The question book

Creative problem solving
Acknowledgements

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- Sh. Ramesh Pokhriyal, 'Nishank', Hon'ble Minister of Human Resource Development, Government of India
- Sh. Sanjay Dhotre, Hon'ble Minister of State, Minister of Human Resource Development, Government of India
- Ms. Anita Karwal, Secretary, Department of School Education and Literacy, Ministry of Human Resource Development

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Every species on this earth faces problem, big or small and finds probable solution(s) for it through various ways as per their capabilities and limitations. Problem solving is a daily activity for most of us, whether we realize it or not. Whenever there is a gap between where you are now and where you want to be and you don’t know how to cross that gap, you have a problem. For example, if you want to go to your friends’ place and there are three paths to take and you don’t know how to find the best path to take, that is a problem and when you figure out the best path after looking into various aspects that’s problem solving.

Learning and understanding the process of problem solving and recognizing patterns in problems is a lifelong activity and a skill that can be applied both in personal and professional lives. This is one of the most essential 21st century skills for anyone to learn.

The question book in your hand will take you through a journey where you will find various kinds of problems and discussions on them to find the probable solutions after looking into pros and cons of various aspects of it. It will also give you an opportunity to identify the problems in your daily life for which you will find best solutions yourself.

- Ankit and Ankita
The question book
Ankita! I think there is some problem with my circuits.

I am designed to solve tough problems but I don't know how!

Humans think that machines is superior to them!

Don't worry Dextro! Let me take a look.

I think this problem solving circuit is broken!

But I can tell you! We machines think humans are and will always be superior!

Why do you think that Dextro?

Well, one thing that you humans can do is think creatively in solving problems!

The good thing is that I can learn! Machines can learn!

...and I am almost done!

Don't be sad Dextro!

Machines or humans, those who can think critically...

... have creativity and imagination, will always be better at problem solving...

While, you are good at doing things fast and solve problems with clear steps.

We humans are good at thinking critically, think creatively and use imagination while solving problems.
So I am going to teach you how to solve problems!

Why do we always come to library?

Because, if you can read, you can learn, if you can learn you think, if you can think you can be the best!
Step by Step
Okay, I am going to give you a problem, and you have to solve it!

There is a series of numbers: 1, 2, 3, 1, 2, 3, ...

What is the 10th, 25th, and 60th number in this series?

10th number is 1, 25th number is 1, 60th number is 3

Wow! You are quick.

All I had to do is to keep repeating the number and look for the 10th, 25th, and 60th number.

This is what you are good at! Now look at this problem!

In a competition to make snowmen, the children made 60 snowmen. The heads of the snowmen followed a particular pattern shown on the diagram below. How do the heads of the 10th, the 25th and the 60th snowman look like?

I don't know! These are the problems I can not solve! There are no clear steps to solve this.

Well! the way humans can think about these kind of problems is by following a simple framework.
**Step 1:** Identifying which pieces of data are relevant when faced with a mass of data, most of which is irrelevant.

**Step 2:** Combining pieces of information that may not appear to be related to give new information.

**Step 3:** Relating one set of information to another in a different form – this involves using experience: relating new problems to ones we have previously solved.

So let's go step by step. What is the information that this problem is giving us?

Hey good timing! Why don't you look at this problem and tell me all the information that you can find?

There are a few things I can immediately notice:

1. The hats go from being a rectangle to a triangle and repeats itself.
2. Similarly, eyes follow a pattern: rectangle, star, circle, triangle, rectangle.
3. Nose also follows a pattern: angle, circle, angle, circle.
4. And smile goes from wide smile, narrow smile, wide, narrow and so on.

If I have to find 10th snowman, I will have to find the hat, the eyes, the nose, and the smile by following these patterns.

These are all the things I can find from the problem.

In a competition to make snowmen, the children made 60 snowmen. The heads of the snowmen followed a particular pattern shown on the diagram below. How do the heads of the 10th, the 25th, and the 50th snowman look like?

That's actually very cool! The first thing to solve any problem is to understand it...
... and for that you have to observe the problem, ask questions to yourself...

... Is there a pattern? Can I break this problem into smaller problems? Can I translate this problem into math using numbers? What does my common sense say?

Let us use the information Ankit found and try to solve this!

I can find patterns! The first information that Ankit found was about hats. The pattern is rectangle, triangle, cylinder.

If I use 1 for rectangle, 2 for triangle, 3 for cylinder then the pattern is 1, 2, 3, 1, 2, 3 and so on.

Since, numbers repeat after 3, the 9th number will be 3 and 10th number will be 1 again.

Since I used 1 for rectangle, the 10th snowman will have a rectangle!

Similarly, 10th snowman will have a star eye, circular nose, and narrow smile.

Yay! You got it! And what about other patterns?

Exercise: You should try to draw the pattern and see for yourself if you get this figure?

Hahaha… This is just one kind of problem and one kind of method to solve a problem! We should see other problems and identify appropriate methods and solve them.

But I can’t think on an empty stomach. Let’s eat first.
Polya's Problem Solving framework

George Pólya was a Hungarian mathematician. He made fundamental contributions to combinatorics, number theory, numerical analysis and probability theory. He is also noted for his work in heuristics and mathematics education.

(December 13, 1887 - July 9th, 1985)

**Questions you should ask to yourself**

**First step**
You have to understand the problem

**Second step**
Find the connection between data and the unknown

**Third step**
Carry out your plan

**Fourth step**
Examine the solution obtained

Have you seen it before or in a slightly different form? Do you know a related problem? Look at the unknown and try to think of a familiar problem having the same or similar unknown.

Carrying out a plan of the solution, check each step. Can you see clearly that the step is correct? Can you prove that it is correct?

Can you check the result? Can you check the argument? Can you derive the results differently? Can you see the result, or method for some other problem?
I need this, I need this not
You guys! Forgot that I don’t eat food... I eat electricity.

You just ate a lot of electricity, Dexter! You stomach is full.

And I am always hungry.

Okay, Arkika, what are the different kinds of problem? What else do I need to learn?

Dexter is smart! He wants to learn so much.

He is!

The daily life problems you would want to solve will require you to do multiple things like selecting the right information from a lot of information...

Processing the data and information given

Finding the right method to solve the problem

In some cases, search for the answer instead of trying to solve them.

There will be some problems where you will find patterns.

But in most of the problems, you will think about breaking it in multiple sub-problems.

This is a lot of information! Is this also a problem?

HaHa... No Dexter... Let me walk you through with examples.

TanZim has a meeting in a town 50 KM away at 3 p.m. tomorrow. He is planning to travel from the town where he lives to the town where the meeting is, by train and walking to and from the station at both ends.

List the pieces of information TanZim needs in order to decide what time he must leave home. Then work out how you would proceed to plan his journey from these pieces of information.
Tanzim needs to leave his house, walk to the station, buy train ticket, go to platform, wait for train and get in, sit inside till he reaches and then walk back.

The pieces of information Tanzim needs are:
1. Time taken to walk till station
2. Time needed to get the ticket
3. Time needed to walk to the platform.
4. The train timetable
5. The time taken to walk from the station to where the meeting is being held.

Rhea has been staying in a hotel on a business trip. When she checks out, the hotel’s computer isn’t working, so the receptionist makes a bill by hand from the receipts, totaling Rs. 3635. Rhea thinks she has been overcharged, so she checks the itemized bill carefully.

**Room:** 4 nights at Rs. 592.00 per night
**Breakfast:** 4 at Rs. 70.00 each
**Dinners:** 9 at Rs. 126.00 each
**Telephone:** 10 units at Rs. 1 per unit
**Restaurant:** Various drinks totaling Rs. 161.00
**Laundry:** 3 dresses at Rs. 100.00 each

It appears that the receptionist miscounted one of the items when adding up the total, which item has Rhea been charged too much for?

Hmmm... Let’s see the total according to the itemized bill is...

![Image](attachment:receipt.png)

Rs 3257

1. 4 x 592 = Rs 2368
2. 4 x 70 = Rs 280
3. 9 x 126 = Rs 378
4. 10 x 1 = Rs 10
5. Rs 161
6. 3 x 100 = Rs 300

You are very quick!

Anyways! This means that Rs 3635 – Rs 3257 = Rs 378

This means, that she has been charged for an extra dinner

Because nothing else could come exactly to Rs 18

Yes! It seems you are right.
Practice Problem

Problem 1

Rajesh is cooking a meal for some friends.

This will involve roasting corns (makka), which will take 2 hours’ cooking time plus 15 minutes resting on removal from the oven. The oven takes 15 minutes to warm up. He will also cook some rice (30 minutes’ soaking plus 15 minutes’ cooking), cabbage (5 minutes to prepare and 5 minutes to cook) and a tomato sauce (10 minutes to prepare and 15 minutes to cook).

What should be the timing of each step of cooking the meal if the friends are to eat at 7 p.m.?
Problem 2

The SuperSave supermarket sells Birma washing powder for Rs 1.20 a bottle. At this price they are charging 50% more than the price at which they buy the item from the manufacturers. Next week SuperSave is having a 'Buy two get a third free' offer on this item. The supermarket does not want to lose money on this offer, so it expects the manufacturers to reduce their prices so SuperSave will make the same actual profit on every three bottles sold.

By how much will the manufacturers have to reduce their prices?

(a) $\frac{1}{6}$  
(b) $\frac{1}{4}$  
(c) $\frac{1}{3}$  
(d) $\frac{1}{2}$  
(e) $\frac{2}{3}$
Itni shakti humein dena data
Aye Aye Captain!

Data is the new gold, it's precious, as long as you are able to make sense of it!

Data can come in various forms:
1. Tables
2. Graphs
3. Words
4. Picture
5. Diagrams

Data can come in various forms like:

One entry in this table is wrong. Which one?

<table>
<thead>
<tr>
<th>Age</th>
<th>Gym</th>
<th>Swimming</th>
<th>Jogging</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-15</td>
<td>14</td>
<td>57</td>
<td>32</td>
<td>103</td>
</tr>
<tr>
<td>16-20</td>
<td>86</td>
<td>92</td>
<td>45</td>
<td>232</td>
</tr>
<tr>
<td>21-25</td>
<td>67</td>
<td>58</td>
<td>44</td>
<td>169</td>
</tr>
<tr>
<td>Total</td>
<td>167</td>
<td>207</td>
<td>130</td>
<td>504</td>
</tr>
</tbody>
</table>

Let's start by looking at data arranged in tabular form.

If I add horizontally there is a problem with row 2. It should be 229 but it is 232. But, I don't know which number.

Well! Then you just do a vertical addition as well.
And you would find that Jogging has a wrong entry. It should be 121 but it is 130.

Dextron, what should be the correct entry? Because 130 – 121 = 9 and 232 – 225 = 9. Hence it must be 9 less than 45 i.e. 36.

That was quite something!

Yeah! I felt like Sherlock Holmes!

Looking for clues! Trying to find hidden information.

I don’t understand human jokes! But it looks like Sherlock Holmes must be a detective!

What is the difference between the lowest average temperature and highest average temperature?
You need to find the lowest average temperature and highest average temperature, then subtract them! I am so proud of both of you.

And the highest average temperature is 34 degrees in April, lowest average temperature is 14 degrees in December.

In an inter-school hockey knockout competition, there are initially 32 teams. Teams are drawn by lots to play each other and the winner of each match goes through to the next round. This is repeated until there are only two teams left, who play each other in the final, and the winner gets a cup. Matches have two halves of 20 minutes each. If the teams are level at the end of normal play, two extra 10-minute periods are played. It is still a draw, teams take penalty shots at goal to decide the winner.

High-school was eventually knocked out in the semi-final (without extra time). In one of the earlier rounds they had to play the two extra periods before they won.

And hence the answer is 20. What about words? How do you find data in words? Let me give you an example of that as well. You guys are getting the hang of it!

Teams are drawn by lots to play each other and the winner of each match goes through to the next round. This is repeated until there are only two teams left, who play each other in the final, and the winner gets a cup.

Okay... Let's start... In an inter-school hockey knockout competition, there are initially 32 teams.

There are 32 teams initially.

Teams play with each other until only two teams play the final.

High-school played till semi-finals.

It might look intimidating and difficult in the beginning... But you need to give it just one good read and make notes.

Matches have two halves of 20 minutes each. If the teams are level at the end of normal play, two extra 10-minute periods are played.

If it is still a draw, teams take penalty shots at goal to decide the winner. High-school was eventually knocked out in the semi-final (without extra time).

High-school played two extra periods i.e. 20 min.

Normal match of 40 min if teams have similar scores, there can be 10 min or 20 min more given.
See! When you read it carefully, it is easier to understand.

That’s actually true! I just need to read part by part.

And now, if you think of it, the problem is such an easy one!

That’s actually true. First we need to know how many matches High-schools played!

So first round had 32 teams then 16, 8 and 4 teams in subsequent rounds. So by fourth round is when the team got knocked out.

So High-school played 4 matches in total!

Each match was 40 minutes, so $40 \times 4 = 160$ minutes.

High school also played two extra periods in one match!

That would mean $2 \times 10 = 20$ min.

And that would mean $160 \text{ min} + 20 \text{ min} = 180 \text{ min}$ or 3 hours.

That’s exactly what the answer is!

As a cardinal rule! Always break problems into sub-problems...

...think in steps, and when you have all the pieces sum them up!

Now tell us about pictorial information!

Okay, when you look at this problem, understand the problem first and don’t jump to your instincts!

There are 24 individual tiles with different printing on them. How many different kind of tiles are required to make this overall pattern?
You will have to think in steps. There are tiles that probably can be rotated and used. Yes! The procedure is to eliminate tiles one by one, noting each time if a new tile can be used or a new tile is needed! Let's start with the first tile. This will be unique!

The second tile is different from the first one. We would need this tile so let's keep it! We can use the pattern in second tile to make third tile! So we will cross it! And if you continue to do this you will get

You will get three unique patterns which can be used to make the 24 tiled pattern.

This was so much fun! What else do you have in your hat?

Hahahah... Well the next thing you both should try is extracting information from a diagram!

What else do you have in your hat is a phrase used to say what is the next thing you will bring up.

I live in Chaukor park and wish to visit a friend in Deer park. I normally go via Friendship park but have discovered (before setting off) that the road between Friendship park and Deer park is blocked by an accident. How much will this add to my journey?
We need to calculate the length of normal route, then we have to consider the alternate route!

The journey normally is 8 KM + 12 KM = 20 KM.

If the road between Friendship park and Deer park is closed, the only other route is via Tikona park.

So the route is Chaukri park, Tikona park to Deer park.

This is 8 KM more than the regular route!

And that is brilliantly done!

And that will make the distance 12 KM + 16 KM = 28 KM.

That was quite a journey! I am already looking for the next challenge.

Okay... Now Let me take you through.

I am so eager to learn more!

Then let's understand some spatial reasoning!
Give me some space!
Okay, I got it.

Okay let me show you a magic trick. Think of a number.

mhm

15 x 3 = 45

Now multiply the number by 3.

Okay

Now add 18 to that number.

45 + 18 = 63

Now divide the answer by 3.

63 / 3 = 21

Now subtract the number you had first thought.

Done!

This is algebra. If one follows your procedure then to get the answer you divide the number you asked me to add later with the number you asked me to add earlier i.e. 18 / 3 = 6

Did you get the 6 as the answer?

Yes!

See this is magic!

Pshh! Why are you so smart!

Okay enough of child's play! We are going to learn about spatial reasoning.

Spatial reasoning is simply reasoning about shapes and spaces.

Take this as an example.
This is a 3 dimensional shape! Which of the following pieces of cardboard shape will fold to make the shape?

A

B

C

D

There are 7 faces in all the options, I think I will have to fold these shapes in my head.

Well A doesn’t work because there is no way to close the sides of squared bottom!

In the option B, the edges won’t meet to form the top.

The sides of shape C is also not going to close. There is no symmetry!

Shape D however, will close from all the sides. This is the answer!

Correct! This is using imagination to do things in your head. Imagination will also help you think about 3D shapes from various angles and folds. Try this!
Some children are making decorations. A square sheet of paper is folded along a diagonal and then again so the two sharp points meet, as shown. A cut is made through all the layers of paper along the dotted line shown and the small pieces removed. The paper is then opened. What does it look like?

I think you will have to cut the top to get side A. So that can not be the answer.

It would also not look like shape B.

However, if you made a side cut, it would look like shape C. That must be the answer!

Well yes! You can try thinking about shape D and E as well just to verify it!

Thinking about shapes can be pretty fun and useful. This thinking will help in various fields such as engineering, architecture and even high levels of Physics!

Now is the time when I taught you about solving problems by searching the answer.
If you can’t solve it, search for it!
So far we have seen that problems that can be solved by a little bit of thinking.

But there are certain kinds of problems where...

You could search for the answers instead of trying to solve them.

This is useful when you don't know a method which can be used!

Take this situation for example:

Amir is helping with a charity collection and has gathered envelopes containing rupee notes from a number of donors. He notes that all the envelopes contain exactly three items but some of them contain one, two or three buttons instead of rupee notes. This is probably from people who did not want to donate. All the notes have denominations of Re 1, Rs 5, Rs 10, Rs 20 or Rs 50. If Amir counted the value of each envelope, what is the smallest amount that he will not find in any envelope?

How would you solve this?

Well, one way to solve this problem is list all the possibilities in a systematic order.

We know that envelopes can contain 0, 1, 2 or 3 notes. The possibilities with one note being: Re 1, Rs 5, Rs 10, Rs 20 or Rs 50.

With two coins, we have to be a little more careful.

We can start with Re 1 and look for possibility for second note.

Explanations: Each envelope can have a different value starting from as low as Re 1 to as high as Rs 50. But there are value that Amir cannot find in the envelope. For example, Amir cannot find Rs 14 in any note as he would need Rs 10 and four Rs 1 note and each envelope has three notes maximum.
If you list down all the values that envelop can have:

Re 1, Rs 2, Rs 3, Rs 4, Rs 5, Rs 6, Rs 7, Rs 8, Rs 9 and so on

Then you can start striking off the numbers that you have found as the problem asks you to find the number that is not possible to get in the envelope. In this case:

Re 1, Rs 2, Rs 3, Rs 4, Rs 5, Rs 6, Rs 7, Rs 8, Rs 9 and so on
I stay in Town B and have to deliver groceries in the other three towns and finally return to Town B. What is the shortest distance I have to drive?

Take another example, where it is much easier if you try searching for the solution.

In such problems, the best way to solve is list all the paths.

Since there are very limited number of paths we should list all of them down along with the total distance.

These are all the possible paths:

- BADCB = 14 + 10 + 8 + 5 + 22 = 59
- BACDB = 14 + 18 + 5 + 8 + 12 = 62
- BDACB = 12 + 10 + 18 + 5 + 22 = 67
- BDCA = 12 + 8 + 5 + 18 + 14 = 62
- BCAD = 22 + 5 + 8 + 10 + 14 = 59
- BCADB = 22 + 5 + 18 + 10 + 12 = 67

Once we have listed all these possibilities, it is easy to find the path. In this case, there are two paths, both of them will take 59 KM.

This strategy is useful when you can not find or remember a conventional method to solve the problem.

Ankita, I have a question! Can we solve all the problems that exist? I mean are there any problems that can not be solved?

That's a very good question! There are certain types of problems which have not been solved yet, but most of the problems we deal with in day-to-day life can be solved through a little bit of data and thinking.

Take this for example.
I use the trip meter on my car to measure the distance driven since I last had the car serviced, so that I know when the next service is due. The trip meter can be set to zero by the press of a button and records the kilometres driven since it was last reset.

I set the trip meter to zero after my last service. The next service is due after 20,000 km have been driven. Some time later, I lent the car to my brother. I forgot to tell him about the trip meter; he pressed the button to zero it and drove 375 km. I then started driving again without realising what he had done.

What should the trip meter read when the next service is due?

The above problem cannot be solved with the information given. What additional piece of information is needed to solve it?

In this example, the data was not sufficient to solve the problem. If you read the problem carefully, you will be able to find if the data is sufficient or not to solve the problem!

Why don’t you try this problem by yourself and see if you can find the answer?

I have a small collection of three different types of old coin (1 paisa coins, 2 paisa coins and 5 paisa coins). The collection contains 15 coins in total. There are more 1 Paisa coin than 2 Paisa coins and more 2 Paisa coins than 5 Paisa coins. Which one of the following single pieces of information would enable you to know exactly how many of each type of coin there was?

(a) There are four more 2 Paisa coin than 5 Paisa coins
(b) There are five more 1 paisa coin than 5 paisa coins
(c) There are three more 1 paisa coin than 2 paisa coins.
(d) There is one less 1 paisa coin than 5 paisa coins and 2 paisa coins together.

You can apply the same strategy we used earlier. Search for the solution.

<table>
<thead>
<tr>
<th>1 paisa coin</th>
<th>2 paisa coin</th>
<th>5 paisa coin</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answer: Eight 1 Paisa coins, Five 2 Paisa coins, Two 5 Paisa coins.
Acknowledgement

We acknowledge the work done by Mr. John Butterworth and Mr. Geoff Thwaites on critical thinking through their book – Thinking Skills (Critical Thinking and Problem Solving)
This book has been a big source of inspiration for this book
Additional resources

Readings

1. Hands on Maths
2. Mathematics Can Be Fun
3. Great Ideas of Modern Mathematics
4. Polya’s Problem Solving strategy

Videos

1. TED-Ed - Riddles
2. TED-Ed - Think like a coder

We thank all the contributors for making the above resources available on the internet. We specially thank Dr. Arvind Gupta for making excellent learning resources available for free on the internet.