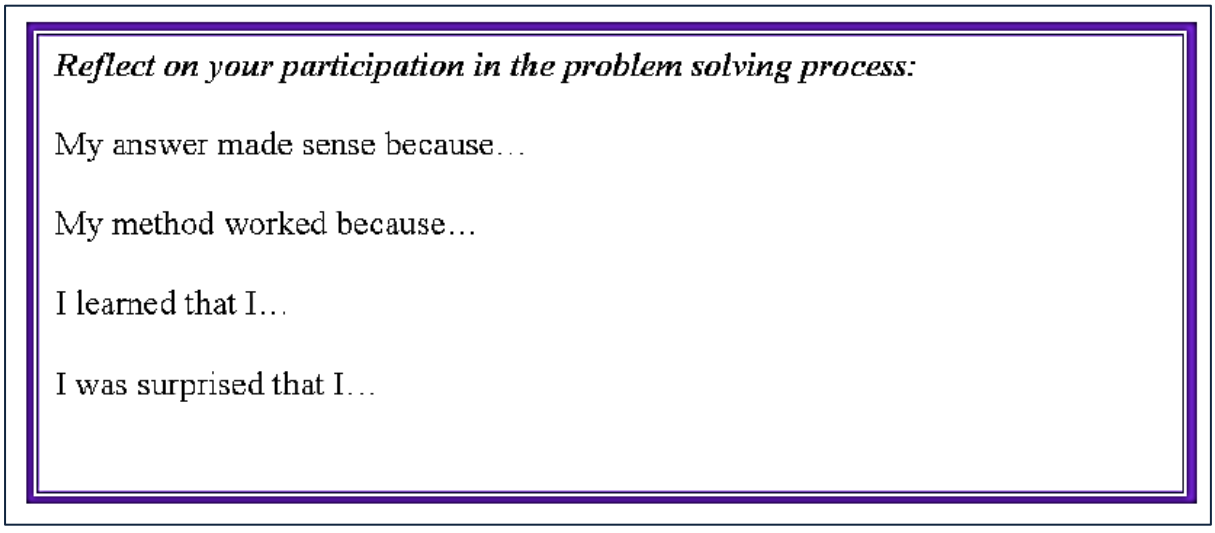
After students solve a problem and come up with a solution, they should be encouraged to reflect on the problem-solving process. To convert problem-solving into a mathematics learning opportunity, it is critical to reflect and look back at what has been done, what worked, and what didn't.

Questions to ask students at this stage may include:

- Have you checked the result?
- Have you checked the argument?
- Does your answer make sense?
- Did you answer all parts of the question?
- What methods worked?
- What methods failed?
- What did you learn from completing this problem?
- Could the problem be solved in another way?
- Was there an easier way to solve this problem?
- If you encountered a similar situation in the future, how could you do it better?

Teachers can use the template as given below to help students collect their thoughts.

Figure 4.1: Examples of Problem Solving Strategies – Interpret and evaluate



Source 4-5: Florida Department of Education. (2010).

http://floridarti.usf.edu/resources/topic/academic_support/kops/class_strategies.pdf

4.4. Concluding remarks

The traditional practices in mathematics focus on the mastery of mathematical facts, algorithms, and proofs. The approach based on the activation of the metacognitive view of mathematics focuses on acquiring mathematical competencies related to real-life. It considers domain knowledge and understanding along with competencies as the goal of mathematics education. Studies have shown that mathematics taught using metacognition strategies like reasoning and problem-solving result in a more profound knowledge of the subject. Metacognitive strategies enable learners to think about their own thinking, resulting in better control of learning.

Cognitive activation strategies help students' master mathematics by connecting mathematical facts, procedures, and ideas. They focus on the method of problem-solving rather than on finding the correct answer. The cognitive skills can be activated using thoughtful mathematics teaching strategies. The core of all mathematical competencies lies in the ability of a student to reason. Reasoning skills of a student can be enhanced through teaching strategies like using multiple mathematical representations of the same concept and establishing a connection between them. Using open-ended classroom activities helps students widen their thinking process; they become aware that a problem can have multiple solutions obtained through multiple methods. Communication is an integral part of mathematics reasoning; strategies like mathematical dialogues help students understand others' mathematical logic and communicate their own reasoning.

Teaching through problem-solving is helpful for students' deep understanding of mathematics. Through this method, learning occurs during the process of solving a problem based on mathematical concepts. Careful thought should be given to the tasks chosen for problem-solving. It should not be too easy or too challenging. Optimal level of challenge helps students to develop problem-solving competencies and enhance their confidence. Few classroom teaching strategies like paraphrasing, visualization, analysis of the information, hypothesizing, eliminating possibilities, logical reasoning, and looking for patterns, which have been discussed in this chapter, inculcate the habit of systematic problem-solving.

The PISA mathematical literacy framework draws competencies from problem-solving strategies like formulating, employing, and interpreting and evaluating. It considers reasoning as the core of all competencies. It helps individuals make well-founded judgments and decisions expected of constructive, engaged, and reflective 21st-century citizens (OECD, 2018).

5. ASSESSING THE COMPETENCIES

5.1. Introduction to proficiency

Mathematical competencies help in applying mathematical skills and understanding in real life. In chapter 4, the importance of mathematical competencies aligned with PISA was discussed along with some exemplar strategies to inculcate these competencies in young learners. In this chapter, the discussion is focused on mathematical proficiencies and progressions, how they are reported on the PISA scale and some examples to illustrate the use of test items to collect evidence about the mathematical capabilities of students in PISA.

It is crucial but challenging for teachers to get information about what students know, can do, and are disposed to do mathematically. This ability is known as mathematical proficiency. The kind of mathematics questions or problems to which a learner is exposed during school years are different from those which he or she faces in higher studies or professional life. Mathematical problems usually used in school curricula can be solved in a few minutes or hours, but the problems in higher studies or professional life may take days, weeks, months, or years to solve (Schoenfeld, 2007). Characteristics other than knowledge of mathematics are also required to solve such problems. Flexibility and efficient use of mathematical knowledge is a must. However, when one way of problem-solving does not work, a solver might require to look at alternatives. He/she needs to be persistent with problem-solving efforts. All of these are fundamental characteristics of being mathematically proficient. Thus mathematical proficiency can be considered a mix of:

- Conceptual understanding
- Strategic competence
- Metacognition
- Beliefs and disposition

Conceptual understanding

According to Schoenfeld (2007) conceptual understanding in mathematics consists of:

- Knowledge and understanding of mathematical facts and procedures
- Application of knowledge in solving problems
- Analysing information
- Ability to use mathematical representations appropriate for a situation
- Evaluation of strategies and solution for the problem

Strategic competence

According to Schoenfeld (2007) strategic competence can be understood as the ability to:

- Formulate mathematical problems
- Represent and solve them

It is similar to the 'formulate and employ' competency used in PISA.

Metacognition

Metacognition in mathematical proficiency leads to effective use of knowledge. Students develop their capacity of logical thinking, reflecting, explaining, and justifying. Metacognition guides

learning, as students think about facts, procedures, concepts, and strategies to look at their compatibility. It is similar to the 'interpret and evaluate' competency used in PISA.

Beliefs and disposition

Beliefs and disposition refer to productive disposition; a "habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy" (NRC, 2001, 5). Belief and disposition play a vital role in developing mathematical competencies. For example, when students' strategic competency is developed while solving non-routine problems, their belief in themselves as learners of mathematics becomes more positive. Similarly solving problems successfully enhances procedural fluency and metacognitive abilities.

Assessing proficiencies

Conceptual understanding plays a central role in mathematical proficiency along with an individual's ability to employ problem-solving strategies, beliefs and dispositions. It is essential to consider all four aspects discussed above to assess mathematical competencies. Traditionally, teachers are more inclined to assess conceptual understanding. With new research in the field of mathematics education, it is possible to assess and report on all four essential components of mathematical proficiencies (OECD, 2019). International tests, especially PISA, assess mathematical proficiency and report the results using a mathematical literacy proficiency scale. Table 5.1 shows summary description of the six levels of mathematics proficiency in PISA 2018.

Table 5.1: Summary description of the eight levels of mathematics proficiency in PISA-D

Level	What students can typically do
6	At Level 6, students can conceptualise, generalise and utilise information based on their investigations and modelling of complex problem situations, and can use their knowledge in relatively non-standard contexts. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understanding, along with a mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for attacking novel situations. Students at this level can reflect on their actions, and can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments and the appropriateness of these to the original situation.
5	At Level 5, students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations, and insight pertaining to these situations. They begin to reflect on their work and can formulate and communicate their interpretations and reasoning.
4	At Level 4, students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic, linking them directly to aspects of real-world situations. Students at this level can utilise their limited range of skills and can reason with some insight, in straightforward contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments and actions.
3	At Level 3, students can execute clearly described procedures, including those that require sequential decisions. Their interpretations are sufficiently sound to be a base for building a simple model or for selecting and applying simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They typically show some ability to handle percentages, fractions and decimal numbers, and to work with proportional relationships. Their solutions reflect that they have engaged in basic interpretation and reasoning.
2	At Level 2, students can interpret and recognise situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures or conventions to solve problems involving whole numbers. They are capable of making literal interpretations of the results.
1a	At Level 1a, students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are almost always obvious and follow immediately from the given stimuli.
1b	At Level 1b, students can respond to questions involving easy to understand contexts where all relevant information is clearly given in a simple representation (for example tabular or graphic) and defined in a short syntactically simple text. They are able to follow clearly prescribed instructions.
1c	At Level 1c, students can respond to questions involving easy to understand contexts where all relevant information is clearly given in a simple, familiar format (for example a small table or picture) and defined in a very short syntactically simple text. They are able to follow a clear instruction describing a single step or operation.

Source 5-1: OECD (2017). PISA for Development Assessment and Analytical Framework: Reading, Mathematics and Science, Preliminary Version, OECD Publishing, Paris.

5.2. Learning progression and PISA proficiency levels

5.2.1. Learning progression

A mathematics curriculum identifies content to be taught and the rate of learning which may prepare students for higher studies or career goals by the end of secondary school, ensuring that the introduction of topics across the different content strands of mathematics are synchronized (OECD, 2019). A typical mathematics classroom consists of students at varying levels of mathematical learning; they may be at the grade level, below or above as per the description of the curriculum for the grade. According to most influential learning theories, effective teaching-learning can happen when instructions are proximal to the learner's current state of understanding. Following curricula may create a gap between what a student actually knows and what he/she is expected to learn as per grade level expectations (OECD, 2019).

The concept of learning progression helps to decode curriculum and its expectation in terms of proficiency levels. A learning progression contains "descriptions of successfully more sophisticated ways of reasoning within a content domain based on research syntheses and conceptual analyses" (Smith, Wiser, Anderson, & Krajcik, 2006, 1). Learning progressions describe what teachers and curricula are trying to achieve along with what students are attending to and learning (Daro, Mosher, & Corcoran, 2011). Learning progression can also be considered as a description of skills, understanding, and knowledge in a developmental order. Alternatively, learning progression reflects the learning progress along a single continuum of increasing expertise, from emerging to mastered (Sáez, Lai, & Tindal, 2012). Table 5.2 shows the developmental progress of learning equal partitioning and unitizing.

Table 5.2: Development of Equal Partitioning and Unitizing

Level	Development	Example
Level 1	Objects	3 objects + 5 objects = 8 objects
Level 2	Pure numbers	3 ones + 5 ones = 8 ones
Level 3	Groups of objects	3 groups of 10 objects + 5 groups of 10 objects = 8 groups of 10 objects = 80
Level 4	Groups of 10 ones are tens	3 tens + 5 tens = 8 tens
Level 5	Equal lengths are units	3 inches + 5 inches = 8 inches
Level 6	A length can be equipartitioned into equal sized units	¼ + ¼ + ¼ + ¼ = 1
Level 7	A part of 1 inch, ¼ inch, can be counted, added, etc. as a unit	3 (1/4 inches) + 5 (1/4 inches) = 8 (1/4 inches)
Level 8	Unit fractions as pure numbers can be counted, added and multiplied	3(1/4) + 5(1/4) = 8(1/4) = 8/4
Level 9	Expressions with letters can be read as uncalculated numbers	3(x + 1) + 5(x+1) = 8(x+1)

Source 5-2: Adapted from Daro, P., Mosher, F.A., & Corcoran, T. B. (2011). Learning Trajectories in Mathematics: A Foundation for Standards, Curriculum, Assessment, and Instruction. CPRE Research Reports. Retrieved from http://repository.upenn.edu/cpre_researchrep

Thus, the concept of learning progression is a way to restructure and rethink the curriculum content in increasing sequence of expertise. It supports teachers to understand individual students' mathematical skill development and assists teachers in developing targeted teaching and learning programs. A learning progression spans levels of increasing domain proficiency from low at the bottom of the progression to high at the top of the progression. Research shows that teachers with an understanding of learning progression help students attain mathematical proficiency better (OECD, 2019). While understanding the structure of learning progressions, it is essential to keep in mind the progression from one level to the next higher level may or may not be reached in the same amount of time and efforts by different students (Battista, 2011). Sometimes a student might take more time to reach to the next level, depending upon the complexity of the topic and the student's mastery of various competencies. Table 5.3 shows qualities of a learning progression.

Table 5.3: Qualities of learning progression (left) and misperceptions (right)

What learning progressions *ARE*	**What learning progressions Are *NOT***
Domain-specific model	General or universal principles
Expected probabilities	Stage theories
Empirical models of student thinking	Logical-mathematical deconstructions
Based on students' thinking	Based on opinions of experts in mathematics
Elicited by rich or novel tasks	Derived from typical exercises
Include strategies, reasons, explanations and cases	Sub-goals of the target
Include exploring misconceptions	A means to avoid errors
Ordered by increasing sophistication	Ordered by difficulty
Connected to big ideas over the long term	Curriculum material
Evolving	Fixed

Source 5-3: Confrey, J., Maloney, A., Shah, M. & Belcher, M. (2019) Future of Education and Skills 2030: Curriculum analysis. https://www.oecd.org/education/2030/A-Synthesis-of-Research-on-Learning-Trajectories-Progressions-in-Mathematics.pdf

5.2.2. Relationship between learning progression and proficiency levels

A learning progression can also be considered as a scale which defines characteristics of a test intended to measure educational progress in a particular domain (for example, mathematics) (Adams, Jackson, & Turner, 2018). This definition requires "one to weave all the components of the learning progression into an integrated system that incorporates careful attention to the connections between learning progression and the curriculum (materials), instruction, classroom assessment (formative and measurement-based), and all forms of professional development, support, and capacity building" (Confrey, Maloney, Shah, & Belcher, 2019).). Learning progressions in mathematics have the potential for designing assessments that can report in educationally meaningful terms (Daro, Mosher, & Corcoran, 2011). Assessments based on learning progression can provide feedback to the instructional process.

Learning progressions illustrate learning from less to more; they can be associated with a scale; lower points on the scale represent less learning, and higher points represent more learning. The

location on a scale can be "described numerically, as proficiency scores, or substantively, as proficiency descriptions" (Adams, Jackson, & Turner, 2018). The descriptions of proficiency are the competencies which learners are expected to exhibit, illustrating what students know and can do. In the next section, PISA proficiency levels are explained with reference to learning progression.

5.2.3. Link between learning progression of PISA competencies and proficiency levels

PISA reports students' mathematical proficiencies in terms of mathematical literacy scale. The proficiency scales describe what students typically know and can do at the pre-defined level of proficiency. Reporting students' proficiency is different from reporting students' performance on a single administered test. PISA reports at the general population-level rather than the results for individual students. To give general population estimates, PISA uses samples of students and items. A representative sample of 15-year-old students is selected from a country or economy. Test items from a large pool of items designed to test mathematical competencies are administered to each sample student. The students' responses are then analysed using statistical models (OECD, 2018). Outcomes are used to estimate students' proficiency with the skills and knowledge being assessed in each domain. For mathematics, skills like formulation, employ, interpretation and evaluation are assessed in the content areas, including space and shape, change and relationships, quantity, and uncertainty and data.

Mathematical and conceptual understanding is crucial in mathematical literacy. With the increase in the level of mathematical literacy possessed by an individual, his capacity to use fundamental mathematical understanding increases (Adams, Jackson, & Turner, 2018). Along with the conceptual knowledge of mathematics, the fundamental mathematical capabilities include: communication, mathematising, representation, reasoning and argument, devising strategies for solving problems, using symbolic, formal and technical language and operations, and using mathematical tools. The fundamental mathematical capabilities form the basis of reporting of mathematical proficiency in the PISA mathematical assessment. Increasing item difficulty (regarding mathematical capabilities) is associated with an increase in mathematical literacy on the PISA proficiency scale. On the scale, the easiest items elicit evidence of students' ability to perform tasks like identifying mathematical information from a mathematical representation and using it in the given problem context. The most difficult items elicit evidence about knowledge of specific mathematical content or procedures. The items require creativity or strategic control to relate the context to the mathematical representation, and substantial mathematical processing or calculation to devise a solution.

Item difficulty on the scale is linked with the proficiency level. Students at the lowest proficiency level can correctly answer well-formulated problems, reproduce well-known mathematical facts or processes, and apply simple computational skills in explicit situations. At the highest proficiency levels, a student takes a more creative approach to solve mathematical problems. It involves carrying out more complex tasks in a systematic way, involving multiple processing steps. The proficiency levels are related to the mathematical progression of the problem-solving process. The problem-solving process involves, formulating situations mathematically; employing mathematical concepts, facts, procedures and reasoning; interpreting, applying and evaluating mathematical outcomes.

The relation between the fundamental mathematical capabilities and the problem-solving process is shown in the table 5.4.

Table 5.4: Relation between fundamental mathematical capabilities and problem-solving process

	Formulating situations mathematically	Employing mathematical concepts, facts, procedures and reasoning	Interpreting, applying and evaluating mathematical outcomes
Communicating	Read, decode, and make sense of statements, questions, tasks, objects or images, in order to form a mental model of the situation	Articulate a solution, show the work involved in reaching a solution and/or summarise and present intermediate mathematical results	Construct and communicate explanations and arguments in the context of the problem
Mathematising	Identify the underlying mathematical variables and structures in the real world problem, and make assumptions so that they can be used	Use an understanding of the context to guide or expedite the mathematical solving process, e.g. working to a context-appropriate level of accuracy	Understand the extent and limits of a mathematical solution that are a consequence of the mathematical model employed
Representation	Create a mathematical representation of real-world information	Make sense of, relate and use a variety of representations when interacting with a problem	Interpret mathematical outcomes in a variety of formats in relation to a situation or use; compare or evaluate two or more representations in relation to a situation
Reasoning and argument	Explain, defend or provide a justification for the identified or devised representation of a real-world situation	Explain, defend or provide a justification for the processes and procedures used to determine a mathematical result or solution Connect pieces of information to arrive at a mathematical solution, make generalisations or create a multi-step argument	Reflect on mathematical solutions and create explanations and arguments that support, refute or qualify a mathematical solution to a contextualised problem
Devising strategies for solving problems	Select or devise a plan or strategy to mathematically reframe contextualised problems	Activate effective and sustained control mechanisms across a multi-step procedure leading to a mathematical solution, conclusion or generalisation	Devise and implement a strategy in order to interpret, evaluate and validate a mathematical solution to a contextualised problem
Using symbolic, formal and technical language and operations	Use appropriate variables, symbols, diagrams and standard models in order to represent a real-world problem using symbolic/formal language	Understand and utilise formal constructs based on definitions, rules and formal systems as well as employing algorithms	Understand the relationship between the context of the problem and representation of the mathematical solution. Use this understanding to help interpret the solution in context and gauge the feasibility and possible limitations of the solution
Using mathematical tools	Use mathematical tools in order to recognise mathematical structures or to portray mathematical relationships	Know about and be able to make appropriate use of various tools that may assist in implementing processes and procedures for determining mathematical solutions	Use mathematical tools to ascertain the reasonableness of a mathematical solution and any limits and constraints on that solution, given the context of the problem

Source 5-4: OECD (2019), PISA 2018 Mathematics Framework, in PISA 2018 Assessment and Analytical Framework, OECD Publishing, Paris, https://doi.org/10.1787/13c8a22c-en.

5.3. Sample tasks

5.3.1. PISA released mathematical literacy items and responses

In this section, the problem-solving process is discussed with the help of PISA released items and are linked with proficiency level descriptors. All the example items below are taken from PISA 2012 (OECD, 2013).

Unit name: Which Car?

WHICH CAR?

Chris has just received her car driving licence and wants to buy her first car.

This table below shows the details of four cars she finds at a local car dealer.

Model:	Alpha	Bolte	Castel	Dezal
Year	2003	2000	2001	1999
Advertised price (zeds)	4800	4450	4250	3990
Distance travelled (kilometres)	105 000	115 000	128 000	109 000
Engine capacity (litres)	1.79	1.796	1.82	1.783

Question 1: WHICH CAR? PM985Q01

Chris wants a car that meets all of these conditions:

- The distance travelled is not higher than 120 000 kilometres.
- It was made in the year 2000 or a later year.
- The advertised price is not higher than 4500 zeds.

Which car meets Chris's conditions?

A. Alpha
B. Bolte
C. Castel
D. Dezal

Scoring

Full Credit

Code 1: B Bolte.

No Credit

Code 0: Other responses.

Code 9: Missing.

This is an example of a task in 'uncertainty and data' strand. The competency required to answer the question is 'interpret'. A student is required to identify data in a table meeting specifications of simple mathematical relationships. 'Which car' is an example of a well-formulated problem. No calculation is required to answer the question; only direct instructions are to be followed in the explicit situation. This item lies at the lowest level of mathematical literacy scale with a scale score of 327.8. The item aligns with level 1 of the proficiency scale.

At Level 1, students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are almost always obvious and follow immediately from the given stimuli.

Question 3: WHICH CAR? PM985Q03 – 0 1 9

Chris will have to pay an extra 2.5% of the advertised cost of the car as taxes.

How much are the extra taxes for the Alpha?

Extra taxes in zeds: .....

Scoring

Full Credit

Code 1: 120.

No Credit

Code 0: Other responses.

- 2.5% of 4800 zeds [Needs to be evaluated.]

Code 9: Missing.

This is an example of an item from the same unit 'Which Car?' in the 'quantity' strand. The competency required to answer the question is 'employ'. A student is required to interpret information on the tax rate for purchasing the car to formulate a simple model, locate and extract data from a table, and calculate a percentage. In this example, a student executes clearly described procedures. He selects and applies a simple problem-solving strategy for calculating percentages. This item lies at a lower level of mathematical literacy scale with a scale score of 552.6. The item aligns with level 3 of the proficiency scale.

At Level 3, students can execute clearly described procedures, including those that require sequential decisions. Their interpretations are sufficiently sound to be a base for building a simple model or for selecting and applying simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They typically show some ability to handle percentages, fractions and decimal numbers, and to work with proportional relationships. Their solutions reflect that they have engaged in basic interpretation and reasoning.

Unit Name: Drip Rate

DRIP RATE

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients.

Nurses need to calculate the drip rate, D, in drops per minute for infusions.

They use the formula $D = \frac{dv}{60n}$ where

d is the drop factor measured in drops per millilitre (mL)

v is the volume in mL of the infusion

n is the number of hours the infusion is required to run.

Question 1: DRIP RATE PM903Q01 – 0 1 2 9

A nurse wants to double the time an infusion runs for.

Describe precisely how D changes if n is doubled but d and v do not change.

.....

.....

Scoring

Full Credit

Code 2: Explanation describes both the direction of the effect and its size.

- It halves
- It is half
- D will be 50% smaller
- D will be half as big

Partial Credit

Code 1: A response which correctly states EITHER the direction OR the size of the effect, but not BOTH.

- D gets smaller [no size]
- There's a 50% change [no direction]
- D gets bigger by 50%. [incorrect direction but correct size]

No Credit

Code 0: Other responses.

- D will also double [Both the size and direction are incorrect.]

Code 9: Missing.

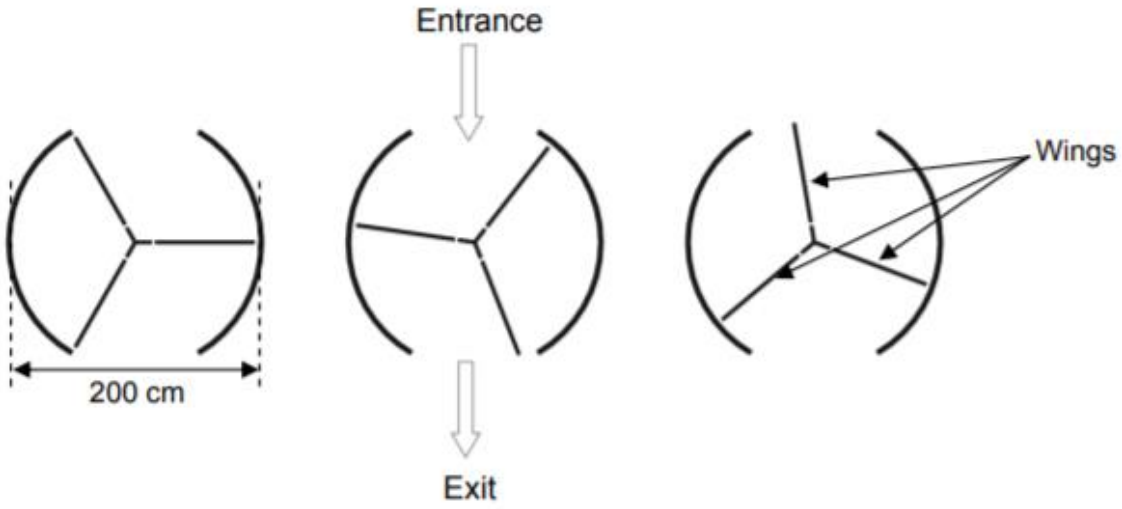
This is an example of an item from the 'change and relationship' strand. The competency required to answer the question is 'employ'. A student is required to interpret text and an equation linking four variables, and explain the effect of a specified change to one variable on a second variable if all other variables remain unchanged. The context can be considered as complex with a concrete situation involving constraints. He is required to select and apply a direct problem-solving strategy to communicate an explanation for the variation. This item was evaluated based on complete and partial explanations. A complete explanation includes size (for example, half, 50%) and change (for example smaller/bigger). A partial explanation includes only either size or change. Complete explanations were placed at a scale score of 657.7, and partial explanations were reported at a scale score of 610.5. The item aligns with level 4 of the proficiency scale.

At Level 4, students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic, linking them directly to aspects of real-world situations. Students at this level can utilize their limited range of skills and can reason with some insight, in straightforward contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments and actions.

Unit Name: Revolving door

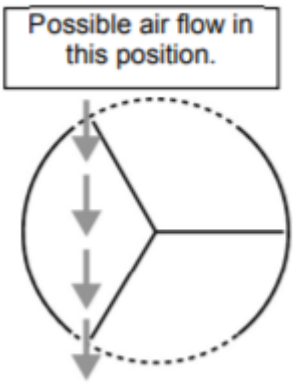
REVOLVING DOOR

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



Question 2: REVOLVING DOOR PM995Q02 – 0 1 9

The two door openings (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram.



What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: ..... cm

Scoring

Full Credit

Code 1: Answers in the range from 103 to 105. [Accept answers calculated as 1/6th of the circumference (100π3. Also accept an answer of 100 only if it is clear that this response resulted from using π = 3. Note: Answer of 100 without supporting working could be obtained by a simple guess that it is the same as the radius (length of a single wing).]

No Credit

Code 0: Other responses.

- 209 [states the total size of the openings rather than the size of "each" opening].

Code 9: Missing.

This is an example of an item from the 'space and shape' strand. The competency required to answer the question is 'formulate'. A student is required to apply knowledge of circle geometry and reasoning to interpret a given geometric model and to formulate it mathematically, enabling a solution. The context is a complex requiring investigation and modelling. It is a relatively non-standard context. The solver is required to link different information sources and translate among them. The situation demands insight, understanding and reflection to find a novel solution. The item lies at the highest level of mathematical proficiency (level 6) and has a scale score of 840.3.

At Level 6, students can conceptualize, generalize and utilize information based on their investigations and modelling of complex problem situations, and can use their knowledge in relatively non-standard contexts. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understanding, along with a mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for attacking novel situations. Students at this level can reflect on their actions, and can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situation.

The level 1 of the PISA proficiency scale is further divided into three parts for PISA-D test. To understand level 1a, 1b and 1c, sample items from PISA-D (OECD, 2017) are discussed below.

Mei-Ling found out that the exchange rate between Singapore dollars and South African rand was

$$1 \text{ SGD} = 4.2 \text{ ZAR}$$

Mei-Ling changed 3 000 Singapore dollars into South African rand at this exchange rate. Choose a correct method from those listed. Then calculate n, the amount of South African rand Mei-Ling received after the exchange.

$$\frac{1}{4.2} = \frac{n}{3000} \qquad \frac{1}{3000} = \frac{4.2}{n} \qquad 4.2n = 3000 \qquad n = 3000(4.2)$$

Conversion of currency in the 21st century can be considered as familiar and easy to understand the context. The exchange rate is provided, i.e., all relevant information is present. There are two tasks in this item, selecting an appropriate method (given as two proportions) and performing a calculation (division or multiplication), i.e., questions are clearly defined. Thus this item can be associated with proficiency level 1 of PISA scale. Two steps are to be taken in order to solve this problem, step one selecting a method, can be associated with level 1b and performing an operation can be associated with level 1c.

1a	At Level 1a, students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are almost always obvious and follow immediately from the given stimuli.
1b	At Level 1b, students can respond to questions involving easy to understand contexts where all relevant information is clearly given in a simple representation (for example tabular or graphic) and defined in a short syntactically simple text. They are able to follow clearly prescribed instructions.
1c	At Level 1c, students can respond to questions involving easy to understand contexts where all relevant information is clearly given in a simple, familiar format (for example a small table or picture) and defined in a very short syntactically simple text. They are able to follow a clear instruction describing a single step or operation.

Nick wants to pave the rectangular patio of his new house with bricks. The patio has a length of 5.25 metres and a width of 3.00 metres. One box of bricks can pave 2 square metres.

Calculate the number of boxes of bricks Nick needs for the entire patio.

This item resembles regular school book problems. In order to solve this problem, two steps are required. In the first step, the area of the rectangular patio is to be found, in the second step numerical value of the area is to be divided by 2, to calculate the number of boxes required. In case a student finds only the area of the rectangular space, proficiency level 1b is addressed as multiple processes in understanding what must be done, devising a strategy and performing the calculations is involved. Proficiency 1a is addressed if a student does both steps correctly.

5.3.2. NCERT Curriculum-based competency items and responses

Sample task 1: School fete muffins

The unit 'School fete muffins' is an example of a task in the 'Change and relationship' strand of the PISA mathematical content domain. In the NCERT curriculum, the mathematical content domain is 'Number and Algebra' and the chapter is 'Comparing Quantities' in class 8.

'School fete muffins' is based on a familiar context where all relevant information is present, and the questions are clearly defined. Students must identify relevant information and carry out routine procedures under constrains given in the problem.

Unit name: School fete muffins

For the school fete, four class 9 students are going to make muffins. A local baker donated 10 kilograms of flour and the school is supplying the other ingredients. The ingredients for one batch, which consists of 12 muffins, is given below:

- 240g of flour
- 60 grams sugar
- 1 egg
- ¼ cup oil
- 1 cup milk

The students want to make as many muffins as possible using the 10kg of flour.

Question 1: How much of each ingredient (excluding flour) is needed?

Scoring

Full Credit

Code 1:

- Sugar -2500 grams or 2.5 kilograms

  Eggs – 41

  Oil – 10 ¼ cups
  Milk – 41 cups

No Credit

Code 0: Other responses

QUESTION INTENT

Description: Use proportional reasoning to find the quantity of each ingredient

Mathematical content area: Change and relationship

Competency: Employing mathematical concepts, facts, procedures and reasoning

The PISA competency required to answer the question is 'Employing mathematical concepts, facts, procedures and reasoning'. A student is required to calculate the number of 240 grams flour lots in 10000 grams flour. Once the number of batches are found then the student can work out the quantity of other ingredients.

Question 2. The cost of the ingredients is estimated to be Rs 250 per batch. How much money can be earned by selling the muffins for Rs 30 each, excluding the cost of the ingredients? Show your working.

Scoring

Full Credit

Code 1:

- Rs 4510

No Credit

Code 0: Other responses

QUESTION INTENT

Description: Use the concept of profit and loss in real life context

Mathematical content area: Change and relationship

Competency: Employing mathematical concepts, facts, procedures and reasoning

Context: Personal

The competency required to answer the question is 'Employing mathematical concepts, facts, procedures and reasoning'. A student is required to calculate the number of muffins that can be made and the cost per muffin. Money earned can be calculated by subtracting the cost of muffins from the money collected by selling them.

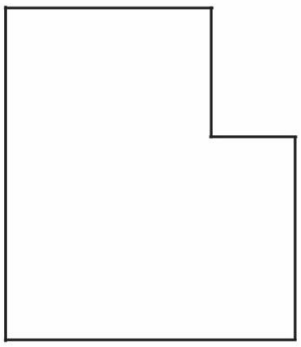
Sample task 2: Asha's Courtyard

The unit 'Asha's Courtyard' is an example of a task in the 'Space and shape' strand of the PISA mathematical content domain. In the NCERT curriculum the mathematical content domain is 'Measurement' and the chapter is 'Mensuration' class 8.

'Asha's Courtyard' is based on a familiar context where a student has to identify the information required to answer the question.

Unit name: Asha's Courtyard

Asha lives in a traditional house. The rooms in the house are constructed around a central courtyard. The shape of the courtyard is shown below.



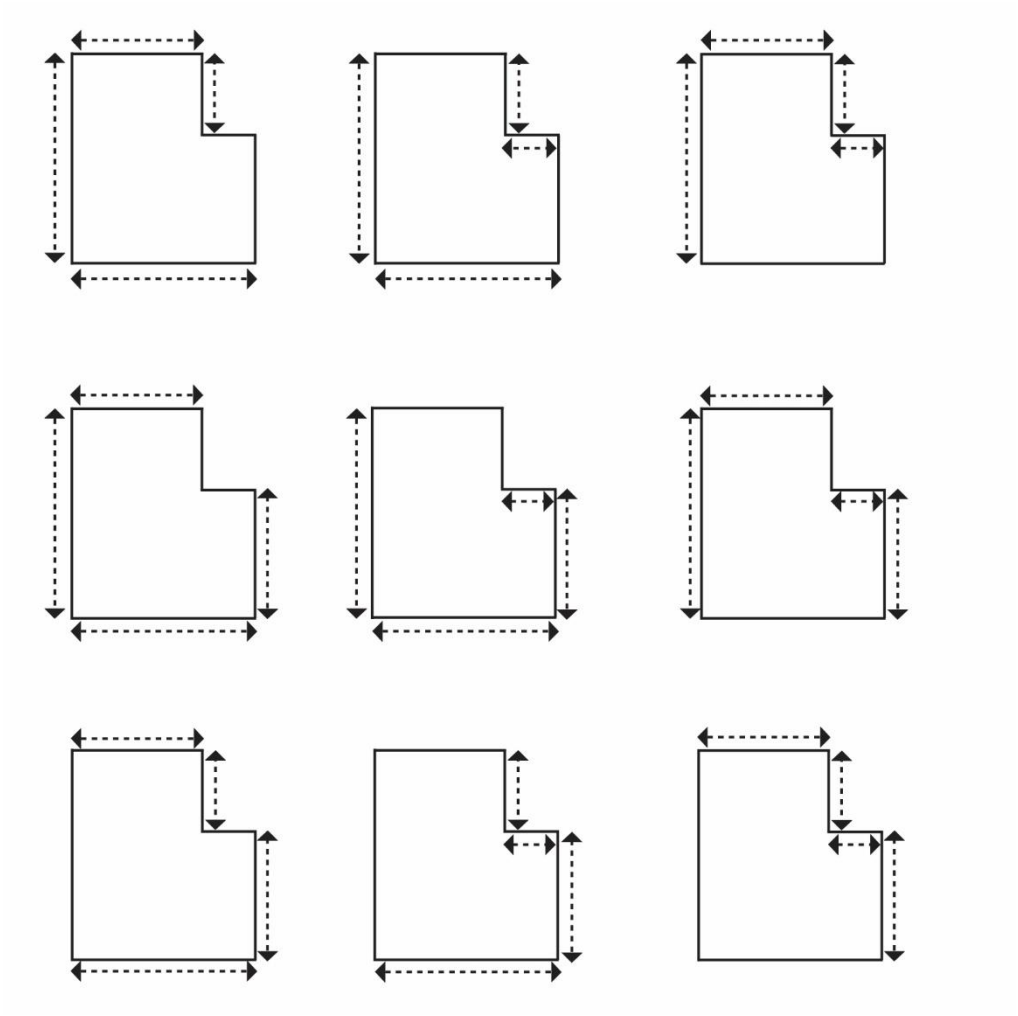
Question 3: To estimate the total floor area of the courtyard, Asha can use different methods like dividing the floor into squares with a side length of 1 metre and adding the number of squares.

However, there is a more efficient method to estimate the total floor area where she only needs to measure the length of four sides. On the figure above mark the four sides that need to be measured to estimate the total floor area of the courtyard.

Scoring

Full Credit

Code 1: Has indicated the four dimensions needed to estimate the floor area of the courtyard on the figure. There are 9 possible solutions as shown in the diagrams below.



No Credit

Code 0: Other responses.

Code 9: Missing.

QUESTION INTENT:

Description: Use spatial reasoning to show on a shape the minimum number of side lengths needed to determine floor area

Mathematical content area: Space and shape

Competency: Formulating situations mathematically

Context: Personal

The competency required to answer the question is 'Formulating situations mathematically'. A student is required to convert a real-life situation into a mathematical structure. Once the student recognises that the area of the floor can be calculated by subtracting the area of the smaller rectangle from the larger rectangle (or another similar method, for example, a student can partition the figure into two rectangles to get the relevant area using four side lengths from the figure), he can identify the four required sides.

Sample task 3: New Year Holidays

The unit 'New Year Holidays' is an example of a task in the 'Change and relationship' strand of the PISA mathematical content domain. In the NCERT curriculum, the mathematical content domain is 'Number and Algebra'. The chapter is 'Pair of Linear Equations in Two Variables' in class 10.

'New Year Holidays' is based on a complex context, in which students are required to identify constraints and specify assumptions. They have to select, compare and evaluate appropriate problem-solving strategies to arrive at the solution. Students have to reflect on their work, formulate and communicate their interpretations and reasoning.

Unit name: New Year Holidays

Giri and his family celebrated the New Year holidays in the mountains. They travelled to a hill station in their car and stayed in a resort. Giri chooses four nearby places connected by motorable roads to visit, and drew them on a map (shown below). He estimated that they need 3 hours each for visiting Adventure Park or Beautiful Falls, excluding travel time.

Question 4: The family plans to drive to Heights Hill for trekking and visit Adventure Park on New Year's Day. At Height Hills they need to walk 3 km uphill and the same distance downhill. The family estimates that they can walk up the mountain at 1.5 kilometres per hour on average, and down at twice that speed. These speeds take into account meal breaks and rest times.

The distance between Adventure Park and Heights Hill can be covered in 30 minutes by car.

Using Giri family's estimated speeds, what is the latest time they can start from resort so that they can start their journey back at 5 pm, from Adventure Park?

Scoring

Full Credit

Code 1: Any time between 9:30 am to 9:48 am

No Credit

Code 0: Other responses

QUESTION INTENT:

Description: Calculate the start time for a trip given two different speeds, a total distance to travel and a finish time

Mathematical content area: Change and relationships

Competency: Formulating situations mathematically

Context: Societal

The competency required to answer the question is 'Formulating situations mathematically'. A student is required to convert a real-life situation into a mathematical structure. Once the student recognises that start time can be calculated by finding the time required for each activity and the constraints involved in it, they can select and use different problem-solving strategies to find the start time.

Question 5: The average speed of a car in the mountain region is 20 km/hr. Instead of planning to visit Heights hill and Adventure Park on New Year's Day, if the family had planned for Adventure Park and Beautiful falls, would the family be able to finish at 5 pm? Justify your answer.

Scoring

Full Credit

Code 1: Yes/No with logical reasoning

- Yes, they have to spend 1 hour 30 minutes in travelling, if they start early they can visit both the places
- Yes, travelling time increased by 48 minutes, they have to take care of it
- No, the travelling time increased by 48 minutes, moreover on New Year's day there can be lot of traffic jams in mountain region if they start at the same time they won't be able to finish by 5:00 pm

No Credit

Code 0: Other responses

QUESTION INTENT:

Description: Analyse the feasibility of mathematical outcome under the specified constraints

Mathematical content area: Change and relationships

Competency: Interpreting, applying and evaluating mathematical outcomes

Context: Societal

The competency required to answer the question is 'Interpreting, applying and evaluating mathematical outcomes'. A student is required to evaluate the feasibility of a mathematical outcome in a real-life scenario. Students have to reflect on their work and communicate their interpretations and reasoning.

Sample task 4: Smart Purchase

The unit 'Smart Purchase Equated monthly instalment' is an example of a task in the 'Uncertainty and Data' strand of the PISA mathematical content domain. In the NCERT curriculum, the mathematical content domain is 'Chance and Data' and the chapter is 'Data Handling' class 8.

'Smart Purchase' is based on a complex context where all relevant information is present, and the questions are clearly defined. Students must understand the complexity of the situation, interpret the information given and compare them to get the result.

Unit name: Smart Purchase

In recent times, a popular method of purchasing expensive items is using Equated Monthly Instalments (EMI). An EMI is a monthly amount paid by a purchaser to a seller for a fixed period of time. Banks offer plans to make payments through debit/credit cards in the form of EMI. The purchaser who opts for buying an item through the EMI option needs to pay the price of the item and the interest charged. These amounts are added together and split into equal parts to arrive at the amount to be paid in each instalment. The total amount a purchaser has to pay is called 'Total amount payable'.

Jai plans to purchase a TV. The TV costs ₹25,999. Jai can pay a maximum of Rs 3000 every month after accounting for all his expenses. He decides to make the purchase using the EMI option.

He has two options to choose from. Delight Bank and Kenstar Bank offer EMI plans on the TV brand Jai chooses, as shown in the tables below.

EMI plans offered by Delight Bank

Time period	Equated monthly installment (EMI)	Rate of interest	Total amount payable
6 months	₹4,499/month	13%	₹26,994
12 months	₹2,322/month	13%	₹27,864
18 months	₹1,598/month	13%	₹28,764
24 months	₹1,236/month	13%	₹29,664

EMI plans offered by Kenstar Bank

Time period	Equated monthly installment (EMI)	Rate of interest	Total amount payable
6 months	₹4,493/month	12.5%	₹26,958
12 months	₹2,316/month	12.5%	₹27,792
18 months	₹1,604/month	13.5%	₹28,872
24 months	₹1,242/month	13.5%	₹29,808

Question 6: Which plan do you think is best for Jai? Why?

Full Credit

Code 2: Student offers a complete explanation of why the plan is best plan using given mathematical facts to support his/her explanation. Sample answers

- 12 months EMI plan offered by Kenstar Bank as it charges lower rate of interest than Delight Bank.

- 12 months EMI plan offered by Kenstar Bank because total payable amount is less.
- 18 months EMI plan offered by Delight Bank as it charges lower rate of interest.
- 18 months EMI plan offered by Delight Bank because total payable amount is less.
- 24 months EMI plan by Delight Bank as instalment is the lowest.

Partial Credit

Code 1: Student offers a partial explanation of why the plan is best. The mathematical reasoning provided is incorrect or incomplete. Sample answers

- 12 months EMI plan offered by Kenstar Bank is best for Jai.
- 24 months EMI plan by Delight Bank is best for Jai.

No Credit

Code 0: Other responses, only name of bank or time period without valid mathematical justification. Sample answers

- Kenstar Bank
- Delight Bank
- 12 months EMI plan
- 24 months EMI plan

QUESTION INTENT:

Description: Analyse a given situation for the best bargain

Mathematical content area: Uncertainty and data

Competency: Interpreting, applying and evaluating mathematical outcomes

Context: Societal

The competency required to answer the question is 'Interpreting, applying and evaluating mathematical outcomes'. A student is required to compare different options in order to find the most suitable plan. They need to interpret the given data and communicate the mathematical reasoning behind their choice.

5.4. Distribution of items in a PISA mathematics test

In a balanced assessment like PISA, approximately equal weightage is given to formulation and interpret, apply and evaluate the outcome. Almost half of the questions in a PISA test are focused on employ competency. Table 5.5 shows the desired distribution of competencies in PISA.

Table 5.5: Distribution of mathematical competencies in PISA

Competency	Percentage of items in PISA
Formulating situations mathematically	25
Employing mathematical concepts, facts, procedures and reasoning	50
Interpreting, applying and evaluating mathematical outcomes	25

Source 5-5: OECD (2018). PISA Mathematics Framework. OECD (2018). https://pisa2021-maths.oecd.org/files/PISA%202021%20Mathematics%20Framework%20Draft.pdf

The mathematics content areas (Change and Relationship, Space and Shape, Quantity, and Uncertainty and Data) are given equal weightage in the PISA. The items are equally distributed in four content areas since knowledge in all domains of mathematics is critical in achieving mathematical literacy. Table 5.5 shows the content-wise distribution of items.

Table 5.6: Distribution of mathematical competencies in PISA

Content	Percentage of items in PISA
Change and Relationship	25
Space and Shape	25
Quantity	25
Uncertainty and Data	25

Source 5-6: OECD (2018). PISA Mathematics Framework. OECD (2018). https://pisa2021-maths.oecd.org/files/PISA%202021%20Mathematics%20Framework%20Draft.pdf

In PISA, multiple questions are asked on one context. The number of items asked on different contexts in a test is evenly distributed, i.e., 25% of items related to personal contexts, 25% of items related to occupational contexts, 25% of items related to societal contexts and rest 25% of items related to scientific contexts.

Table 5.7: Distribution of contexts in PISA

Context	Percentage of items in PISA
Personal	25
Occupational	25
Societal	25
Scientific	25

Source 5-7: OECD (2018). PISA Mathematics Framework. OECD (2018). https://pisa2021-maths.oecd.org/files/PISA%202021%20Mathematics%20Framework%20Draft.pdf

Three types of item formats are used to assess mathematical literacy in PISA.

- Open constructed-response
- Closed constructed-response
- Selected-response (multiple-choice) items

A PISA test contains an approximately equal distribution of item format.

5.5. Concluding remarks

In this chapter, the discussion was around mathematical proficiencies, capabilities and progressions. It also explains how all three of them are interrelated in PISA. Mathematical proficiency was considered the sum of conceptual knowledge, effective use of strategies, metacognition and an individual disposition towards mathematics. The level of mathematical proficiency can indicate the mathematical literacy acquired by an individual. As some of the characteristics of mathematical proficiency cannot be measured directly, it is required to link it with the curriculum, which is traditionally assessed by education systems, leading to the concept of learning progressions.

"A learning progression is a continuum that maps key stages in the development of a learning domain from simple beginnings through to complex interpretations and applications" (ACER Centre for Global Education Monitoring, 2020). For reporting students' mathematical proficiency, the level of learning in a learning progression is reported on a scale. The PISA reporting scales are made of learning progression from foundational skills to upper secondary education. They describe developmental progress of key skills, knowledge and understanding that are readily observable and practical to assess in a standardised manner (ACER Centre for Global Education Monitoring, 2020).

Several examples of the PISA mathematical literacy proficiency scale were discussed to illustrate the concept of learning progressions and proficiencies further, in the chapter. The PISA proficiency scale is divided into six levels, based on item difficulty and students' performances. Each successive level of this scale is associated with tasks of increased difficulty. These levels are used to interpret students' performance scores and compare the performance of students at school, national, or international level over the years.

Along with PISA released items, examples based on NCERT curriculum are discussed in the chapter. The examples illustrate how the PISA competencies can be assessed through curriculum topics.

6. USING ASSESSMENT IN THE TEACHING LEARNING PROCESS

6.1. Introduction to rubrics

To optimise the effect of competency-based education in the teaching-learning process, teachers should accept the approach of competency-based assessments and interpret results based on the approach (Guskey, 2003). In the traditional approach, most teachers consider assessment and teaching-learning as two different processes. Traditionally, assessments are viewed as evaluation of learning, rather than the information about what students know at the time of the assessment and how they can be guided to learn more. A competency-based system emphasises that assessment is an integral part of the teaching-learning process, which can help teachers define students' current learning level and guide them along with their learning continuum.

Competency-based assessments can pave the path of learning improvement, provided that they are not used as a mere indicator to rank schools and students. Competency-based assessments, particularly formative assessment, provide diagnostic information about both teaching and learning when administered regularly. Assessments, when aligned with learning outcomes, help students achieve their learning goals. Diagnostic information from the assessment can be extracted without the use of sophisticated statistical techniques. A simple counting method, like how many students missed each assessment question or failed to meet specific criteria, can give information about the challenges. In case half of the class was unable to answer or have not reached the desired criteria, corrective instructions are required (Guskey, 2003). Another way of getting information is to use rubrics.

The key idea of competency-based assessment is utilising rubrics for assessments. Rubrics indicate what is expected from students, what are they required to do, and how they will be graded. If this is communicated to students, they can participate actively in the assessment. They become aware of the critical elements being assessed and are able to demonstrate their achievements and learning productively.

Involving students in the development of rubrics enables them to take responsibility for their learning by ensuring that all requirements for the assignment are met. Rubrics can serve as a tool to promote student accountability for their learning. When rubrics are used only for giving grades in the assessment, it deprives students the opportunity for self-learning as they become mere recipients rather than active partners in the assessment.

Research confirms that rubrics can be used for evaluation as well as teaching. Teachers who use assessment as a tool of instruction, and provide corrective feedback help students learn better. Teachers can explicitly list the assessment criteria to enhance the alignment of learning, instruction, and assessment (Andrade, 2005). When student performances best indicate the intended learning outcomes—things students would do, make, say, or write, then rubrics are the best way to assess them (Brookhart, 2013). Rubrics can be defined for processes like mathematical communication, reasoning, problem-solving as well as a product like mathematical models. For example, in a collaborative task, the rubric can be used to provide consistency and description of the performance. The results can be used to provide feedback for teachers and students.

The quality of assessment heavily depends upon the tool used for it. A robust assessment tool, coupled with a well-defined rubric, can provide precise information about the learning level of students. Rubrics, when used accurately, can serve as a link between assessment and learning.

6.2. Rubric for learning

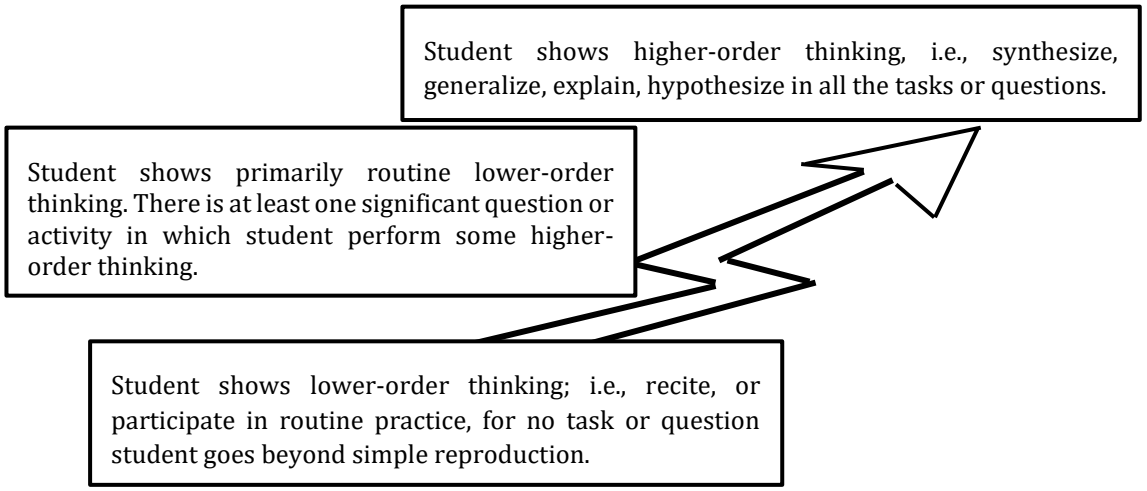
In general, rubrics evaluate the series of tasks or criteria required to produce a quality finished product. A well-written rubric defines the quality of and proficiency of students required for an assessment task. It describes the qualitative aspect of a task and provides benchmarks to attain it. Students can use a rubric to plan their work, along with its qualitative characteristics. A rubric is also a set of guidelines for giving scores or grades. It can provide feedback in terms of marks or direction for further improvement. A typical rubric consists of several possible performance levels and a scale to help raters assign marks or grades to given work properly (Jandris, 2001, 84).

Using a rubric for scoring can provide diagnostic information to students. However, it can also be judgemental. A well-defined assessment rubric provides information about how students are progressing towards excellence in their learning. An assessment rubric is expected to include clear level descriptions to convey the desired learning or competencies at a particular level in mathematics and should include preselected criteria. Table 6.1 shows an example of a learning rubric in mathematics.

Figure 6.1: Demonstration of progression in higher order thinking skills

Learning rubric for higher-order thinking skills

Higher-order thinking requires students to manipulate information and mathematical knowledge flexibly to produce new meaning and implications. This process requires students to combine facts and ideas in order to synthesise, generalise, explain, hypothesise or arrive at some solution or interpretation. Manipulating information and ideas through these processes allows students to solve problems and discover new (for them) meanings and understandings. Lower-order thinking occurs when students retrieve previously acquired knowledge of mathematical facts, rules and algorithms. Previously acquired knowledge can range from simple facts and information to more complex concepts.



Source 6-1: Adapted from A guide to productive pedagogies. Queensland Government Education, Queensland.[1]

This rubric describes a path for achieving higher-order thinking skills for students. It can also be used by teachers to evaluate whether a student has achieved a particular level of higher-order thinking skills.

6.3. Types of rubrics

Effective rubrics are useful for both students and teachers, and they can be customised to serve specific purposes. Rubrics, which provide information about single criteria or multiple criteria at a time, can be general or task-specific. Each composition serves for both summative and formative purposes. In a holistic rubric, a task is described in terms of the overall quality. They are often used to describe the overall performance of a student (Whittaker, Salend, and Duhaney, 2001). When a task is broken into subparts or components of performance and includes the various levels of each subpart or component, the rubric used is called an analytical rubric. Both holistic and analytical rubric can be task-specific or general.

Rubrics to measure student progress in learning and inform teaching are often used for formative assessment. Rubrics with marking scheme can be used in awarding grades in summative assessment (Jackson and Larkin 2002). Some researchers (Brookhart, 2013) argue that focusing on the criteria one at a time provides accurate feedback for teachers and students. But developing extensive rubric for each criteria is time-consuming and impractical for classroom assessments. Holistic rubrics, reporting the overall performance of student in a task are useful when an assessment is administered over a large group, and faster results are required.

A task-specific rubric is also useful when scoring is required. It helps in maintaining consistency in grading assignments. A task is divided into various components defining the task, and a qualitative description of the performance for each component is provided. General rubrics use criteria and descriptions of performance appropriate for different tasks related to the same competencies, for example, mathematical problem-solving. The criteria point to different subcomponents of a competency not related to one specific task.

An example of a rubric on using mathematical strategies showing four levels of proficiency is given in the table 6.1.

Table 6.1: Four levels on use of mathematical strategies

Criteria	Emerging	Developing	Proficient	Exemplary
Identification of strategy	Strategies used are not appropriate for the problem.	Strategies are appropriate but not the most efficient for the task.	Appropriate and efficient strategies are used.	Innovative and insightful strategies are used.
Justification of processes	Work is not supported by	No explanation of for using a strategies.	All steps are supported by	Solution is proved and

[1] Retrieved from https://digitised-collections.unimelb.edu.au/bitstream/handle/11343/115606/scpp-00431-qld.pdf?sequence=1#:~:text=Productive%20Pedagogies%20are%20classroom%20strategies,knowledges%20and%20skills%20than%20others.

Criteria	Emerging	Developing	Proficient	Exemplary
	mathematical reasoning.		mathematical reasoning.	approach is valid.
Appropriateness of mathematical representation	There was no apparent relationship between mathematical representations and the given task.	Not all mathematical representations are related to given task.	Mathematical representation(s) fits the task.	Examples and counter examples are provided
Logical order of justification	Steps were not logical	Not all steps are supported by logic.	All steps are defined in logical way	Sophisticated approach is used to complete the task.

Source 6-2: Arter, J., & McTighe, J. (2001). Scoring rubrics in the classroom: Using performance criteria for assessing and improving student performance. Corwin Press.

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In this rubric, the descriptions of performance are general, so students get feedback about general competencies and not task-specific skills. This type of rubric has an advantage over task-specific rubrics in that it can be shared with students at the beginning of an assessment to help them plan their work. Although rubric creation is time consuming if this approach is used, teachers do not need to rewrite the rubric for every assignment.

6.4. Steps for constructing a learning rubric

Rubrics have the potential to guide the teaching-learning process, provided specific criteria are kept in mind while developing them. Rubrics which are not aligned to competencies required for a task may create confusion among students; they become unsure of expectations from a given task. Sometimes, it may lead to students' development of dependence on rubrics rather than developing the art of learning from it (Glasgow & Hicks, 2003).

Focus of writing or selecting rubrics should be on learning rather than the way a topic is taught, or a specific task can be done. Since rubric creation is a time-consuming task, it is recommended that most rubrics be designed for repeated use on several tasks. The criteria and performance-level descriptions in rubrics should indicate the desired performance and what can be the next step to enhance the quality of performance (Brookhart, 2013).

Each evaluative criterion of a rubric must represent a key attribute of the skill being assessed. Rubrics that are aligned with learning outcomes provides a useful way to measure progress toward the standards. However, teachers may find it beneficial to develop additional rubrics for specific purposes.

Table 6.2: Questions to consider

Before starting a rubric development for a task the following questions should be considered

- What is the desired outcome of the task?
- What is the best possible performance on the task?
- How to decide what is the best performance on the task?
- What contributes to it being the best?

Source 6-3: Adapted from Educational Research Service. (2014). Developing and Using Instructional Rubrics. Arlington, VA 22201-2908.

There are no hard and fast rules for designing a rubric; however, the steps provided below can act as general guidelines. (https://champlain.instructure.com/courses/200147/pages/rubric).

- Determine the elements or criteria used to evaluate the work. (Performance Elements)
- List all of the criteria or traits that must be present in the student's work to ensure that it is high in quality.
- Identify the difference between good work and weaker work. (Performance Levels)
- Use three to -five levels (Beginning, Fundamental, Practicing, Inspiring; Unacceptable, Developing, Acceptable, Exemplary; Novice, Developing, Proficient, Expert)
- Describe the procedures used for making judgments (or assigning scores). (level descriptions)
- Write clear descriptions of the types of work assigned to each category or level of achievement. These descriptions will help users apply the rubric consistently over time, increasing the reliability and perceived fairness of the evaluation process.

6.5. Constructing rubric: examples

The guidelines given above are good indicators of the structure of a rubric. However, writing the 'language' of the descriptors may be the most challenging aspect of developing a rubric. The language should make fine discriminations between performance and products without compromising reliability. Another point required in a rubric is a balance between specificity and generalisation. It should be related to the mathematics curriculum as well as to the specific aspect of the performance or product. To make rubrics usable for students the language in the description should aid self-correction (Wiggins, 1998). The examples illustrate how these guidelines are useful in developing rubrics.

6.5.1. Designing Problem Solving Rubrics

Problem solving is a competency that does not need to be task specific but can be generalised. Therefore, a rubric developed for this competency can be used multiple times or in multiple areas. Problem-solving is a critical component of mathematics learning. This competency is required to be proficient on number & algebra, measurement & geometry, data and probability; thus, a robust problem-solving rubric can be used for teaching instruction as well as evaluation in mathematics.

Determining elements of problem-solving

In general, problem-solving is a systematic process involving five stages, namely, identifying a problem, defining and representing the problem, exploring possible strategies to solve the

problem, acting on the strategies, and looking back and evaluating the effect of the chosen strategies. Thus, the performance elements can be

- Understanding the problem and translating it into mathematical representations (Formulate)
- Using strategies to solve the problem (Strategies, Reasoning, Procedures)
- Interpretation & evaluation (Communication)

Defining performance levels

Different criteria could be used for defining performance levels, for example in the problem solving rubric criteria could be

- Unacceptable
- Developing
- Acceptable
- Exemplary

Describing performance levels

Once the performance levels have been decided upon, a description for each level for the various competencies can be developed. Table 6.3 shows example of problem solving rubric.

Table 6.3: Example of problem solving rubric

Competency	Unacceptable	Developing	Acceptable	Exemplary
Formulate	• The mathematical representation of the task does not match the expression mentioned or mathematical representation is not available. • Inappropriate concepts are applied in representation.	• The solution indicates incomplete understanding of the problem statement and an attempt is made to represent it with appropriate symbols and notations. • The solution includes some, but not all of the mathematical components presented in the task.	• Mathematical representation is appropriate to the problem and indicates an attempt to use correct mathematical symbols and notations with some errors. • The solution includes or implies use of all of the mathematical components presented in the task.	• The solution shows a deep understanding of the problem, including the ability to identify the appropriate mathematical concepts and the information necessary for its solution. • The solution includes or implies use of the underlying mathematical concepts upon which the task is designed.
Strategies, Reasoning, Procedures	• No evidence of a strategy or procedure, or uses a strategy that does not help solve the problem. • No evidence of mathematical reasoning. • There were so many errors in implementing mathematical procedures	• Uses a strategy that is partially useful, leading partly towards a solution, but not to a full solution to the problem. • Some evidence of mathematical reasoning. • Could not completely carry out mathematical procedures.	• Uses a strategy that leads to a solution to the problem. • Uses effective mathematical reasoning. • Mathematical procedures used.	• Uses efficient and sophisticated strategy leading directly to a solution. • Supports all the stages in the solution with mathematical reasoning and use of logical steps • Applies procedures accurately to solve the problem and verify the results correctly.

Competency	Unacceptable	Developing	Acceptable	Exemplary
	that the problem could not be solved.			
Communication	• There is no explanation of the solution, the explanation cannot be understood, or it is unrelated to the problem. • There is no use, or incorrect use, of mathematical terminology and notation.	• There is an incomplete explanation. • There is some use of mathematical terminology and notation appropriate of the problem.	• There is a clear explanation. • There is an appropriate use of mathematical representation.	• There is a clear, compelling explanation detailing how the problem is solved. All of the steps are included; a reader does not need to infer how and why decisions were made. • Mathematical representation is actively used as a means of communicating ideas related to the solution of the problem. There is a precise and appropriate use of mathematical terminology and notation.

6.5.2. Designing rubric for deep understanding of a topic (Numbers)

In a particular mathematical topic, knowledge is in-depth when students can develop relatively complex understandings of the central concepts. Instead of being able to reproduce the mathematical terminologies and procedures learned in a classroom, students develop relatively systematic, integrated or holistic understandings. Proficiency is demonstrated by their success in producing new knowledge by discovering relationships, solving problems, constructing explanations, and drawing conclusions.

Determining performance elements

While learning about numbers including, natural numbers, whole numbers, decimals, integers, rational numbers, students follow a sequential approach, they learn to identify a number, define it, and know its properties, give examples and counterexamples, can perform a mathematical operation, solve problems, provide proofs to statements with reasoning. Thus performance elements can be

- Identify & compare numbers
- Perform mathematical operations
- Solve problems

Defining performance levels

Different criteria could be used for defining performance levels, for example, in the problem solving rubric. A list of suggested criteria are provided below:

- Novice
- Developing
- Proficient
- Master

Describing performance levels

Once the performance levels have been decided upon, a description for each level for the various competencies can be developed. Table 6.4 shows example of rubric for deep understanding of numbers.

Table 6.4: Example of rubric for deep understanding of numbers

Performance	Novice	Developing	Proficient	Master
Identify & compare numbers	Identifies and compares numbers taught in the classroom	Understanding of numbers is uneven. Identifies or compares numbers only in a few instances.	Understanding of numbers is even. Identifies or compares correctly most of the times.	Can give examples and counterexamples of numbers (based on definitions of types of numbers).
Perform mathematical operations	An incorrect algorithm is used.	A correct algorithm is used, but there are many computational errors.	A correct algorithm is used but with one or two computational errors.	Demonstrates own ways of performing algorithms.
Solve problems	Does not show any evidence of understanding problems.	Solves problems with some help from peers/teacher.	Solves familiar problems independently.	Demonstrates complex understanding by arriving at a reasoned, supported conclusion; or explains how a complex problem was solved.

Once a rubric is developed, it is a good idea to assess its quality. Quality can be assessed with the help of peers or students. The following considerations should be kept in mind while reviewing a rubric.

- Criteria being assessed are clear, appropriate and non-overlapping
- The distinction between levels is apparent; each level is distinct and progresses in exact, logical order.

The rubric should be used as a reference point for discussion and guidance for a course or assignment as well as evaluation of the assignment. In an ideal situation, peers and learners should be involved in the process of development.

6.6. Concluding remarks

In this chapter, rubrics are projected as a bridge between assessment and the teaching-learning process. When students are provided with a rubric at the beginning of a unit of instruction or task, they can work accordingly. It helps them receive feedback, practice, revise or do another task till they complete the unit or receive a grade, resulting in complete alignment of learning outcomes, the teaching-learning process, and assessment.

A rubric can be general or task-specific, holistic or analytical, process or product oriented. The two main components of a rubric are criteria and descriptions of levels of performance. It has been emphasised in this chapter that the focus of writing criteria should be on learning outcomes, not aspects of the task itself. The descriptions of performance levels should not overlap and should help teachers in evaluation and instruction as well as guide students to self-evaluate and improve their performance. The evaluation aspect of assessment should be done by comparing student work with the description of the performance rather than categorising and labelling students.

7. Chapter-wise mapping of NCERT learning outcomes toward PISA competencies

7.1. Grade 6

Chapter No.	Chapter/Topic Name	Content	Strand	LO Code	Learning Outcome	Measuring the LOs	Learning Objectives	Suggested Learning Objectives	PISA competency			Suggestive example of pedagogical strategies
									Formulating situations mathematically	Employing mathematical concepts, facts, procedures and reasoning	Interpreting, applying and evaluating mathematical outcomes	
1	**Knowing our numbers**											Mathematical Dialogues
1.1	Introduction		Number & Algebra									
1.2	Comparing numbers	Place value	Number & Algebra	N.6.1	Solves problems involving large numbers by applying appropriate operations (addition, subtraction, multiplication and division)	Applies appropriate operations (addition, subtraction, multiplication and division) in order to solves problems involving large numbers	Find the place value of the digit and numbers	Find the place value of a digit	No	Yes	No	
							List the total numbers which can be made from the given digits and know the place value of the digit in the number	Apply the concept of place value	No	Yes	No	
							Arrange the digits of a given number and make smaller or bigger number		Yes	No	No	
							Add 1 to the greatest 1 digit,2-digit, 3-digit number and so on and get the smallest next digit number	Apply the concept of place value	No	Yes	No	
							Expand the given number and know the place value of a given digit in a particular number	Apply the concept of place value to write numbers in expanded form	Yes	No	No	
							Write the 6 digits number in expanded form and write its number name	Apply the concept of place value to write numbers in expanded form	Yes	No	No	
							Add and subtract one from number and find predecessor and successor of a given number	Add and subtract numbers	No	Yes	No	

						Add bigger digits numbers and understand the situations dealing with larger numbers	Add and subtract numbers	No	Yes	No
						Use places of the digits of a particular number and read it easily	Apply the concept of place value	No	Yes	No
1.3	Large numbers in practice	Estimation	Number & Algebra	N.6.1		Read the given situation and find the approximately estimated number		No	No	Yes
						Estimate the number to the nearest tens and round off		No	No	Yes
						Estimate the outcome of a number and get a quick round off number		No	No	Yes
						Round off the numbers and find their sum and difference easily		No	No	Yes
						Round off the numbers and find their product easily		No	No	Yes
1.4	Using brackets	Order of operation	Number & Algebra	N.6.1		Use bracket to solve the problem and make calculation quick and to avoid confusion	Use order of operation on whole numbers	No	Yes	No
1.5	Roman numerals	Roman Numerals	Number & Algebra	N.6.1		Write numbers in the form of roman numerals and represent and interpret the numbers written in a clock, timetable etc.		Yes	No	No
						Apply the rules of roman numbers operations and perform arithmetic operation on them	Apply mathematical operations on Roman numerals	No	Yes	No

1.6	Summary		Number & Algebra									
2	**Whole Numbers**											Metacognitive strategies
2.1	Introduction	Number sense	Number & Algebra				Use the understanding of the predecessor of one and know the whole number	Identify predecessor and successor of a given number	Yes	No	No	
2.2	Whole numbers	Number sense	Number & Algebra				Explain the whole number and know the predecessor of 1 and the subtraction of the two same number	Expand knowledge of natural numbers to understand concept of whole numbers	No	No	Yes	
2.3	The number line	Whole numbers on number line	Number & Algebra				Define 'unit distance' and construct the number line	Draw a number line	Yes	No	No	
							Draw the Number line and represent the whole number	Represent a whole number on number line	Yes	No	No	
							Draw a number line and find the predecessor and successor of a given number		Yes	No	No	
							Represent the Numbers on Number line and perform number operation	Use number line to perform mathematical operations	Yes	Yes	No	
2.4	Properties of whole numbers	Properties of mathematical operations on whole numbers	Number & Algebra				Apply properties of whole number and simplify arithmetic expression		No	Yes	No	
2.5	Patterns in whole numbers	Number pattern	Number & Algebra				Represent numbers and form line, rectangle, triangle and a square	Represent numbers with dots and form dot patterns	No	No	Yes	
							Form number patterns and verbal calculation and to understand numbers better		No	No	Yes	
2.6	Summary		Number & Algebra									

3	**Playing with Numbers**											Multiple mathematical representations of same concept
3.1	Introduction	Factors and multiples	Number & Algebra	N.6.2	Recognises and appreciates (through patterns) the broad classification of numbers as even, odd, prime, co-prime, etc. Recognises and appreciates (through patterns) the broad classification of numbers as even, odd, prime, co-prime, etc.	Identifies number patterns through factorization in order to recognise and appreciate (through patterns) the broad classification of numbers as even, odd, prime, co-prime, etc.	Arrange the numbers in a row and determine the factors of a given number	Use arrays to understand factors	No	No	Yes	
3.2	Factors and multiples	Factors and multiples	Number & Algebra				Write the factors of a given number and determine prime and composite numbers	Write the factors of a given number	Yes	No	No	
3.3	Prime and composite numbers	Prime and composite numbers	Number & Algebra					Identify and write prime and composite numbers	No	Yes	No	
3.4	Tests for divisibility of numbers	Divisibility rules	Number & Algebra				Apply the rules of divisibility and find the factors of a number quickly	Apply the rules of divisibility to find the factors of a number	No	Yes	No	
3.5	Common factors and common multiples	Common factors and multiples of two or more numbers	Number & Algebra					Identify co-prime numbers	No	Yes	No	
							Evaluate the factors of given two or more numbers and find the common factors and multiples		No	Yes	Yes	
3.6	Some more divisibility rules	Divisibility rules	Number & Algebra				Apply the rules of divisibility and find the factors of a number quickly		No	Yes	No	
3.7	Prime factorisation	Prime factorisation	Number & Algebra				Factories a number through prime factorization and list the primes factors	Factorise a number through prime factorization and list the primes factors	No	Yes	No	

3.8	Highest common factors	Highest common factors	Number & Algebra	N.6.3	Applies HCF or LCM in a particular situation	Applies the concept of HCF or LCM in order to solve problems in a real-life situation	List down the common factors of given numbers and determine their HCF		Yes	No	No	
3.9	Lowest common multiples	Lowest common multiples	Number & Algebra				List down the common multiples of given numbers and determine their LCM		Yes	No	No	
3.10	Some problems on HCF and LCM	Word problems on HCF and LCM	Number & Algebra				Apply the concept of HCF and solve related real-life problems		Yes	Yes	Yes	
							Apply the concept of LCM and solve related real-life problems		Yes	Yes	Yes	
3.11	Summary		Number & Algebra									
4	**Basic Geometrical Ideas**											Multiple mathematical representations of same concept
4.1	Introduction		Geometry	G.6.1	Describes geometrical ideas like line, line segment, open and closed figures, angle, triangle, quadrilateral, circle, etc., with the help of examples in surroundings	Provides examples from surround in order to describes geometrical ideas like line, line segment, open and closed figures, angle, triangle, quadrilateral, circle, etc.						
4.2	Points	Points	Geometry				Give example(s) and explain the importance of a point		Yes	No	No	
4.3	A line segment	Line segment	Geometry				Give example(s) and describe a line segment		Yes	No	No	
4.4	A line	Line	Geometry				Give example(s) and describe a line		Yes	No	No	
4.5	Intersecting lines	Intersecting lines	Geometry				Examine the given lines and identify intersecting lines among them	Identify intersecting lines	No	Yes	No	
4.6	Parallel lines	Parallel lines	Geometry				Examine the given lines and identify parallel lines among them	Identify parallel lines	No	Yes	No	
4.7	Ray	Ray	Geometry				Describe a ray and identify it from the given figures		Yes	Yes	No	

						Compare the given figures and identify a ray, line, line segment among them		No	No	Yes	
4.8	Curves	Curves	Geometry			Give example(s) and demonstrate an understanding of a simple curve and a curve that is not simple		No	Yes	No	
						Describe an open curve and a closed curve and distinguish between the two		No	No	Yes	
4.9	Polygons	Polygons	Geometry			Discuss the parts of a closed curve and determine the position of a point with respect to it		Yes	No	No	
						Examine the given curves and identify polygons and non-polygons		No	No	Yes	
						Draw rough sketch of a polygon and label and describe its elements		Yes	No	No	
4.10	Angles	Angles	Geometry			Identify the elements of an Angle (Vertex, arm, interior and exterior angles) for the given angles		No	Yes	No	
						Give example(s) and name an angle in the given figure		No	Yes	No	
4.11	Triangles	Triangles	Geometry			Describe the elements of a triangle and identify it among the given figures		No	Yes	No	
4.12	Quadrilaterals	Quadrilaterals	Geometry			Describe the elements of a quadrilateral and identify it among the given figures		No	Yes	No	
4.13	Circles	Circles	Geometry			Describe the parts of a circle and identify them in the given circle		No	Yes	No	
						Draw a rough sketch of a circle and label and describe its elements		No	Yes	No	

							Determine the parts of closed curves and identify the position of a point with respect to a polygon and a circle		No	Yes	No	
4.14	Summary		Geometry									
5	**Understanding Elementary Shapes**											Open ended activities
5.1	Introduction		Geometry									
5.2	Measuring Line Segments	Line Segments	Geometry	G.6.1	Describes geometrical ideas like line, line segment, open and closed figures, angle, triangle, quadrilateral, circle, etc., with the help of examples in surroundings	Provides examples from surround in order to describes geometrical ideas like line, line segment, open and closed figures, angle, triangle, quadrilateral, circle, etc.	Measure the given line segments and compare them		No	Yes	No	
5.3	Angles	Angles	Geometry	G.6.2	Demonstrates an understanding of angles by o identifying examples of angles in the surroundings o classifying angles according to their measure o estimating the measure of angles using 45°, 90°, and 180° as reference angles	In order to demonstrate an understanding of angles: a) Identifies examples of angles in the surrounding b) Classifies angles according to their measure c) Estimates the measure of angles using 45°, 90°, and 180° as reference angles	Examine the rotation of angles and classify angles based on the amount of rotation		Yes	No	No	
5.4	Angles	Angles	Geometry				Compare the given angles and classify them as a right angle, straight angle or a complete angle		Yes	No	No	
							Compare the given angles and classify them as an acute angle, obtuse angle or a reflex angle according to their measure		Yes	No	No	

							Identify the different types of angles in our surroundings and demonstrate an understanding of angles		Yes	No	No
5.5	Measuring Angles	Angles	Geometry				Use a protractor and measure the given angle and classify its type Use a protractor and draw an angle of the given measure		No	Yes	No
5.6	Perpendicular Lines	Perpendicular lines	Geometry				Describe perpendicular and a perpendicular bisector and identify the same in the given figure				
							Give example(s) of perpendicular lines and demonstrate an understanding of the same				
5.7	Classification of Triangles	Triangles	Geometry	G.6.4	Classifies triangles into different groups/types on the basis of their angles and sides. For example- scalene, isosceles or equilateral on the basis of sides, etc.	Classifies triangles with different measurements in order to show different types of triangle based on their angles and sides	Observe the measure of sides of a triangle and classify it into different types (scalene, isosceles, equilateral) based on its sides		Yes	No	No
							Observe the measure of angles of a triangle and classify it into different types (acute, obtuse, right) based on its angles		Yes	No	No
5.8	Quadrilaterals	Quadrilaterals - rectangle, square, parallelogram, rhombus, trapezium	Geometry	G.6.5	Classifies quadrilaterals into different groups/types on the basis of their sides/angles	Classifies quadrilaterals with different measurements in order to show different types of quadrilaterals based on their sides and internal angles	Examine the given figures and classify type quadrilaterals based on their properties		Yes	No	No
5.9	Polygons	Polygons	Geometry				Examine the given figures and identify polygons		Yes	No	No
							Describe polygons and classify them based on their number of sides and angles. (Up to 8 sides)		Yes	No	No
							Give example(s) and distinguish between regular and irregular polygons		No	No	Yes

5.10	Three dimensional shapes	Three dimensional shapes	Geometry	G.6.6	Identifies various (3-D) objects like sphere, cube, cuboid, cylinder, cone from the surroundings with the help of examples from surroundings	Classifies commonly found 3-d objects from the surroundings in order to find sphere, cube, cuboid, cylinder, cone etc.	Describe solid shapes and distinguish them from flat shapes		Yes	No	No	
				G.6.7	Describes and provides examples of edges, vertices and faces of 3-D objects	Labels different parts of a 3-d objects in order to explain edges, vertices and faces of the given 3-d object	Examine the given solid shapes and identify their type (Cubes, Cuboids, cylinder, sphere, cone, prism, pyramid)		Yes	No	No	
							Describe the faces, edges and vertices of a 3D shape and discuss the various aspects of the given 3D object		Yes	No	Yes	
5.11	Summary		Geometry									
6	**Integers**											
6.1	Introduction	Integers	Number & Algebra	N.6.4	Solves problem involving addition and subtraction of integers	Applies addition and subtraction rules involving positive and negative integers and solve real life problems	Represent integers with their signs and differentiate positive number, negative number and zero from each other		Yes	No	No	
							Denote numbers with their signs and represent real life situations like temperature scale, credit, debit etc.		Yes	No	No	
6.2	Integers	Positive and negative Integers Integers on number line Ordering of integers	Number & Algebra				Represent the integer on Number Line and determine its position with respect to other integers		Yes	Yes	No	
							Determine one more and one less of a given integers and find its predecessor and successor		No	Yes	No	
							Determine the order of integers and represent them on a number line and draw comparison between them		No	Yes	No	

6.3	Addition of integers	Addition of two positive integers Addition of two negative integers Addition of one positive and one negative integer Addition of integers on a number line Additive Inverse	Number & Algebra				Represent the integers on number line and perform arithmetic operations on them		No	Yes	No
6.4	Subtraction of integers with the help of a number line	Subtraction of a negative integer from another negative integer Subtraction of a positive integer from another positive integer Subtraction of a negative integer from a positive integer Subtraction of a positive integer from a negative integer	Number & Algebra				Use the rules to perform arithmetic operations on integers		No	Yes	No
6.5	Summary		Number & Algebra								
7	**Fractions**											Metacognitive strategies
7.1	Introduction	Fraction	Number & Algebra	N.6.5	Uses fractions and decimals in different situations which	Calculates fractions and decimals in different real-	Represent a number as a part of the whole and determine the fraction		Yes	No	No	

7.2	A fraction	Fraction	Number & Algebra		involve money, length, temperature etc. For example, 7 metres of cloth, distance between two places is 112.5 km etc.	life situations in order to identify the appropriate quantity of money, length, temperature etc.	Determine part and whole and label numerator and denominator of a fraction		Yes	No	No	
7.3	Fraction on the number line	Fraction	Number & Algebra				Draw equal parts between the whole numbers and represent fractions on a number line	Represent fractions on a number line	No	Yes	No	
7.4	Proper fractions	Fraction	Number & Algebra				Write proper fractions and deduce that they are always less than one/numerator is less than denominator	Identify proper fraction	Yes	No	No	
7.5	Improper and mixed fractions	Fraction	Number & Algebra				Write fractions where numerator is greater than denominator and determine improper fractions	Identify improper fraction	Yes	No	No	
							Write the improper fraction in the form of mixed fraction and represent it as a combination of whole and a part	Identify mixed fraction	Yes	No	No	
7.6	Equivalent fractions	Fraction	Number & Algebra				Multiply /Divide the numerator and denominator with the same number and find equivalent fractions	Find equivalent fractions	No	Yes	No	
							Perform cross multiplication among two fractions and verify their equivalence	Verify equivalent fractions	No	No	Yes	
7.7	Simplest form of a fraction	Fraction	Number & Algebra				Reduce the fraction and determine its simplest form		No	Yes	No	
7.8	Like fractions	Fraction	Number & Algebra					Recognize like and unlike fractions	No	Yes	No	
7.9	Comparing fractions	Fraction	Number & Algebra				Inspect the numerators of the like fractions and determine larger and smaller fraction(s)		No	No	Yes	
							Determine the LCM of the unlike fractions and compare them		Yes	No	No	

7.10	Addition and subtraction of fractions	Fraction	Number & Algebra	N.6.6	Solves problems on daily life situations involving addition and subtraction of fractions/decimals	Calculates addition and subtraction of fractions and decimals in order to solve daily life problems involving quantities that measure between two integers	Solve (addition /subtraction) the numerator and retain the denominator of the like fractions and perform addition and subtraction on the given fraction		No	Yes	No	
							Convert the given fractions into its equivalent fractions and perform addition and subtraction on them		No	Yes	No	
7.11	Summary		Number & Algebra									
8	**Decimals**											Metacognitive strategies
8.1	Introduction	Decimals	Number & Algebra	N.6.5	Uses fractions and decimals in different situations which involve money, length, temperature etc. For example, 7 metres of cloth, distance between two places is 112.5 km etc.	Calculates fractions and decimals in different real-life situations in order to identify the appropriate quantity of money, length, temperature etc.	Write rupees and paisa in decimal form and know the meaning and relevance of dot point		Yes	No	No	
8.2	Tenths	Decimals	Number & Algebra				Represent number in its unit and tenth part in order to write it in decimal form		Yes	Yes	No	
							Determine the place value of decimal numbers up to tenth and write the number in expanded form		No	Yes	No	

							Divide the numbers into ten equal parts and represent decimal numbers up to tenth place		No	Yes	No
8.3	Hundredths	Decimals	Number & Algebra				Determine the part and whole of a given decimal number and represent it in the form of fractions.		No	Yes	No
8.4	Comparing decimals	Decimals	Number & Algebra				Compare the units and parts of the decimal numbers and compare them as a whole		Yes	No	No
8.5	Using decimals	Decimals	Number & Algebra				Represent /Convert the money, length and weight into smaller units and represent it into decimal form		Yes	Yes	No
8.6	Addition of numbers with decimals	Decimals	Number & Algebra	N.6.6	Solves problems on daily life situations involving addition and subtraction of fractions/decimals	Calculates addition and subtraction of fractions and decimals in order to solve daily life problems involving quantities that measure between two integers	Add and subtract the whole and parts of decimal numbers and find their sum and difference		No	Yes	No
8.7	Subtraction of decimals	Decimals	Number & Algebra								
8.8	Summary		Number & Algebra								
9	**Data Handling**											Mathematical dialogues
9.1	9.1 Introduction	Data organisation		C.6.1	Arranges given/collected information such as expenditure on different items in a family in the last six months, in the form of table, pictograph and bar graph and interprets them	Arranges given /collected information such as expenditure on different items in a family in the last six months, in the form of table, pictograph and bar graph in order to interpret them	Observe different tables and gather the information recorded in the table of data	Organise the given information in tabular format	Yes	No	No	
9.2	Recording Data	Data organisation	Chance & Data				Group and compare raw data systematically and infer the relevant information quickly		Yes	Yes	No	
9.3	Organisation of Data	Tally marks	Chance & Data				Organize raw data into a table using tally marks and organize the given data		Yes	No	No	

9.4	Pictograph	Pictograph	Chance & Data				Observe and understand pictograph representation of data and answer the question on data at a glance		No	Yes	No	
9.5	Interpretation of a Pictograph	Pictograph	Chance & Data				Analyse pictograph and reason the information presented		No	No	Yes	
9.6	Drawing a pictograph	Pictograph	Chance & Data				Draw a pictograph and represent the given information using appropriate symbols		Yes	Yes	No	
9.7	A bar graph	Bar graph	Chance & Data				Observe bar graph and reason the information presented		No	No	Yes	
							Choose an appropriate scale and represent a given information in the form of a bar graph		Yes	Yes	No	
							Interpret bar graph and find the relevant information represented by the bar graph		No	Yes	No	
9.8	Summary		Chance & Data									
10	**Mensuration**											Mathematical dialogues
10.1	Introduction		Measurement	M.6.1	Finds out the perimeter and area of rectangular objects in the surroundings like floor of the class room, surfaces of a chalk box etc.	Calculates perimeter and area of rectangular 2-d and 3-d objects to measure them for real life objects	Give example(s) and define perimeter of closed figures		Yes	No	No	
10.2	Perimeter	Perimeter	Measurement				Deduce and apply the formula to determine the perimeter of a rectangle		No	Yes	No	
							Deduce and apply the formula to determine the perimeter of a square		No	Yes	No	

							Deduce and generalize the formula to determine the perimeter of a regular polygon		No	Yes	No	
							Give examples and defend that different shapes can have the same perimeter		Yes	Yes	Yes	
10.3	Area	Area	Measurement		Finds out the perimeter and area of rectangular objects in the surroundings like floor of the class room, surfaces of a chalk box etc.	Finds out the perimeter and area of the rectangular objects in order to calculate them for commonly found objects from the surroundings like floor of the class room, surfaces of a chalk box etc.	Count the squares and estimate the area of the given closed curve in the squares grid sheet		Yes	Yes	Yes	
							Deduce and apply the formula and determine the area of a rectangle		No	Yes	No	
							Deduce and apply the formula and determine the area of a square		No	Yes	No	
10.4	Summary		Measurement									
11	**Algebra**											Metacognitive strategies
11.1	Introduction		Number & Algebra	N.6.7	Uses variable with different operations to generalise a given situation. For example, Perimeter of a rectangle with sides x units and 3 units is 2(x+3) units	Involves use of variables with different operations to generalize a given situation and find a solution to a given problem	Describe algebraic expressions and distinguish them from arithmetic expressions		No	No	Yes	
11.2	Matchstick patterns	Patterns	Number & Algebra				Examine patterns and identify relationship in patterns		Yes	Yes	Yes	
11.3	The idea of a variable	Variables	Number & Algebra				Use variable with different operations and generalize a given situation		Yes	Yes	Yes	
11.4	More matchstick patterns	Patterns	Number & Algebra				Introduce a variable and form a rule for the given pattern		Yes	Yes	Yes	
11.5	More examples of variables	Variables	Number & Algebra				Use variable with different operations and form an algebraic expression		Yes	Yes	Yes	

11.6	Use of variables in common rules	Variables	Number & Algebra				Use variable(s) and express some mathematical rules and formulae		Yes	Yes	No	
11.7	Expressions with variables	Algebraic expression	Number & Algebra				Use variable with different operations and form an algebraic expression		Yes	Yes	No	
11.8	Using expressions practically	Algebraic expression	Number & Algebra				Change the given algebraic expression in statements and describe the situation in ordinary language		Yes	Yes	Yes	
11.9	What is an equation?	Algebraic equation	Number & Algebra				Explain the meaning of an equation and identify equations from the given options		No	No	Yes	
11.10	Solution of an equation	Algebraic equation	Number & Algebra				Use trial and error and find the solution of the given equation		No	Yes	No	
							Evaluate the given values of variable as possible solution of the equation		No	Yes	Yes	
11.11	Summary		Number & Algebra									
12	**Ratio and Proportion**											Analyse the information
12.1	Introduction		Number & Algebra	N.6.8	Compares quantities using ratios in different situations. For example the ratio of girls to boys in a particular class in 3:2	Represents the measurement as ratios in order to compare two quantities in real life	Represent two quantities in same unit and compare them		Yes	No	Yes	
							Compare two quantities and find their ratio		No	Yes	No	
12.2	Ratio	Ratio	Number & Algebra				Multiply /divide numerator and denominator by same number and find equivalent		No	Yes	No	
12.3	Proportion	Proportion	Number & Algebra				Compare ratio and determine whether they are in proportion		No	Yes	Yes	
							Solve the proportion and find out the missing term		No	Yes	No	

12.4	Unitary method	Unitary method	Number & Algebra	N.6.9	Uses unitary method in solving various word problems. For example, if the cost of a dozen notebooks is given she finds the cost of 7 notebooks by first finding the cost of 1 notebook	Uses unitary method in problem solving to calculate the quantity for one unit in order to calculate the total quantity for larger quantities	Solve daily life problems with the help of Unitary method and compute the value of one article, given the value of many		Yes	Yes	No	
12.5	Summary		Number & Algebra									
13	**Symmetry**											Eliminating possibilities
13.1	Introduction		Geometry	G.6.3	Demonstrates an understanding of line symmetry by o identifying symmetrical 2-Dimensional (2-D) shapes which are symmetrical along one or more lines o creating symmetrical 2-D shapes	In order to demonstrate an understanding of line symmetry: a) Identifies symmetrical 2-dimensional (2-D) shapes which are symmetrical along one or more lines b) Creates symmetrical 2-D shapes	Explain the meaning of symmetry and identify symmetric figures in our surrounding		Yes	No	No	
13.2	Making symmetric figures: Ink-blot devils	Symmetry	Geometry				Identify symmetrical 2-Dimensional shapes which are symmetrical along one line and demonstrate an understanding of the same		No	Yes	No	
13.3	Figures with two Lines of symmetry	Symmetry	Geometry				Draw line(s) of symmetry and classify the given shapes as shapes with no symmetry, one line of symmetry, two lines of symmetry or multiple lines of symmetry		Yes	Yes	No	
13.4	Figures with multiple (more than two) lines of symmetry	Symmetry	Geometry				Draw line(s) of symmetry and classify the given shapes as shapes with no symmetry, one line of symmetry, two lines of symmetry or multiple lines of symmetry		Yes	Yes	No	
13.5	Reflection and symmetry	Reflection and symmetry	Geometry				Draw the mirror image of the given 2D shapes or objects and identify objects with reflection symmetry		No	Yes	No	

							Give example(s) and discuss the applications of reflection symmetry in real life		Yes	No	Yes
13.6	Summary		Geometry								
14	**Practical Geometry**										
14.1	Introduction		Geometry				Discuss the different tools of construction and describe their uses		Yes	No	No
14.2	The circle	Construction of a circle	Geometry				List and execute steps of construction and construct a circle when its radius is known	Construct a circle when its radius is known	No	Yes	No
14.3	A line segment	Construction of a line segment	Geometry				List and execute steps of construction and construct a line segment when its length is known		No	Yes	No
							List and execute steps of construction and construct a copy of the given line segment		No	Yes	No
14.4	Perpendiculars	Construction of perpendicular lines	Geometry				List and execute steps of construction in order to construct a perpendicular to a line through a point on it		No	Yes	No
							List down and execute steps of construction and construct a perpendicular to a line through a point not on it		No	Yes	No
								Construct the perpendicular bisector of a line segment	No	Yes	No
14.5	Angles	Constructing of angles and angle bisectors	Geometry				Use a protractor and ruler and construct an angle of the given measure		No	Yes	No
							List and execute steps of construction and construct a copy of the given angle of unknown measure using a compass		No	Yes	No

							List and execute steps of construction and construct the bisector of an angle and construct angles of measures 30-degree, 45 degree and so on		No	Yes	No	
							List and execute steps of construction and construct angles of measures 60-degree, 90 degree and 120 degree		No	Yes	No	
14.6	Summary		Geometry									

7.2. Grade 7

Chapter No.	Chapter/Topic Name	Content	Strand	LO Code	Learning Outcome	Measuring the LOs	Learning Objectives	Suggested Learning Objectives	PISA competency			Suggestive example of pedagogical strategies
									Formulating situations mathematically	Employing mathematical concepts, facts, procedures and reasoning	Interpreting, applying and evaluating mathematical outcomes	
1	**Integers**											Logical reasoning
1.1	Introduction	Integers	Number & Algebra	N.7.1 (Pre-requisite)	Multiplies/divides two integers	Applies rules for multiplication and division in order to solve problems involving two integers with same or different signs	Differentiate between whole numbers and integers and give concrete examples		No	Yes	No	
1.2	Recall	Integers	Number & Algebra				Represent numbers with positive and negative signs and apply to various situations	Identify and represent integers on a number line	No	Yes	No	
1.3	Properties of Addition and Subtraction of Integers	Integers - Properties of operations on integers	Number & Algebra	N.7.1	Multiplies/divides two integers	Applies rules for multiplication and division in order to solve problems involving two integers with same or different signs	Represent integers on a number line and perform operations and verify properties of integers		No	Yes	No	
1.3.1	Closure under Addition	Integers - Properties of operations on integers	Number & Algebra				Apply properties of addition and subtraction of integers and simplify arithmetic expressions		No	Yes	No	
1.3.2	Closure under Subtraction	Integers - Properties of operations on integers	Number & Algebra				Apply properties of addition and subtraction of integers and simplify		No	Yes	No	

						arithmetic expressions				
1.3.3	Commutative Property	Integers - Properties of operations on integers	Number & Algebra			Apply properties of addition and subtraction of integers and simplify arithmetic expressions	Apply properties of addition and subtraction of integers to simplify arithmetic expressions	No	Yes	No
1.3.4	Associative Property	Integers - Properties of operations on integers	Number & Algebra			Apply properties of addition and subtraction of integers and simplify arithmetic expressions		No	Yes	No
1.3.5	Additive Identity	Integers - Properties of operations on integers	Number & Algebra			Apply properties of addition and subtraction of integers and simplify arithmetic expressions		No	Yes	No
1.4	Multiplication of Integers	Integers - Multiplication	Number & Algebra			Apply rules of multiplication of integers and solve various arithmetic expressions and contextual problems		Yes	Yes	No
1.4.1	Multiplication of a positive and a negative integer	Integers - Multiplication	Number & Algebra			Apply rules of multiplication of integers and solve various arithmetic expressions and contextual problems		Yes	Yes	No
1.4.2	Multiplication of two negative integers	Integers - Multiplication	Number & Algebra			Apply rules of multiplication of integers and solve various arithmetic expressions and contextual problems		Yes	Yes	No

1.4.3	Product of three or more negative integers	Integers - Multiplication	Number & Algebra			Apply rules of multiplication of integers and solve various arithmetic expressions and contextual problems		Yes	Yes	No
1.5	Properties of Multiplication of Integers	Integers - Properties of operations on integers	Number & Algebra			Apply properties of multiplication of integers and simplify arithmetic expressions		No	Yes	No
1.5.1	Closure under Multiplication	Integers - Properties of operations on integers	Number & Algebra			Apply properties of multiplication of integers and simplify arithmetic expressions		No	Yes	No
1.5.2	Commutativity of Multiplication	Integers - Properties of operations on integers	Number & Algebra			Apply properties of multiplication of integers and simplify arithmetic expressions		No	Yes	No
1.5.3	Multiplication by Zero	Integers - Properties of operations on integers	Number & Algebra			Apply properties of multiplication of integers and simplify arithmetic expressions		No	Yes	No
1.5.4	Multiplicative Identity	Integers - Properties of operations on integers	Number & Algebra			Apply properties of multiplication of integers and simplify arithmetic expressions		No	Yes	No
1.5.5	Associativity for Multiplication	Integers - Properties of operations on integers	Number & Algebra			Apply properties of multiplication of integers and simplify arithmetic expressions		No	Yes	No
1.5.6	Distributive Property	Integers - Properties of operations on integers	Number & Algebra			Apply properties of multiplication of integers and simplify	Apply properties of multiplication of integers to simplify arithmetic expressions	No	Yes	No

							arithmetic expressions					
1.5.7	Multiplication using these properties	Integers - Multiplication	Number & Algebra				Apply properties of addition, subtraction and multiplication of integers and devise methods for easier calculation and solve problems based on real life related to integers	Solve problems using properties of multiplication of integers	Yes	Yes	No	
1.6	Division of Integers	Integers - Division	Number & Algebra				Infer division of integers as inverse operation of multiplication and write multiplication statement into corresponding division statement		Yes	Yes	No	
1.7	Properties of Division of Integers	Division is not commutative For any integer a, a ÷ 0 is not defined but 0÷ a = 0 for a ≠ 0 Any integer divided by 1 gives the same integer Division is not associative	Number & Algebra				Apply properties of division of integers and simplify arithmetic expressions	Apply properties of division of integers to simplify arithmetic expressions	No	Yes	No	
1.8	Summary		Number & Algebra									
2	**Fractions and Decimals**											Metacognitive strategies
2.1	Introduction		Number & Algebra	N.7.2 (Pre-requisite)	Interprets the division and multiplication of fractions. for example interprets × as of . Also ÷ is	Applies repeated addition and subtraction in order to interpret the division and	Define proper, improper and mixed fractions and distinguish between them		No	Yes	No	
2.2	How well have you learnt	Proper fraction,	Number & Algebra					Add and subtract like and unlike fraction	No	Yes	No	

	about fractions?	improper fraction and mixed fraction, equivalent fractions and daily life examples			interpreted as how many make ?	multiplication of fractions.	Multiply (or divide) numerator and denominator with the same number and write equivalent fractions		No	Yes	Yes
							Convert unlike fractions into like fractions and compare them		No	Yes	Yes
2.3	Multiplication of fractions		Number & Algebra	N.7.2	Interprets the division and multiplication of fractions. · for example interprets × as of . Also ÷ is interpreted as how many make ?	Applies repeated addition and subtraction in order to interpret the division and multiplication of fractions.	Convert unlike fractions into like fractions and compare them		Yes	Yes	No
2.3.1	Multiplication of a fraction by a whole number	Multiplying a fraction by a whole number, Fraction as an operator 'of'	Number & Algebra	N.7.3	Uses algorithms to multiply and divide fractions/decimals	Applies algorithms for multiplication and division in order to multiply and divide fractions /decimals	Extend concept of multiplication as repetitive addition for fraction and multiply a fraction and a whole number Multiply fractions involving the term 'of' Multiply fractions and calculate the total number of parts Multiply fractions and compare the value of the product with the original fractions		No	Yes	No

2.3.2	Multiplication of a fraction by a fraction	Multiplying a fraction by a fraction	Number & Algebra				Extend concept of multiplication as repetitive addition for fraction and multiply a fraction and a whole number Multiply fractions involving the term 'of' Multiply fractions and calculate the total number of parts Multiply fractions and compare the value of the product with the original fractions		No	Yes	No	
2.4	Division of Fractions	Dividing fractions	Number & Algebra				Invert a given fraction and find its reciprocal Divide two fractions and find the smaller parts of the fraction		No	Yes	No	
2.4.1	Division of whole number by a fraction	Dividing a whole number by a fraction and reciprocal of a fraction	Number & Algebra				Invert a given fraction and find its reciprocal Divide two fractions and find the smaller parts of the fraction		No	Yes	No	
2.4.2	Division of a fraction by a whole number	Dividing a fraction by a whole number	Number & Algebra				Invert a given fraction and find its reciprocal Divide two fractions and find the smaller parts of the fraction		No	Yes	No	

2.4.3	Division of a fraction by another fraction	Dividing a fraction by another fraction	Number & Algebra			Invert a given fraction and find its reciprocal Divide two fractions and find the smaller parts of the fraction		No	Yes	No	
2.5	How well have you learnt about decimal numbers	Place Value and daily life examples	Number & Algebra			Recall and apply concept of decimal representation and expansion and perform mathematical operations on decimal		No	Yes	No	
2.6	Multiplication of decimal numbers	Examples on multiplication of decimal numbers	Number & Algebra			Find the intersection of 2 decimal numbers on the grid and represent their product		Yes	Yes	No	
2.6.1	Multiplication of decimal numbers by 10, 100 and 1000	Multiplication by 10, 100 and 1000	Number & Algebra			Multiply decimal numbers by 10, 100 and 1000 and infer right shift in decimal point		No	Yes	No	
2.7	Division of decimal numbers		Number & Algebra			Divide decimal numbers by 10, 100 and 1000 and infer left shift in decimal point Divide decimal number by a whole number and solve real life problems related to decimals Convert decimals into fractions and divide decimal number by another decimal number		No	Yes	No	

2.7.1	Division by 10, 100 and 1000	Dividing decimal number by 10, 100 and 1000	Number & Algebra				Divide decimal numbers by 10, 100 and 1000 and infer left shift in decimal point Divide decimal number by a whole number and solve real life problems related to decimals Convert decimals into fractions and divide decimal number by another decimal number		No	Yes	No	
2.7.2	Division of a decimal number by a whole number	Dividing decimal number by a whole number	Number & Algebra				Divide decimal number by a whole number and solve real life problems related to decimals		Yes	Yes	Yes	
2.7.3	Division of a decimal number by another decimal number	Dividing decimal number by another decimal number	Number & Algebra				Divide decimal numbers by 10, 100 and 1000 and infer left shift in decimal point Divide decimal number by a whole number and solve real life problems related to decimals Convert decimals into fractions and divide decimal number by another decimal number		Yes	Yes	Yes	
2.8	Summary		Number & Algebra									
3	**Data Handling**											

3.1	Introduction	Data representation	Chance & Data	C.7.1	Represents data pictorially in order to interpret data using bar graph	Represents data pictorially in order to interpret data using bar graph						Mathematical dialogues
3.2	Collecting Data	Data representation	Chance & Data				Collect, record and present data and organize experiences or information and draw inferences from them		Yes	No	No	
3.3	Organisation of Data	Data representation	Chance & Data				Organize raw data into tabular form and make data easier to interpret		Yes	Yes	No	
3.4	Representative Values	Central tendencies	Chance & Data	C.7.2	Calculates mean, median and mode in order to find various representative values for simple data from her /his daily life	Calculates mean, median and mode in order to find various representative values for simple data from her /his daily life	Calculate average and represent the central tendency of the data		No	Yes	No	
3.5	Arithmetic Mean	Central tendencies	Chance & Data				Calculate arithmetic mean and find its position in the data		No	Yes	No	
3.5.1	Range	Central tendencies	Chance & Data				Calculate range of the data and know the spread of the data		No	Yes	No	
3.6	Mode	Central tendencies	Chance & Data				Calculate mode of the data and find the observation that occurs most often in the data set		No	Yes	No	
3.6.1	Mode of Large Data	Central tendencies	Chance & Data									
3.7	Median	Central tendencies	Chance & Data				Calculate median of the data and find the observation that lies in the middle of the data set		No	Yes	No	
3.8	Use of Bar Graph	Data representation	Chance & Data	C.7.1	Represents data pictorially in order to interpret data using bar graph	Represents data pictorially in order to interpret data using bar graph	Represent data in a bar graph using appropriate scale and represent given information in form of a bar graph		Yes	Yes	No	

3.8.1	Choosing a scale	Data representation	Chance & Data				Represent data using double bar graph and compare and discuss two collection of data at a glance		Yes	Yes	No	
3.9	Chance and Probability	Chance and Probability	Chance & Data	C.7.3	Calculates the variability in real life situation and appreciate the variation observed in real life situations	Calculates the variability in real life situation and appreciate the variation observed in real life situations	Calculate probability and find the chance of occurring /not-occurring of the event/s		Yes	Yes	Yes	
3.9.1	Chance	Chance and Probability	Chance & Data									
3.10	Summary		Chance & Data									
4	**Simple Equation**											Metacognitive strategies
4.1	A Mind-Reading Game	Algebraic expressions	Number & Algebra	N.7.6	Represents daily life situations in the form of a simple equation and solves it	Translates a real-life situation in the form of a simple algebraic equation in order to arrive at a generalized problem and solution for the situation	Use number and variable with different operations and express a real-life situation in the form of a simple linear equation		Yes	No	No	
4.2	Setting up of an equation	Algebraic equations	Number & Algebra				Use number and variable with different operations and express a real-life situation in the form of a simple linear equation		Yes	No	No	
4.3	Review of what we know	Algebraic equations	Number & Algebra				Convert the given equation in words and express it in statement form		Yes	No	No	
4.4	What equation is?	Algebraic equations	Number & Algebra				Use trial and error method and determine the solution of a simple equation		Yes	Yes	Yes	
4.4.1	Solving an equation	Algebraic equations	Number & Algebra				Create a strategy and solve the given simple equation		Yes	Yes	No	

4.5	More equations	Algebraic equations	Number & Algebra				Explain the first step to be taken and separate the variable while solving the given equation		Yes	Yes	Yes	
4.6	From solution to equation	Algebraic equations	Number & Algebra				Use the given solution and construct equations from it		Yes	Yes	Yes	
4.7	Applications of simple equations to practical situations	Algebraic equations	Number & Algebra				Construct simple equations and solve them for the given problems /puzzles in the familiar or unfamiliar contexts		Yes	Yes	Yes	
4.8	Summary		Number & Algebra									
5	**Lines and Angles**											Open ended activities
5.1	Introduction	Point, line, line segment, ray and angle	Geometry	G.7.1 (Pre-requisite)	Classifies pairs of angles based on their properties as linear, supplementary, complementary, adjacent and vertically opposite and finds value of the one when the other is given	Classifies pairs of angles based on their properties in order describe linear, supplementary, complementary, adjacent and vertically opposite angles	Recall the concept of line, line segment and angles and identify them in the given figure(s)		Yes	No	No	
5.2	Related Angles			G.7.1	Classifies pairs of angles based on their properties as linear, supplementary, complementary, adjacent and vertically opposite and finds value of the one when the other is given.		Examine different angles and determine the measure of their complement and supplement	Find the measure of an unknown angle	No	Yes	No	
5.2.1	Complementary Angles	Definition and examples	Geometry				Examine different angles and identify complementary angles		No	Yes	No	
5.2.2	Supplementary Angles	Definition and examples	Geometry				Examine different angles and		No	Yes	No	

							identify supplementary angles				
5.2.3	Adjacent Angles	Definition and examples	Geometry				Describe adjacent angles and identify a pair of adjacent angles in the given angles	Identify a pair of adjacent angles	Yes	No	No
5.2.4	Linear Pair	Definition and examples	Geometry				Examine different angles and identify linear pair		Yes	No	No
5.2.5	Vertically Opposite Angles	Definition and examples	Geometry				Describe vertically opposite angles and their property and identify them in the given figure		Yes	No	No
5.3	Pairs of lines		Geometry	G.7.2 (Pre-requisite)	Verifies the properties of various pairs of angles formed when a transversal cuts two lines.	Applies the properties of linear, supplementary, complementary etc. Angle in order to find the value of one angle when the other one is given.	Compare the given lines and distinguish between intersecting and parallel lines		Yes	No	No
5.3.1	Intersecting Lines	Definition and examples	Geometry	G.7.2	Verifies the properties of various pairs of angles formed when a transversal cuts two lines		Discuss the different angles made by a transversal and intersecting lines and identify them in the given figure		Yes	Yes	No
5.3.2	Transversal	Definition and examples	Geometry				Discuss the different angles made by a transversal and intersecting lines and identify them in the given figure		Yes	Yes	No

5.3.3	Angles made by a Transversal	Definition and examples	Geometry			Verifies the properties of various pairs of angles formed when a transversal cuts two lines in order demonstrate the properties of angles when two lines are parallel	Discuss the different angles made by a transversal and intersecting lines and identify them in the given figure		Yes	Yes	No	
5.3.4	Transversal of Parallel Lines	Definition and examples	Geometry				Use the properties of angles made by a transversal of parallel lines and determine the measure of unknown angles		Yes	Yes	No	
5.4	Checking for Parallel lines	Conditions to be pair of parallel line	Geometry				Create a strategy and determine whether the given lines are parallel or not		Yes	Yes	Yes	
5.5	Summary		Geometry									
6	**The triangle and its properties**											Visualisation
6.1	Introduction	Triangle	Geometry	G.7.3 (Pre-requisite)	Finds unknown angle of a triangle when its two angles are known	Applies angle sum property of a triangle to calculate unknown angles of a triangle when its two angles are known	Compare different triangles and classify them on the basis of their sides and angles Recall the parts of a triangle and describe it for the given triangle		Yes	Yes	No	
6.2	Medians of a triangle	Triangle	Geometry				Describe median of a triangle and identify it for the given triangle		Yes	No	No	
6.3	Altitude of a triangle	Triangle	Geometry				Describe altitude of a triangle and identify it for the given triangle		Yes	No	No	

6.4	Exterior angle of a triangle and its property	Triangle	Geometry	G.7.3	Finds unknown angle of a triangle when its two angles are known		Apply the exterior angle property of a triangle and find the measure of the unknown angle in the given triangle		Yes	Yes	No
6.5	Angle sum property of a triangle	Triangle	Geometry				Apply the angle sum property of a triangle and find the measure of unknown angle		Yes	Yes	No
6.6	Two special triangle: equilateral and isosceles	Triangle	Geometry				Use appropriate property and determine the measure of the unknown angle(s) in the given figure		Yes	Yes	No
6.7	Sum of lengths of two sides of a triangle	Triangle	Geometry				Apply the property of lengths of sides of a triangle and determine whether a triangle is possible for the given side lengths or not		No	Yes	Yes
6.8	Right-angles triangle and Pythagoras property	Triangle - Pythagoras property in right-angled triangle	Geometry				Apply the Pythagoras property and verify whether the triangle for the given side lengths will be right angled triangle or not Apply the Pythagoras property and fine the length of the unknown side in a right-angled triangle Use appropriate properties and defend whether		No	Yes	Yes

							the given triangle is possible or not					
6.9	Summary		Geometry									
7	**Congruence of Triangles**											Mathematical dialogues
7.1	Introduction	Congruence	Geometry	G.7.4 (Pre-requisite)	Explains congruency of triangles on the basis of the information given about them like (SSS, SAS, ASA, RHS)	Applies the similarity rules in order to explains the congruency of triangles on the basis of the information given about them (like SSS, SAS, ASA, RHS)	Experiment superposition of different figures and verify congruence of two figures	Identify congruence by superposition of two figures	Yes	No	No	
7.2	Congruence of plane figure	Congruence - Plane figure	Geometry				Experiment superposition of different figures and verify congruence of two figures		Yes	No	No	
7.3	Congruence among line segments	Congruence - Line segments	Geometry				Experiment superposition of different lengths and understand congruence of two-line segments and vice versa		Yes	No	No	
7.4	Congruence of angles	Congruence - Angles	Geometry				Experiment superposition of different angles and understand congruence of two angles and vice versa		Yes	No	No	
7.5	Congruence of triangles	Congruence - Triangles	Geometry	G.7.4	Explains congruency of triangles on the basis of the information given		Give example(s) and discuss the congruence of triangles and its corresponding		Yes	Yes	No	

					about them like (SSS, SAS, ASA, RHS)		parts under a given correspondence				
7.6	Criteria for congruence of triangle	Congruence - Triangles	Geometry				Use SSS Congruence criterion and examine whether the given triangles are congruent or not Use SAS Congruence criterion and examine whether the given triangles are congruent or not Use ASA Congruence criterion and examine whether the given triangles are congruent or not Use any appropriate criterion of congruency and check whether the given triangles are congruent or not		Yes	Yes	No
7.7	Congruence among right-angled triangles	Congruence - Triangles	Geometry				Apply RHS congruence criterion and check the congruence of given right triangles		Yes	Yes	No
7.8	Summary		Geometry								

8	**Comparing Quantities**											Metacognitive strategies
8.1	Introduction	Ratio	Number & Algebra	N.7.8 (Pre-requisite)	Distinguishes quantities that are in proportion. For example, tells that 15,45,40,120 are in proportion as 15/45 is the same as 40/120	Applies algorithm to calculate percentages in order to calculate profits, loss and rate of interest in simple interest calculation	Compare quantities and represent them as ratio Compare the units of the quantities and represent them in ratio		Yes	Yes	No	
8.2	Equivalent Ratio	Ratio	Number & Algebra	N.7.8	Distinguishes quantities that are in proportion. For example, tells that 15,45,40,120 are in proportion as 15/45 is the same as 40/120		Convert ratios into like fractions and compare them and identify equivalent ratios Equate ratios and represent them in proportion Represent equal ratios in proportion and find missing term(s)		Yes	Yes	No	
8.2.1	Keeping things in proportion and getting solutions	Proportion and unitary method	Number & Algebra									
8.3	Percentage: Another way of comparing quantities	Percentage	Number & Algebra	N.7.9	Solves problems related to conversion of percentage to fraction and decimal and vice versa	Applies algorithm to calculate percentages in order to calculate profits, loss and rate of interest in simple interest calculation	Convert denominators of fractions into 100 and represent them in percentages		Yes	Yes	No	
8.3.1	Meaning of percentage	Percentage	Number & Algebra				Convert denominators of fractions into 100 and represent them in percentages		Yes	Yes	No	
8.3.2	Converting Fractional Numbers to Percentage	Percentage	Number & Algebra				Convert fractional numbers to percentage and make comparing of quantities easier		Yes	Yes	No	
8.3.3	Converting Decimals to Percentage	Percentage	Number & Algebra				Convert decimal numbers to percentage and make comparing		Yes	Yes	No	

							of quantities easier				
8.3.4	Converting Percentages to Fractions or Decimals	Percentage	Number & Algebra				Convert percentages to fractions or decimals and solve real life problems		Yes	Yes	Yes
8.3.5	Fun with Estimation	Percentage	Number & Algebra				Represent shaded part of a figure in the form of percentage and estimate the part of an area		Yes	Yes	Yes
8.4	Use of percentage	Percentage	Number & Algebra				Interpret percentage given in a statement and infer meaning of the statement		Yes	No	No
8.4.1	Interpreting percentages	Percentage	Number & Algebra				Interpret percentage given in a statement and infer meaning of the statement		Yes	No	No
8.4.2	Converting Percentages to “How Many”	Percentage	Number & Algebra				Convert percentage into number and know how many of a given situation		Yes	Yes	No
8.4.3	Ratios to Percent	Percentage	Number & Algebra				Convert ratios to percentages and solve problems based on real life		Yes	Yes	Yes
8.4.4	Increase or Decrease as Per Cent	Percentage	Number & Algebra				Calculate increase or decrease in quantity as percentage and examine change in quantity based on real life problems		Yes	Yes	Yes
8.5	Prices related to an item or	Profit and loss percent	Number & Algebra	N.7.10	Calculates profit/loss percent	Applies algorithm to	Calculate cost and selling price and		Yes	Yes	Yes

	buying and selling				and rate percent in simple interest	calculate percentages in order to calculate profits, loss and rate of interest in simple interest calculation	determine profit /loss percentage					
8.5.1	Profit or Loss as a Percentage	Profit and loss percent	Number & Algebra				Calculate cost and selling price and determine profit /loss percentage		Yes	Yes	Yes	
8.6	Charges given on borrowed money or simple interest	Simple interest	Number & Algebra				Understand the concept of simple interest and interpret real life problems	Apply the concept of simple interest to solve problems	Yes	Yes	Yes	
8.6.1	Interest for Multiple Years	Simple interest	Number & Algebra				Make use of percentage and calculate simple interest for multiple years		Yes	Yes	Yes	
8.7	Summary		Number & Algebra									
9	**Rational Numbers**											Metacognitive strategies
9.1	Introduction	Rational numbers	Number & Algebra	N.7.4 (Pre-requisite)	Solves problems related to daily life situations involving rational numbers	Applies appropriate mathematical operations on rational numbers in order to solve problems related to daily life situations	Represent integers in the form of numerator /denominator where denominator is non-zero and define rational numbers		Yes	No	No	
9.2	Need for rational numbers	Rational numbers	Number & Algebra				Represent integers in the form of numerator /denominator where denominator is non-zero and define rational numbers		Yes	No	No	

9.3	What are rational numbers?	Rational numbers	Number & Algebra			Define rational numbers and classify a number as a rational number Multiply numerator and denominator by same non-zero integer and find equivalent rational numbers		Yes	Yes	No
9.4	Positive and negative rational numbers	Rational numbers	Number & Algebra			Define positive and negative rational numbers and classify a number as either of them		Yes	No	No
9.5	Rational numbers on a number line	Rational numbers	Number & Algebra			Construct a number line and represent rational numbers on it		Yes	Yes	No
9.6	Rational numbers in standard form	Rational numbers	Number & Algebra			Simplify rational number such that there is no common factor between numerator and denominator and represent the number in standard form	Reduce a rational number to its simplest form	Yes	Yes	No
9.7	Comparison of rational numbers	Rational numbers	Number & Algebra			Determine the distance of a rational number from 0 and compare them		No	Yes	Yes
9.8	Rational numbers between two rational numbers	Rational numbers	Number & Algebra			Calculate and find rational numbers between any two rational numbers and infer that there are infinite rational numbers between any two given rational numbers		No	Yes	No

9.9	Operations on rational numbers	Rational numbers - Operations		N.7.4	Solves problems related to daily life situations involving rational numbers	Applies appropriate mathematical operations on rational numbers in order to solve problems related to daily life situations	Apply the rules of rational numbers operations and simplify arithmetic operations		No	Yes	No	
9.9.1	Addition	Rational numbers - Operations	Number & Algebra						No	Yes	No	
9.9.2	Subtraction	Rational numbers - Operations	Number & Algebra						No	Yes	No	
9.9.3	Multiplication	Rational numbers - Operations	Number & Algebra						No	Yes	No	
9.9.4	Division	Rational numbers - Operations	Number & Algebra						No	Yes	No	
9.1	Summary		Number & Algebra									
10	**Practical Geometry**											
10.1	Introduction	Construction	Geometry	G.7.5 (Pre-requisite)	Using ruler and a pair of compasses constructs, a line parallel to a given line from a point outside it and triangles	Uses ruler and a pair of compasses in order to construct a line parallel to a given line from a point outside the line and the triangles	Use a ruler and compass and construct a line parallel to another line through a point not on the line		No	No	No	
10.2	Construction of a line parallel to a given line, through a point not on the line	Construction - Line	Geometry	G.7.5	Using ruler and a pair of compasses constructs, a line parallel to a given line from a point outside it and triangles		Use a ruler and compass and construct a line parallel to another line through a point not on the line		Yes	No	No	
10.3	Construction of triangles	Construction - Triangle	Geometry				List and execute steps and construct a triangle given the measures of its three sides		Yes	No	No	

10.4	Constructing a triangle when the lengths of its three sides are known (SSS criterion)	Construction - Triangle	Geometry				List and execute steps and construct a triangle given the measures of its three sides		Yes	Yes	No	
10.5	Constructing a triangle when the lengths of two sides and the measure of the angle between them are known (SAS criterion)	Construction - Triangle	Geometry				List and execute steps and construct a triangle when any of its two lengths and an angle between them is given		Yes	Yes	No	
10.6	Constructing a triangle when the measure of two angles and the length of the side included between them is known (ASA criterion)	Construction - Triangle	Geometry				List and execute steps and construct a triangle when any of its two angles and the side included between them is given		Yes	Yes	No	
10.7	Constructing a right-angled triangle when the length of one leg and its hypotenuse are given (RHS criterion)	Construction - Triangle	Geometry				List and execute steps and construct a right-angled triangle when the length of one leg and its hypotenuse are given Examine the given information and determine if construction of a triangle from it is possible or not		Yes	Yes	No	
10.8	Summary		Geometry									
11	**Perimeter and Area**											Open ended activities
11.1	Introduction	Perimeter and area	Measurement	M.7.2	Calculates areas of the regions enclosed in a rectangle and a square	Uses unit square grid /graph sheet in order to approximate	Use unit square grid sheets and determine the perimeter and		Yes	No	No	

						the area of a closed shape	area square and rectangles				
11.2	Squares and Rectangles	Perimeter and area of squares and rectangles	Measurement	M.7.2	Calculates areas of the regions enclosed in a rectangle and a square	Applies properties of simple shape in order to calculate the areas of the regions enclosed in a rectangle and a square	Describe the area and perimeter of plane figures and find the same for square and rectangle Give example(s) and explain that increase in perimeter of a plane figure does not always mean that area will also increase		Yes	Yes	Yes
11.2.1	Triangles as Parts of Rectangles	Perimeter and area of triangles	Measurement				Develop and apply a formula and determine the area of triangle as half of the area of a rectangle		Yes	Yes	No
11.2.2	Generalising for other Congruent Parts of Rectangles	Perimeter and area of rectangles	Measurement				Recall the concept of congruent figures and generalize the area of congruent parts of rectangles		Yes	No	No
11.3	Area of a parallelogram	Area of a parallelogram by using square grid	Measurement	M.7.1	Finds out approximate area of closed shapes by using unit square grid/ graph sheet	Uses unit square grid /graph sheet in order to approximate the area of a closed shape	Use unit square grid sheets and find the perimeter and estimate the area of parallelogram Develop and apply a formula and determine the area of a parallelogram		Yes	Yes	Yes
11.4	Area of a triangle	Area of a triangle	Measurement				Compare the area of a triangle and its corresponding parallelogram and		Yes	Yes	Yes

							discuss their relation				
11.5	Circles	Circle	Measurement				Use direct or indirect measurements and describe the relationships among radius, diameter, and circumference of circles		Yes	No	No
11.5.1	Circumference of a Circle	Circle	Measurement				Use direct or indirect methods to find the circumference of circle, semicircle Investigate different circumference of circles and compare them with their respective diameter and relate circumference to Pi		Yes	No	No
11.5.2	Area of a circle	Area of a circle	Measurement				Develop and apply the formula and find the area of a circle and semicircle		Yes	Yes	No
11.6	Conversion of units	Unit conversion	Measurement				Convert units and measure area or perimeter in other units		Yes	Yes	No
11.7	Applications	Real life problems on perimeter and area	Measurement	M.7.2	Calculates areas of the regions enclosed in a rectangle and a square	Applies properties of simple shape in order to calculate the areas of the regions enclosed in a rectangle and a square	Examine area and perimeter of different figures and find solution for real life problems		Yes	Yes	Yes

11.8	Summary											
12	**Algebraic Expressions**											Metacognitive strategies
12.1	Introduction	Algebraic expressions intro	Number & Algebra	N.7.6	Represents daily life situations in the form of a simple equation and solves it	Translates a real-life situation in the form of a simple algebraic equation in order to arrive at a generalized problem and solution for the situation	Describe algebraic expressions and distinguish them from arithmetic expressions		Yes	Yes	No	
12.2	How are expression formed?	Formation of an expression with variable and constant	Number & Algebra	N.7.7 (Pre-requisite)	Adds/subtracts algebraic expressions	Translates a real-life situation in the form of a simple algebraic equation in order to arrive at a generalized problem and solution for the situation	Combine variables and constants in order to form an algebraic expression for the given statement		Yes	Yes	No	
12.3	Terms of an expression	Terms, factors and coefficients of an expression	Number & Algebra				Examine the given algebraic expression and determine its terms and their factors Examine the given algebraic expressions and distinguish between the terms which are constants and those which are not Examine the given algebraic expression and determine the numerical coefficient of the given variable		No	Yes	No	
12.4	Like and unlike terms	Identifying like and unlike terms	Number & Algebra				Examine the algebraic factors of the given terms and distinguish		No	Yes	No	

							between like and unlike terms				
12.5	Monomials, Binomials, Trinomials and Polynomials	Definitions and examples	Number & Algebra				Examine the given algebraic expressions and classify them as monomial, binomial, trinomial, polynomial		No	Yes	No
12.6	Addition and Subtraction of Algebraic Expressions	Adding and subtracting like terms and general algebraic expressions	Number & Algebra	N.7.7	Adds/subtracts algebraic expressions	Applies algebraic properties in order to add /subtract two algebraic expressions	Combine like terms and simplify the given algebraic expression Add algebraic expressions and determine their sum Subtract the given algebraic expressions and determine their difference		Yes	Yes	No
12.7	Finding the value of an expression	By substituting value	Number & Algebra				Use the given value of variable(s) and evaluate the algebraic expression		Yes	Yes	Yes
12.8	Using algebraic expression- formulas and rules	Perimeter formula, area formula and rules for number patterns	Number & Algebra	N.7.6	Represents daily life situations in the form of a simple equation and solves it	Applies algebraic properties in order to add /subtract two algebraic expressions	Use the given algebraic expression and complete the table of number patterns or find its nth term Examine the pattern and verify whether the given algebraic expression satisfies the shown pattern or not		Yes	Yes	Yes

12.9	Summary		Number & Algebra									
13	**Exponents and Powers**											Multiple mathematical representations of same concept
13.1	Introduction	Exponents and Powers	Number & Algebra	N.7.5 (Pre-requisite)	Uses exponential form of numbers to simplify problems involving multiplication and division of large numbers	Applies properties of exponential numbers in order to simplify problems involving multiplication and division of large numbers	Describe exponential form of numbers and express numbers in exponential notation	Express numbers in exponential form	Yes	Yes	No	
13.2	Exponents	Exponents	Number & Algebra	N.7.5	Uses exponential form of numbers to simplify problems involving multiplication and division of large numbers		Examine the exponential form of the given number and identify its base and exponent Examine the numbers given in exponential form and compare and represent them in an order Find prime factors of numbers and express them as the product of powers of prime factors		Yes	Yes	No	
13.3	Laws of exponents	Laws of exponents	Number & Algebra				Apply laws of exponents and simplify a given expression		Yes	Yes	No	
13.3.1	Multiplying powers with the same base	Laws of exponents	Number & Algebra				Apply laws of exponents and simplify a given expression		Yes	Yes	No	
13.3.2	Dividing powers with the same base	Laws of exponents	Number & Algebra				Apply laws of exponents and simplify a given expression		Yes	Yes	No	
13.3.3	Taking power of a power	Laws of exponents	Number & Algebra				Apply laws of exponents and simplify a given expression		Yes	Yes	No	

13.3.4	Multiplying powers with the same exponents	Laws of exponents	Number & Algebra				Apply laws of exponents and simplify a given expression		Yes	Yes	No	
13.3.5	Dividing powers with the same exponents	Laws of exponents	Number & Algebra				Apply laws of exponents and simplify a given expression		Yes	Yes	No	
13.4	Miscellaneous examples using the laws of exponents	Laws of exponents	Number & Algebra				Write numbers using powers of 10 and express them in standard form	Write numbers using laws of exponents and express them in standard form	Yes	Yes	No	
13.5	Decimal number systems	Laws of exponents	Number & Algebra				Expand the given number using powers of 10 and express it in the exponent form		Yes	Yes	No	
13.6	Expressing large numbers in the standard form	Laws of exponents	Number & Algebra				Represent large numbers in exponential form and read, understand and compare them easily		Yes	Yes	No	
13.7	Summary		Number & Algebra									
14	**Symmetry**											
14.1	Introduction	Symmetry in daily life and line of symmetry	Geometry				Give examples and non-examples and describe symmetrical figures		No	No	Yes	
14.2	Lines of symmetry for regular polygons	Finding number of line of symmetry in regular polygon	Geometry				Determine lines of symmetry for the given figures and classify them on the basis of number of lines of symmetry Examine regular polygons and determine their lines of symmetry		No	Yes	Yes	

							Complete the mirror reflection of the given figure(s) along the mirror line (ie, the line of symmetry) and identify the figure				
14.3	Rotational Symmetry	Definition and examples, centre of rotation and angle of rotation	Geometry				Give example(s) for rotational symmetry and describe their centre of rotation and the direction of rotation Examine the given figure and determine its angle of rotation Examine the given figure and determine its order of rotation		No	Yes	Yes
14.4	Line symmetry and rotational symmetry	Line symmetry and rotational symmetry in daily life	Geometry				Examine the given figures and identify figures which have both line symmetry as well as rotational symmetry		No	Yes	Yes
14.5	Summary		Geometry								
15	**Visualising solid shapes**										
15.1	Introduction: plane figures and solid shapes	2-D and 3-D shapes with their names	Geometry				Discuss and give examples and differentiate between plane figures and solid shapes		No	Yes	Yes

15.2	Faces, edges and vertices	Faces, edges and vertices of solid shapes	Geometry				Examine different solid shapes and identify and count their number of faces, edges and vertices		No	Yes	Yes
15.3	Nets for building 3-D shapes	Making nets for solid shapes	Geometry				Build nets of 3D shapes and understand their properties		Yes	Yes	No
15.4	Drawing solids on a flat surface	Distort images to look 3-D	Geometry								
15.4.1	Oblique Sketches	Oblique sketches to draw 3-D shapes	Geometry				Examine oblique sketches and visualize all the faces of a solid shape		No	Yes	Yes
15.4.2	Isometric Sketches	Using isometric dots to draw 3-D shapes	Geometry				Use isometric dot sheet and draw isometric sketches of a 3D shape		Yes	Yes	No
15.4.3	Visualising Solid Objects	Visualising solid objects in different arrangements	Geometry				Draw 3D objects in 2D and visualize solid objects from different perspectives		Yes	No	Yes
15.5	Viewing different sections of a solid		Geometry								
15.5.1	One Way to View an Object is by Cutting or Slicing	Cross-section view	Geometry				Examine cross sections of different solid shapes and interpret and visualize different planes		No	Yes	Yes
15.5.2	Another Way is by Shadow Play	Shadow view	Geometry				Examine the different figures formed by changing the angle of shadows formed and visualize solid figures		No	Yes	Yes

15.5.3	A Third Way is by Looking at it from Certain Angles to Get Different Views	Looking from different angles	Geometry				Examine solid figures from different angles and view different sections of solids		No	Yes	Yes	
15.6	Summary		Geometry									

7.3. Grade 8

Chapter No.	Chapter/Topic Name	Content	Strand	LO Code	Learning Outcome	Measuring the LOs	Learning Objectives	Suggested Learning Objectives	PISA competency			Suggestive example of pedagogical strategies
									Formulating situations mathematically	Employing mathematical concepts, facts, procedures and reasoning	Interpreting, applying and evaluating mathematical outcomes	
1	**Rational Numbers**											
1.1	Introduction	Rational numbers	Number & Algebra	N.8.1 (Pre-requisite)	Generalises properties of addition, subtraction, multiplication and division of rational numbers through patterns	Explores patterns in operations in order to generalise properties of addition, subtraction, multiplication and division for rational numbers	Define rational number in order to identify whether the given number is a rational number or not	Define and identify rational number	Yes	Yes	No	
1.2	Properties of Rational Numbers	Rational numbers - Properties	Number & Algebra				Apply the properties of natural numbers, whole numbers and integers with respect to all the arithmetic operations and extend them for rational numbers	Apply properties of rational numbers	No	Yes	No	
1.2.1	Closure	Closure Property	Number & Algebra	N.8.1	Generalises properties of addition, subtraction, multiplication and division of rational numbers through patterns							
1.2.2	Commutativity	Commutative Property	Number & Algebra									
1.2.3	Associativity	Associative Property	Number & Algebra									
1.2.4	The role of zero (0)	Role of zero	Number & Algebra				Define the additive and multiplicative identity of rational numbers using prior knowledge	Define the additive identity of rational numbers	Yes	No	No	
1.2.5	The role of 1	Role of one	Number & Algebra				Define the additive and multiplicative inverse of rational numbers using prior knowledge of integers and fractions	Define the multiplicative identity of rational numbers	Yes	No	No	
1.2.6	Negative of a number	Negative or additive inverse	Number & Algebra					Define the additive inverse of rational	Yes	No	No	

								numbers			
1.2.7	Reciprocal	Reciprocal or multiplicative inverse	Number & Algebra				Define the additive and multiplicative inverse of rational numbers using prior knowledge of integers and fractions	Define multiplicative inverse of rational number	Yes	No	No
1.2.8	Distributivity of multiplication over addition for rational numbers	Distributive Property	Number & Algebra				Apply Distributive property of multiplication over addition for rational numbers and simplify a given expression		No	Yes	Yes
1.3	Representation of Rational Numbers on the Number Line	Rational numbers on the number line	Number & Algebra	N.8.2	Finds out as many rational numbers as possible between two given rational numbers.	Calculates rational numbers between any two given rational numbers in order to prove that there are infinite rational numbers between two rational numbers	Extend the concepts of number line and represent rational number on the number line	Represent rational number on the number line	Yes	Yes	No
1.4	Rational Numbers between Two Rational Numbers	Infinite rational numbers	Number & Algebra				Calculate and find rational numbers between any two rational numbers and prove that there are infinite rational numbers between any two given rational numbers	Find rational number between two rational numbers	Yes	Yes	No
1.5	Summary		Number &Algebra								
2	**Linear Equations in One Variable**										Eliminating possibilities
2.1	Introduction	Linear equations	Number & Algebra	N.8.6 (Pre-requisite)	Solves puzzles and daily life problems using variables.	Uses variables in order to solve puzzles and daily life problems	Identify the variable(s) and the highest power of the variable in a given algebraic equation and distinguish whether it is a linear equation in one variable or not	Identify the variable(s) and the highest power of the variable in a linear equation in one variable	Yes	No	No
2.2	Solving Equations which have Linear Expressions on	Linear equations	Number & Algebra	N.8.6	Solves puzzles and daily life problems using variables.		Substitute the given values of variable and verify whether it is the solution of the equation or not	Substitute the given values of variable in an equation to find the solution	No	Yes	No

	one Side and Numbers on the other Side						Transpose terms to the other side and solve linear equations which have linear expression on one side and numbers on the other side		Yes	Yes	No	
2.3	Some Applications	Linear equations - Word problems	Number & Algebra				Write simple contextual problems as linear equations in one variable and find its solution		Yes	Yes	No	
2.4	Solving Equations having the Variable on both Sides	Linear equations	Number & Algebra				Transpose terms to the other side and solve linear equations in one variable which have variable on both sides	Solve linear equation which have variable on both sides by transposing terms	Yes	Yes	No	
2.5	Some More Applications	Linear equations - Word problems	Number & Algebra				Write simple contextual problems as linear equations in one variable and find its solution		Yes	Yes	No	
2.6	Reducing Equations to Simpler Form	Linear equations	Number & Algebra				Simplify the given linear equation in one variable and solve them		No	Yes	No	
2.7	Equations Reducible to the Linear Form	Linear equations	Number & Algebra				Use cross multiplication and reduce certain equations into their linear form		Yes	Yes	No	
2.8	Summary		Number & Algebra									
3	**Understanding Quadrilaterals**											Open ended activities
3.1	Introduction	Polygons	Geometry	G.8.1 (Pre-requisite)	Solves problems related to angles of a quadrilateral using angle sum property	Uses angle sum property in order to solve problems related to angles of quadrilateral						
3.2	Polygons	Polygons	Geometry				List the properties of a polygon and classify the given figures as a polygon		No	Yes	No	
3.2.1	Classification of polygons	Polygons - Classification	Geometry									

3.2.2	Diagonals	Polygons - Diagonals	Geometry				List the properties of different types of polygons and classify them as regular or irregular, concave or convex	List the properties of different types of polygons	Yes	No	No
3.2.3	Convex and concave polygon	Polygons - Types	Geometry				List the properties of different types of polygons and classify them as regular or irregular, concave or convex	List the properties of different types of polygons and classify them as concave or convex	Yes	No	No
3.2.4	Regular and irregular polygons	Polygons - Types	Geometry				List the properties of different types of polygons and classify them as regular or irregular, concave or convex	List the properties of different types of polygons and classify them as regular or irregular	Yes	No	No
3.2.5	Angle sum property	Polygons - Angle sum property	Geometry	G.8.1	Solves problems related to angles of a quadrilateral using angle sum property		Recall the angle sum property of triangle and extend it for quadrilaterals	Extend angle sum property of triangle to quadrilaterals	No	Yes	Yes
							Relate the angle sum property of triangle and quadrilateral and extend it for an n-sided polygon		No	Yes	Yes
							Apply angle sum property of a quadrilateral and find the measure of the unknown angle in a given quadrilateral		No	Yes	No
3.3	Sum of the Measures of the Exterior Angles of a Polygon	Polygons - Angle sum property	Geometry				Apply exterior angle property of a polygon and find the measure of the unknown angle in a given figure		No	Yes	No
3.4	Kinds of Quadrilaterals	Quadrilaterals	Geometry	G.8.2	Verifies properties of parallelograms and establishes the relationship between them	Applies reasoning through activities such as constructing parallelograms, drawing their diagonals and measuring their sides and angles in order to	List the properties of quadrilaterals and classify them as trapezium, kite and parallelogram		Yes	No	No
3.4.1	Trapezium	Quadrilaterals - Trapezium	Geometry								
3.4.2	Kite	Quadrilaterals - Kite	Geometry								

3.4.3	Parallelogram	Quadrilaterals - Parallelograms	Geometry		through reasoning.	verify properties of parallelograms					
3.4.4	Elements of a parallelogram	Naming of components	Geometry				Discuss the properties of a parallelogram in order to describe the relation between its opposite sides, angles and diagonals		Yes	Yes	No
3.4.5	Angles of a parallelogram	Parallelogram - Properties	Geometry								
3.4.6	Diagonals of a parallelogram	Parallelogram - Properties	Geometry								
3.5	Some Special Parallelograms	Parallelogram - Types	Geometry				Discuss the properties of a square and show it as special case of parallelogram, rhombus and rectangle		Yes	Yes	No
3.5.1	Rhombus	Parallelogram - Rhombus	Geometry				Discuss the properties of a rhombus and classify it as special case of kite and parallelogram		Yes	Yes	No
3.5.2	A rectangle	Parallelogram - Rectangle	Geometry				Discuss the properties of a rectangle and show that it is a special case of parallelogram		Yes	Yes	No
3.5.3	A square	Parallelogram - Square	Geometry				Discuss the properties of a square and show it as special case of parallelogram, rhombus and rectangle		Yes	Yes	No
3.6	Summary		Geometry								
4	**Practical Geometry**										
4.1	Introduction	Quadrilateral - Constructions	Geometry	G.8.5 (Pre-requisite)	Constructs different quadrilaterals using compasses and straight edge.	Uses compasses and straight edge in order to construct a given quadrilateral	Discuss and list the minimum number of elements required and construct a unique quadrilateral		No	No	Yes
4.2	Constructing a Quadrilateral	Quadrilateral - Constructions	Geometry	G.8.5	Constructs different quadrilaterals using compasses and straight edge.		Discuss and list the minimum number of elements required and construct a unique quadrilateral		No	Yes	No
4.2.1	When the lengths of four sides and a	Quadrilateral - Constructions	Geometry				List and execute steps of construction and construct a		No	Yes	No

	diagonal are given						quadrilateral length if its four sides and a diagonal are given				
4.2.2	When two diagonals and three sides are given	Quadrilateral - Constructions	Geometry				List and execute steps of construction and construct a quadrilateral given the length of its three sides and two diagonals		No	Yes	No
4.2.3	When two adjacent sides and three angles are known	Quadrilateral - Constructions	Geometry				List and execute steps of construction and construct a quadrilateral if length of two adjacent sides and measures of three angles are known		No	Yes	No
4.2.4	When three sides and two included angles are given	Quadrilateral - Constructions	Geometry				List and execute steps of construction and construct a quadrilateral given the length of three sides and measures of two included angles are known		No	Yes	No
4.3	Some Special Cases	Quadrilateral - Constructions	Geometry				Identify the minimum number of elements required and construct special cases of quadrilaterals		No	Yes	No
4.4	Summary		Geometry								
5	**Data Handling**										
5.1	Looking for Information	Data handling	Chance & Data	C.8.1	Draws and interprets bar charts and pie charts.	Draws and interpret bar graphs and pie charts in order to answer a variety of questions based on them	Represent the given data using the most suitable representation and interpret them applying the knowledge of different types of graphical representation (namely pictograph, bar graph and double bar graph) of data		Yes	Yes	Yes
5.2	Organising Data	Data tabulation	Chance & Data				Use tally marks and organize the given raw		Yes	Yes	No

							data in a frequency distribution table				
5.3	Grouping Data	Frequency table	Chance & Data				Use tally marks and prepare a grouped frequency distribution table for large ungrouped data		Yes	Yes	No
5.3.1	Bars with a difference	Histogram	Chance & Data				Construct histogram and represent the given grouped data		Yes	Yes	No
							Explain the elements of the given histogram and interpret it		No	Yes	No
5.4	Circle Graph or Pie Chart	Pie chart	Chance & Data				Infer a variety of information from a given circle graph		No	Yes	Yes
5.4.1	Drawing pie charts	Pie chart	Chance & Data				Construct a circle graph with the given data		Yes	Yes	No
5.5	Chance and Probability	Probability	Chance & Data	C.8.2	Makes hypotheses on chances of future events on the basis of its earlier occurrences or available data like, after repeated throws of dice and coins.	Conducts activities in order to makes hypotheses on chances of future events on the basis of its earlier occurrences or available data like, after repeated throws of dice and coins	List all the possible outcomes of an experiment and define the equally likely outcomes		Yes	No	No
5.5.1	Getting a result	Probability	Chance & Data				List all the possible outcomes of an experiment and define the equally likely outcomes		Yes	No	No
5.5.2	Equally likely outcomes	Probability	Chance & Data				List all the possible outcomes of an experiment and define the equally likely outcomes		Yes	No	No
5.5.3	Linking chances to probability	Probability	Chance & Data				List all the possible outcomes of an event and calculate the probability of a given event		Yes	Yes	No
5.5.4	Outcomes as events	Outcomes of an event	Chance & Data				List all the possible outcomes of an event and calculate the probability of a given event		Yes	Yes	No

5.5.5	Chance and probability related to real life	Chance and probability	Chance & Data				List all the possible outcomes of an event and calculate the probability of a given event		Yes	Yes	No	
5.6	Summary		Chance & Data									
6	**Squares and Square Roots**											
6.1	Introduction	Squares	Number & Algebra	N.8.4 (Pre-requisite)	Finds squares, cubes and square roots and cube roots of numbers using different methods.	Applies different methods in order to find the squares, cubes, square roots and cube roots of a given number	Define perfect squares and classify the given numbers as perfect squares or non-perfect squares		Yes	Yes	No	
6.2	Properties of Square Numbers	Squares - Properties	Number & Algebra				Observe the number and find the unit place of its square		Yes	Yes	No	
							Observe different number pattern and deduce square numbers		Yes	Yes	No	
6.3	Some More Interesting Pattern	Squares - Pattern	Number & Algebra				Observe different number pattern and deduce square numbers		Yes	Yes	No	
6.4	Finding the Square of a Number	Squares	Number & Algebra	N.8.4	Finds squares, cubes and square roots and cube roots of numbers using different methods.	Applies different methods in order to find the squares, cubes, square roots and cube roots of a given number	Use the rule that a perfect square number (n^2) can be written as the sum of first n odd natural numbers and distinguish between square and non-square numbers		Yes	Yes	No	
6.4.1	Other patterns in squares	Squares	Number & Algebra				Observe different number pattern and deduce square numbers		Yes	Yes	No	
6.4.2	Pythagorean triplets	Pythagorean triplets	Number & Algebra				Use Pythagoras theorem and find the Pythagorean triplet	Find the Pythagorean triplets	No	Yes	No	
6.5	Square Roots	Square roots	Number & Algebra				Apply inverse operations on a given perfect square and deduce square root of this number		No	Yes	Yes	

6.5.1	Finding square root	Square roots	Number & Algebra		
6.5.2	Finding square root through repeated subtraction	Square roots	Number & Algebra		
6.5.3	Finding square root through prime factorisation	Square roots	Number & Algebra		
6.5.4	Finding square root by division method	Square roots	Number & Algebra		

Apply inverse operations on a given perfect square and deduce square root of this number		No	Yes	Yes
Use method of repeated subtraction and find the square root of the given square number		Yes	Yes	No
Use prime factorization method and find the square root of the given perfect square		Yes	Yes	No
Use prime factorization method and determine whether the given number is a perfect square or not		Yes	Yes	No
Use prime factorization method and find the smallest number to be operated (all the four arithmetic operations) on given number to get a perfect square and then find the square root of the new number		Yes	Yes	Yes
Use long division method and find the square root of the given perfect square number		No	Yes	No
Use long division method and find the smallest number to be operated (all the four arithmetic operations) on given number to get a perfect square and then find the square root of the new number		No	Yes	Yes

6.6	Square Roots of Decimals	Square roots - Decimal numbers	Number & Algebra				Use long division method and find the square root of the given decimal number		No	Yes	No	
6.7	Estimating Square Root	Square roots - Estimations	Number & Algebra				Use estimation and approximate the value of the square root of the given number to the nearest whole		No	Yes	Yes	
6.8	Summary		Number & Algebra									
7	**Cubes and Cube Roots**											
7.1	Introduction		Number & Algebra	N.8.4	Finds squares, cubes and square roots and cube roots of numbers using different methods.	Applies different methods in order to find the squares, cubes, square roots and cube roots of a given number	Define perfect cube /cube number and classify the given numbers as cube numbers or non-cube numbers		Yes	Yes	No	
7.2	Cubes	Explanation with example	Number & Algebra									
7.2.1	Some interesting patterns	Examples	Number & Algebra				Observe the pattern of cube of even numbers and generalize that cubes of even numbers are even		Yes	Yes	No	
							Observe the pattern of cube of numbers with one's digit as 1, 2, 3, 4… etc. and explore the one's digit of their perfect cubes and comment on it		Yes	Yes	No	
7.2.2	Smallest multiple that is a perfect cube	Perfect cube	Number & Algebra				Use prime factorization and rule out a number as a perfect cube		Yes	Yes	No	
							Use prime factorization on the given number and find the smallest number to be operated (all the four arithmetic operations) on given number to get a perfect cube		Yes	Yes	Yes	

7.3	Cube Roots	Symbol	Number & Algebra				Add n consecutive odd numbers and get the sum equal to n3		No	Yes	No	
7.3.1	Cube root through prime factorisation method	Prime factorisation	Number & Algebra				Use prime factorization and find the cube root of a given number		Yes	Yes	No	
7.3.2	Cube root of a cube number	Cube and cube root	Number & Algebra				Use estimation and find the cube root of a given perfect cube		No	Yes	Yes	
7.4	Summary		Number & Algebra									
8	**Comparing Quantities**											Metacognitive strategies
8.1	Recalling Ratios and Percentages	Ratios and percentages	Number & Algebra	N.8.9	Applies the concept of per cent in profit and loss situation in finding discount, VAT and compound interest. e.g. calculates discount per cent when marked price and actual discount are given or finds profit per cent when cost price and profit in a transaction are given.	Observes a given context in order to apply the concepts of profit and loss, discount, vat, simple and compound interest	Convert ratios to percentage and solve the given questions	Convert ratios to percentage and solve the given questions and vice versa	Yes	Yes	No	
8.2	Finding the Increase or Decrease Per cent	Increase or decrease per cent	Number & Algebra				Convert ratios to percentage and solve the given questions		Yes	Yes	No	
8.3	Finding Discounts	Discount	Number & Algebra				Calculate the discount in given situations and comment whether the seller has made a profit /loss in the given transaction	Use the concept of discount to identify profit and loss	Yes	Yes	No	
8.3.1	Estimation in percentages	Percentages					Convert ratios to percentage and solve the given questions	Solve problems using estimations in percentage	Yes	Yes	No	
8.4	Prices Related to Buying and Selling (Profit and Loss)	Profit and Loss	Number & Algebra				Convert ratios to percentage and solve the given questions		Yes	Yes	No	
8.4.1	Finding cost price/selling price, profit %/loss%	Profit and Loss	Number & Algebra				Apply the formula for discount and discount percentage and solve the given problem on discount		No	Yes	No	
8.5	Sales Tax/Value Added Tax	Sales Tax/Value Added Tax	Number & Algebra				Apply the formula for discount and discount percentage and solve		No	Yes	No	

							the given problem on discount				
8.6	Compound Interest	Simple and compound interest	Number & Algebra				Define and compare simple interest and compound interest and comment on the situations where either of the two are applied	Define and compare simple interest and compound interest and identifies the situations where either of the two are applied	Yes	Yes	No
							Calculate the simple interest and find the total amount to be paid by the debtor		No	Yes	No
8.7	Deducing a Formula for Compound Interest	Compound interest	Number & Algebra				Use formula of simple interest and deduce the formula to calculate the compound interest		No	Yes	Yes
							Calculate the compound interest and find the total amount to be paid by the debtor		Yes	Yes	No
8.8	Rate Compounded Annually or Half Yearly (Semi Annually)	Impact of rate on compound interest	Number & Algebra				Define the terms 'compounded annually', 'compounded half yearly' and 'compounded quarterly' and give examples and differentiate between the three		Yes	No	No
8.9	Applications of Compound Interest Formula	Compound interest	Number & Algebra				Use formula of compound interest and solve problems related to increase (or decrease) in population		Yes	Yes	Yes
							Use formula of compound interest and solve problems related to increase (or decrease) in the price		Yes	Yes	Yes

							of an item in intermediate years					
8.10	Summary		Number &Algebra									
9	**Algebraic Expressions and Identities**											
9.1	What are Expressions?	Algebraic expressions	Number & Algebra	N.8.7 (Pre-requisite)	Multiplies algebraic expressions. e.g. expands (2x-5)(3x2+7).	Apply distributive property in order to multiply two algebraic expressions						
9.2	Terms, Factors and Coefficients	Algebraic expressions	Number & Algebra				Count the number of terms in an algebraic expression and classify them as monomial, binomial, trinomial or polynomial in general		Yes	No	No	
9.3	Monomials, Binomials and Polynomials	Algebraic expressions	Number & Algebra									
9.4	Like and Unlike Terms	Algebraic expressions	Number & Algebra				Identify like and unlike terms in algebraic expressions and add or subtract the given algebraic expressions		Yes	Yes	No	
9.5	Addition and Subtraction of Algebraic Expressions	Algebraic expressions - Addition and subtraction	Number & Algebra				Identify like and unlike terms in algebraic expressions and add or subtract the given algebraic expressions		Yes	Yes	No	
9.6	Multiplication of Algebraic Expressions: Introduction	Algebraic expressions - Multiplication	Number & Algebra	N.8.7	Multiplies algebraic expressions. · e.g. expands (2x-5)(3x2+7).		Use pattern of dots to understand the multiplication of algebraic expressions		Yes	No	No	
9.7	Multiplying a Monomial by a Monomial	Algebraic expressions - Multiplication	Number & Algebra				Use rules of exponents and powers and multiply a monomial by a monomial		No	Yes	No	
9.7.1	Multiplying two monomials	Algebraic expressions - Multiplication	Number & Algebra				Extend the multiplication of monomial by a monomial and obtain the product of any number of monomials	Find the product of monomials	No	Yes	No	
9.7.2	Multiplying three or more monomials	Algebraic expressions - Multiplication					Extend the multiplication of monomial by a monomial and obtain		No	Yes	No	

							the product of any number of monomials				
9.8	Multiplying a Monomial by a Polynomial	Algebraic expressions - Multiplication	Number & Algebra					Use distributive property of multiplication over addition and subtraction and obtain the product	No	Yes	No
9.8.1	Multiplying a monomial by a binomial	Algebraic expressions - Multiplication	Number & Algebra				Use distributive property of multiplication over addition and subtraction and obtain the product of a monomial and a binomial		No	Yes	No
9.8.2	Multiplying a monomial by a trinomial	Algebraic expressions - Multiplication	Number & Algebra				Use distributive property of multiplication over addition and subtraction and obtain the product of a monomial and a trinomial		No	Yes	No
9.9	Multiplying a Polynomial by a Polynomial	Algebraic expressions - Multiplication					Use distributive law of multiplication and obtain the product of two binomials	Use distributive property of multiplication and obtain the product	No	Yes	No
9.9.1	Multiplying a binomial by a binomial	Algebraic expressions - Multiplication	Number & Algebra				Use distributive law of multiplication and obtain the product of two binomials		No	Yes	No
9.9.2	Multiplying a binomial by a trinomial	Algebraic expressions - Multiplication	Number &Algebra				Use distributive law of multiplication and obtain the product of a binomial and a trinomial		No	Yes	No
9.10	What is an Identity?	Algebraic identities	Number & Algebra	N.8.8	Uses various algebraic identities in solving	Use various algebraic identities in order to solve problems of daily life	Define and compare equation and identity and classify a given question into either of the two		Yes	No	No

9.11	Standard Identities	Algebraic identities	Number & Algebra		problems of daily life		Use multiplication of binomials and explore and verify the standard identities for squares of binomials		No	Yes	No	
9.12	Applying Identities	Algebraic identities	Number & Algebra				Use identities and find the product of the given numbers		No	Yes	No	
							Use identities and simplify the given algebraic expressions		No	Yes	No	
9.13	Summary		Number & Algebra									
10	**Visualising Solid Shapes**											Multiple mathematical representations of same concept
10.1	Introduction	Solid shapes	Geometry	G.8.3	Represents 3D shapes on a plane surface such as sheet of paper, black board etc.	Visualizes 3-D shapes in order to represent them in a plane surface such as sheet of paper, black board, etc.	Compare 2D shapes and 3D shapes and classify a given shape into either	Identify and compare 2D and 3D shapes	Yes	Yes	No	
10.2	Views of 3D-Shapes	Nets and views	Geometry				Visualize 3D objects and draw them from different perspectives		Yes	Yes	No	
							Identify different shapes in nested objects and match the object with its shape		Yes	No	No	
							Discuss the given front, top and side view of an object and identify the object		Yes	Yes	No	
10.3	Mapping Space Around Us	Maps	Geometry				Discuss the elements in a map and differentiate between a map and a picture		Yes	No	No	
							Read and interpret simple map and answer questions based on them		Yes	No	Yes	
							Choose appropriate scale and use symbols to denote landmarks and draw a simple map		Yes	Yes	No	
10.4	Faces, Edges and Vertices	Faces, edges and vertices	Geometry	G.8.4			Identify faces, edges and vertices in a given		Yes	Yes	No	

					Verifies Euler's relation through pattern	Analyses patterns in order to verify Euler's relation	solid and classify it as a polyhedron or a non-polyhedron				
							Count vertices, edges and faces in 3D figures with flat faces and verify Euler's formula	Verify Euler's formula	No	Yes	Yes
10.5	Summary		Geometry								
11	**Mensuration**										
11.1	Introduction	Plane shapes		M.8.1	Estimates the area of shapes like trapezium and other polygons by using square grid/graph sheet and verifies using formulas.	Uses square grid /graph sheet in order to estimate the areas of various polygons	Calculate area and perimeter of circle, square, rectangle, triangle and calculate area and perimeter of adjoint shapes	Calculate area and perimeter of circle, square, rectangle, triangle and combined shapes	Yes	Yes	No
11.2	Let us Recall	Area of a quadrilateral	Measurement				Calculate area and perimeter of circle, square, rectangle, triangle and calculate area and perimeter of adjoint shapes		Yes	Yes	No
11.3	Area of Trapezium	Area of a quadrilateral	Measurement	M.8.2	Finds the area of a polygon.	Uses appropriate methods to find the area of a polygon	Breakdown a given trapezium into known figures (triangles, squares, rectangles) and derive the formula for the area of a trapezium		Yes	Yes	No
11.4	Area of a General Quadrilateral	Area of a quadrilateral	Measurement				Calculate the area of a given polygon after breaking down the polygon in multiple ways and compare the values and comment on it	Calculate the area of a given polygon by decomposing it in multiple ways and compare results	Yes	Yes	No
11.4.1	Area of special quadrilaterals	Area of a quadrilateral	Measurement								
11.5	Area of a Polygon	Area of polygon by dividing the shape	Measurement								
11.6	Solid Shapes	Solid shapes	Measurement	M.8.3	Finds surface area and volume of cuboidal and cylindrical object.	Uses appropriate formulae in order to find surface area and volume of cuboidal and cylindrical object	Illustrate 2-D representation of a cuboid, cube and cylinder and compute the surface areas by breaking them in to areas of known figures	Illustrate 2-D representations of shapes such as cuboid, cube and cylinder	Yes	Yes	No

11.7	Surface Area of Cube, Cuboid and Cylinder	Surface area of cube, cuboid and cylinder	Measurement				Illustrate 2-D representation of a cuboid, cube and cylinder and compute the surface areas by breaking them in to areas of known figures		Yes	Yes	No
11.7.1	Cuboid	Surface area of cube, cuboid and cylinder	Measurement				Illustrate 2-D representation of a cuboid, cube and cylinder and compute the surface areas by breaking them in to areas of known figures		Yes	Yes	No
11.7.2	Cube	Surface area of cube, cuboid and cylinder	Measurement				Illustrate 2-D representation of a cuboid, cube and cylinder and compute the surface areas by breaking them in to areas of known figures		Yes	Yes	No
11.7.3	Cylinder	Surface area of cube, cuboid and cylinder	Measurement				Illustrate 2-D representation of a cuboid, cube and cylinder and compute the surface areas by breaking them in to areas of known figures		Yes	Yes	No
							Calculate the surface area of a cube, cuboid and cylinder to determine the cost of painting/covering their surface		No	Yes	Yes
11.8	Volume of Cube, Cuboid and Cylinder	Volume of cube, cuboid and cylinder	Measurement								
11.8.1	Cuboid	Volume of cube, cuboid and cylinder	Measurement				Calculate the volume of a given cube, cuboid, cylinder and infer the quantity of any substance it can hold	Calculate the volume and capacity of cube, cuboid, and cylinder	No	Yes	Yes
11.8.2	Cube	Volume of cube, cuboid and cylinder	Measurement				Calculate the volume of a given cube, cuboid, cylinder and infer the quantity of any substance it can hold		No	Yes	Yes

							Modify the values of l, b, h and examine the effect it has on the value of the surface area/volume of a cuboid	Find the effect of change in length, breadth and height on the surface area /volume of a cuboid	No	Yes	Yes	
11.8.3	Cylinder	Volume of cube, cuboid and cylinder	Measurement				Modify the values of r, h and examine the effect it has on the value of the surface area/volume of a cylinder		No	Yes	Yes	
11.9	Volume and Capacity	Volume and capacity	Measurement				Calculate the volume of a given cuboid, cylinder and determine the time taken to fill it with a liquid at a given rate		No	Yes	No	
11.10	Summary		Measurement									
12	**Exponents and Powers**											
12.1	Introduction	Exponents and powers	Number & Algebra	N.8.5 (Pre-requisite)	Solves problems with integral exponents.	Applies rules of exponents in order to solve problems with integral exponents						
12.2	Powers with Negative Exponents	Exponents and powers	Number & Algebra	N.8.5	Solves problems with integral exponents.		Simplify powers with negative exponents and calculate the multiplicative inverse of a number		No	Yes	No	
12.3	Laws of Exponents	Exponents and powers	Number & Algebra				Apply the first law of exponents and principles of negative exponents and derive the rest of the laws of exponents		No	Yes	Yes	
							Apply laws of exponents and simplify a given expression. Give different examples of application of the laws		No	Yes	Yes	
12.4	Use of Exponents to Express Small Numbers in Standard Form	Exponents and powers	Number & Algebra				Express very large and very small numbers in the standard form and compare and estimate quantities		Yes	No	Yes	

12.4.1	Comparing very large and very small numbers	Exponents and powers	Number & Algebra				Express very large and very small numbers in the standard form and compare and estimate quantities		Yes	No	Yes	
12.5	Summary		Number & Algebra									
13	**Direct and Inverse Proportions**											Mathematical dialogues
13.1	Introduction	Example	Number & Algebra	N.8.10	Solves problems based on direct and inverse proportions	Solves problems based on direct or inverse proportions in order to establish how one quantity depends on other	Observe the relationship between the given two quantities and solve to find constant of proportionality		Yes	Yes	No	
13.2	Direct Proportion	Real world problems	Number & Algebra				Complete a given table showing two proportional quantities and answer questions based on them		No	Yes	Yes	
							Examine situations and decide whether two quantities are proportional to each other or not		No	No	Yes	
13.3	Inverse Proportion	Real world problems	Number & Algebra				Observe the table and determine which pair of variables are inversely proportional		No	No	Yes	
							Convert the given statement on relationship (directly or inversely proportional) between two quantities into a table and identify the missing quantity and solve for its value	Calculate a missing value by constructing proportionality table	Yes	Yes	Yes	
							Create a scale using a suitable proportionality constant and draw a given figure with large dimensions		No	Yes	Yes	
13.4	Summary		Number & Algebra									

14	**Factorisation**											Multiple mathematical representations of same concept
14.1	Introduction	Factorisation	Number & Algebra									
14.1.1	Factors of natural numbers	Factors	Number & Algebra				Express each term as a product of irreducible factors and find the common factors of the given terms		Yes	Yes	No	
14.1.2	Factors of algebraic expressions	Factors	Number & Algebra				Express each term as a product of irreducible factors and find the common factors of the given terms		Yes	Yes	No	
14.2	What is Factorisation?	Factorisation	Number & Algebra									
14.2.1	Method of common factors	Factorisation	Number & Algebra				Use the method of common factors and factorize the given algebraic expression		Yes	Yes	No	
14.2.2	Factorisation by regrouping terms	Factorisation	Number & Algebra				Regroup the terms and factorize the given algebraic expressions		Yes	Yes	No	
14.2.3	Factorisation using identities	Factorisation	Number & Algebra				Apply the standard algebraic identities and factorize the given algebraic expressions (for perfect squares)		Yes	Yes	Yes	
14.2.4	Factors of the form (x + a) (x + b)	Factorisation	Number & Algebra				Factorize algebraic expressions in the form and express it as a product of its irreducible factors of the form		Yes	Yes	No	
14.3	Division of Algebraic Expressions	Algebraic expressions - Division	Number & Algebra									
14.3.1	Division of a monomial by another monomial	Algebraic expressions - Division	Number & Algebra				Use the common factor method and divide a monomial by a monomial		Yes	Yes	No	
14.3.2	Division of a polynomial by a monomial	Algebraic expressions - Division	Number & Algebra				Use the common factor method and divide a polynomial by a monomial		Yes	Yes	No	

							Divide each term in the numerator by the denominator and divide a polynomial by a monomial		No	Yes	No	
14.4	Division of Algebraic Expressions Continued (Polynomial ÷ Polynomial)	Algebraic expressions - Division	Number & Algebra				Use the common factor method and divide a polynomial by a polynomial		Yes	Yes	No	
14.5	Can you Find the Error?	Algebraic expressions - Error in operation	Number & Algebra				Check the given mathematical statements and find and give reasons for the possible errors in them		No	Yes	Yes	
14.6	Summary		Number & Algebra									
15	**Introduction to Graphs**											
15.1	Introduction			C.8.1 (Pre-requisite)	Draws and interprets bar charts and pie charts.	Draws and interpret bar graphs and pie charts in order to answer a variety of questions based on them						
15.1.1	A Bar graph	Data analysis graphs	Number & Algebra	C.8.1	Draws and interprets bar charts and pie charts.		Recall the interpretation of the bar graph and double bar graph.					
15.1.2	A Pie graph (or a circle-graph)	Data analysis graphs	Number & Algebra				Interpret the Pie graph to answer the given questions		No	Yes	No	
15.1.3	A histogram	Data analysis graphs	Number & Algebra				Interpret and draw the histogram to answer the given questions		No	Yes	No	
15.1.4	A line graph	Data analysis graphs	Number & Algebra				Draw a line graph and represent the given data that changes continuously over periods of time		Yes	Yes	Yes	
15.2	Linear Graphs	Linear graphs	Number & Algebra				Interpret the given line graph and answer the given questions		No	Yes	Yes	
15.2.1	Location of a point	Location of a point	Number & Algebra				Plot a point on the graph and describe its coordinates		Yes	Yes	No	

15.5	Coordinates	Coordinates of a point	Number & Algebra				Plot a point on the graph and describe its coordinates		Yes	Yes	No	
							Plot the given points on the graph and verify if they lie on the same line or not		Yes	Yes	Yes	
							Choose an appropriate scale and plot a graph for the given data		Yes	Yes	No	
15.5.1	Some Applications	Graphs in real life scenario	Number & Algebra				Construct the line graph and discuss the relationship between independent and dependent variable in a given mathematical or a real-life situation		Yes	Yes	Yes	
15.6	Summary		Number & Algebra									
16	**Playing with Numbers**											Metacognitive strategies
16.1	Introduction		Number &Algebra									
16.2	Numbers in General Form	Generalisation of numbers	Number & Algebra				Use the concepts of place value and express the given numbers in their generalized form		Yes	Yes	No	
16.3	Games with Numbers	Variable	Number & Algebra				Use the concepts of place value and express the given numbers in their generalized form		Yes	Yes	No	
							Add or subtract a two-digit number and its reverse and check whether it is divisible by 9 or not		Yes	Yes	Yes	
							Subtract a three-digit number and its reverse and verify that it is divisible by 99		Yes	Yes	Yes	
							Form all possible three-digit numbers using the given 3 digits and verify that the sum		Yes	Yes	Yes	

							of these numbers will be divisible by 37				
16.4	Letters for Digits	Variable	Number & Algebra				Use addition and multiplication and find the values of the letters in the given puzzles		Yes	Yes	No
16.5	Tests of Divisibility		Number & Algebra								
16.5.1	Divisibility by 10	Divisibility test	Number & Algebra	N.8.3	Proves divisibility rules of 2, 3, 4, 5, 6, 9 and 11	Observes patterns and use algebraic operations in order to derive the divisibility rules of 2, 3, 4, 5, 6, 9 & 11	Apply the divisibility rule of 10 and check whether a given number is divisible by 10 or not		No	Yes	Yes
16.5.2	Divisibility by 5	Divisibility test	Number & Algebra				Apply the divisibility rule of 5 and check whether a given number is divisible by 5 or not		No	Yes	Yes
16.5.3	Divisibility by 2	Divisibility test	Number & Algebra				Apply the divisibility rule of 2 and check whether a given number is divisible by 2 or not		No	Yes	Yes
16.5.4	Divisibility by 9 and 3	Divisibility test	Number & Algebra				Apply the divisibility rule of 3 and 9 and check whether a given number is divisible by them		No	Yes	Yes
16.6	Summary		Number & Algebra								

7.4. Grade 9

Chapter No.	Chapter/Topic Name	Content	Strand	LO Code	Learning Outcome	Measuring the Los	Learning Objectives	Suggested Learning Objectives	PISA competency			Suggestive example of pedagogical strategies
									Formulating situations mathematically	Employing mathematical concepts, facts, procedures and reasoning	Interpreting, applying and evaluating mathematical outcomes	
1	**Number System**											
1.1	Introduction	Rational numbers between two rational numbers, p/q format	Number & Algebra	N.9.1	Applies logical reasoning in classifying real numbers, proving their properties and using them in different situations	Applies logical reasoning in classifying real numbers in order to prove their properties and use them in different situations	Recall natural numbers, whole numbers, integers and Rational numbers and classify a given number as either of them	Classify real numbers as natural numbers, whole numbers, integers and rational numbers	No	Yes	No	
1.2	Irrational Numbers	Definition, proof, location on number line	Number & Algebra				Represent a given number in the form p/q in order to show whether the given number is rational or not		Yes	No	No	
							Calculate and find rational numbers between any two rational numbers in order to prove that there are infinite rational numbers between any two given rational numbers		No	Yes	No	
							Modify a given non-terminating decimal number in the form of p/q and comment whether this number is irrational		No	Yes	No	
							Use Pythagoras' theorem and create a Pythagorean triplet and construct the length equivalent to root of a given number		No	Yes	No	

1.3	Real Numbers and their Decimal Expansions	Decimal representation, terminating and non-terminating decimals	Number & Algebra				Deduce the value of a given fraction in its decimal form and infer if the decimal number is terminating or non-terminating		Yes	Yes	No	
1.4	Representing Real Numbers on the Number Line	Terminating and non-terminating decimals on number line	Number & Algebra				Use successive magnification and represent a given decimal number on a number line		Yes	Yes	No	
1.5	Operations on Real Numbers	Operations on irrational numbers, rationalisation of denominator	Number & Algebra				Use the commutative, associative and distributive laws for addition and multiplication for irrational numbers and determine whether the sum, difference, quotients and products of irrational numbers are irrational or not		No	Yes	No	
							Rationalize the denominator of a given expression with a square root term in the denominator and convert it to an equivalent expression whose denominator is a rational number		Yes	Yes	No	
1.6	Laws of Exponents for Real Numbers	Rational power and operations	Number & Algebra				Extend the laws of exponents and simplify a given expression	Applies laws of exponents to simplify a given expression	No	Yes	No	
1.7	Summary		Number & Algebra									
2	**Polynomials**											
2.1	Introduction		Number & Algebra									

2.2	Polynomials in One Variable	Linear and quadratic expressions	Number & Algebra	N.9.2	Identifies/classifies polynomials among algebraic expressions and factorises them by applying appropriate algebraic identities	Identifies /classifies polynomials among algebraic expressions in order to apply appropriate algebraic identities to factorise them	Recognize variables and their degree in a given algebraic expression and differentiate whether given expression is a polynomial in one variable or not		No	Yes	No
							Express real-life situations into a polynomial		Yes	No	Yes
								Identify the terms of a given polynomial and classify an expression as constant polynomial, zero polynomial, monomial, binomial, and trinomial	No	Yes	No
							Identify the degree of a given polynomial and classify an expression as zero, linear, quadratic and cubic polynomials		No	Yes	No
							Substitute the value of 'a' in a given expression p(x) and find the value of polynomial at 'a' i.e. p(a)	Finds the value of a polynomial {p(x)} by substituting the value of variable x	No	Yes	No
2.3	Zeroes of a Polynomial	Zeroes of a Polynomial	Number & Algebra				Use given values for the variable 'x' in a polynomial p(x) and identify if the given value is a zero of the polynomials	Finds zeros of a polynomial	No	Yes	No
2.4	Remainder Theorem	Remainder theorem	Number & Algebra				Using Remainder Theorem, calculate division of p(x) by a linear polynomial 'x – a' and find that the remainder is p(a) and		No	Yes	Yes

							verify using long division method					
2.5	Factorisation of Polynomials	Factorisation of Polynomials	Number & Algebra				Apply factor theorem and determine if a linear polynomial 'x-a' is a factor of the given polynomial P(x)		No	Yes	No	
							Apply factor theorem and determine the value of an unknown constant 'k' in Polynomial P(x) when a linear polynomial 'x-a' is a known factor of P(x)		No	Yes	No	
							Apply factor theorem and factories a given polynomial	Apply factor theorem to factories a given polynomial	Yes	Yes	No	
							Factories a given polynomial using splitting middle-term method and factor theorem and compare the results of the two		Yes	Yes	No	
2.6	Algebraic Identities	Algebraic identities	Number & Algebra				Point out to an algebraic identity that can be used and factorize a given expression	Use algebraic identity to factorise a given polynomial	Yes	Yes	Yes	
							Select appropriate algebraic identities and evaluate the values of given expressions		Yes	Yes	No	
2.7	Summary		Number & Algebra									
3	**Coordinate Geometry**											
3.1	Introduction	Coordinates of a point	Number & Algebra	G.9.4	Develops strategies to locate points in a Cartesian plane	Develops strategies from understanding of coordinate geometry in	Determine the x & y co-ordinate of a point from a graph and write the co-ordinates of the		Yes	Yes	No	

						order to locate points in a Cartesian plane	point as an ordered pair					
3.2	Cartesian System	Cartesian system	Number & Algebra				Plot a point on the Cartesian plane and determine QUADRANT of the point		Yes	Yes	No	
3.3	Plotting a Point in the Plane if its Coordinates are given	Plotting of a point	Number & Algebra				Observe a given ordered pair and comment on its location	Identify location of a given ordered pair	Yes	No	No	
							Apply concepts of coordinate geometry and simplify given real life problems	Apply concepts of coordinate geometry to solve real life problems	Yes	Yes	Yes	
3.4	Summary		Number & Algebra									
4	**Linear Equations in two variables**											Metacognitive strategies
4.1	Introduction	Linear equation in two variables	Number & Algebra	N.9.3	Relates the algebraic and graphical representations of a linear equation in one/two variables and applies the concepts to daily life situations	Relates the algebraic and graphical representations of a linear equation in one /two variables in order to apply the concepts to daily life situations	Recall concepts of coefficients and variables and construct a linear equation from a given statement		Yes	Yes	No	
4.2	Linear Equations	Linear equation in two variables	Number & Algebra				Compare a given linear equation to the standard form ax + by + c + 0 and deduce the values of a, b and c	Compare a given linear equation to the standard form ax + by + c = 0 to deduce the values of a, b and c	Yes	Yes	No	
4.3	Solution of a Linear Equation	Linear equation in two variables	Number & Algebra				Use substitution method and deduce whether the ordered pair is solution to a given linear equation		No	Yes	No	

4.4	Graph of a Linear Equation in Two Variables	Linear equation in two variables	Number & Algebra				Plot the points on a graph and represent a linear equation in two variables		Yes	No	No	
4.5	Equations of Lines Parallel to x-axis and y-axis	Linear equation in two variables	Number & Algebra				Solve an equation and represent it on a number line and a Cartesian plane		Yes	Yes	No	
							Using principles of linear equations, formulate and solve variety of problems in real-life situations		Yes	Yes	Yes	
4.6	Summary	Linear equation in two variables	Number & Algebra									
5	**Introduction to Euclid's Geometry**											Logical reasoning
5.1	Introduction	History of Euclid	Geometry	G.9.2 (Pre-requisite)	Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. by applying axiomatic approach and solves problems using them	Applies axiomatic approach and derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. in order to solve problems using them	Give examples of theorems, postulates and axioms and differentiate between them with examples		No	No	Yes	

5.2	Euclid's Definitions, Axioms and Postulates	Euclid's Definitions, Axioms and Postulates	Geometry	G.9.2	Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. by applying axiomatic approach and solves problems using them	Applies axiomatic approach and derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. in order to solve problems using them	Reproduce Euclid's axioms in your own words and give examples for each		No	Yes	No	
							List Euclid's five postulates and visualize and illustrate them through a diagram		Yes	Yes	No	
							Analyse given statements/postulates and determine if they are extensions of Euclid's postulates		No	Yes	Yes	
							Apply Euclid's postulates and prove basic geometrical concepts about lines, points, planes, shapes, etc.		No	Yes	Yes	
5.3	Equivalent Versions of Euclid's Fifth Postulate	Euclid's fifth postulate	Geometry				Illustrate the equivalent of Euclid's fifth postulate through a diagram and list conditions for two lines to be parallel		No	Yes	Yes	
5.4	Summary		Geometry									
6	**Lines and Angles**											Mathematical dialogues
6.1	Introduction	Lines and angles	Geometry	G.9.2 (Pre-requisite)	Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals,	Applies axiomatic approach and derives proofs of mathematical statements particularly related to geometrical	Define segment, ray, collinear points, non-collinear points, acute angle, right angle, obtuse angle, straight angle, reflex angle, complementary angles, Supplementary angles					

					circles etc. by applying axiomatic approach and solves problems using them	concepts, like parallel lines, triangles, quadrilaterals, circles etc. in order to solve problems using them	and identify them in a given figure				
6.2	Basic Terms and Definitions	Type of angle pairs	Geometry				Define segment, ray, collinear points, non-collinear points, acute angle, right angle, obtuse angle, straight angle, reflex angle, complementary angles, Supplementary angles and identify them in a given figure				
6.3	Intersecting Lines and Non-intersecting Lines	Intersecting and parallel lines	Geometry	G.9.2	Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. by applying axiomatic approach and solves problems using them			Identify the types of lines such as intersecting lines or non-intersecting (parallel) lines	Yes	No	No
6.4	Pairs of Angles	Pair of angles simple proofs	Geometry				Label angles created by two intersecting lines and identify vertically opposite pairs, adjacent angles, linear pairs, complementary /supplementary pairs of angles		Yes	Yes	No
							Apply the concepts of linear pairs of angles and vertically opposite angles and establish relationships between the angles in a given figure and solve for missing values		Yes	Yes	Yes

6.5	Parallel Lines and a Transversal	Parallel lines and a transversal	Geometry				Label angles created by a transversal intersecting two parallel lines and identify corresponding angles, alternate angles, interior angles and define relationship between these angles		Yes	Yes	No	
6.6	Lines Parallel to the same Line	Parallel lines	Geometry				Find out the unknown angles created by a transversal in a given figure and infer if the lines are parallel or not		Yes	Yes	No	
6.7	Angle Sum Property of a Triangle	Triangle angle sum proofs	Geometry				Define relationship between angles formed when a triangle is placed between two parallel lines and prove that exterior angle of a triangle is the sum of the two opposite interior angles		Yes	Yes	Yes	
6.8	Summary		Geometry									
7	**Triangles**											Mathematical dialogues
7.1	Introduction	Triangles	Geometry	G.9.2	Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. by applying axiomatic approach and solves problems using them	Applies axiomatic approach and derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. in order to solve problems using them	Observe the angles and sides of the given figures and find out whether they are congruent or not congruent		No	Yes	No	
7.2	Congruence of Triangles	Congruence of triangles	Geometry				Apply concepts of linear pairs of angles, vertically opposite angles, corresponding angles, alternate angles, transversal angles & exterior angles of a triangle and prove congruence between two triangles in a given figure		No	Yes	No	

7.3	Criteria for Congruence of Triangles	Congruence of triangles	Geometry				Illustrate the criteria of congruencies of triangles through diagrams (ASA, SAS, SSS, RHS) and prove relationships between given angles, sides and triangles of a given figure	Illustrate the criteria of congruencies of triangles through diagrams (SAS, ASA, and AAS) and prove relationships between given angles, sides and triangles of a given figure	Yes	Yes	Yes	
7.4	Some Properties of a Triangle	Congruence of triangles	Geometry				Apply criteria for congruence in a triangle with two congruent sides and prove that the angle opposite to the sides are equal and apply it in a given figure to solve for the measure of an angle		Yes	Yes	Yes	
7.5	Some More Criteria for Congruence of Triangles	Congruence of triangles	Geometry					Use SSS or RHS criteria to prove triangle congruency	Yes	Yes	Yes	
7.6	Inequalities in a Triangle	Triangle inequality proof and application	Geometry				Using properties of inequalities in triangles prove the relationship between any given sides or angles in a given figure		Yes	Yes	Yes	
7.7	Summary		Geometry									
8	**Quadrilaterals**											Metacognitive strategies
8.1	Introduction	Quadrilaterals	Geometry	G.9.2	Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines,	Applies axiomatic approach and derives proofs of mathematical statements particularly		Recall the different shapes made by joining the non-collinear points such as triangles				

8.2	Angle Sum Property of a Quadrilateral	Quadrilaterals - Angle sum property proofs	Geometry		triangles, quadrilaterals, circles etc. by applying axiomatic approach and solves problems using them	related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. in order to solve problems using them	Apply angle sum property of quadrilateral and find the value of the unknown angle		No	Yes	No	
8.3	Types of Quadrilaterals	Quadrilaterals - Theorem, proofs and simple problems	Geometry				List the properties of parallelogram and identify if a given quadrilateral is a parallelogram	List the properties of quadrilaterals and classify real life objects into different types of Quadrilaterals	No	Yes	No	
8.4	Properties of a Parallelogram	Parallelograms - Theorem, proofs and simple problems	Geometry				List the properties of parallelogram and identify if a given quadrilateral is a parallelogram		No	Yes	No	
8.5	Another Condition for a Quadrilateral to be a Parallelogram	Quadrilaterals and parallelogram – Proofs	Geometry				Apply properties of parallelogram and find a) an unknown angle b) an unknown side	Apply properties of parallelogram to find the measure of an unknown angle and side	No	Yes	No	
8.6	The Mid-point Theorem	Mid-point theorem	Geometry				Prove the midpoint theorem of triangles using concepts of congruency and transversal angles and extend the application to quadrilaterals		No	Yes	Yes	
8.7	Summary		Geometry									
9	**Areas of parallelograms and triangles**											Mathematical dialogues
9.1	Introduction	Area of congruent figures	Geometry	G.9.2	Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines,	Applies axiomatic approach and derives proofs of mathematical statements particularly	Identify the planar region and area associated and show that area of non-overlapping planar region formed is the sum of their areas		No	Yes	No	

9.2	Figures on the same Base and Between the same Parallels	Figures on the same base and between the same parallels	Geometry		triangles, quadrilaterals, circles etc. by applying axiomatic approach and solves problems using them	related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. in order to solve problems using them	Identify if given figures lie on the same base and between the same parallels and write the common base and the two parallels		No	Yes	Yes	
9.3	Parallelograms on the same Base and between the same Parallels	Parallelograms on the same base and between the same parallels - Theorems, proofs and application	Geometry				Extend the understanding of congruency of triangle and prove that: Parallelograms on the same base and between the same parallels are equal in area		Yes	Yes	No	
9.4	Triangles on the same Base and between the same Parallels	Triangles on the same base and between the same parallels - Theorems and proofs	Geometry				Extend prior knowledge from this chapter and prove that when a triangle and a parallelogram are on the same base and between the same parallels, the area of the triangle is equal to half the area of the parallelogram		Yes	Yes	No	
							Extend prior knowledge and prove that Two triangles on the same base (or equal bases) and between the same parallels are equal in area		Yes	Yes	No	
							Extend prior knowledge to prove that Two triangles having the same base (or equal bases) and equal areas lie between the same parallels		Yes	Yes	No	
9.5	Summary		Geometry									
10	**Circles**											Logical reasoning
10.1	Introduction	Circles	Geometry									

10.2	Circles and its Related Terms : A Review	Circles	Geometry	G.9.2	Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. by applying axiomatic approach and solves problems using them	Applies axiomatic approach and derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. in order to solve problems using them	Construct a circle of a given radius and verify that the length of multiple segments drawn from the centre of the circle to the circumference is equal		Yes	Yes	Yes
							Define radius, chord, diameter, segment (major and minor), arc (major and minor), interior or exterior of a circle and illustrate and label them on a given circle	Define radius, chord, diameter, segment (major and minor), arc (major and minor), interior or exterior of a circle and label them on a given circle	Yes	No	No
10.3	Angle Subtended by a Chord at a Point	Circles theorems	Geometry				Apply theorems regarding angle subtended by a chord in a circle and find the measure of an angle in the given figure		Yes	Yes	No
10.4	Perpendicular from the Centre to a Chord	Circles theorems	Geometry				Apply the property of perpendicular from the centre to the chord and solve for the missing values (lengths and angles) in a given figure		Yes	Yes	No
10.5	Circle through Three Points	Circles theorems	Geometry				Construct circle passing through 1, 2 & 3 non-collinear points and comment on how many circles can be constructed passing through them		Yes	Yes	No
10.6	Equal Chords and their Distances from the Centre	Circles theorems and application	Geometry				Use the value of radius and perpendicular to the chord and compute the length of a chord		No	Yes	No
10.7	Angle Subtended by an Arc of a Circle	Circles theorems	Geometry				Interpret and apply theorems on the angles subtended by arcs of a circle and solve for unknown		No	Yes	Yes

							values in given examples					
10.8	Cyclic Quadrilaterals	Circles theorems	Geometry				Apply the relation between angles of a cyclic quadrilateral and solve for the value of a given angle		No	Yes	No	
10.9	Summary		Geometry									
11	**Constructions**											Open ended activities
11.1	Introduction		Geometry									
11.2	Basic Constructions	Constructions - Angle, perpendicular bisector	Geometry	G.9.3	Constructs different geometrical shapes like bisectors of line segments, angles, and triangles under given conditions and provides reasons for the processes of such constructions	Constructs different geometrical shapes like bisectors of line segments, angles, and triangles under given conditions in order to analyse and provide reasons for the processes of such constructions	List and execute steps of construction and bisect a given angle	Construct an angle bisectors and list steps of constructions	Yes	No	No	
							List and execute steps of construction and draw the perpendicular bisector of a given line segment		Yes	No	No	
							List and execute steps of construction and construct an angle of any given measurement		Yes	No	No	
11.3	Some Constructions of Triangles	Constructions - Triangles with given conditions	Geometry				List and execute steps of construction and construct a triangle given its base, a base angle and the sum of the other two sides		Yes	No	No	
							List and execute steps of construction and construct a triangle given its base, a base angle and the difference of the other two sides		Yes	No	No	
							List and execute steps of construction and construct a triangle given its perimeter and its two base angles		Yes	Yes	No	

11.4	Summary		Geometry									
12	**Heron's Formula**											Logical reasoning
12.1	Introduction	Area formula for right triangle	Measurement	M.9.1	Finds areas of all types of triangles by applying appropriate formulae	Applies appropriate formulae in order to find areas of all types of triangles	Calculate area of a given triangle to state the limitation of the Standard formula (Area of Triangle = ½ b.h)		No	Yes	Yes	
12.2	Area of a Triangle – by Heron's Formula	Area of a triangle – Heron's Formula	Measurement				Apply Heron's formula and calculate the area of a Triangle		No	Yes	No	
12.3	Application of Heron's Formula in finding Areas of Quadrilaterals	Application of Heron's formula in finding areas of quadrilaterals	Measurement				Breakdown a given polygon into triangles and find the area of a given polygon as a sum of areas of those triangles		Yes	Yes	No	
12.4	Summary		Measurement									
13	**Surface area and volume**											Metacognitive strategies
13.1	Introduction		Measurement									
13.2	Surface Area of a Cuboid and a Cube	Surface area of a cuboid and a cube	Measurement	M.9.2	Derives formulas for surface areas and volumes of different solid objects like, cubes, cuboids, right circular cylinders/ cones, spheres and hemispheres and applies them to objects found in the surroundings	Derives formulas for surface areas and volumes of different solid objects like, cubes, cuboids, right circular cylinders / cones, spheres and hemispheres in order to apply them to objects found in the surroundings	Visualize a cube and cuboid in its 2-D form and calculate the surface area		Yes	Yes	No	
							Calculate the surface area (lateral and total) of the cube or cuboid and determine the cost of painting /covering the given surface		No	Yes	No	
13.3	Surface Area of a Right Circular Cylinder	Surface area of a right circular cylinder	Measurement				Visualize a cylinder in its 2-D form and calculate the curved surface area and total surface area		Yes	Yes	No	
							Calculate the surface area (curved and total) of a cylinder to determine the cost of painting /covering the given surface		No	Yes	No	

13.4	Surface Area of a Right Circular Cone	Surface area of a right circular cone	Measurement			Visualize a right circular cone in 2-D and calculate the surface area (curved and total)	Visualize a right circular cone in its 2-D form and calculate the surface area (curved and total)	Yes	No	No	
						Calculate the surface area (curved and total) of a cone to determine the cost of painting /covering the given surface		No	Yes	No	
13.5	Surface Area of a Sphere	Surface area of a sphere	Measurement			Calculate the surface area of a sphere /hemisphere to determine the cost of painting /covering the given surface of a sphere /hemisphere		No	Yes	No	
13.6	Volume of a Cuboid	Volume of a cuboid	Measurement			Calculate the volume of a given cube and infer the quantity of any substance it can hold	Calculate the volume of a given cuboid and infer the quantity of any substance it can hold	No	Yes	Yes	
						Calculate the volume of a given cuboid and infer the quantity of any substance it can hold	Calculate the volume of a given cube and infer the quantity of any substance it can hold	No	Yes	Yes	
13.7	Volume of a Cylinder	Volume of a cylinder	Measurement			Calculate the volume of a given cylinder and infer the quantity of any substance it can hold		No	Yes	Yes	
13.8	Volume of a Right Circular Cone	Volume of a right circular cone	Measurement			Calculate the volume of a given cone and infer the quantity of any substance it can hold		No	Yes	Yes	
13.9	Volume of a Sphere	Volume of a sphere	Measurement			Calculate the volume of a given sphere and infer the quantity of any substance it can hold		No	Yes	Yes	

							Calculate the volume of a given hemisphere and infer the quantity of any substance it can hold		No	Yes	Yes	
13.1	Summary		Measurement									
14	**Statistics**											Multiple mathematical representations of same concept
14.1	Introduction	Data set	Data & Chance	C.9.2 (Pre-requisite)	Analyses data by representing it in different forms like, tabular form (grouped or ungrouped), bar graph, histogram (with equal and varying width and length), and frequency polygon	Represents given data in different forms like, tabular form (grouped or ungrouped), bar graph, histogram (with equal and varying width and length), and frequency polygon in order to analyse given data						
14.2	Collection of Data	Primary or secondary data collection	Data & Chance					Understands primary and secondary data				
14.3	Presentation of Data	Presentation of data in frequency tables	Data & Chance	C.9.2	Analyses data by representing it in different forms like, tabular form (grouped or ungrouped), bar graph, histogram (with equal and varying width and length), and frequency polygon		Record and label a given data set and create a frequency table	Create a frequency table for a data set	Yes	Yes	No	
14.4	Graphical Representation of Data	Bar graph, histograph, frequency polygons	Data & Chance				Identify an appropriate scale and labels and represent given data through a bar graph	Represent given data through a bar graph by choosing appropriate scale	Yes	Yes	No	
							Read a given bar graph and infer a variety of information from it	Read and interpret a given bar graph	No	Yes	No	
							Compare the values and correlate two data points from the graph		No	Yes	Yes	
							Read the given data and create a histogram for continuous and discontinuous data sets		Yes	Yes	No	
							Read a given histogram and infer a variety of information from it		No	Yes	Yes	

							Read the given data and create a frequency polygon for given data sets		Yes	Yes	No	
							Read a given frequency polygon and infer a variety of information from it		No	Yes	Yes	
14.5	Measures of Central Tendency	Mean, median and mode for grouped and ungrouped data	Data & Chance	C.9.1	Identifies and classifies the daily life situations in which mean, median and mode can be used	Identifies daily life situations in order to classify them as situations where mean, median and mode can be used	Differentiate between mean, median and mode with examples and understand most effective measure of central tendency in various cases	Identify and differentiate between +measure of central tendencies	No	Yes	Yes	
							Apply appropriate formula and calculate the mean and median of even and odd number of data points		No	Yes	No	
							Recall and use the formula for mean in order find the value of a missing observation	Recall and use the formula for mean in order to find the value of a missing observation	No	Yes	Yes	
14.6	Summary		Data & Chance									
15	**Probability**											Metacognitive strategies
15.1	Introduction	Probability	Data & Chance	C.9.3 (Pre-requisite)	Calculates empirical probability through experiments	Conducts experiments and analyses data in order to calculate empirical probability						
15.2	Probability – an Experimental Approach	Probability – an Experimental Approach	Data & Chance	C.9.3	Calculates empirical probability through experiments		With the formula for Empirical probability, calculate the probability for a simple event	Calculate the probability for a simple event by using the formula of empirical probability	Yes	Yes	No	
							Create a flow chart of all the terms related to random experiments (coins, dice, cards) and calculate the total number of trials of a given experiment and		Yes	Yes	Yes	

							calculate the Empirical Probability				
							Compute the total number of trials and trials for a given event E represent in various forms (table, histogram, pie-charts, etc.) to solve for the value of Empirical Probability P(E)		Yes	Yes	Yes
							Calculate empirical probability of a situation and predict the likelihood of an event		Yes	Yes	Yes
							Arrange events from least likely to most likely and predict outcomes in a given experiment		Yes	No	Yes
							Calculate the sum of probabilities of all events and prove that the sum of the probability of all events in a single experiment is 1		Yes	Yes	Yes
15.3	Summary		Data & Chance								

7.5. Grade 10

Chapter No.	Chapter/Topic Name	Content	Strand	LO Code	Learning Outcome	Measuring the LOs	Learning Objectives	Suggested Learning Objectives	PISA competency			Suggestive example of pedagogical strategies
									Formulating situations mathematically	Employing mathematical concepts, facts, procedures and reasoning	Interpreting, applying and evaluating mathematical outcomes	
1	**Real Numbers**											
1.1	Introduction	HCF, LCM	Number & Algebra	N.10.1	Generalises properties of numbers and relations among them studied earlier to evolve results, such as, Euclid's division algorithm, Fundamental Theorem of Arithmetic, and applies them to solve problems related to real life contexts	Generalises properties of numbers and relations among them studied earlier, to evolve results, such as, Euclid's division algorithm, fundamental theorem of arithmetic in order to apply them to solve problems related to real life contexts	Apply Euclid Division Algorithm and obtain HCF of two positive integers in the context of the given problem		Yes	Yes	No	
1.2	Euclid's Division Lemma	Euclid's division lemma	Number & Algebra				Apply Euclid Division Algorithm and prove results of positive integers in the form of ax + b where a and b are constants		No	Yes	Yes	
1.3	The Fundamental Theorem of Arithmetic	Fundamental theorem of arithmetic	Number & Algebra				Use the Fundamental Theorem of Arithmetic and calculate HCF and LCM of the given numbers in the context of the given problem		Yes	Yes	No	
1.4	Revisiting Irrational Numbers	Irrational numbers	Number & Algebra				Apply theorems of irrational number and prove whether a given number is irrational or not		No	Yes	Yes	
1.5	Revisiting Rational Numbers and Their Decimal Expansions	Terminating and non-terminating decimals	Number & Algebra				Apply theorems of rational numbers and find out about the nature of their decimal representation and their factors		No	Yes	No	
1.6	Summary		Number & Algebra									

2	**Polynomials**										
2.1	Introduction	Linear, quadratic and cubic polynomials	Number & Algebra					Recall degree of polynomial and identify the type of polynomial as linear, quadratic or cubic			
2.2	Geometrical Meaning of the Zeroes of a Polynomial	Zero of polynomial on graph	Number & Algebra	N.10.2	Develop a relationship between algebraic and graphical methods of finding the zeroes of a polynomial	Uses algebraic and graphical method of finding zeroes of a polynomial in order to establish a relationship between them	Recall degree of polynomial and find the number of zeroes of polynomial	Find zeroes of a polynomial	No	Yes	No
							Analyse the graph of the polynomials and find the number of zeroes of polynomial	Find zeroes of a polynomial by analysing its graph	No	Yes	No
2.3	Relationship between Zeroes and Coefficients of a Polynomial	Sum and product of roots	Number & Algebra	N.10.2	Develop a relationship between algebraic and graphical methods of finding the zeroes of a polynomial	Uses algebraic and graphical method of finding zeroes of a polynomial in order to establish a relationship between them	Compute zeroes of the polynomials and verify the relationship between zeroes and the coefficients		No	Yes	No
				N.10.4	Demonstrates strategies of finding roots and determining the nature of roots of a quadratic equation	Demonstrates knowledge of application of various strategies in order to find roots and determine the nature of roots of a given equation	Compute the sum and product of zeroes of the polynomial and find the quadratic polynomial		No	Yes	No
2.4	Division Algorithm for Polynomials	Division of polynomials	Number & Algebra	N.10.2	Develop a relationship between algebraic and graphical methods of finding the zeroes of a polynomial	Uses algebraic and graphical method of finding zeroes of a polynomial in order to establish a relationship between them	Divide the two given polynomials and verify the division algorithm		No	Yes	Yes
							Divide the given polynomial with its known zero and find all the other zeroes of that polynomial		No	Yes	No

2.5	Summary		Number & Algebra									
3	**Pair of Linear Equations in Two Variables**											Metacognitive strategies
3.1	Introduction	Setting up equation in two variables	Number & Algebra	N.10.3	Finds solutions of pairs of linear equations in two variables using graphical and different algebraic methods	Uses graphical and other methods in order to find solutions of pairs of linear equations in two variables	State the properties of linear equation and classify the given equations as linear or nonlinear	Represent statements related to real life situations as linear equations	Yes	No	No	
3.2	Pair of Linear Equations in Two Variables	Pair of linear equation in two variables	Number & Algebra				State the properties of linear equation and classify the given equations as linear or nonlinear		No	Yes	No	
3.3	Graphical Method of Solution of a Pair of Linear Equations	Graphical solution of simultaneous equations	Number & Algebra				Interpret the concepts of linear equations and represent any given situation algebraically and graphically		Yes	No	Yes	
							Plot the lines corresponding to the given two linear equations and comment on the nature /behaviour of the lines representing the linear equations		Yes	No	Yes	
3.4	Algebraic Methods of Solving a Pair of Linear Equations	Algebraic methods of solving a pair of linear equations	Number & Algebra				Use different algebraic methods and solve a pair of linear equation		No	Yes	No	
3.4.1	Substitution Method	Algebraic methods of solving a pair of linear equations - Substitution method	Number & Algebra				Use the most appropriate algebraic method and solve the given pair of linear equations		No	Yes	Yes	
3.4.2	Elimination Method	Algebraic methods of solving a pair of linear equations - Elimination method	Number & Algebra				Use the concepts of pair of linear equations in two variables and represent any given situation algebraically and find its solution	Represent a situation algebraically and find its solution	Yes	Yes	Yes	

3.4.3	Cross-Multiplication Method	Algebraic methods of solving a pair of linear equations - Cross-multiplication method	Number & Algebra				Calculate the ratio of coefficients of linear equations and discuss the nature of pair of linear equations		No	Yes	Yes	
3.5	Equations Reducible to a Pair of Linear Equations in Two Variables	Equations reducible to a pair of linear equations in two variables	Number & Algebra				Rewrite the given equations (using substitution method) which are reducible to a pair of linear equations and find the solution of those equations	Reduce the pair of linear equation in its simplest form and find solution	Yes	Yes	No	
3.6	Summary		Number & Algebra									
4	**Quadratic Equations**											Analyse the information
4.1	Introduction	Formation of quadratic equation	Number & Algebra	N.10.4	Demonstrates strategies of finding roots and determining the nature of roots of a quadratic equation	Demonstrates knowledge of application of various strategies in order to find roots and determine the nature of roots of a given equation	In the form of Quadratic Equation represent the given situation algebraically	Represent a situation algebraically in the form of quadratic equation	Yes	No	No	
4.2	Quadratic Equations	Quadratic equations	Number & Algebra				Rewrite the given equations in the standard form and check whether they are quadratic or not		Yes	Yes	No	
4.3	Solution of a Quadratic Equation by Factorisation	Solution of quadratic equation - Factorisation	Number & Algebra				Solve quadratic equations through factorization and find its roots		No	Yes	No	
							Solve quadratic equations through middle term splitting and find its roots		No	Yes	No	
4.4	Solution of a Quadratic Equation by Completing the Square	Solution of quadratic equation - Completing square	Number & Algebra				Solve quadratic equations by completing the square and find its roots		No	Yes	No	
							Use the quadratic formula and find the roots of quadratic equation		No	Yes	No	

							Substitute the value of the roots of a given quadratic equation and verify them		Yes	No	No	
4.5	Nature of Roots	Nature of Roots	Number & Algebra				Examine the discriminant of quadratic equation and find out the nature of its roots		Yes	No	Yes	
							Describe the nature of the roots of a quadratic equation and determine that whether a given situation is possible or not		Yes	Yes	Yes	
4.6	Summary		Number & Algebra									
5	**Arithmetic Progressions**											Look for a pattern
5.1	Introduction	Patterns and relations	Number & Algebra	N.10.5	Develops strategies to apply the concept of A.P to daily life situations	Develops strategies in order to apply the concept of AP to daily life situations	Produce patterns and observe that succeeding terms are obtained by adding a fixed number to the preceding terms		No	Yes	No	
5.2	Arithmetic Progressions	Pattern with constant difference	Number & Algebra				Distinguish between finite and infinite AP and determine the nature and write the last term of the given AP		No	Yes	No	
5.3	nth Term of an AP	Particular term in AP	Number & Algebra				Calculate the nth term of a given AP and find its terms and their nature		No	Yes	Yes	
							Calculate the nth term of a given AP and solve real-life word problems		Yes	Yes	Yes	
5.4	Sum of First n Terms of an AP	Sum in AP	Number & Algebra				Calculate the sum of a given AP and get the solution of real-life word problems		Yes	Yes	Yes	

							Calculate the sum of a given AP and solve contextual problems		Yes	Yes	Yes	
							Use appropriate formula to calculate the last term of the given AP		No	Yes	No	
5.5	Summary		Number & Algebra									
6	**Triangles**											Mathematical dialogues
6.1	Introduction	Triangles	Geometry	G.10.1	Works out ways to differentiate between congruent and similar figures	Uses reasoning in order to differentiate between congruent and similar figures	Distinguish between congruency and similarity and understand the concept of similar figures		No	No	Yes	
6.2	Similar Figures	Similarity in geometric shapes	Geometry	G.10.1	Works out ways to differentiate between congruent and similar figures	Uses different geometric criteria established earlier such as basic proportionality theorem etc. in order to establish properties for similarity of two triangles	Compute the angles and ratio of sides of polygons and determine their similarity		Yes	Yes	No	
6.3	Similarity of Triangles	Similarity in triangles	Geometry	G.10.2	Establishes properties for similarity of two triangles logically using different geometric criteria established earlier such as Basic Proportionality Theorem etc.	Uses different geometric criteria established earlier such as basic proportionality theorem etc. in order to establish properties for similarity of two triangles	Compute the angles and ratio of sides of triangles and determine their similarity		Yes	Yes	No	
							Apply basic proportionality theorem and its converse and determine the ratio of sides in the given triangle(s)		No	Yes	No	
6.4	Criteria for Similarity of Triangles	Criteria for similarity	Geometry				Apply various criteria of similarity and prove whether given triangles are similar or not		Yes	Yes	No	

							Show similarity of triangles and solve real life problems	Use similarity of triangles to solve real life problems	Yes	Yes	Yes	
6.5	Areas of Similar Triangles	Area of similar triangles	Geometry				Compute the square of the ratio of the corresponding sides of triangles and find the area of similar triangles		Yes	Yes	No	
							Compute the area of similar triangles and find the relation between their sides, medians, mid points of the triangles	Find the area of similar triangles and establish the relation between their sides, medians and mid points of the triangles	No	Yes	Yes	
6.6	Pythagoras Theorem	Pythagoras theorem - proof	Geometry				Apply the theorem that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and prove Pythagoras Theorem		Yes	Yes	Yes	
							Prove Pythagoras theorem and its converse and solve real life problems		Yes	Yes	Yes	
							Apply Pythagoras theorem and its converse and determine that whether a given triangle is a right-angled triangle or not		Yes	Yes	Yes	
6.7	Summary		Geometry									
7	**Coordinate Geometry**											
7.1	Introduction	Coordinate geometry	Geometry	G.10.3	Derives formulae to establish relations for	Derives formulae to establish	Identify x and y coordinate and plot points on the graph		Yes	Yes	No	

7.2	Distance Formula	Distance Formula	Geometry		geometrical shapes in the context of a coordinate plane, such as finding the distance between two given points, to determine coordinates of a point between any two given points, to find area of a triangle etc.	relations for geometrical shapes in the context of a coordinate plane, such as finding the distance between two given points, in order to determine coordinates of a point between any two given points, to find area of a triangle etc.	Apply and derive distance formula and determine the distance between two coordinates on the graph		Yes	Yes	No	
							Apply distance formula and solve various mathematical and real-life problems graphically		Yes	Yes	Yes	
7.3	Section Formula	Section Formula	Geometry				Apply and derive section formula and divide the line segment in a given ratio	Derive and apply section formula and divide the line segment in a given ratio	No	Yes	No	
							Apply distance and section formula and determine the vertices/diagonals/mid points of given geometrical shapes		No	Yes	No	
7.4	Area of a Triangle	Area of a Triangle	Geometry				Apply and derive the formula of area of triangle geometrically and determine the area of quadrilateral/triangle		No	Yes	No	
7.5	Summary		Geometry									
8	**Introduction to Trigonometry**											
8.1	Introduction	Similar right triangles	Trigonometry	G.10.4	Determines all trigonometric ratios with respect to a given acute angle (of a right triangle) and uses them in solving problems in daily life contexts like finding heights of different structures or distances from them	Determines all trigonometric ratios with respect to a given acute angle (of a right triangle) in order to use them in solving problems in daily life contexts like finding heights of different structures or distances from them	Describe trigonometry and study the relationship between side and angle of a triangle		Yes	No	No	
8.2	Trigonometric Ratios	Trigonometric ratios	Trigonometry				Define and distinguish various trigonometric ratios and describe and verify sine, cosine, tangent, cosecant, secant, cotangent of an angle		Yes	No	Yes	
							Use given trigonometric ratio(s) and find and verify other trigonometric		No	Yes	Yes	

							ratios/angles of the triangle					
8.3	Trigonometric Ratios of Some Specific Angles	Trigonometric ratios - Specific angles	Trigonometry				Compute the trigonometric ratio 0°, 30°, 45°, 60° and 90° and use these for different angles		No	Yes	No	
8.4	Trigonometric Ratios of Complementary Angles	Trigonometric ratios - Complementary angles	Trigonometry				Compute the trigonometric ratio of complimentary angles and apply the values in solving contextual problems		Yes	Yes	Yes	
8.5	Trigonometric Identities	Trigonometric identities	Trigonometry				Compute and apply trigonometric identities and simplify and solve mathematical problems		No	Yes	No	
8.6	Summary		Trigonometry									
9	**Some Applications of Trigonometry**											Metacognitive strategies
9.1	Introduction	Trigonometry	Trigonometry	G.10.4	Determines all trigonometric ratios with respect to a given acute angle (of a right triangle) and uses them in solving problems in daily life contexts like finding heights of different structures or distances from them	Determines all trigonometric ratios with respect to a given acute angle (of a right triangle) in order to use them in solving problems in daily life contexts like finding heights of different structures or distances from them						
9.2	Heights and Distances	Trigonometry - Heights and distances	Trigonometry				Identify line of sight and determine angle of elevation and angle of depression		Yes	Yes	No	
							Apply trigonometric ratios (of specific angles) and determine heights and distances of the objects in the real-life context		Yes	Yes	Yes	
9.3	Summary		Trigonometry									
10	**Circles**											Logical reasoning
10.1	Introduction	Circles - Chord, segment, sector, arc	Geometry				Draw, identify and differentiate between secant and tangent of a circle and prove and	Define radius, chord, diameter, segment (major and minor), arc (major and minor),	Yes	No	No	

							apply various theorems related to circles	interior or exterior of a circle			
10.2	Tangent to a Circle	Tangent to a circle	Geometry				Draw, identify and differentiate between secant and tangent of a circle and prove and apply various theorems related to circles		Yes	Yes	No
10.3	Number of Tangents from a Point on a Circle	Tangent to a circle	Geometry				Prove and apply theorems related to tangent of a circle and determine number of tangents from the given point(s)		Yes	Yes	No
							Prove and apply theorem related to tangent of a circle and determine length of the tangent		Yes	Yes	No
10.4	Summary		Geometry								
11	**Constructions**										
11.1	Introduction	Constructions	Geometry	G.10.5	Constructs A) A triangle similar to a given triangle as per a given scale factor B) A pair of tangents from an external point to a circle and justify procedures Examines the steps of geometrical constructions and reason out each step	Examines each step and reasons out each step, in order to: A) Construct a triangle similar to a given triangle as per a given scale factor B) Construct a pair of tangents from an external point to a circle and justify procedures					
11.2	Division of a Line Segment	Division of a line segment	Geometry				List and execute steps of construction and divide a line segment in a given ratio		Yes	Yes	No
							List and execute steps of construction and construct a similar triangle as per a given scale factor		Yes	Yes	No
11.3	Construction of Tangents to a Circle	Construction of tangent	Geometry				List and execute steps of construction and construct tangent(s) to a given circle		Yes	Yes	No
11.4	Summary		Geometry								
12	**Areas Related to Circles**										
12.1	Introduction		Measurement								

12.2	Perimeter and Area of a Circle — A Review	Perimeter and area of a circle	Measurement				Describe the relationship between circumference and diameter of a circle and define pie		Yes	No	No
							Apply the concepts of circumference and area of and solve in for various circular objects in real life	Solve problems involving circumference and area of circular objects in real life problems	Yes	Yes	Yes
12.3	Areas of Sector and Segment of a Circle	Areas of sector and segment of a circle	Measurement				Describe sector and segment of a circle and differentiate between the two		Yes	No	No
							Describe minor and major sector of a circle and differentiate between the two		Yes	No	No
							Describe minor and major segment of a circle and differentiate between the two		Yes	No	No
							Apply the formula of area of sector and segment of a circle, and compute the area of a specified region		Yes	Yes	No
							Calculate the length of an arc of a circle and comment whether it is the major arc or minor arc		No	Yes	No
12.4	Areas of Combinations of Plane Figures	Areas of combinations of plane figures	Measurement				Calculate the area of various combinations of plane figures and apply the concepts of circles, quadrilaterals and triangles		Yes	Yes	Yes
12.5	Summary		Measurement								
13	**Surface Areas and Volumes**										
13.1	Introduction		Measurement	M.10.1	Finds surface areas and volumes of objects in the surroundings by	Visualizes objects in surrounding as a combination of		Recall different solids such as cuboid, cone, cylinder and sphere and visualise the	Yes	No	No

					visualising them as a combination of different solids like cylinder and a cone, cylinder and a hemisphere, combination of different cubes etc.	different solids like cylinder and a cone, cylinder and a hemisphere, combination of different cubes etc.in order to find their surface areas and volumes		combination of these shapes				
13.2	Surface Area of a Combination of Solids	Surface area of a combination of solids	Measurement				Apply formulae of surface area of different 3D solids and derive the surface area of combination of these solid objects		No	Yes	Yes	
13.3	Volume of a Combination of Solids	Volume of a combination of solids	Measurement				Apply formulae of volume of different 3D solids and derive the volume of the combination of these solid objects		No	Yes	Yes	
13.4	Conversion of Solid from One Shape to Another	Conversion of cuboid, sphere, cone, cylinder	Measurement				Combine different solid shapes to create a new solid form		No	Yes	Yes	
13.5	Frustum of a Cone	Frustum of a cone	Measurement				Apply the formula of surface area of a cone and derive the total surface area of the frustum		No	Yes	Yes	
							Apply the formula of volume of a cone and derive the volume of the frustum		No	Yes	Yes	
13.6	Summary		Measurement									
14	**Statistics**											Mathematical dialogues
14.1	Introduction		Data & Chance	C.10.1	Calculates mean, median and mode for different sets of data related with real life contexts	Calculates mean, median and mode in order to apply them to real life contexts						
14.2	Mean of Grouped Data	Derivation of formula and problems based on it	Data & Chance				Apply direct method and calculate the mean of the grouped data	Calculate the mean of the grouped data	No	Yes	No	
							Apply assumed mean method and calculate the mean for a grouped data	Calculate the mean of the grouped data using assumed mean method	No	Yes	No	
								Calculate the mean of the grouped data using step deviation method	No	Yes	No	
14.3	Mode of Grouped Data	Derivation of formula and problems based on it	Data & Chance				Compute the mean and mode of the given data and interpret these two measures of central tendency		No	Yes	No	

14.4	Median of Grouped Data	Derivation of formula and problems based on it	Data & Chance				Apply formula for the median of a given grouped data and calculate missing values of frequency		No	Yes	No	
							Differentiate between mean, median and mode with examples and use most effective measure of central tendency in various cases		No	Yes	Yes	
14.5	Graphical Representation of Cumulative Frequency Distribution	Ogives	Data & Chance				Derive the co-ordinates to plot a graph and represent the two ogives		Yes	Yes	No	
							Graph both ogives for the data obtained and determine the median of the given grouped data		Yes	Yes	No	
14.6	Summary		Data & Chance									
15	**Probability**											Metacognitive strategies
15.1	Introduction	Probability	Data & Chance	C.10.2	Determines the probability of an event	Calculates in order to determine the probability of a given event		Recall formula for finding experimental/empirical probability				
15.2	Probability — A Theoretical Approach	Probability	Data & Chance				Differentiate between Empirical Probability and Theoretical Probability and find the two for a variety of cases	Differentiate between Empirical Probability and Theoretical Probability	No	Yes	No	
							Calculate the probability of given events in an experiment and comment whether they are Complementary Events /Sure Events /Impossible Events		No	Yes	No	
							Represent using organized lists, tables, or tree diagrams and		Yes	Yes	No	

							list the sample space for compound events					
							Calculate the probability of various events and rank them from most probable to least probable events		No	Yes	No	
15.3	Summary		Data & Chance									

8. CONCLUSION

8.1. Summary

This learning framework contributes to Indian education by introducing and unpacking competency-based education which is a valid approach to prepare students for the rapidly changing future.

Chapter 1 introduces competency-based teaching and learning. It outlines how technological advancement has drastically altered the landscape of education. The focus of education is gradually shifting from rote learning and acquisition of skills to problem solving rooted in critical thinking and creativity. Competency-based teaching and learning, with its student-centric paradigm, is identified as a useful alternative to the traditional curriculum-based pedagogy in helping students to navigate the dynamic challenges that they are expected to encounter in a technology-driven world. PISA is introduced as an internationally accepted best practice in competency-based assessment. Participation in PISA, with its strong foundation in competency-based learning framework, is identified as a useful starting point for India to embark on competency-based pedagogic practices in its classrooms.

Chapter 2 in this competency-based learning framework, explains the term 'literacy' in mathematics. It states that mathematical literacy has two components – development of mathematical understanding and communication of mathematical reasoning. The chapter highlights the importance of mathematical literacy in the 21st century. It is vital for the teachers to encourage their students to think critically, reason mathematically, and possess problem-solving skills. The chapter elaborates upon the PISA definition of mathematical literacy and discusses competencies like formulating, employing, and interpreting and evaluating. Understanding and application of mathematical facts, terminologies and procedures are considered the start of the journey towards acquiring mathematical literacy. Reasoning is considered the basis of all mathematical competencies. The chapter emphasises that if a student is capable of reasoning, he/she is more likely to follow a systemic approach towards problem-solving in real life. A mathematically literate person can contribute productively in society. The chapter also suggests that mathematical literacy is in line with the principles defined for quality education in India's National Education Policy 2020 and can help in attaining national education goals.

Chapter 3 of the competency-based learning framework focuses on the importance of competency-based learning among students and how it equips them to be better and resourceful future citizens. This chapter includes a detailed discussion of the PISA mathematical competencies. The three competencies assessed in PISA 2021 are: formulating situations mathematically; employing mathematical concepts, facts, procedures and reasoning; and interpreting, applying and evaluating mathematical outcomes. All these competencies are discussed in relation to problem-solving tasks.

The first competency discussed in chapter 3 is 'Formulating situations mathematically'. Students possessing this competency, are expected to identify and organise the information given in a problem and translate the given information from real-life situations to mathematical representations. A competent student should be able to simplify the problem and identify its mathematical aspects, recognise the mathematical structure and represent the problem mathematically, translate and decode the problem to get a better understanding of what has been asked in the problem.

The second competency discussed in the chapter is 'Employing mathematical concepts, facts, procedures and reasoning'. In order to display this competency, after translating the given information into mathematical representations, students can proceed to the next step of selecting and using appropriate strategies to solve the problem. Students need to perform a simple calculation, draw a simple conclusion, make an argument and justify their actions in a problem situation.

The third competency discussed in the chapter is 'Interpreting, applying and evaluating mathematical outcomes'. This competency involves the next step of the problem-solving cycle which is to evaluate the mathematical solution for its validity in real-life situations. Students are also required to communicate solutions to the outer world. They should be able to use mathematically correct language and terminology to communicate their solutions.

Students possessing these competencies can evaluate a mathematical outcome and interpret the mathematical result of the problem. They can also understand the extent and limits of mathematical concepts, identify the limitations of mathematical models, make predictions and provide supporting evidence for mathematical arguments.

Chapter 4 is based on the development of mathematical competencies in the classroom. The first section of the chapter provides the answer to the question of why it is important to develop competencies in a classroom. The second section is devoted to strategies which teachers can use to develop mathematical competencies like reasoning and problem-solving.

Chapter 5 of this framework focuses on mathematical proficiencies, capabilities and progressions. It explains how all three are interrelated by using the example of proficiency assessments. Mathematical proficiencies are considered as capabilities which are built on conceptual knowledge, effective use of strategies, metacognition and an individual disposition towards mathematics. The chapter explains how mathematical literacy can be measured in terms of mathematical proficiencies. Conceptual understanding is the basis of mathematical proficiencies, and mathematical concepts are taught to young learners through curriculum designed by different agencies of various states. However, some of the constituents of mathematical proficiency are not captured in the domain curriculum; thus, it is challenging to assess mathematical proficiency through the traditional assessment of conceptual understanding. To ease the process of proficiency assessment and make it curriculum free, learning progressions in mathematics were developed. A learning progression maps key stages in the development of learning mathematics from basic concepts to complex interpretations and applications.

This chapter also discusses the importance and use of learning progression in teaching and learning. It emphasises that a teacher with a clear understanding of learning progressions can help students in building their mathematical competencies. The chapter further explains the levels of proficiencies in terms of proficiency scales using the PISA proficiency scale as an example and reviews some of the items used in the assessment to assess mathematical literacy. Further it provides guidance to teachers how competency based items associated with NCERT curriculum can assess mathematical literacy competencies.

Chapter 6 of this framework discusses the use of assessment in the teaching-learning process. The chapter projects rubrics as a bridge between assessment and teaching-learning process. It highlights the importance of providing rubrics to students at the beginning of a unit of instruction or task to integrate learning outcomes, the teaching-learning process and assessment.

The chapter further explains the various types of rubrics and provides guidelines to develop rubrics for specific purposes. It is suggested in the chapter that general rubrics based on specific

competencies can help in developing teaching strategies that guide students towards mathematical proficiencies. Examples of how to develop rubrics are discussed to illustrate how criteria and descriptions of performance levels can be written. It has been emphasised in the chapter that the focus of writing criteria should be on learning outcomes, not aspects of a specific task. The descriptions of levels of performance should not overlap and should help teachers in evaluation and instruction as well as aid students in self-evaluation and in improving their proficiency.

Chapter 7 of this framework provides a mapping of subtopics in the NCERT curriculum with the PISA competencies. The mapping shows the alignment of mathematical topics, learning outcomes, learning objectives, and the PISA competencies. The purpose of alignment is to help teachers are aware of the mathematical literacy competencies associated with particular mathematical content in the NCERT curriculum.

8.2. Suggestions for teaching mathematics

Traditional practices in mathematics focus on the mastery of mathematical facts, algorithms, and proofs. The improvisation of this approach, with the help of competency-based learning suggested in this framework, can help prepare young learners for future needs. Throughout the framework, stress is on helping students to attain mathematical competencies. In chapters 4, 5 and 6, strategies to prepare a mathematically literate generation are suggested.

Recommendation 1: Teaching metacognition, reasoning and problem-solving strategies to improve proficiency in mathematics classrooms

There are three categories of teaching strategies suggested in the framework:

- Metacognition activation strategies
- Reasoning strategies
- Problem solving strategies

It is recommended that teachers activate metacognitive skills in students. Studies have shown that mathematics taught using metacognitive strategies result in a more profound knowledge of the subject. Metacognitive strategies enable learners to think about their own thinking, resulting in better control of the learning. They help students' master mathematics by connecting mathematical facts, procedures, and ideas. They focus on the method of problem-solving rather than on finding the correct answer. One of the results of activation of metacognitive skills in mathematics is enhancement of the capability of a student to reason.

Reasoning is considered as a core skill in being mathematical literate. The framework contains specific strategies to enhance mathematical reasoning in students. Reasoning skills can be activated through teaching strategies like using multiple mathematical representations of the same concept and establishing a connection between them. Open-ended classroom activities help students widen their thinking process. Students become aware that a problem need not have just one solution; there can be multiple solutions obtained through multiple methods. Thus, a student realises the importance of the process of problem-solving rather than finding a solution.

In competency-based assessments, the focus is on getting information about young learners' preparedness to solve real-life problems. By shifting students' perspective to the process of solving a problem, teachers can strengthen their problem-solving capabilities. Communication of the logic used to solve problems and defending the solution is a critical component of problem-solving. Strategies like mathematical dialogues help students understand others' mathematical logic and communicate their reasoning.

Recommendation 2: Using learning progressions

One of the key recommendations of the framework is to use learning progressions to bridge the gap between student knowledge and curriculum expectations. Learning progressions help teachers to see a bigger picture of mathematical education and guide students throughout their developmental process. In this way, the learning progression defined by the PISA proficiency levels help teachers and students to picture the learning pathway to become a mathematically literate citizen in the 21st century. The PISA proficiency levels depict progression in mathematics learning from the lowest level of problem solving which involves familiar contexts where all relevant information is present and the questions are clearly defined to the highest level of problem solving which involves conceptualising, generalising and utilising information based on their investigations and modelling of complex problem situations.

Recommendation 3: Using rubrics to inform the teaching learning process

It is recommended that teachers use rubrics to collect evidence of student progress as well as receive feedback on instructional practices implemented in the classroom. It is suggested that teachers develop rubrics based on learning outcomes rather than specific tasks as it will help them to synchronise teaching-learning and assessment with the desired outcomes. A well-defined rubric should have criteria linked with the proficiency required to complete the task and non-overlapping performance levels indicating the progress of capabilities.

It is clear that society needs to educate the new generation of learners in a different way to prepare for the great uncertainty they will face in a technology-driven world. We hope that the details of competency-based education unpacked in this learning framework will help India achieve this objective.

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ANNEXURE

Linking examples of teaching strategies to PISA competencies through NCERT curriculum

Learning Outcome: Applies the concept of per cent in profit and loss situation in finding discount, VAT and compound interest. E.g. calculates discount per cent when marked price and actual discount are given or finds profit per cent when cost price and profit in a transaction are given.

Sample tasks showing the application of problem-solving strategy in classrooms to foster problem solving competency. The task proposed can be adapted by the teacher to suit their context. For example, they can change 'muffin' to a suitable local word; caution should also be exercised in regard to problems about pricing and other similar calculations.

Example 1: Applying Problem Solving Strategies		
Problem	**School fete muffins** For the school fete, four Year 9 students are going to make muffins. A local baker donated 10 kilograms of flour and the school is supplying the other ingredients. The ingredients for one batch, which consists of 12 muffins, is given below: • 240g of flour • 60 grams sugar • 1 egg • ¼ cup oil • 1 cup milk The students want to make as many muffins as possible using the 10kg of flour. Work out how much of each ingredient is needed and the number of muffins that can be made. If the cost of the ingredients is estimated to be Rs 250 per batch, excluding flour, how much will they make if they sell the muffins for Rs 30 each?	
Formulate the problem mathematically. Strategy - **Analyse the Information**	**Modelling** - The teacher can model (unpacking the problem and recording important information) on the board. **Discussion -** The teacher can initiate a discussion about what information is provided and what is to be found	**Example work** Each batch needs 240 grams flour and 10 kg flour in total. Each kilogram is 1000 grams, so we need to work out how many

Example 1: Applying Problem Solving Strategies		
	Summarising - Classroom discussion could result in a summary of this information on the board. Students can then be asked to summarise the rest of the information in the problem, recording what they know.	lots of 240 grams are in 1000 grams.
Employing mathematical concepts, facts, procedures and reasoning. Strategy - **Hypothesising**	Ask students what assumptions they need to make (the assumption is that each batch will produce 12 muffins). Teachers can use questioning and discussion techniques to ascertain the proportional relationship when scaling up recipes and ask students to discuss mathematical approaches which might be helpful here. Students then need to solve the problem and record their reasoning to communicate how they have solved the problem.	**Example work** Work out number of batches Work out the quantity/cost of other ingredients Calculate the cost for the number of batches Work out the number of muffins Calculate the cost Calculate the profit
Interpreting, applying and evaluating mathematical outcomes. Strategy- **Asking questions**	Teachers can discuss the logical sequence for solving the problem. Questioning and discussion will lead students to record the order in which they might need to perform mathematical calculations and their strategies for solving. Students should provide a statement of the answer at the end of the report. Encourage re-reading of the problem to check that all information has been used and that the problem has been answered.	**Example work** 41 batches of muffins and 12 muffins in each gives a total of 492 muffins To make 41 batches, it costs Rs 250 x 41 = Rs10, 250 If 492 muffins are sold for Rs 30 each, it amounts to Rs 14,760 Therefore, 492 muffins can be made Profit = income - cost to produce muffins Profit is Rs 14,760 – Rs 10,250 = Rs 4510

Example 1: Applying Problem Solving Strategies

If all the muffins are sold, the school will have a profit of Rs 4510.

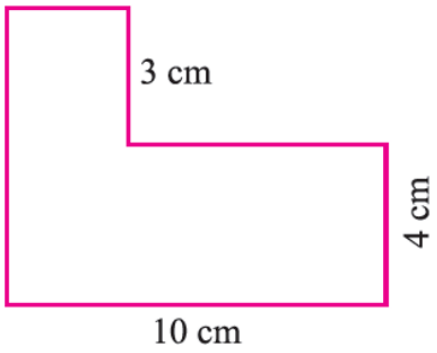
Learning outcome: Finds the area of a polygon.

Sample task showing use of applying reasoning strategy in the classroom to foster problem solving competency. A simple task can be used to illustrate different solving methods.

Example 2: Applying Reasoning Strategies

Problem (Based on multiple methods single solution)

Find the area of the given figure.



Strategy - Mathematical Dialogues

Teacher can divide the students into small, heterogeneous groups

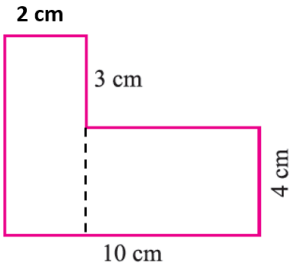
Teacher can ask students to divide the given figure into parts by drawing lines in a way area can be calculated. Encourage students to test conjectures and hear the justification of peers

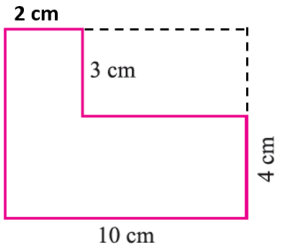
Encourage students to develop discursive and convincing arguments for themselves.

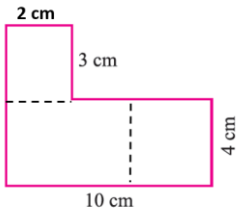
Use language structures in the form of sentence starters like, I think this because…, This can't be because…

Give students time to articulate and reflect on their ideas.

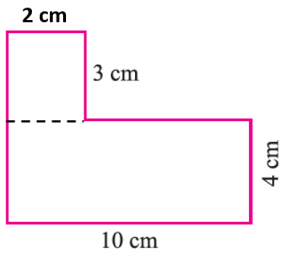
Example work







Example 2: Applying Reasoning Strategies



Interpreting, applying and evaluating mathematical outcomes- **Asking questions**	Teachers can discuss the division of the given figure. Questioning and discussion will lead students to division should lead to parts with known dimensions.	**Example work** The figure can be divided into two rectangles. Area of rectangle 1 is 2 cm x 3 cm = 6 cm^2 Area of rectangle 2 is 10 cm x 4 cm = 40 cm^2 Thus, area of the complete figure = 6 + 40 = 46 cm^2