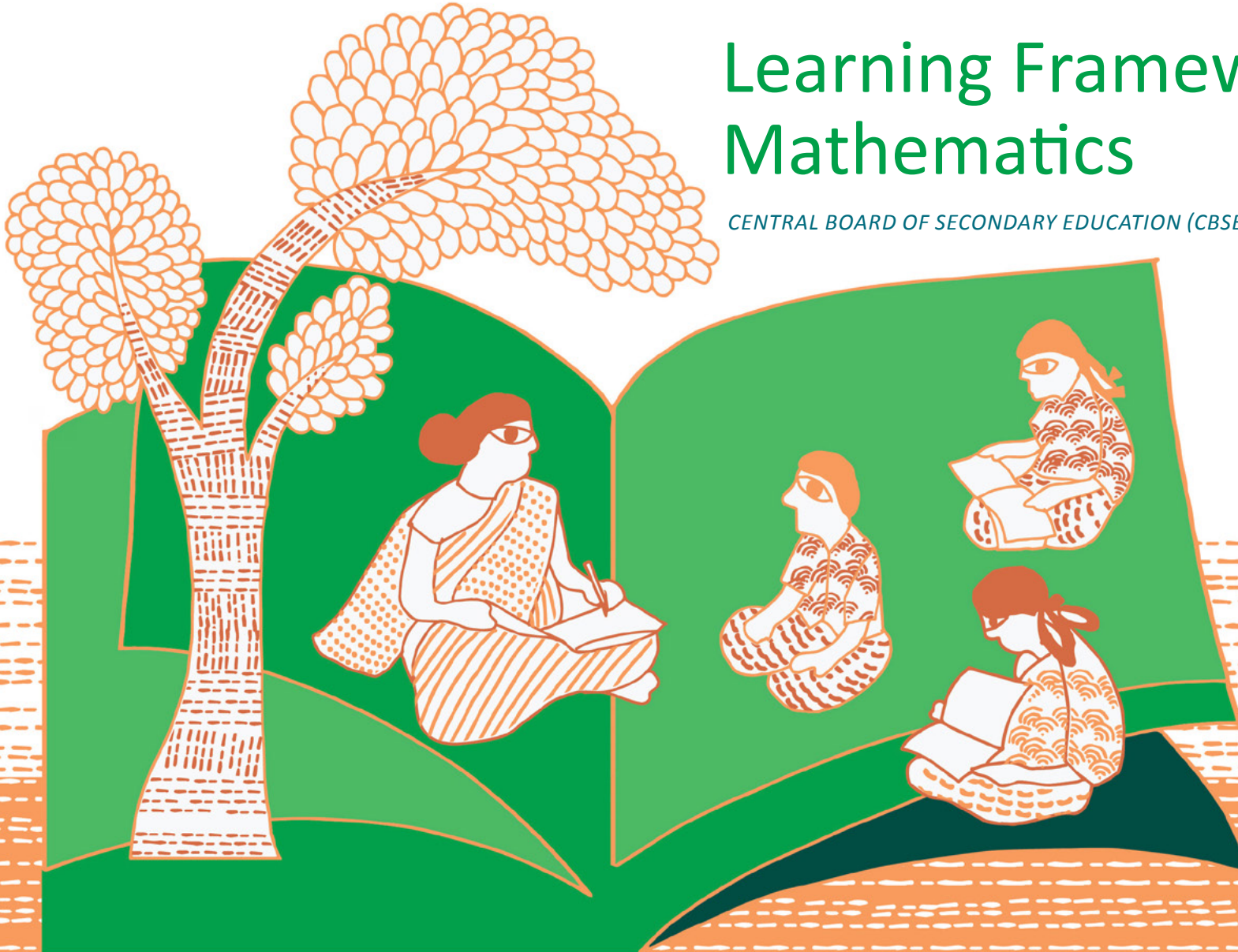# Learning Framework - Mathematics

CENTRAL BOARD OF SECONDARY EDUCATION (CBSE)

Foreword

The National Education Policy (NEP) 2020 released by the Government of India, has given a clear mandate for competency-based education (CBE) to enhance acquisition of critical 21st century skills by the learners. The first determinant for implementing CBE is a curriculum which is aligned to defined learning outcomes and that clearly states the indicators to be achieved.

The Central Board of Secondary Education (CBSE) has collaborated with Azim Premji University, Bengaluru, to develop the Learning Framework for Science, Mathematics, Social Science, English and Hindi for classes 9 and 10. The Learning Frameworks comprise explicitly stated knowledge, skills and dispositions that an education system should try to achieve. These frameworks would help develop a common shared understanding among teachers, students and other stakeholders and would serve as a common benchmark for teaching, learning and assessment across the country. These frameworks present indicators that are aligned to the CBSE curriculum and the NCERT Learning Outcomes. They further outline samples of pedagogical processes and assessment strategies to encourage curiosity, objectivity, creativity with a view to nurture scientific temper. This framework would be a key resource for the teacher as he/ she transacts the curriculum. They have been developed to ensure that the teacher aligns the teaching learning to meet the set quality standards and also use it to track learning levels of students. The effort has been to synchronize focus on quality education with uniformity in quality of standards across CBSE schools.

We hope, these frameworks would not only become a reference point for competency-based education across the country but also facilitate planning and design of teaching-learning processes and assessment strategies by teachers and other stakeholders

Team CBSE

Preface

The shift towards competency-based teaching and learning in the National Education Policy 2020 will be an important basis for curricular and pedagogical transformation in schools. The learning goals defined by the Policy, namely holistic and integrated development of students, and the acquisition of higher order cognitive capacities such as analysis, critical thinking and problem solving will be enabled by this shift. Such indicators will also enable equitable educational experiences through ensuring equivalence of pedagogical approaches and learning outcomes across schools and Boards of Examination.

In keeping with the thrust on indicator based teaching-learning proposed in the National Education Policy, 2020, Azim Premji University has supported the Central Board of Secondary Education to develop a 'Learning Framework'. The learning framework is a comprehensive package which provides learning outcomes, indicators, assessment frameworks, samples of pedagogical processes, tools and techniques for formative assessment, blueprint, assessment items and rubrics. Five such frameworks have been developed for English, Hindi, Science, Social Science and Mathematics at the secondary stage.

This document details the Learning Framework for Mathematics. The National Education Policy 2020 emphasizes "Mathematical thinking, problem solving, connecting 'classroom mathematics with 'real life mathematics' and recreational mathematics will be incorporated throughout the school curriculum …. in order to excite children about mathematics and develop the logical skills that are critical throughout school years and indeed throughout life."

As per the National Focus group Position Paper on Teaching of Mathematics, at the secondary stage, mathematics is perceived as a discipline in which argumentation and proof take a central stage. The concepts and techniques learnt in the elementary stage should be utilized at this stage to solve problems which require understanding of more than one content area in an integrated manner.

Mathematics education at this stage is also used to solve problems not only in daily life but those encountered in physical and social sciences. Hence, mathematical tasks should be designed in such a manner that a student's mathematical knowledge helps provide logical solutions to realistic societal problems thereby helping in informed, justified and democratic decision making. This document presents outcomes and indicators that focus on these Mathematics specific skills that students need to attain through different concepts addressed in the syllabus. In addition to this, sample pedagogical processes, formative assessment strategies and summative assessment items are also provided to enable teachers to derive principles for making the alignment between learning outcomes-pedagogy and assessment in their classrooms.

Team Azim Premji University

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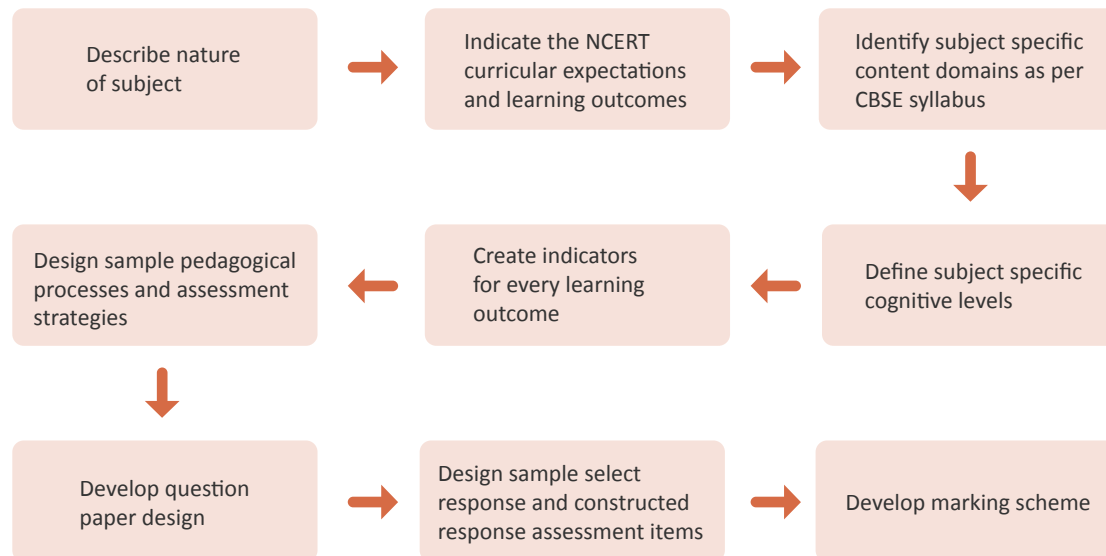- Ms. Sapna Makan, TGT, Bal Bharati Public School, Sector -14, Rohini, Delhi

Contents

1. Introduction

Development of quality standards is paramount for any education board to understand and track the learning level of learners in the country. These standards comprise explicitly stated knowledge, skills and dispositions that the education system should strive to achieve. The quality standards are expected to serve as common benchmarks for teaching, learning and assessment. It is hoped that this framework would enable teachers, schools and other boards to design teaching-learning processes and develop assessment tools. The learning framework is a comprehensive package which delineates indicators, assessment frameworks, samples of pedagogical processes, tools and techniques for formative assessment purpose, blueprint, assessment items and rubrics.

2. Process of Developing the Learning Framework Document

An academically rigorous process has been undertaken to develop this document for various subjects, namely, English, Hindi, Mathematics Social Sciences, and Science. The flow diagram below depicts the steps in the process of developing the Learning Framework.

| Describe nature of subject | → | Indicate the NCERT curricular expectations and learning outcomes | → | Identify subject specific content domains as per CBSE syllabus |

| Design sample pedagogical processes and assessment strategies | ← | Create indicators for every learning outcome | ← | Define subject specific cognitive levels |

| Develop question paper design | → | Design sample select response and constructed response assessment items | → | Develop marking scheme |

2.1 Describe Nature of Subject

The school curriculum as per National Curriculum Framework 2005, is a broad plan for facilitating certain capabilities in learners which are guided by the larger aims of school education. These aims of school education are an inter-play of various variables – philosophical, psychological, existing socio-cultural context and the needs of the society, the nature of learning and many more informed by research etc. Various school subjects enable in pursuing these aims through their distinct disciplinary focus and methods. This section clearly states the processes followed for constructing knowledge in a subject, and the tools and methods of enquiry used in each subject. Further, the relevance of engaging with the given subject in school education is highlighted. The research affirms that an in-depth understanding of the nature of subject would enable a teacher to design quality pedagogy and assessment in alignment to the subject specific skills.

2.2 Indicate the NCERT Curricular Expectations and Learning Outcomes

The broad aims of school education stated in National Education Policy (NEP) 2020 are to-
- Develop good human beings capable of rational thought and action, possessing compassion and empathy, courage and resilience, a scientific temper and creative imagination, with sound ethical moorings and values

- Build character and enable learners to be ethical, rational, compassionate and caring

- Achieve full human potential – through developing all aspects and capabilities of learners

- Develop the creative potential of each individual

- Enable learners to learn to think critically and solve problems, and use a multidisciplinary perspective

- Enable learners to innovate, adapt and absorb new material in novel and changing fields

- Enable learners to not only learn, but more importantly learn how to learn

- Develop engaged, productive and contributing citizens for building an equitable, inclusive and plural society as envisaged by our Constitution

- Achieve economic and social mobility, inclusion and equality by ensuring all learners can access quality education with particular focus on historically marginalised, disadvantaged, and underrepresented groups

- Prepare a workforce with multidisciplinary abilities across the sciences, social sciences, arts and humanities through experiential, holistic, integrated, inquiry-driven, discovery-oriented, learner-centred, discussion-based, flexible and enjoyable education

The curricular expectations related to different age-groups are spelt out stage-wise for various levels- primary, upper primary, secondary and senior secondary. These are stated at a broader level and pitched at an appropriate level of learner's understanding. In this document, curricular expectations as defined in NCERT secondary stages learning outcomes document are referred. These curricular expectations form the basis for organising the age-appropriate content based on learners' prior knowledge. Curricular expectations for a subject are ascertained through a range and variety of specific learning outcomes defined for each class. The NCERT learning outcomes at the secondary stage are the primary point of reference for this document. Learning outcomes provide a benchmark on which learning progress can be tracked in both quantitative and qualitative manner. The NCERT learning outcomes linked with curricular expectations are indicator based and help to determine the pedagogical processes for their development. The indicator based learning encourages learners to not only acquire knowledge but also apply knowledge and skills to successfully perform tasks in real life situations.

2.3 Identify Subject Specific Content Domains as per CBSE Syllabus

The learning outcomes for each subject are expressed in terms of cognitive skill to be demonstrated and the content to be acquired by the learners. In accordance with the nature of the subject, the CBSE curricula aligns content domains to various textbook chapters for every subject. To support teachers see the connection between syllabus, textbook and learning outcomes, relevant NCERT learning outcomes are mapped to each of these content domains/ themes.

2.4 Define Subject Specific Cognitive Levels

Cognitive levels describe the thinking processes that learners are expected to engage in when encountering the indicators. Revised Bloom's taxonomy provides a useful frame of reference to classify the learning outcomes. It also helps in aligning the classroom pedagogy and assessment practices with the learning outcomes. In this document, an attempt is made to define the subject specific cognitive levels that essentially draw from the Revised Bloom's Taxonomy. Such subject specific cognitive levels are able to capture the disciplinary knowledge and skills in a concrete manner. Assessment frameworks developed by Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) have been referred to while conceptualising these subject specific cognitive levels.
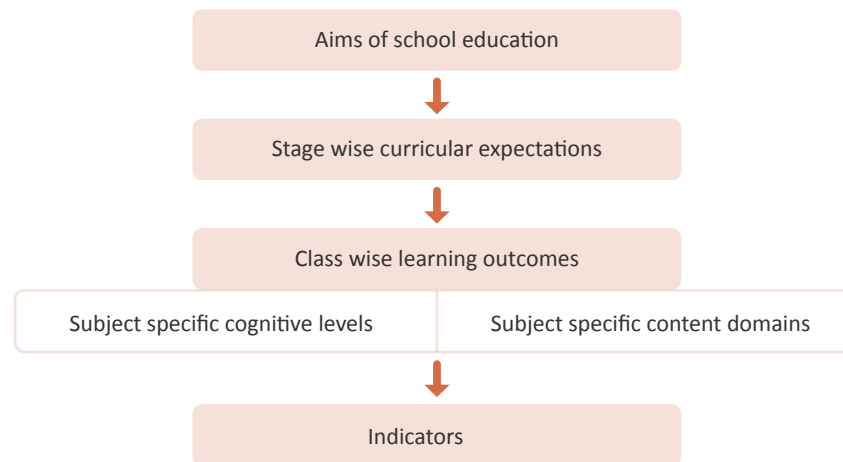
2.5 Devise Indicators for Every Learning Outcome

The overarching learning outcomes are further broken down into specific indicators aligned to the subject specific skills and the content in related chapters in the textbook. They are focused on subject specific skills that learners need to attain through different concepts addressed in the CBSE curriculum. A clear understanding of the scope of these learning outcomes for each concept dealt within a textbook chapter will be immensely helpful for both teachers and learners to plan their teaching and learning in a better way. Therefore, indicators aligned to subject specific skills are defined for every learning outcome. These indicators are -

• Aligned to the goals of teaching the subject at the secondary stage

• Defined as specific, measurable and demonstrable indicators

• Distributed across all content domains/ themes as per the prescribed CBSE syllabus

• Address 21st century skills including collaboration, communication, creativity, and critical thinking across school subjects

• Range from simple to complex cognitive processes for all subjects

• Supplemented with sample assessment strategies and provide suggestions to concretise these in the classrooms

2.6 Design Sample Pedagogical Processes and Assessment Strategies

NCERT's secondary stage learning outcomes document provides a common set of pedagogical processes for each subject. The specific indicators are further supported with sample pedagogical and assessment strategies. This has been done to enable teachers to derive principles for making the alignment between learning outcomes, pedagogical practices and assessment in their classrooms. The key principles considered while designing the pedagogical processes and assessment strategies are:

• Student-centred

  o Since new knowledge is built over existing knowledge, both pedagogy and assessment shall focus on learners' pre-requisite knowledge, skills, attitudes, and beliefs that they bring into a classroom setting.

  o Constructivist approaches to learning, with the learners being at the center of the learning process as an active constructor of knowledge shall be emphasised.

  o Since learners effectively learn by doing, classroom processes shall involve activities and analysis on issues surrounding the environment.

  o Cooperative and peer-supported teaching learning activities shall be used to empower learners to take charge of their own learning.

• Indicators centred

  o The focus shifts to measurable and observable behavior of what learners do at the end of an instruction.

  o Learners comprehend better when the method of teaching, learning activities and assessment strategies are all aligned well to the indicators. Pedagogical processes and assessment strategies shall be aligned to both content domains and cognitive skills as indicated in indicator statements.

• Assessment centred

  o Assessments should be viewed as an integral part of pedagogy and focus on giving timely individualised feedback to learners. Quality formative assessments shall be designed as it helps to modulate learners understanding of their own learning and help teachers adapt their pedagogy based on learner's performances.

  o Multiple modes of assessment, including portfolios, project work, presentations, written and oral assignments, shall be used to provide a scope to reflect individual capacities of a learner.

  o Peer assessment involving learners assessing the work of their peers against set assessment criteria shall be used.

2.7 Develop Question Paper Design

Question paper design also called as blueprint is a map and a detailed specification that ensures that all aspects of the curriculum are covered by the question paper. It translates the design in operational terms and all the dimensions of an item (content domain and cognitive level) become clear to the question paper developer. The information in the question paper design pertains to the content domains and; cognitive levels that will be tested, number of assessment items; aggregation of the percentages of content domains and cognitive levels. A balanced design adequately represents all the content domains and uniformly addresses a range of cognitive skills. This is a planning document where all the relevant information of a question paper is listed in the form of a table.

2.8 Design Sample Select Response and Constructed Response Assessment Items

A test developer may decide on the item format/item type that will be used in the test. Broadly, item formats are of two kinds-
Selected response questions like multiple choice questions, true/false where learner has to select the correct response from the options provided and Constructed response questions- where the learner has to produce the correct response. These could be as short as one-word responses to long essay type questions. A sample set of assessment items of both select response type and constructed response type is included in this document.

2.9 Develop Marking Scheme

A marking scheme is as important as the assessment item. For assessment items, particularly those assessing understanding and other cognitive levels, it is necessary to include variations in learner's responses. There should be scope for fully correct, partially correct, levels of partially correct responses. Also, the marking scheme should be aligned to the cognitive level of the item. If an item is testing application of a concept, the marking scheme should illustrate the possible responses that could be representing application. At any given point of time the marking scheme would only be indicative – it may not include an exhaustive summary of all possible responses.

3. Potential Users/ Uses of the Document

- **Classroom teaching, learning and assessment:** This document will be helpful for classroom teachers and learners to see the connection between syllabus, textbook and learning outcomes. The indicators can be used to effectively plan classroom pedagogy and design formative assessment. The indicators can also be used by learners to self-assess and plan their learning better.
- **Pre-service and In-service Teacher Professional Development:** This document can be utilized for developing content for professional development programs for teachers both at the pre-service and in-service level. The sample pedagogical processes and assessment strategies will provide learner teachers as well as in-service teachers an insight on how to concretize the learning outcomes in the classroom.
- **Textbook development:** Clearly stated indicators could also facilitate the review and revision of the textbooks used for secondary stage and in the designing of new textbooks.
- **Developing Assessment Frameworks and tools for large scale assessments/surveys:** This document may be used for developing assessment frameworks and question papers for Board examinations of different states leading to uniformity in quality standards across states. It could be used as a framework for drawing key indicators for National Achievement Survey (NAS). Reporting the assessment data against these outcomes will provide a comprehensive picture of the overall health of the education system of the country.

4. Nature of subject

"Mathematics, as an expression of the human mind, reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality"- Courant and Robbins

Mathematics has been a part of everyone's life, be it estimates we make in our routine activities or precise calculations for various transactions and fairness in sharing or in describing objects around us. The relevance of mathematics is more than its utilitarian value. It helps us to think and reason about the world around us and take informed decisions, be it at the individual level to cope with life in various spheres of activity or at the societal level to contribute to technological and socio-economic development.

Given these reasons, it is not surprising that mathematics education has been made compulsory at the school level and is one of the first subjects encountered by the learner entering formal schooling. The focus of school mathematics is developing the problem solving and reasoning skills needed to have an organised and progressing society. This includes reflecting on and studying problems and topics which may be perceived as more of an intellectual exercise and not immediately useful at this stage. However, these have unforeseen far-reaching benefits. It must be emphasized that the selection of such study material must be made in a manner such that mathematics will not be a burden to the learner but an engaging and joyful activity.

Mathematical objects and ideas are abstract – created by humans from the needs of science, economics, statistics and any kind of quantitative analysis needed in daily life. That is, they have no physical properties such as size, colour, smell, taste, texture, sound and so on. Mathematical ideas are formed by classifying similarly related and commonly noticed properties. This leads to the pedagogical challenge of making these ideas experiential. For example, Number, which is a root concept is derived by providing experiences of collections of the same number of objects. The concept of addition is built on the concept of number, and it then becomes the pre-requisite concept for viewing multiplication as repeated addition. This in turn builds on to the understanding of higher concepts. Thus, mathematics builds up from the bottom i.e., from axioms and definitions in a structured and hierarchical way as a vast network of interlinked concepts.

It is well recognized how rigid mathematics is, i.e., 2 + 2 = 4 and not 5 or 22. However, new mathematics can and has been discovered based on 2 + 2 = 22 i.e., based on how the rules are modified. At the same time, this 'rigid' structure is free from perspectives and subjectivity. Mathematical truth, once established and consistent with existing results, lasts forever. Therefore, after the meaning of various mathematical objects and ideas are understood, one can engage with these, and discover their properties. Thus, with proper facilitation, a learner can be mostly self-dependent in learning mathematics and can even be given a glimpse of how math progresses or branches off if rules are changed.

To communicate mathematical ideas, mathematicians have, over the years, developed the language of mathematics which has vocabulary, symbols, and sentence structure and is characterised by both precision and concision. Mathematical language supports in communicating mathematical ideas or concepts during discussions in the classroom and leads to representing ideas, observing and generalizing patterns, communicating thought processes and justifying their discoveries and learning. However, it does bring its own pedagogical challenges when learners are not conversant in this language or when the language used is not age appropriate.

Assessment in mathematics has to encompass both the nature of mathematics and the difficulties which the learner faces because of it. When assessment is cognizant of the limitations of the learner and the constraints and affordances of the nature of the subject, it enables the learner to harness the power of mathematics and the teacher to enable the learner to do this.

5. Stage Specific Curricular Expectations

At this stage learners are expected to develop ability and attitude for —

| SNo.	NCERT Secondary Level Curricular Expectations
CE1	Mathematisation (ability to think logically, formulate and handle abstractions) rather than knowledge of procedures (formal and mechanical)
CE2	Mathematical vocabulary
CE3	Consolidation and generalisation of the concepts learnt so far
CE4	Understanding and proving mathematical statements
CE5	Addressing problems that come from other domains such as, science and social sciences
CE6	Integration of concepts and skills that the children have learnt into a problem-solving ability
CE7	Analysing and constructing the processes involved in mathematical reasoning
CE8	Establishing linkages between mathematics and daily life experiences and across the curriculum.

6. Content domains

The mathematics content up to Class X has been organised into seven content domains which have deep inter-connections and are not to be viewed as independent silos. An understanding of the whole is impossible without understanding the parts and vice-versa. The nature of mathematics permeates each content domain and provides the unifying thread which holds the strands together.

Content Domain	Class IX Textbook Chapter	Class X Textbook Chapter
1. Number Systems	Chapter 1: Number Systems	Chapter 1: Real Numbers
2. Algebra	Chapter 2: Polynomials Chapter 4: Linear Equations in Two Variables	Chapter 2: Polynomials Chapter 3: Pairs of Linear Equations in Two variables Chapter 4: Quadratic Equations Chapter 5: Arithmetic Progressions
3. Trigonometry	-	Chapter 8: Introduction to Trigonometry Chapter 9: Some Applications of Trigonometry
4. Coordinate Geometry	Chapter 3: Coordinate Geometry	Chapter 7: Coordinate Geometry
5. Geometry	Chapter 5: Introduction to Euclid's Geometry Chapter 6: Line and Angles Chapter 7: Triangles Chapter 8: Quadrilaterals Chapter 9: Areas of Parallelograms and Triangles Chapter 10: Circles Chapter 11: Constructions	Chapter 6: Triangles Chapter 10: Circles Chapter 11: Constructions
6. Mensuration	Chapter 12: Heron's Formula Chapter 13: Surface Area and Volume	Chapter 12: Areas Related to Circles Chapter 13: Surface Areas and Volumes
7. Statistics and Probability	Chapter 14: Statistics Chapter 15: Probability	Chapter 14: Statistics Chapter 15: Probability

7. Subject Specific Cognitive Levels

At the secondary stage, the content domain of the mathematics curriculum serves a two-fold process. The first is of course, mastery of content. The second is the development of process skills in the learner, using the content as a vehicle to this end. The learning outcomes dwell on both the content as well as the cognitive domain. Based on them, the cognitive dimension may be viewed as a spectrum of three broad levels, and these define the learner's engagement with the content domain.

Knowing → Applying → Reasoning
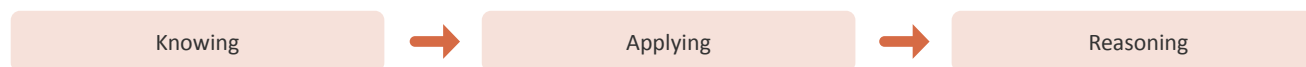
1. **Knowing** – One of the key curricular expectations for mathematics is the consolidation and generalisation of the concepts learnt so far. This cognitive domain primarily addresses the learner's ability to recall definitions, concepts, formulas, axioms, postulates and theorems which form the building blocks of the hierarchical structure of mathematics**.** A learner of mathematics must have a strong foundational knowledge and understanding of mathematical language and of the properties of mathematical objects. Teaching in the mathematics classroom often focuses on procedural understanding – where learners compute answers or prove results using taught procedures. Armed with these and with a set of heuristics, the learner is able to push the levels of understanding, prove new results and solve unfamiliar problems and develop confidence in mathematical communication as well as exploration. Some of the aspects of 'Knowing' are detailed below.

Aspects of Knowing	Examples of indicators (Class X)
Define	Defines sine, cosine, tangent, cotangent, secant and cosecant of an acute angle (in a right triangle) as ratios of appropriate sides.
Recall and provide examples, illustrations	Provides examples and non-examples of similar triangles.
Recall symbols, vocabulary, notation	Uses appropriate symbols ((~, $\cong$, $\perp$ etc.) and representation in writing the proof. Uses vocabulary related to heights and distances problems – for example: line of sight, angle of elevation/depression.
Recall facts, formulas	Recalls nature of pi. Recalls circumference formula.
Recognise patterns	Identifies that for any A.P. ($a_1, a_2, ... a_k$), $a_1 + a_k = a_2 + a_{k-1}$ = ... etc.
Relate two concepts	Relates the value of the discriminant to the nature of the roots.
Reproduce (including measure, compute, construct) procedures and proofs	Determines sine, cosine, tangent, cotangent, secant and cosecant of standard angles such as 0°, 30°, 45°, 60° and 90°. Obtains a term in a given arithmetic progression by identifying the first term, the common difference and the number of terms Constructs a pair of tangents to a circle. Proves Basic Proportionality theorem (and its converse).
Recall and explain	Explains the conditions for two triangles to be similar.
Differentiate between/among (based on recall of properties)	Differentiates between the sum of n terms and the nth term of an arithmetic progression.
Retrieve information	Retrieves median from the intersection of less than ogive and the line $y = N/2$.
Reframe using recall of definitions	Rephrases the division of a polynomial by another polynomial in the form divisor × quotient + remainder.

2. **Applying –** This cognitive domain focuses on using knowledge to determine strategies and represent, model or construct objects. Here, learners are required to engage in applying knowledge of facts, relationships, processes, concepts to solve problems in real life contexts and expand the dimensions of their acquired knowledge.

Aspects of Applying	Examples of indicators (Class X)
Identify by connecting known properties and definitions to given context or situation	Identifies variables given a real life situation related to linear relationships between two variables.
Classify by connecting properties or using information from given data	Distinguishes between congruent and similar figures.
Locate by using properties or computation	Plots the given equations of a pair of lines on the cartesian plane and identifies their point of intersection, if it exists.
Reframe using definitions, properties and by connecting two or more facts	Expresses the prime factorization of a number in exponential form. Draws labelled diagrams (involving the right triangle and the acute angle) representing the given situation.
Explain by connecting two or more facts	Interprets the meaning of the computed mode of a given dataset.
Compute or process by selecting strategy, operation(s) and operand(s)	Utilises the relationship between the zeroes and coefficients of a polynomial of degree less than 4.
Prove an unfamiliar statement using one or two known facts	Proves trigonometric identities based on Pythagoras theorem.

3. **Reasoning** – In this domain, learners are engaged in reasoning to analyse given information, drawing conclusions, and extending their understanding to deal with new situations. In contrast to the more direct applications of mathematical facts and concepts exemplified in the applying domain. Learning outcomes in the reasoning domain involve unfamiliar or more complicated contexts. Mathematical reasoning also encompasses pattern recognition, multiple connections, conjecture and proof.

Aspects of Reasoning	Examples of indicators (Class X)
Distinguish using logical reasoning	Classifies the roots of a quadratic equation as distinct/repeated/not real and rational/irrational.
Analyse	Models the change in the zero of a linear polynomial when the coefficients are changed.
Evaluate and choose	Selects the most efficient method of solving a given quadratic equation.
Extend properties, laws and theorems using reasoning and understanding constraints	Analyses conditions to prove similarity of triangles.
Prove using logical reasoning and understanding of properties, laws and theorems	Proves results/riders based on similarity of triangles.
Create situations based on given constraints	Designs steps to construct composite geometric shapes.

8. Learning outcomes

The following learning outcomes for secondary stage developed by the National Council for Educational Research and Training (NCERT) states important knowledge, skills and dispositions learners need to attain at the end of an academic year for Class IX and X Mathematics.
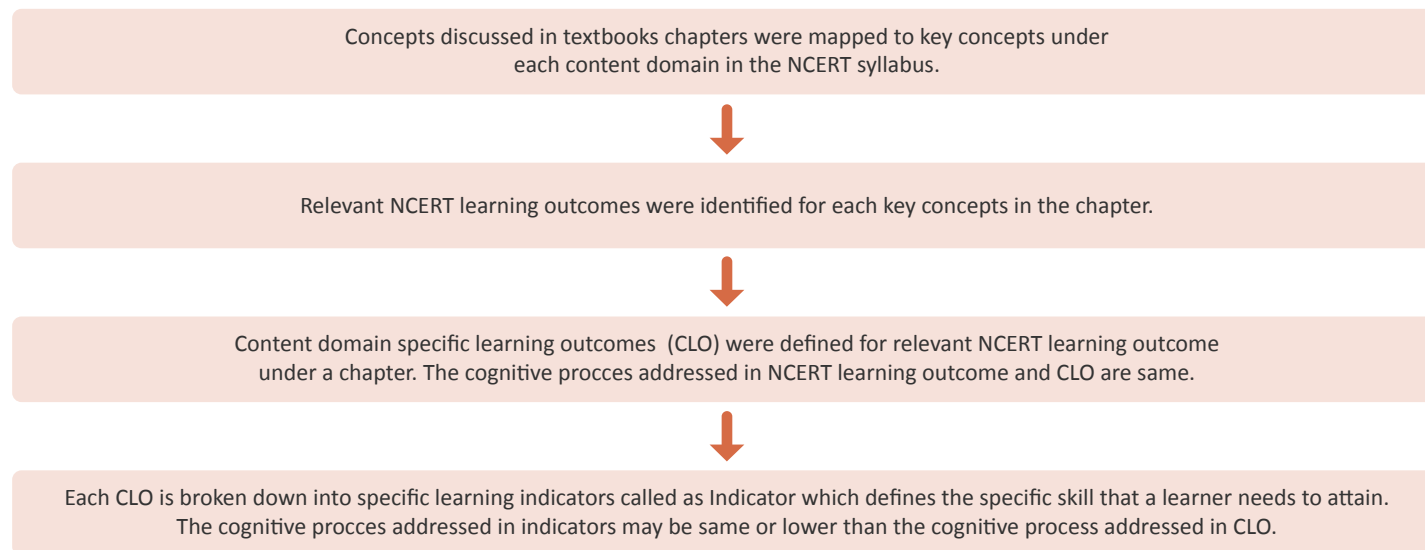
SNo.	Class IX – NCERT Secondary Level Learning Outcomes The Learner —
LO1.	Applies logical reasoning in classifying real numbers, proving their properties, and using them in different situations.
LO2.	Identifies/classifies polynomials among algebraic expressions and factorises them by applying appropriate algebraic identities.
LO3.	Relates the algebraic and graphical representations of a linear equation in one or two variables and applies the concept to daily life situations.
LO4.	Identifies similarities and differences among different geometrical shapes.
LO5.	Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles, etc., by applying axiomatic approach and solves problems using them.
LO6.	Finds areas of all types of triangles by using appropriate formulae and apply them in real life situations.
LO7.	Constructs different geometrical shapes like bisectors of line segments, angles and triangles under given conditions and provides reasons for the processes of such constructions.
LO8.	Develops strategies to locate points in a Cartesian plane.
LO9.	Identifies and classifies the daily life situations in which mean, median and mode can be used.
LO10.	Analyses data by representing it in different forms like, tabular form (grouped or ungrouped), bar graph, histogram (with equal and varying width and length), and frequency polygon.
LO11.	Calculates empirical probability through experiments and describes its use in words.
LO12.	Derives formulae for surface areas and volumes of different solid objects like, cubes, cuboids, right circular cylinders/cones, spheres and hemispheres and applies them to objects found in the surroundings.
LO13.	Solves problems that are not in the familiar context of the child using above learning. These problems should include the situations to which the child is not exposed earlier.

SNo.	Class X – NCERT Secondary Level Learning Outcomes The learner —
LO1.	Generalises properties of numbers and relations among them studied earlier to evolve results - such as, Euclid's division algorithm, Fundamental Theorem of Arithmetic - and applies them to solve problems related to real life contexts.
LO2.	Develops a relationship between algebraic and graphical methods of finding the zeroes of a polynomial.
LO3.	Finds solutions of pairs of linear equations in two variables using graphical and different algebraic methods.
LO4.	Demonstrates strategies of finding roots and determining the nature of roots of a quadratic equation.
LO5.	Develops strategies to apply the concept of A.P. to daily life situations.
LO6.	Works out ways to differentiate between congruent and similar figures.
LO7.	Establishes properties for similarity of two triangles logically using different geometric criteria established earlier such as, Basic Proportionality Theorem, etc.
LO8.	Derives formulae to establish relations for geometrical shapes in the context of a coordinate plane, such as, finding the distance between two given points, to determine the coordinates of a point between any two given points, to find the area of a triangle, etc.
LO9.	Determines all trigonometric ratios with respect to a given acute angle (of a right triangle) and uses them in solving problems in daily life contexts like finding heights of different structures or distance from them.
LO10.	Derives proofs of theorems related to the tangents of circles
LO11.	Constructs — A triangle similar to a given triangle as per a given scale factor. A pair of tangents from an external point to a circle And justifies the procedures.
LO12.	Examines the steps of geometrical constructions and reasons out each step
LO13.	Finds surface areas and volumes of objects in the surroundings by visualising them as a combination of different solids like cylinder and a cone, cylinder and a hemisphere, combination of different cubes, etc.
LO14.	Calculates mean, median and mode for different sets of data related with real life contexts.
LO15.	Determines the probability of an event and applies the concept in solving daily life problems.

9. Content Domain Specific Learning Outcomes and Indicators

The learning outcomes defined by NCERT are generic and broadly defined. They are broadly linked to the content domains but also address mathematics specific skills that learners need to attain through different concepts addressed in the syllabus. A clear understanding of the scope of these learning outcomes for each concept dealt in a textbook chapter will be immensely helpful for both teachers and learners to plan their teaching and learning better. To enable this, the following process has been followed:

Concepts discussed in textbooks chapters were mapped to key concepts under each content domain in the NCERT syllabus.

↓

Relevant NCERT learning outcomes were identified for each key concepts in the chapter.

↓

Content domain specific learning outcomes (CLO) were defined for relevant NCERT learning outcome under a chapter. The cognitive procces addressed in NCERT learning outcome and CLO are same.

↓

Each CLO is broken down into specific learning indicators called as Indicator which defines the specific skill that a learner needs to attain. The cognitive procces addressed in indicators may be same or lower than the cognitive process addressed in CLO.

Class IX

Content Domain, Chapter, Key Concepts	Learning Outcomes – NCERT	Content Domain Specific Learning Outcomes	Indicators
Content domain: Number Systems **Chapter 1: Number System** Key concepts: 1. Review of representation of natural numbers, integers, rational numbers on the number line. Representation of terminating / non-terminating recurring decimals on the number line through successive magnification. Rational numbers as recurring/ terminating decimals. Operations on real numbers.	LO1: Applies logical reasoning in classifying real numbers, proving their properties and using them in different situations.	CLO1: Applies logical reasoning in classifying real numbers.	C1. Explains the need for each new set of numbers. C2. Provides examples and non-examples of rational, irrational, real different types of numbers. C3. Classifies numbers into natural numbers, whole numbers, integers, rational and irrational numbers using their definitions. C4. Places numbers according to the organizational hierarchy of the real number system.
		CLO2: Justifies the properties of real numbers.	C5. Places rational numbers on the number line. C6. Finds rational numbers in between any two given rational numbers numerically and graphically on the number line. C7. Identifies cyclic patterns in remainders when the numerator of a rational number is divided by the denominator. C8. Differentiates between terminating and non-terminating decimals. C9. Defines irrational numbers. C10. Differentiates between rational and irrational numbers. C11. Locates irrational numbers on the number line (using Pythagoras theorem). C12. Locates irrational numbers in between any two given numbers. C13. Recognises rational numbers on either side of an irrational number. C14. Differentiates between rational and irrational numbers. C15. Locates irrational numbers on the number line (using Pythagoras theorem). C16. Locates irrational numbers in between any two given numbers.

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4. Rationalization (with precise meaning) of real numbers of the type and (and their combinations) $1/(a + b\sqrt{x})$ and $1/(\sqrt{x} + \sqrt{y})$ where x and y are natural number and a and b are integers. 5. Recall of laws of exponents with integral powers. Rational exponents with positive real bases (to be done by particular cases, allowing learner to arrive at the general laws.)		CLO3: Applies logical reasoning in using real numbers in different situations.	C17. Performs arithmetic operations on real numbers. C18. Simplifies expressions with real numbers using identities. C19. Simplifies expressions with real numbers using rationalization of denominator. C20. Determines the nature of sum, difference, product and quotient of rational numbers and irrational numbers or a combination of both. C21. Describes the meaning of the nth root of a real number. C22. Expresses the nth root of a real number using a rational exponent and vice-versa. C23. Extends the rules of exponents for rational powers.
Content domains: Algebra **Chapter 2: Polynomials** Key concepts: Definition of a polynomial in one variable, with examples and counter examples. Coefficients of a polynomial, terms of a polynomial and zero polynomial. Degree of a polynomial. Constant, linear, quadratic and cubic polynomials. Monomials, binomials, trinomials. Factors and multiples. Zeros of a polynomial. Motivate and State the Remainder Theorem with examples. Statement and proof of the Factor Theorem. Factorization of $ax^2 + bx + c$, $a \neq 0$ where a, b and c are real numbers, and of cubic polynomials using the Factor Theorem. Recall of algebraic expressions and identities. Verification of identities: $(a \pm b)^2$; $(a \pm b)^3$; $(a \pm b \pm c)^2$; $x^3 + y^3 + z^3 - 3xyz =$ $(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$ and their use in factorization of polynomials.	LO2: Identifies/classifies polynomials among algebraic expressions and factorises them by applying appropriate algebraic identities.	CLO4: Classifies polynomials based on number of terms and on degree.	C24. Differentiates between general algebraic expressions and polynomials. C25. Classifies polynomials on the basis of terms and degree: linear, quadratic and cubic and number of terms: monomial, binomial, trinomial. C26. Interprets the zeroes (roots) of a polynomial (polynomial equated to zero). C27. Relates the degree of a polynomial to the number of zeroes.
		CLO5: Applies remainder and factor theorems to polynomials.	C28. Uses the remainder theorem to find remainders or unknowns in division of a polynomial by a linear polynomial. C29. Relates the factor theorem to the remainder theorem. C30. Relates the factors of a polynomial with its zeroes and vice-versa. C31. Given a zero or a factor of a polynomial, expresses the polynomial in terms of the quotient, divisor and remainder. C32. Uses given information about the zero or factors of a polynomial to factorise it.
		CLO6: Applies algebraic identities to polynomials.	C33. Selects appropriate identity to simplify a calculation C34. Maps a polynomial to known identity/identities. C35. Factorises a polynomial using the appropriate identity (including splitting the middle term). C36. Factorises a polynomial by rephrasing it using the factor theorem and an appropriate identity.

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Content domain: Coordinate Geometry **Chapter 3: Coordinate Geometry** Key concepts: The Cartesian plane, coordinates of a point, names and terms associated with the coordinate plane, notations, plotting points in the plane.	LO8: Develops strategies to locate points in a Cartesian plane.	CLO7: Models the representation of points in the Cartesian plane.	C37. Explains mathematical vocabulary – origin, x-axis, y-axis, quadrants, abscissa, ordinate, ordered pair. C38. Selects the quadrant or the half of the axes in which a given point lies or vice-versa. C39. Marks points in the 2D coordinate system given their coordinates.
		CLO8: Relates the position of a point to the horizontal and vertical movement in the Cartesian plane.	C40. Differentiates between points corresponding to (x, y) and (y, x) (where y ≠ x). C41. Determines the coordinates of a point which is shifted/ translated by given units or reflected in the x-axis, the y-axis or the origin. C42. Builds a table of coordinates of points when given a shift along either horizontal or vertical axes (or both).
Content domain: Algebra **Chapter 4: Linear Equations in Two Variables** Key concepts: Recall of linear equations in one variable. Introduction to the equation in two variables. Focus on linear equations of the type ax + by + c = 0. Explain that a linear equation in two variables has infinitely many solutions and justify their being written as ordered pairs of real numbers, plotting them and showing that they lie on a line. Graph of linear equations in two variables. Examples, problems from real life, including problems on Ratio and Proportion and with algebraic and graphical solutions being done simultaneously.	LO3: Relates the algebraic and graphical representations of a linear equation in one or two variables and applies the concept to daily life situations.	CLO1: Relates the linearity of a line to the coordinates of a point on it.	C43. Checks if a point is on a given line by substituting the coordinates in the equation of the line. C44. Plots points in the cartesian plane to check if they lie in a line.
		CLO10: Demonstrates that a line has an infinite number of points.	C45. Finds the points of intersection of a given line with either axis. C46. Finds either abscissa or ordinate of a point on a line given one of them and the equation of the line. C47. Finds coordinates of points on a given line. C48. Finds one more point on a line given n collinear points.
		CLO11: Solves (algebraically and graphically) a linear equation based on a real life context.	C49. Identifies the variables in a given real life situation. C50. Explains the linear relationship between two variables in a real life situation. C51. Expresses the algebraic relationship between two variables (using the given data).

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		CLO12: Poses problems based on linear equations in two variables.	C52. Uses algebraic substitution to find the value of one variable given the other in a linear equation.
			C53. Uses algebraic substitution to find the algebraic expression for one variable given the other in a linear equation.
			C54. Reads the graph of the line to find one variable given the other.
			C55. Creates a word problem based on a real life context in which two variables have a linear relationship.
			C56. Creates a word problem from a given linear equation.
			C57. Creates a word problem by reading the graph of a linear equation and identifying a relationship between the two variables.
Content domain: Geometry **Chapter 5: Introduction to Euclid's Geometry** Key concepts: History – Geometry in India and Euclid's geometry. Euclid's method of formalizing observed phenomenon into rigorous Mathematics with definitions, common/obvious notions, axioms/postulates and theorems. The five postulates of Euclid. Equivalent versions of the fifth postulate. Showing the relationship between axiom and theorem, for example: (Axiom) 1. Given two distinct points, there exists one and only one line through them. (Theorem) 2. (Prove) Two distinct lines cannot have more than one point in common.		CLO13: Describes axiomatic structure of Euclidean Geometry.	C58. Recalls mathematicians' (including Euclid's) contributions to the development of geometry.
			C59. Illustrates the meaning of definitions, axioms, postulates and theorems.
			C60. Recognises the difference between axioms and postulates.
			C61. Recognises the difference between theorems and axioms/postulates.
			C62. Demonstrates the relationship between axioms and theorems using examples.
		CLO14: Represents geometrical concepts – points, lines, line segments, rays, angles, collinear points, non collinear points and circles with reference to Euclidean geometry.	C63. Defines point, lines, line segment, ray, angles, collinear points, non collinear points and circles with reference to Euclidean geometry.
			C64. Uses the language (symbols and representation) needed to understand, identify and discuss different geometrical concepts such as lines and angles.

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Content domain: Geometry **Chapter 6: Lines and Angles** Key concepts: 1. (Motivate) If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and the converse. 2. (Prove) If two lines intersect, vertically opposite angles are equal. 3. (Motivate) Results on corresponding angles, alternate angles, interior angles when a transversal intersects two parallel lines. 4. (Motivate) Lines which are parallel to a given line are parallel. 5. (Prove) The sum of the angles of a triangle is 180°. 6. (Motivate) If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.	LO4: Identifies similarities and differences among different geometrical shapes.	CLO15:Classifies lines, line segments, rays and pair of lines.	C65. Differentiates between lines, line segments and rays. C66. Identifies the relevant transversal in a given situation (where the other lines may or may not be parallel).
		CLO16:Draws conclusions based on angles (angle pairs and angles formed when a transversal intersects two parallel lines).	C67. Recognises pairs of angles (vertically opposite angles, linear-pair angles, adjacent angles, supplementary angles and complementary angles). C68. Identifies relationships between two angles (vertically opposite angles, linear pair angles, adjacent angles, supplementary angles and complementary angles). C69. Proves vertically opposite angles are equal. C70. Applies relationships between two angles (vertically opposite angles, linear pair angles, adjacent angles, supplementary angles and complementary angles) to find unknown values. C71. Classifies different angles (pairs of corresponding angles and interior angles) formed when a transversal intersects two lines. C72. Develops relationships using different pairs of angles formed when a transversal intersects two parallel lines to find unknown values. C73. Deduces whether two lines are parallel based on the relationship between two angles formed by these lines and the transversal.
	LO5: Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles, etc., by applying axiomatic approach and solves problems using them	CLO17:Proves theorems relating to lines, angles and triangles.	C74. Recalls the corresponding angles axiom. C75. Proves theorems related to parallel lines (alternate interior angles equal, interior angles on the same side of the transversal are supplementary). C76. Proves two lines are parallel when (i) alternate interior angles are equal (ii) interior angles on the same side of the transversal are supplementary C77. Applies theorems and axioms related to lines and angles or parallel lines to prove a result. C78. Proves angle sum property of a triangle. C79. Proves exterior angle property of a triangle. C80. Uses exterior angle property of a triangle to prove results. C81. Proves results using lines and angles theorems.

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		CLO18: Solves problems using mathematical statements involving lines and angles.	C82. Applies theorems and axioms related to lines and angles, parallel lines to find unknown values. C83. Applies angle sum property of a triangle to find unknown angles. C84. Applies exterior angle property of a triangle to find unknown angles.
Content domain: Geometry Chapter 7: Triangles Key concepts: 1. (Motivate) Two triangles are congruent if any two sides and the included angle of one triangle is equal to any two sides and the included angle of the other triangle (SAS Congruence). 2. (Prove) Two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle (ASA Congruence). 3. (Motivate) Two triangles are congruent if the three sides of one triangle are equal to three sides of the other triangle (SSS Congruence). 4. (Motivate) Two right triangles are congruent if the hypotenuse and a side of one triangle are equal (respectively) to the hypotenuse and a side of the other triangle. (RHS Congruence) 5. (Prove) The angles opposite to equal sides of a triangle are equal. 6. (Motivate) The sides opposite to equal angles of a triangle are equal. 7. (Motivate) Triangle inequalities and relation between 'angle and facing side' inequalities in triangles.	LO4: Identifies similarities and differences among different geometrical shapes	CLO19: Analyses similarities and differences between parts of shapes (lines, angles and triangles).	C85. Expresses arguments related to triangles in the process of proof using mathematical language and symbols. C86. Identifies corresponding parts of two triangles (which may or may not be congruent to each other). C87. Analyses equality of corresponding parts of the congruent shapes. C88. Identifies common parts (shapes, lines, angles and vertices) of two shapes.
		CLO20: Proves theorems related to triangle congruency.	C89. Applies SAS rule to prove congruency of triangles. C90. Derives equivalent conditions for congruent triangles (ASA, AAS, SSS and RHS). C91. Proves results related to pair of sides and pair of angles of a triangle. C92. Proves that the sum of any two sides of a triangle is greater than the third side. C93. Proves results using triangle inequality theorems.
		CLO21: Applies theorems and axioms related to triangle congruency.	C94. Applies the concept of congruent triangles in solving problems.

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		CLO22: Solves problems using congruency theorems.	C95. Applies congruency theorems to find sides and angles in triangles.
		CLO23: Utilizes triangle inequality theorem.	C96. Recognises side and angle facing side of a triangle. C97. Applies triangle inequality theorems to order sides (shortest, medium, longest) and angles (smallest, medium, largest) in triangles.
Content domain: Geometry **Chapter 8: Quadrilaterals** Key concepts: 1. (Prove) The diagonal divides a parallelogram into two congruent triangles. 2. (Motivate) In a parallelogram opposite sides are equal, and conversely. 3. (Motivate) In a parallelogram opposite angles are equal, and conversely. 4. (Motivate) A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal. 5. (Motivate) In a parallelogram, the diagonals bisect each other and conversely. 6. (Motivate) In a triangle, the line segment joining the mid points of any two sides is parallel to the third side and in half of it and (motivate) its converse.	LO4: Identifies similarities and differences among different geometrical shapes.	CLO24: Analyses similarities and differences between different quadrilaterals based on properties.	C98. Derives similarities and differences between given types of quadrilaterals C99. Classifies quadrilaterals based on properties. C100. Derives the hierarchical relation among selected quadrilaterals.
	LO5: Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles, etc., by applying axiomatic approach and solves problems using them.	CLO25: Proves and applies theorems related to quadrilaterals.	C101. Proves properties of different types of quadrilaterals. C102. Derives the type of a quadrilateral given some of its characteristics. C103. Proves that the diagonals of a rhombus bisect each other at right angles. C104. Represents geometric configurations (draws a diagram) given a description. C105. Draws conclusions about angles and sides in a geometric configuration based on the given information. C106. Proves that a quadrilateral is a parallelogram, rhombus, rectangle or square based on given information. C107. Proves the mid-point theorem. C108. Proves results based on the mid-point theorem.
		CLO26: Solves problems based on quadrilateral theorems.	C109. Uses theorems on quadrilaterals to find unknowns in the given problem.

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Content domain: Geometry **Chapter 9: Area of Parallelograms and Triangles** Key concepts: 1. (Prove) Parallelograms on the same base and between the same parallels have equal area. 2. (Motivate) Triangles on the same base (or equal bases) and between the same parallels are equal in area.	LO5: Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles, etc., by applying axiomatic approach and solves problems using them.	CLO27: Proves and applies theorems on areas of parallelogram.	C110. Recognizes parallelograms on the same/equal base and between the same parallels. C111. Proves that parallelograms on the same/equal base and between same parallels have the same area. C112. Derives the formula for area of a parallelogram using a rectangle sharing the same/equal base and between the same parallel lines. C113. Proves results based on area of parallelograms. C114. Applies theorems on area of parallelograms to find unknown parts.
		CLO28: Proves and applies theorems on areas of triangles.	C115. Proves that the area of a triangle is half of the area of a parallelogram when both have the same/equal base and are between the same parallel lines. C116. Derives the formula for area of a triangle using a parallelogram that shares the same/equal base and between the same parallel lines. C117. Proves results related to the areas of triangles and parallelograms between the same parallel lines. C118. Finds areas of triangles and parallelograms using theorems based on triangles and parallelograms between same parallel lines.
Content domain: Geometry **Chapter 10: Circles** Key concepts: Through examples, arrive at definition of circle and related concepts-radius, circumference, diameter, chord, arc, secant, sector, segment, subtended angle. 1. (Prove) Equal chords of a circle subtend equal angles at the centre and (motivate) its converse.	LO5: Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles, etc., by applying axiomatic approach and solves problems using them.	CLO29: Proves and applies theorems related to circles.	C119. Defines circle, radius, diameter, arc (minor arc and major arc), chord, segment (minor segment and major segment), central angle and subtended angles. C120. Identifies interior, boundary and exterior of a circle. C121. Proves equal chords subtend equal angles at the centre (and the converse). C122. Proves a perpendicular from the centre of a circle to a chord bisects the chord. C123. Proves a line drawn through the centre of a circle to bisect a chord is perpendicular to the chord. C124. Demonstrates that there exists a unique circle passing through three non-collinear points. C125. Proves equal chords are equidistant from the centre (and its converse). C126. Proves results based on chord theorems. C127. Recalls equal arcs of circle subtend equal angles at the centre.

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2. (Motivate) The perpendicular from the centre of a circle to a chord bisects the chord and conversely, the line drawn through the centre of a circle to bisect a chord is perpendicular to the chord. 3. (Motivate) There is one and only one circle passing through three given non-collinear points. 4. (Motivate) Equal chords of a circle (or of congruent circles) are equidistant from the centre (or their respective centres) and conversely. 5. (Prove) The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle. 6. (Motivate) Angles in the same segment of a circle are equal. 7. (Motivate) If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, the four points lie on a circle. 8. (Motivate) The sum of either of the pair of the opposite angles of a cyclic quadrilateral is 180° and its converse.	LO5: Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles, etc., by applying axiomatic approach and solves problems using them.	CLO29: Proves and applies theorems related to circles.	C128. Proves that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle. C129. Proves that angles in the same segment of a circle are equal. C130. Proves that angle in a semicircle is a right angle. C131. Applies theorems related to angles of a circle to prove results. C132. Proves that if a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e. they are concyclic). C133. Proves that the sum of opposite angles of a cyclic quadrilateral is 180° (and its converse). C134. Proves results based on cyclic quadrilaterals.
		CLO30: Solves problems using circle theorems	C135. Applies chord theorems to find unknown values of lengths and angles. C136. Applies theorems related to angles of a circle to find unknown values. C137. Solves problems that use one or more circle theorems. C138. Applies theorem based on cyclic quadrilateral to find unknown values of angles.
Content domain: Geometry **Chapter 11: Constructions** Key concepts: 1. Construction of bisectors of line segments and angles of measure 60°, 90°, 45° etc., equilateral triangles.	LO7: Constructs different geometrical shapes like bisectors of line segments, angles and triangles under given conditions and provides reasons for the processes of such constructions.	CLO31: Constructs lines – bisectors, angular bisector (60°, 90°, 45°), equilateral triangles and triangles with given conditions (such as base, sum/difference of other two sides, perimeter and one or two base angles given).	C139. Constructs bisectors of given line segment or sides. C140. Recalls construction steps of replicating an angle. C141. Constructs angles (60°, 90°, 45°). C142. Constructs angular bisector of any angle. C143. Reasons and applies construction of line bisectors and angle bisectors in the construction of triangles.

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2. Construction of a triangle given its base, sum/difference of the other two sides and one base angle. 3. Construction of a triangle of given perimeter and base angles			C144. Constructs triangles with given conditions (such as base, sum/difference of other two sides, perimeter and one or two base angles given). C145. Reasons out the steps for construction with given conditions (such as base, sum/difference of other two sides, perimeter and one or two base angles given) connecting them to congruent triangle theorems.
		CLO32: Justifies processes of constructions.	C146. Analyses and relates to theorems or properties of shapes that lead to construction. C147. Analyses steps of construction by relating to theorems and properties of shapes that lead to construction. C148. Justifies each step of a construction.
Content domain: Mensuration **Chapter 12: Heron's Formula** Key concepts: Area of a triangle using Heron's formula (without proof) and its application in finding the area of a quadrilateral	LO6: Finds areas of all types of triangles by using appropriate formulae and apply them in real life situations.	CLO33: Applies Heron's formula.	C149. Identifies whether half base times height can be used to find area of a triangle. C150. Applies Heron's formula to find areas of triangles and composite figures.
		CLO34: Solves real life problems on areas of triangles.	C151. Solves real life problems on areas of triangle and other composite figures using appropriate formula.
Content domain: Mensuration **Chapter 13: Surface Areas and Volumes** Key concepts: Surface areas and volumes of cubes, cuboids, spheres (including hemispheres) and right circular cylinders/cones	LO12: Derives formulae for surface areas and volumes of different solid objects like, cubes, cuboids, right circular cylinders/cones, spheres and hemispheres and applies them to objects found in the surroundings	CLO35: Derives formula to find total surface area, curved surface area and volume of cubes, cuboids, cylinders and cones, spheres and hemispheres using hands on activity (such as nets) and uses these to solve problems on surface area and volumes of solids.	C152. Visualizes the net of a 3D shape. C153. Relates area of the net to surface area of the corresponding 3D shape. C154. Differentiates between lateral surface area (LSA) and curved surface area (CSA). C155. Identifies the shapes for which LSA is equal to CSA C156. Recalls formulas for the lateral surface area, total surface area and volume of cuboid and cube. C157. Explains formulas for the lateral surface area, total surface area of cylinder and cone. C158. Recalls formulas for volumes of cylinder and cone. C159. Recalls formulas for the curved surface area, total surface area and volume of sphere and hemi-sphere. C160. Uses appropriate units of surface areas and volumes.

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		CLO36: Applies formulas of total surface area, curved surface area and volume of cubes, cuboids, cylinders and cones, spheres and hemispheres to solve problems on surface area and volumes of solids.	C161. Calculates surface areas of solids using appropriate formula. C162. Calculates volumes of solids using appropriate formula. C163. Uses formulas to find unknown dimensions (height, radius etc).
		CLO37: Analyses and solves problems on surface area and volume formulas in real world situations.	C164. Solve real world problems by applying surface area and volume formulas of solid shapes. C165. Estimates and approximates measures of area and volumes of solid in real world context.
Content domain: Statistics and Probability **Chapter 14: Statistics** Key concepts: Introduction to Statistics: Collection of data, presentation of data – tabular form, ungrouped / grouped, bar graphs, histograms (with varying base lengths), frequency polygons. Mean, median and mode of ungrouped data	LO10: Analyses data by representing it in different forms like, tabular form (grouped or ungrouped), bar graph, histogram (with equal and varying width and length), and frequency polygon	CLO38: Analyses data by representing it in a tabular form (grouped or ungrouped).	C166. Recognizes vocabulary related to types of data, related to frequency distributions and class intervals. C167. Constructs frequency table (ungrouped) from raw data (qualitative and quantitative). C168. Constructs frequency table (grouped) from raw quantitative data. C169. Classifies data that can be grouped vs that which cannot. C170. Retrieves frequency of given data-value(s) (ungrouped or grouped) from a given frequency table. C171. Retrieves data-values (ungrouped and grouped) with a given frequency from a given frequency table. C172. Retrieves data-values with highest or lowest frequency from a given frequency table. C173. Compares frequency of two data-values or two class-intervals from a given frequency table.
		CLO39: Analyses data by graphically representing it as bar graph, histogram (with equal and varying width and length), and frequency polygon.	C174. Constructs bar graph from ungrouped tabulated (qualitative) data. C175. Constructs histogram for grouped data with equal class-size. C176. Constructs histogram for grouped data with varying class-size. C177. Recognizes that any grouped data can be represented through a histogram and a frequency polygon. C178. Constructs frequency polygon for grouped data.

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			C179. Classifies data based on the type of graphical representation – bar chart vs histogram. C180. Analyses a distribution given its bar chart. C181. Analyses a distribution given its histogram. C182. Analyses a distribution given its frequency polygon. C183. Compares two distributions based on their frequency polygons (drawn on the same axes).
	LO9: Identifies and classifies the daily life situations in which mean, median and mode can be used	CLO40: Computes mean, median and mode daily life situations.	C184. Recognizes vocabulary related to range, mean, median and mode. C185. Recognizes notations for data values (x_1, x_2... x_n), frequencies (f_1, f_2... f_n), summations ($\sum_{i=1}^{n} x_i$, $\sum_{i=1}^{n} f_i$ and $\sum_{i=1}^{n} f_i x_i$), and mean ($\bar{x}$). C186. Computes mean for raw data. C187. Computes mean from ungrouped frequency table. C188. Computes median for raw data. C189. Computes mode for raw data.
		CLO41: Identifies daily life situations in which mean, median and mode can be used.	C190. Classifies situations in which mean can be computed vs where it cannot be (based on type of data). C191. Distinguishes situations in which mean can be computed and is an appropriate measure of central tendency from where it is computable but inappropriate. C192. Provides example of situation in which mean can be computed and is an appropriate measure of central tendency. C193. Provides example of situation in which mean can be computed but is inappropriate. C194. Distinguishes situations in which median is an appropriate measure of central tendency. C195. Provides example of situation in which median is an appropriate measure of central tendency. C196. Classifies situations in which mode can be computed vs where it cannot be computed (multimodal situations).

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Content domain: Statistics and Probability **Chapter 15: Probability** Key concepts: History, Repeated experiments and observed frequency approach to probability. Focus is on empirical probability. (A large amount of time to be devoted to group and to individual activities to motivate the concept; the experiments to be drawn from real-life situations, and from examples used in the chapter on statistics)	LO11: Calculates empirical probability through experiments and describes its use in words.	CLO42:Calculates empirical probability through experiments.	C197. Computes the number of all possible outcomes of a random experiment in a given situation or from a frequency table. C198. Recognizes the outcome(s) favourable to a given event C199. Computes the number of outcomes favourable to a given event in a given situation or from a frequency table. C200. Computes the empirical probability of a given event from the given frequency table. C201. Recognizes empirical probability as relative frequency. C202. Recalls that probability must be from 0 to 1.
		CLO43:Describes the use of empirical probability.	C203. Describes the use of empirical probability.

Class X

Content Domain, Chapter, Key Concepts	Learning Outcomes – NCERT	Content Domain Specific Learning Outcomes	Indicators
Content domain: Number Systems Chapter 1: Real Numbers Key concepts: Euclid's division lemma, Fundamental Theorem of Arithmetic – statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, Decimal representation of rational numbers in terms of terminating/non-terminating recurring decimals	LO1: Generalises properties of numbers and relations among them studied earlier to evolve results – such as, Euclid's division algorithm, Fundamental Theorem of Arithmetic – and applies them to solve problems related to real life contexts.	CLO1: Generalizes properties of numbers and relations among them to evolve results – such as, Euclid's division algorithm, Fundamental Theorem of Arithmetic.	C1. Observes that any two integers a and b (a > b) can be expressed as a = b × quotient + remainder. C2. States Euclid's division Lemma. C3. Uses Euclid's division Lemma to find HCF of two numbers. C4. Applies Euclid's lemma to arrive at general forms of numbers (such as odd numbers are of the form 4q + 1, 4q + 3). C5. Recalls that the fundamental theorem of arithmetic is used to find HCF/LCM for any two positive integers. C6. Uses the fundamental theorem of arithmetic to find either one of LCM/HCF of two positive integers given the other. C7. Applies the identity that HCF (a, b) × LCM (a, b) = a × b to find the LCM of any two positive integers a and b. C8. Justifies the irrationality of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and other surds such as $m + p\sqrt{n}$ using the fundamental theorem of arithmetic. C9. Expresses the prime factorization of a number in exponential form. C10. Uses the fundamental theorem of arithmetic to check the digit in the place of unit in the power of a number. C11. Justifies the classification of rational numbers into terminating and non-terminating (recurring) decimals by prime factorization of the denominator of its simplest form.
		CLO2: Applies properties of numbers and results (as stated above) to solve problems related to real life contexts.	C12. Solves HCF and/or LCM problems based on real life contexts.

Content Domain, Chapter, Key Concepts	Learning Outcomes – NCERT	Content Domain Specific Learning Outcomes	Indicators
Content domains: Algebra Chapter 2: Polynomials Key concepts: Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials. Statement and simple problems on division algorithm for polynomials with real coefficients	LO2: Develops a relationship between algebraic and graphical methods of finding the zeroes of a polynomial.	CLO3: Applies division algorithm to polynomials.	C13. Divides a polynomial by a linear, quadratic or cubic polynomial and identifies the quotient and remainder. C14. Rephrases the division of a polynomial by another polynomial in the form divisor × quotient + remainder. C15. Identifies by long division whether a given polynomial is a factor of another polynomial. C16. Factorises a polynomial with long division given some of its zeroes.
		CLO4: Utilizes the relationship between zeroes and the coefficients of a polynomial.	C17. Utilises the relationship between the zeroes and coefficients of a polynomial of degree less than 4. C18. Models the change in the zero of a linear polynomial when the coefficients are changed. C19. Forms at least one quadratic polynomial given the sum and product of its zeroes.

Content Domain, Chapter, Key Concepts	Learning Outcomes – NCERT	Content Domain Specific Learning Outcomes	Indicators
Content domains: Algebra **Chapter 3: Pair of Linear Equations in Two Variables** Key concepts: Pair of linear equations in two variables and graphical method of their solution, consistency / inconsistency. Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically – by substitution, by elimination and by cross multiplication method. Simple situational problems. Simple problems on equations reducible to linear equations.	LO3: Finds solutions of pairs of linear equations in two variables using graphical and different algebraic methods.	CLO5: Models a real life context using a pair of linear equations.	C20. Identifies variables given a real life situation related to linear relationships between two variables. C21. Forms two linear equations in two variables using given data.
		CLO6: Solves problems on linear equations in two variables graphically or algebraically.	C22. Plots the given equations of a pair of lines on the cartesian plane and identifies their point of intersection, if it exists. C23. Finds one variable in terms of the other using either equation of the given pair. C24. Substitutes in the other equation of the given pair and finds both variables. C25. Uses algebraic procedures on a pair of linear equations to eliminate one variable and solve for both variables. C26. Reduces a given pair of equations or a situational problem to a pair of linear equations using an appropriate substitution and solves for the unknowns.

		CLO7: Classifies linear equations as consistent or inconsistent, and dependent or independent and interprets their geometric meaning, the type of solutions and the relationship between the coefficients.	C27. Selects graphs of equations of pairs of lines for which there is no point of intersection.
			C28. Differentiates between equations of pairs of lines which are coincident and those which are parallel (by graphing).
			C29. Organizes pairs of lines into lines which are:
			  i. intersecting
			  ii. parallel
			  iii.coincident
			C30. Connects relationship between coefficients of terms in the equations of a pair of lines with the three categories:
			  i. intersecting
			  ii. parallel
			  iii.coincident
			C31. Uses general formulae for both variables in terms of the coefficients of the equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ to arrive at condition for existence of point of intersection.
			C32. Uses the ratio of coefficients to identify if the solution to a given pair of lines is unique, non-existent, or infinitely many.

Content Domain, Chapter, Key Concepts	Learning Outcomes – NCERT	Content Domain Specific Learning Outcomes	Indicators

Content domains: Algebra
 	
Chapter 4: Quadratic Equations
 	
Key concepts:
 	
Standard form of a quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$. Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.
 	
Situational problems based on quadratic equations related to day to day activities to be incorporated.
	LO4: Demonstrates strategies of finding roots and determining the nature of roots of a quadratic equation.	CLO8: Explores the graph of a quadratic function.	C33. Plots graph of quadratic $y = x^2$.
			C34. Compares graphs of the functions:
			  i. $y = ax^2$ for positive and negative 'a'
			  ii. $y = x^2 + c$ for positive and negative 'c'
			  iii. $y = (x + d)^2$ for positive and negative 'd'
			C35. Compares the graphs of the function $y = a(x + d)^2 + c$ for positive and negative values of 'a', 'c' and 'd'
			C36. Draws conclusions about the shape of the graph, its vertex, zero(es) and line of symmetry
			C37. Expresses each of the above types in the form $y = ax^2 + bx + c$

		CLO9: Solves a quadratic equation $ax^2 + bx + c = 0$.	C38. Expresses a quadratic equation $ax^2 + bx + c = 0$ in the form $(x + m)(x + n) = 0$. C39. Expresses a quadratic equation $ax^2 + bx + c = 0$ in the form $a(x + p)^2 + q = 0$. C40. Finds the roots of a quadratic equation by any method. C41. Classifies the roots of a quadratic equation as distinct/repeated/not real and rational/irrational. C42. Finds the general formula for the roots of a quadratic equation. C43. Selects the most efficient method of solving a given quadratic equation.
		CLO10: Explores the relationship between the nature of roots and the discriminant of a quadratic equation.	C44. Identifies the discriminant of a quadratic equation. C45. Relates the value of the discriminant to the nature of the roots. C46. Solves problems based on the relationship between the nature of the roots and the discriminant of a quadratic equation.
		CLO11: Solves situational problems based on quadratic equations.	C47. Forms a quadratic equation given a real life context. C48. Solves quadratic equations based on mathematical and/or real life contexts.

Content Domain, Chapter, Key Concepts	Learning Outcomes – NCERT	Content Domain Specific Learning Outcomes	Indicators
Content domains: Algebra **Chapter 5: Arithmetic Progressions** Key concepts: Motivation for studying Arithmetic Progression Derivation of the nth term and sum of the first n terms of A.P. and their application in solving daily life problems	LO5: Develops strategies to apply the concept of AP to daily life situations.	CLO12: Applies the formula for nth term and the sum of first n terms of an arithmetic progression.	C49. Differentiates between arithmetic progressions and other sequences. C50. Obtains a term in a given arithmetic progression by identifying the first term, the common difference and the number of terms. C51. Uses the notation of T_n and S_n for term and sum of n terms. C52. Generalizes this procedure by arriving at the nth term given the first term and common difference C53. Identifies that for any A.P. $(a_1, a_2, ... a_k)$, $a_1 + a_k = a_2 + a_{k-1} = ...$ etc. C54. Generalizes to find the sum of n terms of an arithmetic progression C55. Differentiates between the sum of n terms and the nth term of an arithmetic progression C56. Given the sums of different consecutive terms of an arithmetic progression, finds the first term and common difference of the progression
		CLO13: Solves real life problems based on Arithmetic Progression.	C57. Verifies if the terms identified in a given real life situation are in A.P. C58. Expresses a given real life problem in terms of an arithmetic progression and identifies the first term, common difference and number of terms. C59. Finds unknowns using the formulas for the sum to n terms and the nth term.

Content Domain, Chapter, Key Concepts	Learning Outcomes – NCERT	Content Domain Specific Learning Outcomes	Indicators
Content domain: Geometry **Chapter 6: Triangles** Key concepts: Definitions, examples, counter examples of similar triangles. 1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. 2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side. 3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar. 4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar. 5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar. 6. (Motivate) If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other. 7. (Prove) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.	LO6: Works out ways to differentiate between congruent and similar figures. LO7: Establishes properties for similarity of two triangles logically using different geometric criteria established earlier such as, Basic Proportionality Theorem, etc.	CLO14: Analyses the concept of similar triangles.	C60. Provides examples and non-examples of similar triangles. C61. Explains the conditions for two triangles to be similar. C62. Distinguishes between congruent and similar figures. C63. Reasons why all congruent figures are similar but two similar figures need not be congruent. C64. Uses appropriate symbols (~, ≅, ⊥ etc) and representation in writing the proof.
		CLO15: Proves theorems related to similar triangles.	C65. Recalls that areas of triangles with same base and between same parallel lines are equal. C66. Proves Basic Proportionality theorem (and its converse). C67. Proves that if the corresponding angles are equal in two triangles, then their corresponding sides are in the same ratio and hence the two triangles are similar. C68. Proves that in two triangles, if the sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar. C69. Proves that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar. C70. Proves results/riders based on similarity of triangles. C71. Proves that the ratios of the areas of two similar triangles equals the ratio of the squares of their corresponding sides. C72. Analyses conditions to prove similarity of triangles
		CLO16: Solves problems related to similar triangles.	C73. Uses basic proportionality theorem to find unknown length(s). C74. Uses theorem on areas of similar triangles to find unknown length(s) and/or area(s) of triangles.
		CLO17: Proves Pythagoras theorem.	C75. Proves Pythagoras theorem (and its converse). C76. Applies Pythagoras theorem to prove results. C77. Proves the relationship that exists when the perpendicular is drawn from the vertex of the right- angle to the hypotenuse.
		CLO18: Solves problems based on the Pythagoras theorem.	C78. Apply Pythagoras theorem to find the unknown side(s) in a right-angle triangle. C79. Establishes that two lines are perpendicular by using converse of Pythagoras theorem.

Content Domain, Chapter, Key Concepts	Learning Outcomes – NCERT	Content Domain Specific Learning Outcomes	Indicators
8. (Prove) In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides. 9. (Prove) In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angles opposite to the first side is a right angle.			
Content domain: Coordinate Geometry **Chapter 7: Coordinate Geometry** Key concepts: Concepts of coordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division). Area of a triangle.	LO8: Derives formulae to establish relations for geometrical shapes in the context of a coordinate plane, such as, finding the distance between two given points, to determine the coordinates of a point between any two given points, to find the area of a triangle, etc.	CLO19: Derives the section formula, the area formula and the distance formula.	C80. Plots two points given coordinates and describes the path from one point to the other along x- and y-axes given coordinates of both points. C81. Derives formula for mid-point of two points (x_1, y_1) and (x_2, y_2) (by finding congruent triangles). C82. Generalizes mid-point formula to section formula (internal division) (by finding similar triangles). C83. Uses Pythagoras theorem to obtain distance between two points (x_1, y_1) and (x_2, y_2). C84. Derives the formula for the centroid of a triangle joining the points (x_1, y_1) (x_2, y_2) and (x_3, y_3). C85. Derives the formula for the area of a triangle joining the points (x_1, y_1) (x_2, y_2) and (x_3, y_3). C86. Given coordinates of points applies reasoning to solve problems such as finding: i. Distances ii. Coordinates of points dividing a line in a particular ratio iii. Types of triangles iv. Types of quadrilaterals v. Collinearity of given points

Content Domain, Chapter, Key Concepts	Learning Outcomes – NCERT	Content Domain Specific Learning Outcomes	Indicators
Content domain: Trigonometry **Chapter 8: Introduction to Trigonometry** Key concepts: Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0° and 90°. Values of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios. Proof and applications of the identity sin2A + cos2A = 1. Only simple identities to be given. Trigonometric ratios of complementary angles.	LO9: Determines all trigonometric ratios with respect to a given acute angle (of a right triangle) and uses them in solving problems in daily life contexts like finding heights of different structures or distance from them.	CLO20: Determines all trigonometric ratios with respect to a given acute angle (of a right triangle).	C87. Explains vocabulary related to a right triangle with reference to one of its acute angles: complementary angle, hypotenuse, opposite side and adjacent side. C88. Recalls that two right triangles with the same acute angle are similar. C89. Recalls that corresponding sides are proportional for any two similar triangles. C90. Explains the concept of constant ratios of sides of a triangle in which all three angles remain the same. C91. Defines sine, cosine, tangent, cotangent, secant and cosecant of an acute angle (in a right triangle) as ratios of appropriate sides. C92. Demonstrates understanding of notations for the six trigonometric ratios and their squares. C93. Given one of the trigonometric ratios of an acute angle, finds the others. C94. Determines sine, cosine, tangent, cotangent, secant and cosecant of standard angles such as 0°, 30°, 45°, 60° and 90°.
		CLO21: Uses trigonometric ratios to solve simple problems.	C95. Solves problems using trigonometric ratios of a standard angle to find angles or sides of a given right triangle. C96. Simplifies an expression based on the values of trigonometric ratios. C97. Proves trigonometric identities based on Pythagoras theorem. C98. Solves simple problems based on the properties of trigonometric ratios of complementary angles. C99. Simplifies expressions based on trigonometric ratios and identities. C100. Justifies expressions based on trigonometric ratios and identities.

Content Domain, Chapter, Key Concepts	Learning Outcomes – NCERT	Content Domain Specific Learning Outcomes	Indicators
Content domain: Trigonometry **Chapter 9: Some Applications of Trigonometry** Key concepts: Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30°, 45°, 60°.	LO9: Determines all trigonometric ratios with respect to a given acute angle (of a right triangle) and uses them in solving problems in daily life contexts like finding heights of different structures or distance from them.	CLO22: Uses the six trigonometric ratios of standard angles to solve problems in daily life contexts.	C101. Uses vocabulary related to heights and distances problems – for example: line of sight, angle of elevation/depression C102. Draws labelled diagrams (involving the right triangle and the acute angle) representing the given situation. C103. Finds the height/depth or length of an object or the distance between two distant objects using trigonometric ratios.
Content domain: Geometry **Chapter 10: Circles** Key concepts: Tangent to a circle at, point of contact 1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact. 2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.	LO10: Derives proofs of theorems related to the tangents of circles.	CLO23: Prove theorems related to tangent.	C104. Identifies the angle made by the radius and the tangent. C105. Proves the tangent at any point of a circle is perpendicular to the radius through the point of contact. C106. Recalls number of tangents that can be drawn from an external point to any given circle. C107. Recalls RHS congruency. C108. Proves the lengths of tangents drawn from an external point to a circle are equal.
		CLO24: Solves problems related to tangent of a circle.	C109. Proves results related to tangents to a circle to solve problems. C110. Uses tangent theorems to find unknown values.
Content domain: Geometry **Chapter 11: Constructions** Key concepts: 1. Division of a line segment in a given ratio (internally). 2. Tangents to a circle from a point outside it. 3. Construction of a triangle similar to a given triangle.	LO11: Constructs — a triangle similar to a given triangle as per a given scale factor a pair of tangents from an external point to a circle and justifies the procedures.	CLO25: Divides a line segment in a given ratio.	C111. Applies properties of similarity to divide a line segment in a given ratio.
		CLO26: Constructs a triangle similar to given triangle as per the given scale factor.	C112. Constructs similar triangle to a given scale factor.
		CLO27: Constructs a pair of tangents to a circle from an external point.	C113. Constructs a pair of tangents to a circle.
	LO12: Examines the steps of geometrical constructions and reasons out each step.	CLO28: Justifies whether given steps of a particular construction are appropriate.	C114. Analyses and relates to theorems or properties of shapes that leads to construction. C115. Justifies each step of a construction. C116. Designs steps to construct composite geometric shapes.

Content Domain, Chapter, Key Concepts	Learning Outcomes – NCERT	Content Domain Specific Learning Outcomes	Indicators
Content domain: Mensuration **Chapter 12: Areas Related to Circles** Key concepts: Motivate the area of a circle; area of sectors and segments of a circle. Problems based on areas and perimeter / circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only. Plane figures involving triangles, simple quadrilaterals and circle should be taken.)	LO16: Derives formulas related to circles, and applies them to solve problems related to daily life.	CLO29: Derives formulas related to circumference and area of a circle, arc length, area of sector and area of segment.	C117. Recognises circumference, arc, sector and segment of a circle. C118. Recalls nature of pi. C119. Recalls circumference formula. C120. Relates circumference to radius. C121. Derives formula for arc length of a circle. C122. Derives formula for area of sector of a circle. C123. Derives formula for area of segment of a circle.
		CLO30: Applies formulas related to circumference and area of a circle, arc length, area of sector and area of segment.	C124. Applies formulas on areas and lengths related to circles to solve problems. C125. Uses formulas on areas and lengths related to circles to find unknown measurements. C126. Estimates and approximates measures of area and lengths. C127. Uses appropriate units to find the measure of areas and lengths related to circles.
		CLO31: Solves problems on areas related to circle, sector and segments.	C128. Applies formulas of areas and lengths related circles to solve mathematical and/or real-world problems.

Content Domain, Chapter, Key Concepts	Learning Outcomes – NCERT	Content Domain Specific Learning Outcomes	Indicators
Content domain: Mensuration **Chapter 13: Surface Areas and Volumes** Key concepts: 1. Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones. Frustum of a cone. 2. Problems involving converting one type of metallic solid into another and other mixed problems. (Problems with combination of not more than two different solids be taken).	LO13: Finds surface areas and volumes of objects in the surroundings by visualising them as a combination of different solids like cylinder and a cone, cylinder and a hemisphere, combination of different cubes, etc.	CLO32: Derives the surface area and volume of combination of shapes such as (cube, cuboid, sphere, hemisphere, cone, and cylinder) and frustum of a cone.	C129. Explains formula of lateral surface areas, total surface areas and volumes of frustum of a cone. C130. Visualizes the net of combined solids. C131. Relates area of the net to the surface area of the corresponding combined solid.
		CLO33: Solves problems based on surface area and volume of different shapes and their combinations.	C132. Synthesizes formulas for the curved surface area, total surface area and volume of solids formed by combination of any two of cuboid, cylinder, cone and hemisphere to find the unknown measures. C133. Uses appropriate units of surface areas and volumes of solids.
		CLO34: Solves problems on surface area and volume formulas in real world situations.	C134. Uses surface area and volume formulas of 3D shapes to solve real life problems related to frustum and combined solids. C135. Identifies situations where two volumes are equal and/or where two surface areas are equal. C136. Uses surface area and volume formulas of solids to solve problems related to conversion of one solid to another. C137. Estimates and approximates measures of area and volumes of solid in real-world context.

Content Domain, Chapter, Key Concepts	Learning Outcomes – NCERT	Content Domain Specific Learning Outcomes	Indicators
Content domain: Statistics and Probability **Chapter 14: Statistics** Key concepts: Mean, median and mode of grouped data (bimodal situation to be avoided). Cumulative frequency graph.	LO14: Calculates mean, median and mode for different sets of data related with real life contexts	CLO35: Calculates and interprets mean for tabulated data.	C138. Computes mean for ungrouped tabulated data. C139. Computes mean for grouped data. C140. Interprets the meaning of the computed mean of a given dataset.
		CLO36: Determines and interprets median for tabulated data.	C141. Determines cumulative frequencies of ungrouped tabulated data. C142. Determines median for ungrouped tabulated data. C143. Computes cumulative frequencies for grouped data. C144. Recalls the median formula and what each part represents. C145. Computes median for grouped data using the formula. C146. Constructs less than and more than ogives. C147. Retrieves median from the intersection of less than ogive and the line y = N/2. C148. Retrieves median from the intersection point of the ogives. C149. Interprets the meaning of the computed median of a given dataset.
		CLO37: Calculates and interprets mode for grouped data.	C150. Recalls the mode formula and what each part represents. C151. Computes mode for grouped data. C152. Interprets the meaning of the computed mode of a given dataset.
Content domain: Statistics and Probability **Chapter 15: Probability** Key concepts: Classical definition of probability. Simple problems on finding the probability of an event.	LO15: Determines the probability of an event and applies the concept in solving daily life problems.	CLO38: Determines the probability of an event.	C153. Recalls the probability of a sure event. C154. Recalls the probability of an impossible event. C155. Computes the probability of an event with equally likely outcomes. C156. Computes the probability of an elementary event. C157. Explains the probability of an event. C158. Computes the probability of a complementary event. C159. Computes the probability of an event using measurement for a continuous sample space. C160. Selects correct probability from given options .
		CLO39: Applies the concept of probability in solving daily life problems.	C161. Recognizes the elementary events in daily life context. C162. Determines the probabilities of the elementary events in daily life context. C163. Computes the probability of an event in daily life context using count (discrete probability i.e., countable sample space). C164. Computes the probability of an event in daily life context using measurement for a continuous sample space.

10. Sample Pedagogical Processes and Assessment Strategies for Classroom Purpose

NCERT's secondary stage learning outcomes document provides a common set of pedagogical processes for each subject. Here we have moved from the general to the particular and furnished specific pedagogical processes and assessment strategies for a selected chapter each from classes 9 and 10. This has been done to enable teachers to derive principles for making the alignment between learning outcomes, pedagogical practices and assessment in their classrooms. The key principles considered while designing the pedagogical processes and assessment strategies are that they should be:
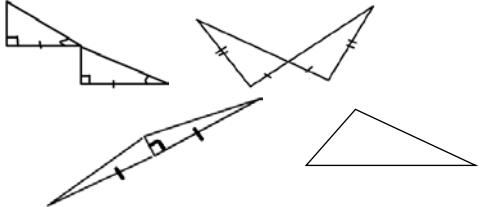
- Learner centred
  - Since new knowledge is built over existing knowledge, both pedagogy and assessment focus on learners' pre-requisite knowledge, skills, attitudes, and the beliefs that they bring into the classroom setting.
  - Emphasis on constructivist approaches to learning with the learner being at the centre of the learning process as an active constructor of knowledge.
  - Learning to be learner-driven with scope for investigation, exploration, conjecture-making and construction of proofs having mathematical rigor and validity.
  - Pedagogical approach and choice of examples to reflect the learner's context and environment.

- Learning outcome centred
  - Learning outcomes indicate what a learner will be able to do at the end of an instruction unit. The destination should be achieved by precisely breaking down the Curricular Expectations and the Vision for teaching school mathematics to more measurable and observable behaviour for each class.
  - Learners learn better when the method of teaching, learning activities and assessment strategies are all aligned well to the learning outcomes. Pedagogical processes and assessment strategies shall be aligned to both content domains and cognitive skills so as to develop the mathematics process skills.

- Assessment centred
  - Assessments should be viewed as an integral part of pedagogy and focus on giving timely individualized feedback to learners. Quality formative assessments to be designed so as to modulate learners understanding of their own learning and help teachers adapt their pedagogy based on learner performances.
  - Multiple modes of assessment including portfolios, project work, presentations, written and oral assignments to be used to provide a scope to reflect individual capacities of a learner.

- Classroom centred
  - Cooperative and peer-supported teaching learning activities to be used to empower learners to take charge of their own learning.
  - Peer assessment involving learners assessing work of their peers against set assessment criteria to be used.

Class IX

Content Domain: Geometry

Chapter 1: Triangles

Learning outcomes and indicators	Pedagogical Processes	Assessment Strategies

LO4: **Identifies** similarities and differences among different geometrical shapes
 	
CLO19. Analyses similarities and differences between parts of shapes (lines, angles and triangles)
 	
Indicators:
C88. Expresses arguments related to triangles in the process of proof using mathematical language and symbols
 	
C89. Identifies corresponding parts of two triangles (which may or may not be congruent to each other)
 	
C90. Analyses equality of corresponding parts of the congruent shapes
 	
C91. Identifies common parts of two shapes
	• Give sets of 2D shapes of different types some of the same size and some of different sizes (triangles, quadrilaterals, pentagons) to small groups and get learners to sort and categorize in all possible ways (based on number of sides, measure of sides and angles, shape and size)
	• Ask learners to explain reasons on sorting done to their peers.
	• Ask them how they would explain their reasons using mathematical language
	• Steer the discussion to focus on
	  a. How some shapes are congruent, some are not congruent but look similar and those which look different
	  b. The language being used while justifying. (Use of terms and symbols, clear and mathematical communication)
	• Get the learners to recall when two shapes are congruent to each other, and the terms and symbols related to congruency.
	
	Questions for Discussion and Suggested Activities:
	1. Ask the learners to identify corresponding parts in the given sets of congruent shapes (triangles and quadrilaterals).
	2. Get learners to define triangle, parts of a triangle and represent it by drawing and naming.
	3. Learners to explore – make cutout of triangle, trace the triangle name it as ΔABC. Now, learners to again trace the cutout to get a second triangle that is placed related to ΔABC in one of the following ways:
	  a. having a vertex common
	  b. having a side common
	  c. having an angle common (but the two triangles don't superimpose on each other)
	
	Note that for each of the three cases, ΔABC (and the second triangle) must be traced separately. In each case, learner to label the new triangle, mark the corresponding parts. Ideally, they should be asked to argue why they consider two sides to be corresponding sides and if they are equal why they are. Same applies to angles.
	
	4. Get learners to analyze equality of corresponding sides of the pair of shapes (label and use mathematical language and symbols to show relations.)
	
		• List the things around you that are equal in shape and size. Draw them and show the equality of corresponding parts.
		• Identify corresponding parts of congruent polygons given a statement on congruency.
		• Draw two congruent triangles label and mark the corresponding parts. State the equalities.

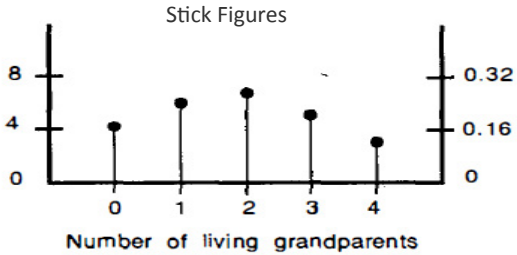| Learning outcomes and indicators	Pedagogical Processes	Assessment Strategies

LO5. Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles, etc., by applying axiomatic approach and solves problems using them
 	
CLO20. Proves theorems related to triangle congruency
 	
Indicators:
 	
C92. Recalls SAS congruence rule
 	
C93. Applies SAS rule to prove congruency of triangles
 	
C94. Derives equivalent conditions for congruent triangles (ASA, AAS, SSS and RHS)
 	
C95. Proves results related to pair of sides and pair of angles of a triangle
 	
C96. Proves that the sum of any two sides of a triangle is greater than the third side.
 	
C97. Proves results using triangle inequality theorems
	• Learners to recall necessary and sufficient conditions for congruency of two triangles
	
	Questions for Discussion and Suggested Activities:
	
	1. Get learners to draw pair of triangles, label and mark corresponding parts to show congruence rules (SAS, ASA, AAS, SSS and RHS)
	2. Game (played in pairs)
	  a. Learner A to construct a triangle with the given measurements (use one of the rules) and name it ΔABC.
	  b. Now, learner A to give minimum instructions to learner B to construct the triangle ΔPQR congruent to the triangle ΔABC. Condition – make use of measures not given in the construction of ΔABC.
	3. Learners to verify congruence rule by construction.
	  a. Draw any triangle ΔPQR. Now construct triangle ΔABC using SAS, ASA, AAS, SSS.
	  b. How would you say ΔABC ≡ ΔPQR?
	4. Learners to verify why AAA and SSA are not congruence rule by construction.
	  a. Draw any triangle ΔPQR. Now construct triangle ΔABC such that $\angle A = \angle P$, $\angle B = \angle Q$, $\angle R = \angle C$ and base of any length.
	  b. How would you show that the two triangles are not the same in size? (Do a similar experiment for SSA)
	5. For the above experiment ask the learners to suggest at least 1 additional condition needed to establish the congruence between two triangles.
	6. Get the learners to justify why the condition suggested establishes congruence.
	7. Introduce SAS congruence rule as axiom.
	8. Now, show learners how with SAS axiom ASA, AAS, SSS and RHS conditions for congruent triangles can be proved by following approach:
	  ⁻ Get the learners to list out the geometrical facts (definition, axioms and theorems) on lines and angles and triangles.
	  ⁻ Get the learners to detail the given information in the mathematical statement (theorem)?
	  ⁻ Let them look from the list the information which led to process/logical steps for proof
	9. Ask the learners to use above approach to prove triangle properties
	  ⁻ Equal angles have equal sides opposite to them.
	  ⁻ Equal sides have equal angles opposite to them.
	
	  by applying congruence rule.
	10. Get learners to explore triangle inequality property by experiment method (construction).
	11. Encourage learners to derive proof by deductive reasoning (following steps mention in point 7) and help them to see the axiomatic structure.
		• Explore under what condition of SSA you could draw a triangle.
		• In a given set of triangles identify congruent triangles and justify.
		• Given jumbled arguments of the proof of a result, sequence them in logical order to complete the proof of the given result.
		• Uses congruence rules to prove given geometrical results.
		• State true/false - Equilateral triangles have all sides equal and all angles equal, hence all equilateral triangles are congruent. Justify your response.

Learning outcomes and indicators	Pedagogical Processes	Assessment Strategies
LO5. Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles, etc., by applying axiomatic approach and solves problems using them CLO21. Applies theorems and axioms related to triangle congruency **Indicators:** C98. Applies the concept of congruent triangles in solving problems	• Get learners to recall triangle congruence rules, triangle equality properties and triangle inequality properties **Questions for Discussion and Suggested Activities:** 1. Show the use of congruency in real – world situation. 2. Guide learners on a project to find how knowledge of congruency has contributed to human development. (Example: ornament making, quilting, building bridges/big structures) 3. Get learners to solve problems related to real world situation by applying congruent triangles.	• Cite examples and describes how congruency is applied in real – world situations. • Construct a triangle congruent to the given triangle.

Learning outcomes and indicators	Pedagogical Processes	Assessment Strategies
LO5. **Derives** proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles, etc., by applying axiomatic approach and **solves** problems using them CLO22. Solves problems using congruency theorems **Indicators:** C99. Applies congruency theorems to find sides and angles in triangles	**Questions for Discussion and Suggested Activities:** 1. Get learners to list out congruency theorems and triangle properties. 2. Show how to apply knowledge of congruent triangles in finding unknown values. 3. Give pairs of congruent triangles with certain parts unknown and get learners to find unknown measure by applying congruency theorems and triangle properties.	• Uses algebra to find unknown measures of sides and angles of congruent polygons. • Finds the value of unknown in given pair of congruent triangles.

Learning outcomes and indicators	Pedagogical Processes	Assessment Strategies
LO5. Derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles, etc., by applying axiomatic approach and solves problems using them CLO23. Utilizes triangle inequality theorem **Indicators:** C100. Recognises side and angle facing side of a triangle C101. Applies triangle inequality theorems to order sides (shortest, medium, longest) and angles (smallest, medium, largest) in triangles	• Ask learners to draw triangle and identify parts of triangle like - side/angle opposite to given side/angle • Recalls facts on triangle inequality theorem **Questions for Discussion and Suggested Activities:** 1. Get learners to solve problems related to sequencing sides or angles in a triangle by applying triangle inequality theorem.	• Uses triangle inequality property to prove results. • With the given perimeter of triangle, find possible integer lengths of triangle.

Class X
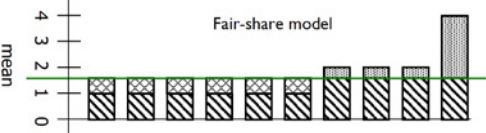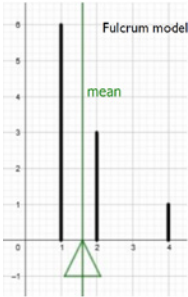
The following pedagogical process for statistics has been designed keeping in mind the fact that the more common pedagogical approach to this chapter is a formula-based approach, which does not explain the reasoning behind the development of the formula. If statistics is to be used with understanding at the higher level, then the latter approach is crucial to lay a solid foundation for this content domain. As some of the ideas mentioned have not been addressed even in the textbook, they are elaborated in the appendix for the benefit of the user of this document.
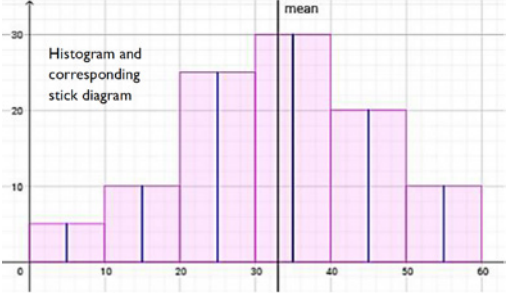
Content Domain: Statistics and Probability

Chapter 14 – Statistics

Learning outcomes and indicators	Pedagogical Processes	Assessment Strategies
LO14: **Calculates** mean, median and mode for different sets of data related with real life contexts	General ideas: • Since the focus is to compute mean, median and mode from grouped data, it doesn't make sense to start with raw data • Select a collection of grouped data such that they include symmetric, left-skewed and right-skewed distributions – these will help in discussing which of mean, median, mode is most appropriate and why • Revisit ungrouped data, graph of the same – stick diagram (bar with zero width), and link it to mean and mode in particular – before discussing mean, median, mode from grouped data • Draw graphs of grouped data – histogram/ogive • Extend the meaning of mean, median and mode to grouped data and the corresponding histogram/ogive • Link the formulas of mean, median and mode to histogram/ogive and derive them Stick Figures Number of living grandparents	• Draw the histogram for the given data • Identify any unknown frequency
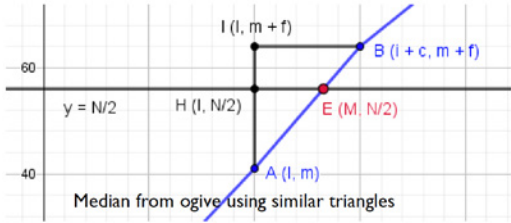
Learning outcomes and indicators	Pedagogical Processes	Assessment Strategies
CLO35. Calculates and interprets mean for tabulated data **Indicators:** C142. Computes mean for ungrouped tabulated data C143. Computes mean for grouped data C144. Interprets the meaning of the computed mean of a given data set	**What is mean?** • Revisit the meaning of mean and the two ways of interpreting it – (i) the fair share model and (ii) the fulcrum model – as illustrated below • For ungrouped distribution, discuss the **fair share model**: If the data values are evenly distributed, what will be the uniform value? • Link the above to non-frequency bar chart where each bar represents a data value, which is proportionate to the height of the corresponding bar (number of bars = total frequency) • Ask: What would be the common height if the heights of the bars are redistributed evenly across all bars? • Derive mean ____ from the bars • Use the stick $= \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$ discuss the **fulcrum model** where the diagram gets balanced • Derive the above formula from ____ where x = m is the position of the fulcrum • (See appendix 'Mean' for a dee $\sum_{i=1}^{n} f_i(x_i - m) = 0$:ween these two models) **Mean for grouped data** • Use the same fulcrum notion for grouped data using a histogram • Since, we don't know the location of each data point within any class, assume uniform distribution within each class • Thus, deduce that the fulcrum for each class to be at the class mark	• Find mean of given data – ungrouped or grouped • Relate mean of grouped data to the location of the fulcrum balancing the histogram

Learning outcomes and indicators	Pedagogical Processes	Assessment Strategies
	• Thus convert the histogram to a stick diagram and then derive the formula to compute mean for grouped data • Note that the stick diagram corresponds to the class marks and their corresponding frequencies and that's all that is used to compute mean of any grouped data Assign different grouped distributions to different groups of learners and get them to find the mean by drawing the histogram and the corresponding stick diagram [Different groups of learners can be asked to find grouped data on their own as well] Appropriateness of mean • Compute mean for various types (symmetric, left-skewed, right-skewed) of grouped distributions • Observe the positions of means (i.e. fulcrum) with respect to the corresponding histograms • Discuss in which cases mean is appropriate and where it is not	
CLO36. Determines and interprets median for tabulated data **Indicators:** C145. Determines cumulative frequencies of ungrouped tabulated data C146. Determines median for ungrouped tabulated data C147. Computes cumulative frequencies for grouped data C148. Recalls the median formula and what each part represents	What is median? • Revisit what is a median of (ungrouped) data, what does it mean (that it halves the data) and how it is computed Why median? • Discuss situations where median may be a better measure of central tendency that mean with suitable (ungrouped) data Cumulative frequencies • Pick an ungrouped frequency tables such that the frequencies are large (at least 3 frequencies > 10) and let learners experience the tediousness of ordering all the data values • Introduce cumulative frequencies – less than and more than – for this ungrouped data and use that to demonstrate how cumulative frequency helps in finding the median of ungrouped data by narrowing down on $N/2^{th}$ or $(N + 1)/2^{th}$ data value (for even or odd N respectively; N = total frequency	• Find cumulative frequencies of given ungrouped data • Find frequencies of ungrouped data given cumulative frequencies of the same • Find median of ungrouped data • Relate median as the data value that halves the histogram • Find cumulative frequencies of given grouped data • Find frequencies of grouped data given cumulative frequencies of the same

Learning outcomes and indicators	Pedagogical Processes	Assessment Strategies

C149. Computes median for grouped data using the formula
 	
C150. Constructs less than and more than ogives
 	
C151. Retrieves median from the intersection of less than ogive and the line y = N/2
 	
C152. Retrieves median from the intersection point of the ogives
 	
C153. Recalls the meaning of the computed median of a given data set
	Median with grouped data
	
	• Pose a situation where median would be a more appropriate measure than mean with grouped data
	• Ask learners how one can find the median for this data
	• Observe that it is impossible to find individual data values from a grouped frequency table, and therefore impossible to find $N/2^{th}$ or $(N + 1)/2^{th}$ data value
	• Discuss how median should be interpreted with respect to histogram –
	  o since median for ungrouped data halves the data values
	  o frequencies are proportionate to the heights of the rectangles
	  o therefore, area of histogram is proportionate to total frequency N
	  o therefore, median is M such that the line $x = M$ halves the histogram in terms of area
	• Find the median from a grouped distribution using above
	• Assign different grouped distributions to different groups of learners and get them to find the median through this method
	  [Different groups of learners can be asked to find grouped data on their own as well]
	• Collate the findings from each group in a table and derive the median formula from there (see reference 'Median from histogram' for details)
	• Revisit histograms of various types (symmetric, left-skewed, right-skewed) by computing both mean and median and discuss which one is more appropriate and why
	
	Ogives
	
	• Introduce ogives as an easier way to find the median and as the graphs that represent cumulative frequencies
	• Discuss how cumulative frequencies should be written (e.g. class upper limit with less than cumulative frequencies and why)
	• Draw ogives – less than and more than – and observe that they are almost mirror of each other and why so (see reference 'Median from ogives' for details)
		• Draw ogives of given data
		• Find the point of intersection of the ogives
		• Find the point of intersection of any ogive and the line $y = N/2$
		• Find the median of given grouped data
		• Consider both mean and median of grouped data with respect to its histogram and conclude which one is more appropriate

Learning outcomes and indicators	Pedagogical Processes	Assessment Strategies
	Median from ogive • Discuss how the less than ogive can be used to find the median – generalize the ungrouped situation to splitting the ogive by $y = N/2$ and finding the x-coordinate of the point of intersection of this horizontal line with the ogive (see reference 'Median from ogives' for details) • The median M can be computed using the similar triangles ΔAIB and ΔAHE, especially using the ratios involving the bases and heights of these triangles Median from ogive using similar triangles • As before, assign different grouped distributions to different groups of learners and get them to find the median by drawing the ogive [Different groups of learners can be asked to find grouped data on their own as well] • Collate the findings from each group in a table and derive the median formula from there (see reference 'Median from ogive' for details) • Discuss how the ogives intersect at $y = N/2$ i.e. at the median – again working out examples from several groups can help in understanding general case (see reference 'Median from ogives' for details) **Optional** • Consider grouped distributions which have outlier(s) • Find both mean and median for such a distribution (i) with outlier(s), and (ii) without outlier(s) • Observe how much the mean changed vs how much the median changed due to the outlier • Discuss (i) how to identify and outlier, and (ii) which is better – mean or median – when there is an outlier	

Learning outcomes and indicators	Pedagogical Processes	Assessment Strategies

CLO37. Calculates and interprets mode for grouped data
 	
Indicators:
 	
C154. Recalls the mode formula and what each part represents
 	
C155. Computes mode for grouped data
 	
C156. Interprets the meaning of the computed mode of a given data set
	• Revisit mode of ungrouped data
	
	Mode for grouped data
	
	• Discuss mode for grouped data –
	  ◦ Start with a histogram
	  ◦ Identify the modal class
	  ◦ Observe how it is impossible to pinpoint exact location of the mode within the modal class
	  ◦ Illustrate with an example – suppose the modal class is 20-30 and the modal frequency is 7, the data values in the modal class can be 21, 21, 21, 21, 22, 22, 23 or 28, 29, 29, 29, 29, 30, 30
	• Discuss why the mode should be derived using the top 4 points (F, G, H and K in the diagram) on the rectangle corresponding to the modal class
	
	(see appendix 'Mode' for details)
	• Draw the slant lines (FH and GK) joining these 4 points and find the x-coordinate of their point of intersection (E)
	• Mode is the x-coordinate of E i.e. m and it can be obtained considering the two pairs of similar triangles
	• ΔFHG and ΔEHJ $\Rightarrow$ FG/EJ = GH/JH $\Rightarrow$ FG × JH = GH × EJ
	• ΔKGH and ΔEGJ $\Rightarrow$ KH/EJ = GH/GJ $\Rightarrow$ KH × GJ = GH × Ej
	• KH × GJ = GH × EJ = FG × JH $\Rightarrow$ $(f_2 - f_1)(m - l) = (f_1 - f_0)(l + c - m)$
	• Demonstrate this with numerical values for lower bound of modal class = l, class-width = c, and the frequencies f_0, f_1 and f_2 and solve for the mode = m
		• Identify the modal class(es) (in all possible situations including consecutive ones)
		• Identify the mode on the histogram
		• Compute mode of given grouped data
		• Identify whether mode is closer to the next class, or to the previous class, based on the frequencies of these classes

Learning outcomes and indicators	Pedagogical Processes	Assessment Strategies

	• As before, assign different grouped distributions to different groups of learners and get them to find the mode by drawing the histogram
	• [Different groups of learners can be asked to find grouped data on their own as well]
	• Collate the findings from each group in a table and derive the mode formula from there (see appendix 'Mode' for details)
	• It is a good idea to vary the frequencies of the classes adjacent to the modal class i.e. f_0 and f_2 to observe how the slant lines change and therefore how the mode shifts
	Optional
	• Discuss how to adopt this formula when modal class is an extreme class by using the histogram
	• Discuss how to adapt this formula if there are more than one consecutive modal classes
	• Outlier(s) and mode:
	  ◦ Get grouped data with outlier(s)
	  ◦ Compute mode for both (i) with outlier(s) and (ii) without outlier(s)
	  ◦ Check how much mode changed due to outlier(s)	

11. Question Paper Design

Class X

Content domain	Marks distribution
Number Systems	6
Algebra	20
Coordinate Geometry	6
Geometry	15
Trigonometry	12
Mensuration	10
Statistics and Probability	11
Total	*80*

Item types	Number of questions	Marks distribution
Multiple Choice Questions	15	15
Constructed Response Questions*	25	65
	40	*80*
*Type of CR questions		
Short Answer 1 (2 marks)	14	28
Short Answer 2 (3 marks)	7	21
Long Answer (4 marks)	4	16
	25	*65*

Cognitive process	Assessment objective	Thinking process	Marks distribution
Knowing	Demonstrate knowledge and understanding of mathematical ideas, techniques and procedures	• Define • Recall and provide examples, illustrations • Recall symbols, vocabulary, notation • Recall facts, formulas • Recognise patterns • Relate two concepts • Reproduce (including measure, compute, construct) procedures and proofs • Recall and explain • Differentiate between/among (based on recall of properties) • Retrieve information • Reframe using recall of definitions	20
Applying	Apply knowledge and understanding of mathematical ideas, techniques and procedures to classroom and real-world situations	• Identify by connecting known properties and definitions to given context or situation • Classify by connecting properties or using information from given data • Locate by using properties or computation • Reframe using definitions, properties and by connecting two or more facts • Explain by connecting two or more facts • Compute or process by selecting strategy, operation(s) and operand(s) • Prove an unfamiliar statement using one or two known facts	35
Reasoning	Analyse data and other information, draw conclusions, recognize patterns, draw conjectures, prove results, and extend their understanding to new situations	• Distinguish using logical reasoning • Analyse • Evaluate and choose • Extend properties, laws and theorems using reasoning and understanding constraints • Prove using logical reasoning and understanding of properties, laws and theorems • Create situations based on given constraints	25
Total			*80*

12. Sample assessment items and marking scheme

Class IX

Multiple Choice Questions (MCQ)

Content Domain (Chapter Name)	Numbers (Number System)	
Content Domain (Learning Outcome)	Applies logical reasoning in classifying real numbers.	
Indicator	Classifies numbers into natural numbers, whole numbers, integers, rational and irrational numbers using their definitions.	
Cognitive Level	Applying	
Thinking Process	Classify by connecting properties or using information from given data	
Difficulty Level	Low	
Marks	1	
Time	1 min	
Item Stem	Choose the irrational number out of the following	
Correct Answer	$\dfrac{1}{\sqrt{2}}$	
Distractor 1	$\dfrac{22}{7}$	This fraction may be confused with π which is an irrational number
Distractor 2	5	Candidate may think that every rational number has to be expressed as a fraction
Distractor 3	$\dfrac{\sqrt{9}}{3}$	Candidate is confused seeing the square root sign

Content Domain (Chapter Name)	Algebra (Polynomials)	
Content Domain (Learning Outcome)	Applies remainder and factor theorems to polynomials.	
Indicator	Relates the factors of a polynomial with its zeroes and vice-versa.	
Cognitive Level	Knowing	
Thinking Process	Relate two concepts	
Difficulty Level	Medium	
Marks	1	
Time	2 min	
Item Stem	A polynomial with zeroes 0 and −5 is	
Correct Answer	$x(x+5)^2$	
Distractor 1	$x(x+5)$	Omits the factor connected with the zero
Distractor 2	$(x-5)$	Omits the factor connected with the zero and confuses the sign of the zero with the sign in the factor
Distractor 3	$x(x-5)$	Confusion between the sign of the zero and the sign in the factor

Content Domain (Chapter Name)	Coordinate Geometry
Content Domain (Learning Outcome)	Relates the position of a point to the horizontal and vertical movement in the Cartesian plane.
Indicator	Builds a table of coordinates of points when given a shift along either horizontal or vertical axes (or both).
Cognitive Level	Applying
Thinking Process	Compute or process by selecting strategy, operation(s) and operand(s)
Difficulty Level	Low
Marks	1
Time	2 min

Item Stem	Which of the following tables shows the points on the given line?
Correct Answer	<table><tr><td>8</td><td>-2</td></tr><tr><td>7</td><td>-1</td></tr><tr><td>4</td><td>2</td></tr><tr><td>3</td><td>3</td></tr></table>

Correct Answer		
Correct Answer	8 / -2, 7 / -1, 4 / 2, 3 / 3	
Distractor 1	2 / 8, 1 / 7, -2 / 4, -3 / 3	Reverses the abscissa and ordinates, wrong sign for abscissa
Distractor 2	8 / -1, 7 / 1, 4 / 3, 3 / 2	Wrong ordinates
Distractor 3	-2 / 8, -1 / 7, 2 / 4, 3 / 3	Reverses abscissa and ordinates

Content Domain (Chapter Name)	Geometry (Introduction to Euclid's geometry)	
Content Domain (Learning Outcome)	Represents geometrical concepts – points, lines, line segment, ray, angles, collinear points, non collinear points and circles with reference to Euclidean geometry.	
Indicator	Uses the language (symbols and representation) needed to understand, identify and discuss different geometrical concepts such as lines and angles.	
Cognitive Level	Knowing	
Thinking Process	Recall symbols, vocabulary, notation	
Difficulty Level	Low	
Marks	1	
Time	1 min	
Item Stem	Which of the following statements is true?	
Correct Answer	BA is a line segment with end points A and B	
Distractor 1	m is the one and only one line through point P	Is not clear with concept of lines through a point
Distractor 2	Lines p and q intersect at point o	Is not clear with convention for naming points
Distractor 3	Exactly two straight lines can be drawn which pass through both the points L and M	Is not clear with concept and naming of lines and line segments and its representation

Content Domain (Chapter name)	Mensuration (Heron's Formula)
Content Domain Learning outcome	Applies Heron's formula.
Indicator	Identifies whether half base times height can be used to find area of a triangle.
Cognitive level	Reasoning
Thinking Process	Distinguish using logical reasoning
Difficulty level	Medium
Marks	1
Time	2 min

Item Stem	The area of which of the following triangles cannot be computed with the formula ½ base × height?	
Correct answer	Triangle with sides 6cm, 7cm and 10cm	
Distractor 1	Triangle with sides 6cm, 8cm and 10cm	Right triangle, ∴ ½ base × height = ½ × 6cm × 8cm
Distractor 2	Triangle with sides 5cm, 5cm and 8cm	Isosceles triangle with height $\sqrt{(5^2 - (8/2)^2)}$ = 3cm and base 8cm
Distractor 3	Triangle with sides 13cm, 13cm and 13cm	Equilateral triangle with height $\sqrt{(13^2 - (13/2)^2)}$ and base 13 cm or use of formula for area of an equilateral triangle

Content Domain (Chapter name)	Statistics and Probability (Statistics)
Content Domain Learning outcome	Analyses data by graphically representing it as bar graph, histogram (with equal and varying width and length), and frequency polygon.
Indicator	Compares two distributions based on their frequency polygons (drawn on the same axes).
Cognitive level	Reasoning
Thinking Process	Analyse
Difficulty level	High
Marks	1
Time	1 min
Item Stem	The following graph shows the frequency polygons of two distributions. Which is the correct statement about them?

Correct answer	The distribution represented by solid line has a higher mode	
Distractor 1	The distribution represented by solid line has a lower mode	Mode is determined by where the peak is, not how high it is
Distractor 2	Both distributions have the same median	The solid line one is clearly more to the right
Distractor 3	The class intervals are different for the two distributions	They are identical

Content Domain (Chapter name)	Statistics and Probability (Probability)
Content Domain Learning outcome	Calculates empirical probability through experiments.
Indicator	Recognizes empirical probability as relative frequency.
Cognitive level	Knowing
Thinking Process	Relate two concepts
Difficulty level	Low
Marks	1
Time	1 min
Item Stem	1500 families with 2 children were selected randomly, and the following data were recorded: | Number of girls in a family	2	1	0
---	---	---	---
Number of families	475	814	211	 Compute the probability of a family, chosen at random, having 2 boys. [Assuming that there are no children of the 3rd gender.]

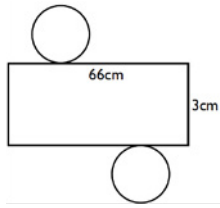| Correct answer	211/1500	
Distractor 1	475/1500	Wrong numerator
Distractor 2	0.211	Wrong denominator OR wrong attempt in trying to convert the frequency into a number between 0 and 1
Distractor 3	0.25	Theoretical probability assuming equal chance for boy and girl

Constructed Response Questions

Content domain (Chapter name)	Numbers (Number System)
Content Domain Learning outcome	Applies logical reasoning in using real numbers in different situations.
Indicator	Expresses the nth root of a real number using a rational exponent and vice-versa.
Cognitive level	Knowing
Thinking Process	Reframe using recall of definitions
Difficulty level	Low
Marks	2
Time	2 min
Item stem	Express $5^{-1/2}$ in the form p/q (where p and q are non-zero real numbers) and rationalise q.
Marking Scheme	

Part	Mark	Answer	Further Information
	½	Expresses $5^{-1/2}$ as $1/\sqrt{5}$	
	½	Identifies the rationalising factor as $\sqrt{5}$	Rationalising factor can be any multiple of $\sqrt{5}$
	½	Multiplies numerator and denominator by $\sqrt{5}$	Follow through by multiplying with above factor
	½	Obtains answer as $\sqrt{5}/5$	Accept only this answer (cancellation expected)
			If candidate rationalises using $5^{1/2}$ accept rationalisation if correct and answer expressed as $5^{1/2}/5$ allot 1 mark

Content domain (Chapter name)	Coordinate Geometry (Coordinate Geometry)
Content Domain Learning outcome	Relates the position of a point to the horizontal and vertical movement in the Cartesian plane. Analyses similarities and differences between parts of shapes (lines, angles and triangles).
Indicator	Determines the coordinates of a point which is shifted/translated by given units or reflected in the x-axis, the y-axis or the origin. Builds a table of coordinates of points when given a shift along either horizontal or vertical axes (or both). Identifies corresponding parts of two triangles (which may or may not be congruent to each other). Analyses equality of corresponding parts of the congruent shapes.
Cognitive level	Reasoning
Thinking Process	Retrieve information Compute or process by selecting strategy, operation(s) and operand(s) Extend properties, laws and theorems using reasoning and understanding constraints
Difficulty level	Medium
Marks	3
Time	2 min
Item stem	Given $\triangle ABC$ with A(-5, 1), B(-4, 3) and C(-1, 1). Shift the given triangle by 2 units right and 3 units up. Write the coordinates of the triangle after the shift. What happens to the area of the new triangle? Why?

Marking Scheme

	Mark	Answer	Further Information
	½	Showing shift is + 2 in x-coordinate and + 3 in the y-coordinate	Done correctly for at least one point
	1½	Writing all the three coordinates	½ + ½ + ½ for accuracy
	½ + ½	Area remains the same + since they are congruent	½ mark for reason can be given if learner indicates in his/her own words that translation is a rigid transformation

Content domain (Chapter name)	Geometry
Content Domain Learning outcome	Constructs lines – bisectors, angular bisector (60°, 90°, 45°), equilateral triangles and triangles with given conditions (such as base, sum/difference of other two sides, perimeter and one or two base angles given).
Indicator	Constructs triangles with given conditions (such as base, sum/difference of other two sides, perimeter and one or two base angles given) .
	Reasons out the steps for construction with given conditions (such as base, sum/difference of other two sides, perimeter and one or two base angles given) connecting them to congruent triangle theorems.
Cognitive level	Reasoning
Thinking Process	Extend properties, laws and theorems using reasoning and understanding constraints
Difficulty level	Medium
Marks	4
Time	7 min
Item stem	Surya says: I have drawn two triangles of perimeter 21 cm. Since the perimeters are the same, they are congruent.
	Show by constructing two triangles each of perimeter 21cm that Surya is wrong.

Marking Scheme

	Mark	Answer	Further Information
	½ + ½	For providing three sides of two valid triangles each of perimeter 21 cm	Identifies in a triangle – perimeter (21 cm) = $a + b + c$ and $a + b > c$, $b + c > a$ and $a + c > b$
	1	For indicating method of SSS construction clearly for at least 1 triangle	Deduct ½ mark for untidy construction, lack of labelling
	1	For accurately constructing two triangles as per data chosen by the candidate	
	1	For showing that corresponding parts are not equal, hence not congruent	Marks to be given if learner measures the parts of constructed triangles and shows that they are not equal

Content domain (Chapter name)	Mensuration (Surface Areas and Volumes)
Content Domain Learning outcome	Derives formula to find total surface area, curved surface area and volume of cubes, cuboids, cylinders and cones, spheres and hemispheres using hands on activity (such as nets) and uses these to solve problems on surface area and volumes of solids. Applies formulas of total surface area, curved surface area and volume of cubes, cuboids, cylinders and cones, spheres and hemispheres to solve problems on surface area and volumes of solids.
Indicator	Relates area of the net to surface area of the corresponding 3D shape. Calculates volumes of solids using appropriate formula.
Cognitive level	Reasoning
Thinking Process	Reproduce (including measure, compute, construct) procedures and proofs Extends properties, laws and theorems using reasoning and understanding constraints
Difficulty level	Medium
Marks	3
Time	5 min
Item stem	Find the volume of the cylinder whose net is given below. [You may approximate π with 22/7] 66cm 3cm

Marking Scheme

Part	Mark	Answer	Further Information
	½	For identifying height = 3cm	
	1	For finding radius = 66/2π = 33/π = 21/2 = 10.5cm	½ for identifying $2\pi r$ = 66cm but not finding radius 33/πcm, 21/2cm, 10.5cm – all are ok
	1	For finding the volume = π × $(33/\pi)^2$ × 3 = 3267/π = 2079/2 = 1039½ = 1039.5	½ for using volume formula and correct substitution of height and/or radius, but incorrect simplification 3267/π, 2079/2, 1039½, 1039.5 – all are ok
	½	For correct unit of volume i.e. cm^3	Not assigned if unit is missing or if it is cm or cm^2

Content domain (Chapter name)	Statistics and Probability (Statistics)
Content Domain Learning outcome	Identifies daily life situations in which mean, median and mode can be used.
Indicator	Distinguishes situations in which mean can be computed and is an appropriate measure of central tendency from where it is computable but inappropriate.
Cognitive level	Reasoning
Thinking Process	Distinguishes using logical reasoning
Difficulty level	High
Marks	2
Time	3 min
Item stem	Give one example of a situation where (i) mean can be computed but (ii) mean is not an appropriate measure of central tendency

Marking Scheme

Part	Mark	Answer	Further Information
	1	For quantitative data	No mark for qualitative data
	1	For data with few very high OR very low values OR a skewed distribution	½ if the data is less skewed

Content domain (Chapter name)	Statistics and Probability (Probability)
Content Domain Learning outcome	Calculates empirical probability through experiments.
Indicator	Computes the number of all possible outcomes of a random experiment in a given situation or from a frequency table. Recognizes the outcome(s) favourable to a given event. Computes the empirical probability of a given event from the given frequency table.
Cognitive level	Applying
Thinking Process	Classify by connecting properties or using information from given data Compute or process by selecting strategy, operation(s) and operand(s)
Difficulty level	Medium
Marks	2
Time	2 min
Item stem	Consider the following distribution for a class of 20 children. If a child is selected at random from this class, how many siblings will s/he have – what are the possibilities? What is the probability of being a single child?

No. of siblings	Frequency
0	13
1	5
3	2

Marking Scheme

Part	Mark	Answer	Further Information
	½	For keeping the range 0, 1, 2, 3	No mark if range includes any negative number or any number > 3
	½	For excluding 2	No mark if 2 is included
	½	For identifying single child corresponds to 0 with frequency 13	No mark if frequency is chosen incorrectly
	½	For equating probability with relative frequency = 13/20	No mark if relative frequency is not used or total frequency is incorrect Allow 0.65

13. Class X

Multiple Choice Questions (MCQ)

Content Domain (Chapter Name)	Number System (Real Numbers)	
Content Domain (Learning Outcome)	Generalizes properties of numbers and relations among them to evolve results – such as, Euclid's division algorithm, Fundamental Theorem of Arithmetic.	
Indicator	Observes that any two integers a and b ($a > b$) can be expressed as $a = b \times$ quotient + remainder.	
Cognitive Level	Knowing	
Thinking Process	Reframe using recall of definitions	
Difficulty Level	Low	
Marks	1	
Time	1 min	
Item Stem	Given two positive integers a and b, where $a > b$. When a is divided by b, the remainder is c. This may be expressed as $a = b \times q + c$. The range of values that c can take is	
Correct Answer	$0 \leq c < b$	
Distractor 1	$0 < c < b$	Thinks that remainder c cannot be zero
Distractor 2	$0 \leq c < a$	Thinks that remainder c is less than the larger number a
Distractor 3	$0 \leq b < c$	Thinks that quotient b has to be less than the remainder c

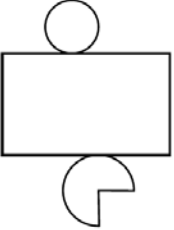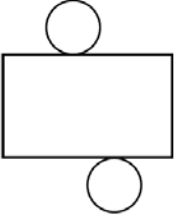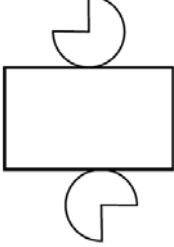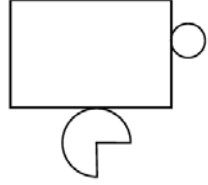| Content Domain (Chapter Name)	Algebra (Quadratic Equations)	
Content Domain (Learning Outcome)	Explores the relationship between the nature of the roots and the discriminant of a quadratic equation.	
Indicator	Solves problems based on the relationship between the nature of the roots and the discriminant of a quadratic equation.	
Cognitive Level	Reasoning	
Thinking Process	Extend properties, laws and theorems using reasoning and understanding constraints	
Difficulty Level	Low	
Marks	1	
Time	1 min	
Item Stem	If the two roots of a quadratic equation are √5 and -√5, then 1] The discriminant is positive 2] The quadratic is a binomial with a constant term 3] The leading coefficient is 1 Which of the following is true?	
Correct Answer	1] and 2] are true	
Distractor 1	1] 2] and 3] are true	Assumes that there is only one quadratic equation having these roots
Distractor 2	1] and 3] are true	Thinks that a quadratic has to be a trinomial, ignores sum of roots
Distractor 3	2] and 3] are true	Thinks that the roots are identical and so the discriminant is zero OR thinks that the roots are irrational and so the discriminant is negative

Content Domain (Chapter Name)	Co-ordinate Geometry (Coordinate Geometry)	
Content Domain (Learning Outcome)	Derives the section formula, the area formula and the distance formula.	
Indicator	Given coordinates of points applies reasoning to solve problems such as finding: i. Distances ii. Coordinates of points dividing a line in a particular ratio iii. Types of triangles iv. Types of quadrilaterals v. Collinearity of given points	
Cognitive Level	Reasoning	
Thinking Process	Extend properties, laws and theorems using reasoning and understanding constraints	
Difficulty Level	Low	
Marks	1	
Time	1 min	
Item Stem	The coordinates of the centroid of the triangle ΔOAB where O is the origin, A is (0, -3) and B is (6, 0) are:	
Correct Answer	(2, -1)	
Distractor 1	(-1, 2)	Reverses abscissa and ordinate
Distractor 2	(2, 1)	Ignores negative sign
Distractor 3	(-2, 1)	Reverses sign

Content Domain (Chapter Name)	Trigonometry (Introduction to Trigonometry)	
Content Domain (Learning Outcome)	Determines all trigonometric ratios with respect to a given acute angle (of a right triangle). Uses trigonometric ratios to solve simple problems.	
Indicator	Demonstrates understanding of notations for the six trigonometric ratios and their squares and reciprocals. Determines sine, cosine, tangent, cotangent, secant and cosecant of standard angles such as 0°, 30°, 45°, 60° and 90°. Justifies expressions based on trigonometric ratios and identities.	
Cognitive Level	Reasoning	
Thinking Process	Recall symbols, vocabulary, notation Recall facts, formulas Proves using logical reasoning and understanding of properties, laws and theorems	
Difficulty Level	High	
Marks	1	
Time	2 min	
Item Stem	If $\sin^2\theta > \sin\theta$, then θ is A] 30° B] 45° C] 90°	
Correct Answer	None of the above	
Distractor 1	All of the above	Assumes that square of a value is always greater than the value itself
Distractor 2	A] and B]	Assumes that square of a value is greater than the value itself, understands that square of 1 is 1 itself
Distractor 3	B]	Assumes that $1/\sqrt{2}$ is less than ½

Content Domain (Chapter Name)	Geometry (Circles)
Content Domain (Learning Outcome)	Solves problems related to tangent of a circle.
Indicator	Uses tangent theorems to find unknown values.
Cognitive Level	Applying
Thinking Process	Compute or process by selecting strategy, operation(s) and operand(s)
Difficulty Level	High
Marks	1
Time	2 min
Item Stem	A pair of tangents AP and AQ drawn from an external point A to a circle with centre O such that the length of chord PQ is equal to the radius. Then $\angle$ PAQ is _____.
Correct Answer	120°
Distractor 1	60°	Calculates $\angle$ POQ and equals it to $\angle$ PAQ
Distractor 2	30°	Considers 60° (equilateral triangle), 90° (tangent and radius) and uses angle sum property of a triangle
Distractor 3	90°	Confuses required angle with $\angle$ APO or $\angle$ AQO

Content Domain (Chapter name)	Mensuration (Surface Areas and Volumes)
Content Domain Learning outcome	Derives the surface area and volume of combination of shapes such as (cube, cuboid, sphere, hemisphere, cone, and cylinder) and frustum of a cone.
Indicator	Visualizes the net of combined solids.
Cognitive level	Reasoning
Thinking Process	Extends properties, laws and theorems using reasoning and understanding constraints
Difficulty level	Medium
Marks	1
Time	1 min
Item Stem	Which of the following is the net of a cylinder surmounted with a cone at one end?

Correct answer		
Distractor 1		Forgot the cone
Distractor 2		Cone at both ends instead of only at one end
Distractor 3		Circle and sector (for cone) not on opposite sides of the rectangle

Content Domain (Chapter name)	Statistics and Probability (Statistics)
Content Domain Learning outcome	Determines and interprets median for tabulated data.
Indicator	Constructs less than and more than ogives.
Cognitive level	Knowing
Thinking Process	Reproduce (including measure, compute, construct) procedures and proofs
Difficulty level	Medium
Marks	1
Time	2 min

Item Stem

Find the coordinates of the points you need to plot to draw the less than ogive from the given distribution.

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35
No. of learners	2	6	13	17	11	4	2

Correct answer	(5, 2), (10, 8), (15, 21), (20, 38), (25, 49), (30, 53), (35, 55)

Distractor 1	(0, 2), (5, 8), (10, 21), (15, 38), (20, 49), (25, 53), (30, 55)	Lower class limits instead of upper class limits
Distractor 2	(0, 55), (5, 53), (10, 47), (15, 34), (20, 17), (25, 6), (30, 2)	Coordinates of points for more than ogive
Distractor 3	(2.5, 2), (7.5, 8), (12.5, 21), (17.5, 38), (22.5, 49), (27.5, 53), (32.5, 55)	Class marks instead of upper class limits

Content Domain (Chapter name)	Statistics and Probability (Probability)	
Content Domain Learning outcome	Applies the concept of probability in solving daily life problems.	
Indicator	Recognizes the elementary events in daily life context.	
Cognitive level	Knowing	
Thinking Process	Retrieve information	
Difficulty level	Low	
Marks	1	
Time	1 min	
Item Stem	Three friends A, B and C appeared for an entrance test to a prestigious college. Consider the possibilities of how many of them passed. Which of the following is an elementary event? a. At least 2 of them passed b. At most 1 of them passed c. At least 1 passed and 1 failed d. None of them passed	
Correct answer	d	
Distractor 1	a	{2, 3}
Distractor 2	b	{0, 1}
Distractor 3	c	{1, 2}

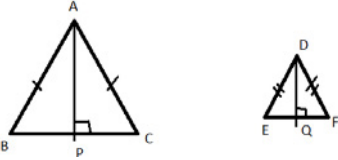**Constructed Response Questions (CR)**

Content domain (Chapter name)	Algebra (Pair of Linear Equations in Two Variables)
Content Domain Learning outcome	Solves problems on linear equations in two variables graphically or algebraically. Classifies linear equations as consistent, inconsistent and dependent and interprets their geometric meaning, the type of solutions and the relationship between the coefficients.
Indicator	Uses algebraic procedures on a pair of linear equations to eliminate one variable and solve for both variables Connects relationship between coefficients of terms in the equations of a pair of lines with the three categories 1. intersecting 2. parallel 3. coincident Uses the ratio of coefficients to identify if the solution to a given pair of lines is unique, non-existent, or infinitely many
Cognitive level	Applying
Thinking Process	Classify by connecting properties or using information from given data Compute or process by selecting strategy, operation(s) and operand(s) Differentiate between/among (based on recall of properties)
Difficulty level	High
Marks	4
Time	10 min
Item stem	The cost of 5 oranges and 3 apples is ₹100 and the cost of 6 apples and 10 oranges is ₹120 a. Is it possible to find the cost of an orange and an apple using this information? Why not? b. If the cost of 6 apples and 10 oranges is to be changed from ₹120, will it be possible to find the cost of an orange and an apple? Why or why not? c. Change **only** the number of apples in either one of the given equations in order to be able to solve the equations and find the cost of each orange and each apple.

Marking Scheme

Part	Mark	Answer	Further Information
a.	1	There are two variables (cost of apple and cost of orange) and the equations connecting them are inconsistent OR any explanation indicating that the total cost of apples and oranges is different in both equations	'Insufficient information' is not a valid explanation. 'Wrong information' is valid but without an explanation earns only ½ mark.
b.	1	No (½ mark) Explanation (½ mark): Any cost other than ₹200 would make the equations inconsistent and ₹200 would make the equations dependent with insufficient information.	Mark to be allotted if learner explains the difficulty in his/her own words even without use of the words 'dependent' or 'inconsistent'
c.	2	½ mark: Number of apples changed in one of the equations 1 mark: Method mark for solving two simultaneous equations ½ mark: Correct values for cost of both orange and apple	Mark is not allotted if the number of apples in both equations or the number of oranges in either equation is changed, or cost is changed Awarded for eliminating either variable; allow any two consistent equations Answer must be consistent with the equations that they formed.

Content Domain (Chapter)	Algebra (Quadratic Equations)
Content Domain Learning outcome	Solves a quadratic equation $ax^2 + bx + c = 0$. Solves situational problems based on quadratic equations.
Indicator	Expresses a quadratic equation $ax^2 + bx + c = 0$ in the form $(x + m)(x + n) = 0$. Solves quadratic equations based on mathematical and/or real life contexts.
Cognitive level	Reasoning
Thinking Process	Reframe using definitions, properties and by connecting two or more facts Extend properties, laws and theorems using reasoning and understanding constraints
Difficulty level	High
Marks	4
Time	10 min
Item stem	The polynomial $25a^2 - 35a + 12$ represents the area of a rectangle. Find the value of a for which the length is 12 units and the breadth is 11 units. Find a value of a which makes the area zero.

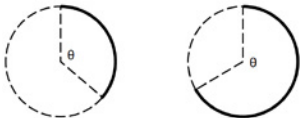| Part	Mark	Answer	Further Information
a.	1	Factorises the polynomial $25a^2 - 35a + 12$ as $(5a - 3)(5a - 4)$	Assign ½ mark if correct process of factorisation using 20a and 15a is used but correct factors are not arrived at.
	1	Uses $5a - 3 = 12$ and $5a - 4 = 11$	Accept $5a - 3 = 11$ and $5a - 4 = 12$, for incorrect factors separately equated to 11 and 12 assign ½ mark
	1	Arrives at $a = 3$	No mark assigned for inconsistent values of a such as 14/5 and/or 16/5
b.	1	$a = 3/5$ or $a = 4/5$	Full mark for correct answers only Assign ½ mark for equating their factors to zero

Content domain (Chapter name)	Geometry (Triangles)
Content Domain Learning outcome	Proves theorems related to similar triangles.
Indicator	Proves results/riders based on similarity of triangles.
Cognitive level	Reasoning
Thinking Process	Prove using logical reasoning and understanding of properties, laws and theorems
Difficulty level	Medium
Marks	3
Time	3 min
Item stem	 Triangles ΔABC and ΔDEF are isosceles, with BC = AP = 4 units and EF = DQ = 2 units. Show that ΔABC is similar to ΔDEF.

Marking Scheme

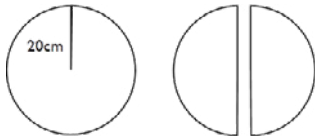| Part	Mark	Answer	Further Information
	½	Shows that EF/ BC = DQ /AP = ½	Accept BC/EF = AP/DQ = ½
	½	Identifies AP and DQ are medians and hence EQ/BP = ½	Showing that ΔABP is similar to ΔACP and that ΔAEQ is similar to ΔDEQ by AA similarity is also sufficient.
	1	Shows ΔDEQ ~ ΔABP (by SAS similarity)	
	1	∠E = ∠B = ∠C = ∠F (corresponding parts of similar triangles)	
		ΔDEF ~ ΔABC (AA similarity)	
	OR	OR	
	½	Indicates that AP and DQ are medians since the triangles are isosceles	Mark is assigned for correct use of formula and correct values of sides, answer not necessarily correct
	1	Calculates AC and DF by using Pythagoras' theorem	
	1	Proves using trigonometry that ∠C = ∠F and ∠B = ∠E	Correct value of trig ratio necessary for at least one angle in each triangle
	½	Hence proves that the triangles are similar by AA	Condition for similarity to be cited

Content domain (Chapter name)	Trigonometry (Some Applications of Trigonometry)
Content Domain Learning outcome	Uses the six trigonometric ratios of standard angles to solve problems in daily life contexts.
Indicator	Uses vocabulary related to heights and distances problems – for example: line of sight, angle of elevation/depression. Draws labelled diagrams (involving the right triangle and the acute angle) representing the given situation. Finds the height/depth or length of an object or the distance between two distant objects using trigonometrical ratios.
Cognitive level	Applying
Thinking Process	Recall symbols , vocabulary , notations Reframe using definitions, properties and by connecting two or more facts Compute or process by selecting strategy, operation(s) and operand(s)
Difficulty level	Medium
Marks	4
Time	10 min

		A man standing at the water's edge of a stream observes on the opposite bank exactly across the stream from where he is standing. He finds the angle of elevation of the top of the tree to be 45°.
Item stem		a. Make a sketch of this information. b. Would the man be able to find the ratio of the height of the tree to the width of the stream at his location, given this information? If so, what is it? (You may assume that the tree is at the water's edge on the opposite bank). c. The man now moves backward (perpendicular to the stream) by a distance of 4 metres from the water's edge. He finds that the angle of elevation of the top of the tree reduces by 15°. Is this information sufficient for the man to determine the height of the tree and the width of the stream? If so, find them.

Marking Scheme

Part	Mark	Answer	Further Information
a.	½	Sketch of a right-angled triangle, indicating tree perpendicular to ground, angle from man's eye to top of tree shown by 45°	Man's eye could be at ground level, his height need not be shown.
b.	1	½ mark: Yes ½ mark: Ratio is 1	Accept any answer which indicates that ratio is 1 and that the height of the tree is equal to the width of the stream
c.	½	Method mark allotted for indicating that tan θ = $$\dfrac{height\ of\ tree}{width\ of\ stream+4}$$	No mark Allot ½ mark only for general formula using ratio of opposite side to adjacent side (without relating to this problem). No mark for incorrect use of height and width without the general formula.
	½	Correct value of tangent of 30°	Accept fraction even if denominator not rationalised or correct value in decimals (2 dp accuracy)
	1	Method mark for solving two simultaneous equations obtained from tan 45° and tan 30°	
	½	Correct value of 2(√3 +1)m for height of tree or width of stream	Full marks to be awarded even if height of person is not accounted for.

Content domain (Chapter name)	Mensuration (Areas Related to Circles)
Content Domain Learning outcome	Derives formulas related to circumference and area of a circle, arc length, area of sector and area of segment.
Indicator	Derives formula for arc length of a circle.
Cognitive level	Reasoning
Thinking Process	Prove using logical reasoning and understanding of properties, laws and theorems
Difficulty level	Medium
Marks	3
Time	3 mins
Item stem	Derive the formula for the length of an arc that subtends an angle of θ (measured in degrees) at the centre.

Marking Scheme

Part	Mark	Answer	Further Information
	1	For recalling arc length = circumference for θ = 360°	
	1	Realizing that arc length is proportionate to the angle subtended at the centre OR Using unitary method or ratio-proportion to find the arc-length for θ = 1°	½ for realizing arc length increases with θ but not getting that they are proportional
	1	For arriving at the formula arc length = θ/360° × 2πr where r = radius	½ if any part is missing e.g. degree, 2, π, radius

Content domain (Chapter name)	Mensuration (Surface Areas and Volumes)
Content Domain Learning outcome	Solves problems on surface area and volume formulas in real world situations.
Indicator	Uses surface area and volume formulas of 3D shapes to solve real life problems related to frustum and combined solids. Identifies situations where two volumes are equal and/or where two surface areas are equal.
Cognitive level	Reasoning
Thinking Process	Compute or process by selecting strategy, operation(s) and operand(s) Distinguish using logical reasoning
Difficulty level	High
Marks	4
Time	10 mins
Item stem	Two children wanted chana for snack. Their dad made them a paper cone to hold the chana by rolling a paper in the shape of a circle of radius 20cm, with a cut along a radius. The resulting cone had two layers of paper. But the children wanted separate cones. So, the dad opened the cone, flattened the paper, halved it, and rolled up each half to a narrower cone with two layers of paper like the original one. Compare the amount of chana in the original cone to that in the two narrower cones.

Part	Mark	Answer	Further Information
	1	For realizing that the radius of the original cone = 10cm and that of the narrower ones = 5cm	½ for realizing circumference (or arc length) of paper = 2 × circumference of the cone = 2π × radius of cone but some calculation error ½ if the radii are incorrect but their ratio is 2 : 1 1 if only one radius is correct Allot this mark even if units are missing
	1	For computing the height of the original cone = $\sqrt{(20^2 - 10^2)}$cm = $\sqrt{300}$cm = $10\sqrt{3}$cm and that of the narrower ones = $\sqrt{(20^2 - 5^2)}$cm = $\sqrt{375}$cm = $5\sqrt{15}$cm	½ for using the Pythagorean triplet relation among slant height, height and radius of a cone, but some calculation error or incorrect simplification of surds $\sqrt{300}$cm or $10\sqrt{3}$cm are ok, $\sqrt{375}$cm or $5\sqrt{15}$cm are ok Allot this mark even if units are missing
	1	For computing the volume of the original cone $V_1 = 1/3 \times 100\pi \times \sqrt{300}$cm³ = $1/3 \times 100\pi \times 10\sqrt{3}$cm³ and that of the two narrower ones $V_2 = 2 \times 1/3 \times 25\pi \times \sqrt{375}$ cm³ = $1/3 \times 50\pi \times 5\sqrt{15}$cm³	½ for using volume formula of a cone, but some calculation error or incorrect simplification of surds Any correct simplification of surds or the lack of it is ok Allot this mark even if units are missing
	1	For realizing that $V_1 > V_2$	The comparison can use any of the following $100\sqrt{300} / 50\sqrt{375} = 2\sqrt{300} / \sqrt{375} = \sqrt{1200} / \sqrt{375} > 1$ $1000\sqrt{3} / 250\sqrt{15} = 4 / \sqrt{5} > 1$ Decimal approximation ½ for partial success reducing the ratio $V_1 : V_2$ but not realizing $V_1 > V_2$

Content domain (Chapter name)	Statistics and Probability (Statistics)
Content Domain Learning outcome	Calculates and interprets mode for grouped data.
Indicator	Recalls the mode formula and what each part represents. Computes mode for grouped data.
Cognitive level	Applying
Thinking Process	Identify by connecting known properties and definitions to given context or situation Reproduce (including measure, compute, construct) procedures and proofs
Difficulty level	High
Marks	4
Time	10 mins
Item stem	Compute the mode of the following data on age distribution of US population of age 65 years and over in 2008.

Age (in years)	Percentage of population
65-70	29.2
70-75	22.6
75-80	18.7
80-85	14.8
85-90	9.2
90-95	4.0
95+	1.5
Total	100.0

Marking Scheme

Part	Mark	Answer	Further Information
	1	For using the correct formula $m = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$	½ if the formula is partly correct
	1	For identifying l = 65, c = 5, f_1 = 29.2 and f_2 = 22.6	½ for getting at least two of the values correct
	1	For identifying f_0 = 0	½ for recalling f_0 is the frequency of the previous class but not realizing its value
	1	For computing the mode based on the above formula $$m = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c = 65 + \frac{29.2 - 0}{2(29.2) - 0 - 22.6} \times 5$$ $$= 69.08$$	69 is acceptable for full credit ½ if there everything is plugged in correctly but there is some error ½ if the answer is within (65, 70) No mark if answer is outside (65, 70)

Content domain (Chapter name)	Statistics and Probability (Probability)
Content Domain Learning outcome	Applies the concept of probability in solving daily life problems.
Indicator	Computes the probability of an event in daily life context using measurement for a continuous sample space.
Cognitive level	Applying
Thinking Process	Compute or process by selecting strategy, operation(s) and operand(s)
Difficulty level	Medium
Marks	3
Time	4 mins
Item stem	A target shown in the figure consists of three concentric circles of radii 4cm, 7cm and 10cm respectively. A dart is thrown and lands on the target. What is the probability that the dart will land on the annular region with outer radius 7cm and inner radius 4cm?

Marking Scheme

Part	Mark	Answer	Further Information
	1	For calculating area of the target = $\pi(10)^2 = 100\pi$	½ for using πr^2 but calculation error
	1	For calculating area of the annular region = $\pi(7^2 - 4^2) = 33\pi$	½ for $\pi(7^2 - 4^2)$ with incorrect simplification
	1	For computing the probability as the ratio of the two areas = $33\pi / 100\pi = 33/100$	½ for $33\pi / 100\pi$ but incorrect simplification or incorrect decimal representation

14. Reference documents

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3. NCERT (2006). National Focus Group, Position Paper on Aims of Education, New Delhi

4. NCERT Mathematics Textbook. http://ncert.nic.in/textbook/textbook.htm

5. PISA 2021 Mathematics Framework (draft)

6. TIMSS 2019 Mathematics Framework

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Learning Framework - Mathematics

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